Abstract:
One of the main causes behind the trade collapse of 2008-09 was a significant fall in the demand for durable goods. This paper develops a small country, overlapping generations model of international trade in which goods durability gives rise to a more than proportional fall in trade volumes, as observed in 2008-09. The model has three goods - two durable, traded goods and one non-durable, non-traded good and two factors of production. The durability of goods affects consumers’ lifetime wealth and their optimal consumption bundle across goods and time periods. A uniform productivity shock reduces consumers’ lifetime wealth inducing a re-optimisation away from durables. This gives rise to a more than proportional effect on international trade, provided the non-traded sector is sufficiently capital intensive. The elasticity of trade flows to GDP is found to be increasing in both the degree of durability and the size of the shock. Thus the model provides micro-foundations for the asymmetric shock to the demand for durable goods observed in recessions and clarifies the link between this endogenous shift in preferences and international trade flows. It also explains the observation that deeper downturns are associated with a higher elasticity of trade to GDP. Furthermore, the greater the degree of durability of traded goods, the larger is the share of domestically produced goods in consumption, for plausible factor intensities. This provides an alternative explanation for the home bias in consumption, and hence another explanation for Trefler's "missing trade ".

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1 Introduction

The global financial crisis of 2008-09 led to a period of recession and slow economic growth in almost every developed economy. At the same time, there was a significant decline in global trade volumes in real terms. From an average growth rate of 7.4 percent per year between 2003 and 2007, export volumes grew by only 2.2 percent in 2008, and fell by 12.2 percent in 2009. However, exports bounced back by growing by 14.5 percent in 2010 (all figures from the World Trade Organisation (WTO)). The trade collapse in 2009 and recovery in 2010 represent the largest percentage changes in trade volumes since the WTO data series began in 1950. In addition, the collapse and recovery of trade volumes is much larger than the comparable fall and rise of world GDP, which grew at 1.6 percent in 2008, decreased by 2.3 percent in 2009, and grew by 3.6 percent in 2010. Figure 1 shows the growth rates of GDP and international trade from 2000 to 2010.

Figure 1: Growth in volume of world trade and GDP, 2000-2010 (annual percentage change). Source: IMF.

The observation that trade fluctuates more than GDP is not unique to the 2008-09 recession. Freund (2009) shows that the elasticity of trade volumes to world GDP has increased from about 2 in the 1960s to over 3 after 1990. In addition, she finds, based on evidence from the previous global
downturns in 1975, 1982, 1991 and 2001, that the trade elasticity is higher in global downturns, so that a global deceleration of 4.8 percent corresponds to a fall in international trade of 19 percent. Similarly, Engel and Wang (2009) show that international trade is about three times as volatile as GDP. This phenomenon can be seen in Figure 1: in 2008 and 2009, when world GDP growth slowed down, the trade response was much larger than the decrease in GDP growth. Similarly, when world GDP recovered in 2010, the recovery of trade flows was much larger than the recovery in GDP.

The reasons for the collapse and recovery of international trade in 2009 and 2010 are manifold. The essays in Baldwin (2009) and Baldwin and Evenett (2009) discuss the key explanations proposed. Since the start of the trade collapse, several empirical papers have emerged seeking to explain the causes of the more-than-proportional collapse in trade volumes. Levchenko et al (2010) compare the contributions of three popular alternative explanations of the trade collapse: vertical production linkages, trade credit, and compositional effects on durables demand. They conclude that the patterns of the trade collapse are consistent with vertical production linkages and durables demand playing important roles, while they did not detect any impact of trade credit.

Similarly, to unpack the determinants of the trade collapse, Eaton et al (2011) develop a multi-sector model of production and trade, calibrated to global data from recent quarters. They consider four exogenous shocks to the model: a shock to final demand, a shock to trade frictions, a productivity shock, and a shock to trade deficits. They find that shocks to manufacturing demand, especially for durable goods, account for the bulk of the decline in international trade. This is similar to the result in Bems et al (2010), who find that final demand shocks can explain 70 percent of the trade collapse, and that a big part of the impact of the demand shock occurs through durables. Behrens et al (2010) using a dataset of Belgian firms find that the fall in global demand explains over half of the fall in exports in 2008-09, and that trade in consumer durables and capital goods fell more severely than trade in other product categories. On the theoretical side, Engel and Wang (2009) develop an international Real Business Cycle (RBC) model incorporating durable goods that is calibrated to observed characteristics of international trade, showing the importance of durables.

However, other work such as Chor and Manova (2011) suggest that credit conditions were an important channel in reducing trade volumes during the crisis. Amiti and Weinstein (2009) show that the health of a bank providing trade finance influences the growth in a firm’s exports. Alessandria et al (2010) show that in the 2008-09 trade collapse, industries with larger inventory adjustments experienced larger trade collapses.
trade in open economy macro models.2

In this paper we develop an overlapping generations model of international trade to capture the role of product durability in the trade collapse of 2009 and recovery of 2010. The model is of a small country with three goods – two durable, traded goods and one non-durable, non-traded good, all produced with constant returns to scale technologies using two factors of production, capital and labour. The small country assumption implies that prices are exogenously determined, and makes the model tractable. Empirically, Hall (2010b) and Levchenko et al (2010) show that prices were much more sticky in the 2008 recession than quantities. The assumption that traded goods are durable whilst non-traded goods are non-durable is strong, but has empirical support from De Gregorio et al (1994), Engel and Wang (2008) and Erceg et al (2008), who show that durables are a much larger share of international trade than they are of the domestic economy (according to Erceg et al (2008), consumer durables and capital goods constitute about three-quarters of US non-fuel imports and exports, but only 20 percent of the production share of the economy). Comparative advantage determines which of the two durable goods the country exports.

A one-period, unanticipated uniform productivity shock is introduced and the model re-solved for the presence of the shock, as well as during recovery from the shock. The unanticipated nature of the shock may appear to be a strong assumption; however, the IMF’s World Economic Outlook as late as October 2008 predicted world economic growth in 2009 to be 3.0 percent (IMF, 2008), well above the actual growth rate of -0.5 percent, suggesting that even the best forecasters were unable to anticipate the magnitude of the shock that hit the global economy. We show that product durability causes the shock to give rise to a more than proportional decline in trade flows, consistent with Figure 1. Moreover, trade flows are found to overshoot their long run level in the period after the shock.

Goods durability affects the consumer’s lifetime wealth and the optimal consumption bundle across goods and time periods. A uniform productivity shock reduces the consumer’s lifetime wealth and hence has a disproportionate effect on his demand for durable goods. In this way the model provides microfoundations for the asymmetric shock to the demand for durable goods identified by Levchenko et al (2010), Engel and Wang (2009), and Eaton et al (2011). This endogenous

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2The fall in GDP during the recession of 2008-09 has been associated with a greater-than-proportional decrease in the demand for consumer durables and business investment; see Hall (2010a) and Wang (2010).
asymmetry arises precisely from the durability of these goods. Moreover, the elasticity of trade flows to GDP is shown to be increasing in both the degree of durability and the size of the shock. The model therefore provides an explanation for the observation that deeper downturns are associated with a higher elasticity of trade to GDP.

Furthermore, traded goods durability means that a country consumes a larger share of domestically-produced goods than would be predicted by the parameters of the utility function, thus providing an alternative explanation for the home bias in consumption (Krugman, 1980), and hence potentially another explanation for Treffer’s missing trade (Treffer, 1995; see also Chung, 2003).

The next section outlines the model. Section 3 analyses the impact of productivity shocks, while Section 4 provides some concluding comments.

2 The model

Consider a small, open economy in which there are three goods: two traded, durable goods, $X$ and $Y$, and one non-traded, non-durable good, $N$. There is an infinite time horizon and in each period, $\tau$, goods $j = \{X, Y, N\}$ are produced with Cobb-Douglas technologies using labour, $L_j$ and capital, $K_j$, as given by

\[ X = \theta K_X^\alpha L_X^{1-\alpha} \]  
\[ Y = \theta K_Y^\beta L_Y^{1-\beta} \]  
\[ N = \theta K_N^\nu L_N^{1-\nu}, \]

where $\alpha, \beta, \nu \in (0, 1)$ and productivity parameter $\theta$ is positive and assumed identical across sectors for simplicity. Let $\beta > \alpha$ such that of the traded goods, $Y$ is relatively capital intensive. We assume the prices of the traded goods, denoted by $P_X$ and $P_Y$, respectively, are quoted on world markets. Let $P_Y = 1$ (the numeraire) and $P_X = p$. Further, $L$ and $K$ denote the economy’s endowment of labour and capital, where these are supplied inelastically and always fully employed. Suppose the economy is relatively capital abundant so good $Y$ is exported and $X$ imported, while

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3 Time subscripts are suppressed here to simplify the exposition of the model.

4 The value of $\nu$ relative to $\beta$ and $\alpha$ is important to the results of the model and is discussed in section 2.2.
parameter values are such that there is incomplete specialisation.

Cost minimisation gives rise to the capital-labour ratios in the production of the three goods, given by

\[
\frac{K_X}{L_X} = \frac{\alpha \ w}{1 - \alpha \ r} \quad (4)
\]

\[
\frac{K_Y}{L_Y} = \frac{\beta \ w}{1 - \beta \ r} \quad (5)
\]

\[
\frac{K_N}{L_N} = \frac{\nu \ w}{1 - \nu \ r} \quad (6)
\]

where \(K_j\) and \(L_j\) denote capital and labour employed in sector \(j\), \(w\) is the wage rate and \(r\) the rental rate. The factor market clearing conditions can be expressed as

\[
a_{LX}X + a_{LY}Y + a_{LN}N = L \quad (7)
\]

\[
a_{KX}X + a_{KY}Y + a_{KN}N = K, \quad (8)
\]

where \(\alpha_{i,j}\) denotes the unit factor requirement of input \(i\) into good \(j\). The unit factor requirements depend on relative factor prices and technological parameters as follows

\[
a_{LX} = \frac{1}{\theta} \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha} \left( \frac{w}{r} \right)^{-\alpha} ; \quad a_{KX} = \frac{1}{\theta} \left( \frac{\alpha}{1 - \alpha} \right)^{1 - \alpha} \left( \frac{w}{r} \right)^{1 - \alpha} \quad (9)
\]

\[
a_{LY} = \frac{1}{\theta} \left( \frac{1 - \beta}{\beta} \right)^{\beta} \left( \frac{w}{r} \right)^{-\beta} ; \quad a_{KY} = \frac{1}{\theta} \left( \frac{\beta}{1 - \beta} \right)^{1 - \beta} \left( \frac{w}{r} \right)^{1 - \beta} \quad (10)
\]

\[
a_{LN} = \frac{1}{\theta} \left( \frac{1 - \nu}{\nu} \right)^{\nu} \left( \frac{w}{r} \right)^{-\nu} ; \quad a_{KN} = \frac{1}{\theta} \left( \frac{\nu}{1 - \nu} \right)^{1 - \nu} \left( \frac{w}{r} \right)^{1 - \nu} \quad (11)
\]

Assuming perfect competition it follows that price equals unit cost in each sector, such that
\begin{align}
a_{LX}w + a_{KX}r &= p \
a_{LY}w + a_{KY}r &= 1 \
a_{LN}w + a_{KN}r &= P_N. \tag{14}
\end{align}

Factor prices and the price of the non-traded good can be determined in terms of \( p \) and technological parameters from equations (9) to (14), and can be expressed as

\begin{align}
w &= \theta \left[ p \alpha^\alpha (1 - \alpha)^{(1-\alpha)} \right]^\frac{\beta}{\beta - \alpha} \\
r &= \theta \left[ \beta^\beta (1 - \beta)^{(1-\beta)} \right]^\frac{1-\alpha}{\beta - \alpha} \\
P_N &= p^{\frac{\beta - \nu}{\beta - \alpha}} \left[ \alpha^\alpha (1 - \alpha)^{(1-\alpha)} \right]^\frac{\beta - \nu}{\beta - \alpha} \left[ \beta^\beta (1 - \beta)^{(1-\beta)} \right]^\frac{\nu - \alpha}{\beta - \alpha}. \tag{17}
\end{align}

Moreover, national income, denoted by \( M \), is the sum of all factor income:

\[ M = wL + rK. \tag{18} \]

To model the impact of durability of goods on consumption decisions and international trade we assume that generations of consumers live for two time periods, denoted by 1 and 2. Consumers own labour and capital, which they supply inelastically in both time periods. Generations are overlapping such that in any \( \tau \) half of consumers are in period 1 of their life i.e. are ‘young’, while the rest are in period 2, i.e. are ‘old’. In fact, let there be one young consumer and one old consumer, each of which owns \( \frac{1}{2} (K + L) \).

Consumers have identical, homothetic preferences and aim to maximise their expected lifetime utility, given by
\[ U = E_1 \sum_{t=1}^{2} \rho^{t-1} u_t, \]  
(19)

where \( \rho > 1 \) is the subjective discount factor and \( u_t \) denotes the consumers’ instantaneous utility function

\[ u_t = \gamma \log C_{N,t} + \frac{1-\gamma}{2} \log C_{X,t} + \frac{1-\gamma}{2} \log C_{Y,t}, \]  
(20)

where \( \gamma \in (0, 1) \) and \( C_{j,t} \) is consumption of good \( j \) in period \( t \) of the consumer’s life. In period \( t \) of their lives, each consumer earns an income \( m_t \), which is half of national income, and are assumed to be unable to borrow or lend:

\[ m_t = \frac{M_t}{2} = \frac{1}{2} (w_tL + r_tK). \]  
(21)

Traded goods \( X \) and \( Y \) are durable, such that a fraction \( d = (1 - \delta) \) of durable purchases by a consumer in period 1 endure and can be enjoyed in consumption in period 2, where \( \delta \in [0, 1] \) denotes the common depreciation rate of durables between periods 1 and 2. Parameter \( d \) therefore reflects the degree of durability of goods \( X \) and \( Y \). Durable goods do not last beyond two periods and, to simplify the analysis, there are no bequests of durable purchases made in year 2 and no second hand market for durables.

Let us distinguish between consumption of durables \( C_{X,t} \) and purchases of durables \( D_{X,t} \). Period 1 consumption of durables is exactly equal to purchases made as there are no bequests, while consumption of durables in period 2 comprises the depreciated stock of durables from period 1 as well as additional purchases in period 2. Since good \( N \) is not durable, consumption is equal to purchases in both periods. The relationships between consumption and purchases are summarised by

\[ C_{X,1} = D_{X,1} ; \quad C_{X,2} = dC_{X,1} + D_{X,2} \]  
(22)

\[ C_{Y,1} = D_{Y,1} ; \quad C_{Y,2} = dC_{Y,1} + D_{Y,2} \]  
(23)

\[ C_{N,1} = D_{N,1} ; \quad C_{N,2} = D_{N,2}. \]  
(24)
Thus consumers chooses $C_{N,1}, C_{X,1}, C_{Y,1}, C_{N,2}, D_{X,2}$ and $D_{Y,2}$ to maximise

$$U = \gamma \log C_{N,1} + \frac{1-\gamma}{2} \log C_{X,1} + \frac{1-\gamma}{2} \log C_{Y,1}$$

$$+ \rho \left[ \gamma \log C_{N,2} + \frac{1-\gamma}{2} \log (dC_{X,1} + D_{X,2}) + \frac{1-\gamma}{2} \log (dC_{Y,1} + D_{Y,2}) \right]$$

subject to income constraints

$$P_N C_{N,1} + p C_{X,1} + C_{Y,1} \leq m_1$$

$$P_N C_{N,2} + p D_{X,2} + D_{Y,2} \leq m_2.$$  \hspace{1cm} (26, 27)

Aggregate demand for good $j$ across both consumers is denoted by $C_j$. Further, we impose the constraint that demand for non-traded goods equals domestic supply,

$$\overline{C_N} = N.$$ \hspace{1cm} (28)

Let $X_Y$ denote exports of good $Y$ and $M_X$ denote imports of $X$, where

$$X_Y = Y - \overline{C_Y}$$ \hspace{1cm} (29)

$$M_X = \overline{C_X} - X,$$ \hspace{1cm} (30)

and trade balances, so

$$pM_X = X_Y.$$ \hspace{1cm} (31)

**2.1 Equilibrium without durability**

As a benchmark we outline the equilibrium if all goods are non-durable and so cannot be consumed beyond the period in which they are purchased. Since consumers cannot accumulate wealth in the form of durable goods when $d = 0$ and are unable to borrow or lend, there is no link between time
periods in the benchmark case. The first order conditions that follow from maximising (25) subject to (26) and (27), setting \( d = 0 \), give the standard result that consumers allocate their income across goods in fixed proportions, according to the preference parameter \( \gamma \):

\[
P_N C_{N,t} = \gamma m_t
\]

\[
p C_{X,t} = C_{Y,t} = \frac{1 - \gamma}{2} m_t.
\]

Aggregate expenditure on each good can be expressed as

\[
P_N C_N = \gamma M
\]

\[
p C_X = C_Y = \frac{1 - \gamma}{2} M,
\]

which combined with equations (7)-(11), (15)-(18) and (28)-(31), allow us to solve for equilibrium trade flows in each period,

\[
p M_X = X_Y = \frac{(1 - \gamma)(1 - \beta) + \gamma(1 - \nu)}{(\beta - \alpha)} + \frac{1 - \gamma}{2} wK - \frac{(1 - \gamma)(1 - \beta) + \gamma(1 - \nu)}{(\beta - \alpha)} + \frac{1 - \gamma}{2} rL.
\]

Since \( w \) and \( r \) are proportional to total factor productivity \( \theta \), then it follows that trade flows are also proportional to \( \theta \) in the non-durable case. Proposition 1 follows directly.

**Proposition 1** If all goods are non-durable \((d = 0)\), then a fall in productivity gives rise to a proportional change in trade flows.

**Proof.** Follows directly from equations (36) and (15)-(16). \( \blacksquare \)

### 2.2 Equilibrium with traded good durability

Now let traded goods have a degree of durability, \( d > 0 \). The first order conditions of the consumer’s optimisation problem are given by equations (37) to (44), where \( \phi \) and \( \mu \) are the lagrangean multipliers for budget constraints (26) and (27), respectively.
\[
\frac{\gamma}{C_{N,1}} - \phi p_N = 0 \quad \text{(37)}
\]
\[
\rho \frac{\gamma}{C_{N,1}} - \mu p_N = 0 \quad \text{(38)}
\]
\[
\frac{1 - \gamma}{2} C_{X,1} + \rho \frac{1 - \gamma}{2} \frac{d}{dC_{X,1} + D_{X,2}} - \phi \rho = 0 \quad \text{(39)}
\]
\[
\frac{1 - \gamma}{2} C_{Y,1} + \rho \frac{1 - \gamma}{2} \frac{d}{dC_{Y,1} + D_{Y,2}} - \phi = 0 \quad \text{(40)}
\]
\[
\rho \frac{1 - \gamma}{2} \frac{d}{dC_{X,1} + D_{X,2}} - \mu \rho = 0 \quad \text{(41)}
\]
\[
\rho \frac{1 - \gamma}{2} \frac{d}{dC_{Y,1} + D_{Y,2}} - \mu = 0 \quad \text{(42)}
\]
\[
P_N C_{N,1} + p C_{X,1} + C_{Y,1} - m_1 = 0 \quad \text{(43)}
\]
\[
P_N C_{N,2} + p D_{X,2} + D_{Y,2} - m_2 = 0 \quad \text{(44)}
\]

It follows from the first order conditions that

\[
p C_{X,2} = C_{Y,2} = \frac{1 - \gamma}{2} (m_2 + d p C_{X,1} + d C_{Y,1}) \quad \text{(45)}
\]

\[
P_N C_{N,2} = \gamma (m_2 + d p C_{X,1} + d C_{Y,1}) \quad \text{(46)}
\]

and \(C_{X,1}\) and \(C_{Y,1}\) satisfy

\[
\frac{1 - \gamma}{2 p C_{X,1}} - \frac{\gamma}{m_1 - p C_{X,1} - C_{Y,1}} + \frac{d \rho}{2 (m_2 + d p C_{X,1} + d C_{Y,1})} = 0 \quad \text{(47)}
\]

\[
\frac{1 - \gamma}{2 C_{Y,1}} - \frac{\gamma}{m_1 - p C_{X,1} - C_{Y,1}} + \frac{d \rho}{2 (m_2 + d p C_{X,1} + d C_{Y,1})} = 0. \quad \text{(48)}
\]

The durability of goods provides consumers with a means of building period 2 wealth through the purchase of durables, which allows higher period 2 consumption of all goods. Equations (45) and (46) show that in period 2 consumers’ expenditure on goods is in fixed proportions of their wealth, in accordance with the preference parameter. The durability of goods \(X\) and \(Y\) generates a tradeoff between period 1 and period 2 utility, such that the consumers’ optimal period 1 expenditure on
each durable good exceeds \(\frac{1-\gamma}{2}m_1\). By skewing consumption towards durable goods when young, consumers can expect to achieve higher lifetime utility through the wealth effect.

In the absence of any productivity shocks, income is constant over consumers’ lifetime, so \(m_1 = m_2 = \frac{M}{2} \equiv m\). Solving (47) and (48) yields

\[
C_{Y,1} = pC_{X,1} = f(\gamma, d, \rho) m > \frac{1-\gamma}{2}m \tag{49}
\]

\[
P_{N}C_{N,1} = (1 - 2f(\gamma, d, \rho)) m < \gamma m \tag{50}
\]

where \(f(\gamma, \cdot) < 0, f_d(\cdot) > 0, f_\rho(\cdot) > 0\),

so consumers’ period 1 expenditure on each durable good is a share\(5 f(\gamma, d, \rho)\) of income. Homotheticity of the utility function implies period 1 expenditure on each good is a constant share of income, but the share spent on durables is greater than when \(d = 0\). Furthermore, \(f(\gamma, d, \rho)\) is decreasing in \(\gamma\) and increasing in \(d\) and \(\rho\). Intuitively, the greater the underlying preference for durable goods, then the greater the income share spent on durables in period 1. Furthermore, the greater the degree of durability, the greater the wealth effect and so the greater the incentive to skew consumption towards durables. Also, the greater is \(\rho\), the more patient are consumers and thus the greater their willingness to sacrifice period 1 utility to build wealth for period 2. If, however, \(\rho = 0\), the incentive to trade-off utility over periods 1 and 2 disappears and \(f(\gamma, d, 0)\) collapses to \(\frac{1-\gamma}{2}\). Consider an example where \(\rho = 0.95\) and \(\gamma = 0.5\); if \(d = 0.5\), then it follows that \(C_{Y,1} = pC_{X,1} = 0.27312m > 0.25m\).

Equations (45), (46) and (49) allows us to express period 2 expenditure as

\[
D_{Y,2} = pD_{X,2} = \left(\frac{1-\gamma}{2} - \gamma df(\gamma, d, \rho)\right) m < \frac{1-\gamma}{2}m \tag{51}
\]

\[
P_{N}C_{N,2} = \gamma (1 + 2d f(\gamma, d, \rho)) m > \gamma m. \tag{52}
\]

The share of period 2 income spent on non-durable purchases is increasing in \(d\), while durable

\[
\frac{1}{2(1+2\rho)} \left( d(1-\gamma) + \frac{1}{2} \rho + \frac{1}{2} \left( 8d + 4d\rho - 8d\gamma - 8d\rho\gamma + 4d^2 + 4d^2\rho - 8d^2\gamma - 4d^2\rho\gamma + d^2\rho^2 + 4d^2\gamma^2 + 4\right) \right) - 1,
\]

for \(d > 0\).
purchases are declining in \(d\). The (discounted) stock of durables from period 1 implies a lower demand for durables in period 2, even though total consumption of durables is a constant share \((1 - \gamma)\) of period 2 wealth.

It follows from equations (49) to (52) that aggregate expenditure on durables and non-durables can be expressed as

\[
p\bar{C}_X = \bar{C}_Y = \frac{1 - \hat{\gamma}(\gamma; d, \rho)}{\gamma} M < \frac{1 - \gamma}{2} M \quad (53)
\]
\[
P_N\bar{C}_N = \hat{\gamma}(\gamma, d, \rho) M > \gamma M \quad (54)
\]

where \(\hat{\gamma}(\gamma, d, \rho) = \frac{1 + \gamma}{2} - f(\gamma, d, \rho) (1 - \gamma d)\) and \(\hat{\gamma}(\cdot) > 0, \hat{\gamma}_d(\cdot) > 0\) and \(\hat{\gamma}_\rho(\cdot) > 0\). That is, aggregate demand for traded durables is lower in equilibrium than if \(X\) and \(Y\) were non-durable. The aggregate share of income spent on durables is decreasing in \(d\), and vice versa for non-durables. In fact, demand in the economy with durability \(d > 0\) and preference parameter \(\gamma\) is identical to when \(d = 0\) and the preference parameter is \(\hat{\gamma}\). In other words, durability of goods in the model endogenously shifts consumption away from durable goods in the aggregate, as if \(\gamma\) were higher. For example, if \(\gamma = 0.5, \rho = 0.95\) and \(d = 0.54337\), then \(P_N\bar{C}_N = 0.55M\). Proposition 2 summarises the results of thus far.

**Proposition 2** The larger the degree of durability, \(d\), then:

(i) the larger is the equilibrium share of income spent on durables by the young,

(ii) the smaller is the equilibrium share of income spent on durables by the old,

(iii) the smaller is the aggregate share of national income spent on durable goods.

**Proof.** Follows from equations (49) to (54) and \(f(\gamma, d, \rho) = \frac{1}{2d(2 + \rho)} \times \left( d(1 - \gamma) + \frac{1}{2}d\rho + \frac{1}{2}(8d + 4d\rho - 8d\gamma - 8d\rho\gamma + 4d^2 + 4d^2\rho - 8d^2\gamma - 4d^2\rho\gamma + d^2\rho^2 + 4d^2\gamma^2 + 4) \right)^{\frac{1}{2}} - 1 \right). \]

It follows from equations (53) and (54) and Proposition 2 that the impact of durability on equilibrium trade flows is the same as that from increasing the preference parameter from \(\gamma\) to \(\hat{\gamma}\). Flam (1985) shows in a generalised model with two traded goods and one non-traded good that the impact on the trade share of an increased preference for the non-traded good depends on the factor intensites of the sectors. This result is confirmed here, since from equation (36) it follows
that

\[
\frac{\partial X_Y}{\partial \gamma} = \frac{\partial (p M_X)}{\partial \gamma} = \frac{\beta + \alpha - 2\nu}{2(\beta - \alpha)} \left( w L + r K \right),
\]

the implications of which are summarised by condition 1.

**Condition 1** If \( \beta + \alpha - 2\nu < 0 \), then trade flows are decreasing in the degree of durability, \( d \).

**Proof.** The result follows directly from Proposition 2 and equation (55).

Condition 1 states that an increase in traded good durability lowers trade flows through the impact of \( d \) on \( \hat{\gamma} (\gamma, d, \rho) \), provided the non-traded sector is not too labour intensive relative to the two traded sectors. The shift in aggregate demand towards the non-durable, non-traded good induces an expansion of domestic production \( N \). If good \( N \) were very labour intensive, e.g. if \( \beta > \alpha > \nu \), then a relatively large quantity of labour would need to be employed to generate this production increase, leaving the residual composition of available resources more capital abundant. This in turn would necessitate an expansion of \( Y \) and contraction of \( X \) for factor markets to clear, increasing trade flows.

How plausible is it that Condition 1 is satisfied? Non-traded goods are largely services, which would include transportation, electricity and telecommunications. Though services are conventionally perceived as being labour intensive, some services such as electricity generation are arguably more capital intensive than some imports from developing countries. It is therefore plausible that imported manufactures can be less capital intensive than some non-traded services. Indeed, Cardi and Restout (2011) document that, across 13 OECD countries from 1970 to 2004, the capital share in the output of non-traded goods is similar to that in traded goods, and in some cases even exceeds the latter.

Consider an example where \( d = 0 \), \( \alpha = \frac{1}{3} \), \( \beta = \frac{2}{3} \), \( \nu = \frac{3}{5} \), \( \gamma = \frac{1}{2} \), \( \rho = 0.95 \), \( p = \theta = 1 \), \( \bar{K} = 900 \) and \( L = 600 \). Substituting into equations (15), (16) and then (36) gives the value of exports (and imports) at \( 75 \times 2^{\frac{2}{3}} \). If durability rises to \( d = 0.54337 \), then \( \hat{\gamma} = 0.55 \), which lowers the value of exports to \( 67.5 \times 2^{\frac{2}{3}} \).

If condition 1 is satisfied, then the findings of the model point to an endogenous home bias in consumption arising from product durability and suggest a new explanation for Trefer’s (1995)
“missing trade”. Greater durability lowers trade flows, thereby expanding the share of domestically produced goods in consumption. The model shows that even with constant returns to scale technologies and homothetic preferences, a home bias can be generated without appealing to transport costs.

**Corollary 1** If \( \beta + \alpha - 2\nu < 0 \) and \( d > 0 \), there is a home bias in consumption, which is increasing in \( d \).

**Proof.** This follows directly from Proposition 2 and condition 1. ■

3 Trade effects of a productivity shock with durability

This section examines the impact of an unanticipated, uniform total factor productivity shock, for a single time period, on consumption decisions and trade flows, both in the period of the shock and in subsequent periods. The pattern of consumption and trade flows are compared to the equilibrium of section 2.2, which we refer to as the ‘steady state’. In what follows superscripts denote the period in which the consumption takes place, while the digit subscript denotes whether the consumer is young or old in that period.

3.1 Trade flows in the shock period

Let \( T \) denote the shock period in which \( \theta \) falls to \( \lambda \theta \), where \( \lambda \in (0, 1) \). From equations (15), (16) and (17) it follows the fall in productivity, uniform across all sectors, lowers equilibrium factor prices \( w \) and \( r \) by the same proportion, but leaves \( P_N \) unchanged. National income thus falls to \( \lambda M \) in period \( T \) and \( m_T = \lambda m \). The shock is unanticipated so \( E_{T-1}(m_T) = m \) and perceived temporary, so \( E_T(m_{T+1}) = m \).

The old consumer in period \( T \) has a (depreciated) stock of durables from \( T - 1 \), purchased with the expectation that period 2 income would also be \( m \). Consumption of the old consumer in \( T \) is in fixed proportion to wealth,
\[ pC^T_{X,2} = C^T_{Y,2} = \frac{1 - \gamma}{2} \left( \lambda m + dpC^T_{X,1} + dC^T_{Y,1} \right) \] (56)
\[ P_NC^T_{N,2} = \gamma \left( \lambda m + dpC^T_{X,1} + dC^T_{Y,1} \right) \] (57)

where \( C^T_{Y,1} = pC^T_{X,1} = f (\gamma, d, \rho) m \). Substituting for \( T - 1 \) consumption levels and subtracting \( dC^T_{Y,1} \) from \( pC^T_{X,2} \) to find durable purchases gives

\[ pD^T_{X,2} = D^T_{Y,2} = \left( \frac{1 - \gamma}{2} - \frac{\gamma df (\gamma, d, \rho)}{\lambda} \right) \lambda m < \lambda D_{Y,2} = \lambda pD_{X,2} \] (58)
\[ P_NC^T_{N,2} = \gamma \left( 1 + \frac{2df (\gamma, d, \rho)}{\lambda} \right) \lambda m > \lambda P_N C_{N,2}. \] (59)

Let \( \hat{\lambda} (\gamma, d, \rho) \) denote the threshold value\(^6\) of \( \lambda \) below which durable purchases of the old consumer fall to zero in the shock period. Assume \( \hat{\lambda} (\gamma, d, \rho) < \lambda < 1 \) so \( pD^T_{X,2} = D^T_{Y,2} > 0 \). Equations (58) and (59) imply that the fall in demand for durables by the old generation is more than proportional to the productivity shock, due to carrying a relatively large stock of durables from \( T - 1 \).

Furthermore, from equations (47) and (48) it follows that consumption of durables by the young consumer in \( T \), \( C^T_{X,1} \) and \( C^T_{Y,1} \), must satisfy

\[ \frac{1 - \gamma}{2pC^T_{X,1}} - \frac{\gamma}{\lambda m - pC^T_{X,1} - C^T_{Y,1}} + \frac{d\rho}{2 \left( m + dpC^T_{X,1} + dC^T_{Y,1} \right)} = 0 \] (60)
\[ \frac{1 - \gamma}{2C^T_{Y,1}} - \frac{\gamma}{\lambda m - pC^T_{X,1} - C^T_{Y,1}} + \frac{d\rho}{2 \left( m + dpC^T_{X,1} + dC^T_{Y,1} \right)} = 0. \] (61)

Solving (60) and (61) gives

\(^6\)\( \hat{\lambda} (\gamma, d, \rho) \) is increasing in \( d \) and \( \rho \) and decreasing in \( \gamma \), since these raise and lower the consumer’s period 1 durable consumption, respectively, through \( f (\gamma, d, \rho) \). For example, if \( \rho = 0.95, \gamma = 0.5 \) and \( d = 0.54337 \), then from \( f (\gamma, d, \rho) \) and equation (58) it follows that \( \lambda = 0.29843 \).
\[ C_{Y,1}^T = pC_{X,1}^T = g(\gamma, d, \rho, \lambda) \lambda m < \lambda C_{Y,1} = \lambda pC_{X,1} \]  \hspace{1cm} (62)

\[ P_N C_{N,1}^T = (1 - 2g(\gamma, d, \rho, \lambda)) \lambda m > \lambda P_N C_{N,1} \]  \hspace{1cm} (63)

where \( g(\cdot) < 0 \), \( g_d(\cdot) > 0 \), \( g_{\rho}(\cdot) > 0 \) and \( g_{\lambda}(\cdot) > 0 \),

so expenditure on each durable good by the young consumer is a share\(^7\) \( g(\gamma, d, \rho, \lambda) \) of income. The fall in demand for durables by the young generation is also more than proportional to the productivity shock. This arises because income is uneven over the consumer’s lifetime. A lower period 1 income reduces the incentive to skew consumption towards durables in period 1, as the sacrifice in period 1 utility from doing so is larger.

Aggregating over the consumers gives national expenditure on each good in period \( T \) as a proportion of national income\(^8\),

\[ pC_{X,T} = \overline{C_{X,T}} = \frac{1 - \hat{\gamma}_T(\gamma, d, \rho, \lambda)}{2} \lambda M < \frac{1 - \hat{\gamma}(\gamma, d, \rho)}{2} \lambda M \]  \hspace{1cm} (64)

\[ P_N C_{N,T} = \overline{C_{N,T}} = \hat{\gamma}_T(\gamma, d, \rho, \lambda) \lambda M > \hat{\gamma}(\gamma, d, \rho) \lambda M, \]  \hspace{1cm} (65)

where \( \hat{\gamma}_T(\gamma, d, \rho, \lambda) = \frac{1 + \gamma}{2} - g(\gamma, d, \rho, \lambda) + \frac{\gamma d f(\gamma, d, \rho)}{\lambda} \) and is increasing in \( \gamma, d, \rho \) and decreasing in \( \lambda \). Since both young and old optimise away from durables, it follows that for given \( d \), a shock \( \lambda \) induces a smaller fraction of national income \( \lambda M \) to be spent on durables.

If \( \beta + \alpha - 2 \nu < 0 \), then the rise in \( \hat{\gamma} \) induced by the productivity shock, induces a more than proportional fall in trade flows. Since \( w_T = \lambda w \) and \( r_T = \lambda r \) it follows that trade flows in \( T \) are given by,

\[ (d\lambda(1 - \gamma) + \frac{1}{2} d\rho\lambda + \frac{1}{2} (8d\lambda + 4d\rho\lambda - 8d\rho\lambda - 8d\rho\lambda + 4d\lambda^2 + 4d^2\rho\lambda^2 - 8d^2\lambda^2\gamma - 4d^2\rho\lambda^2\gamma + d^2\rho^2\lambda^2 + 4d^2\lambda^2\gamma^2 + 4) - 1) \],

for \( d > 0 \).

\(^7\)The share of national income spent on durables is the average of the shares of the young and old consumers.
\[ pM_X^T = X_Y^T = \lambda \left( \frac{(1 - \hat{\gamma}_T)(1 - \beta) + \hat{\gamma}_T(1 - \nu)}{(\beta - \alpha)} + \frac{1 - \hat{\gamma}_T}{2} \right) rK \]

\[ - \lambda \left( \frac{(1 - \hat{\gamma}_T)\beta + \hat{\gamma}_T\nu}{(\beta - \alpha)} - \frac{1 - \hat{\gamma}_T}{2} \right) wL < \lambda pM_X = \lambda X_Y \]

(66)

Trade flows are thus scaled down by \( \lambda \), then lowered further by the preference shift from \( \hat{\gamma} \) to \( \hat{\gamma}_T (\gamma, d, \rho, \lambda) \). The findings are summarised in Proposition 3.

**Proposition 3** If \( \beta + \alpha - 2\nu < 0 \) and \( d > 0 \), then an unanticipated fall in productivity for one period gives rise to a more than proportional decline in trade flows in that period.

**Proof.** This follows from equations (58) to (66) and condition 1. ■

For example, let us return to the setting where \( \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \nu = \frac{3}{5}, \gamma = \frac{1}{2}, d = 0.54337, \rho = 0.95, p = \theta = 1, K = 900 \) and \( L = 600, \) for which \( \hat{\gamma} = 0.55 \) and the value of exports is \( 67.5 \times 2^\frac{3}{2} \).

If productivity falls to \( \lambda \theta \) in \( T \), then \( \hat{\gamma}_T (\lambda) \) can be determined by substituting the parameters into \( f (\gamma, d, \rho) \), equation (58) and then \( g (\gamma, d, \rho, \lambda) \). For \( \lambda = 0.5 \), then \( \hat{\gamma}_T (\lambda = 0.5) = 0.63516 \), which exceeds \( \hat{\gamma} \). Trade flows are computed from equation (66) to be \( 27.363 \times 2^\frac{3}{2} \), lower than a proportional decline in trade flows to \( 33.75 \times 2^\frac{3}{2} \).

### 3.2 The elasticity of trade to the shock

A corollary of Proposition 3 is that the elasticity of trade to the shock exceeds 1, if \( \beta + \alpha - 2\nu < 0 \) and \( d > 0 \). The elasticity of exports to the shock (and thus to GDP) follows from equation (66) and can be expressed as:

\[ \varepsilon_{X,T} \equiv \frac{\partial X_Y^T}{\partial \lambda} \frac{\lambda}{X_Y^T} \]

\[ = 1 + \frac{-\frac{\partial \hat{\gamma}_T (\gamma, d, \rho, \lambda)}{\partial \lambda} \lambda (2\nu - \beta - \alpha) (rK + wL)}{2rK - [\beta + \alpha + \hat{\gamma}_T (2\nu - \beta - \alpha)] (rK + wL)} > 1, \]

(67)

where \( \varepsilon_{X,T} \) is decreasing in \( \lambda \) and \( K \) and increasing in \( d, \gamma, L, \) and \( \rho \). The elasticity of trade to GDP is thus greater the larger the degree of durability and the larger the shock. Returning to the example where \( \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \nu = \frac{3}{5}, \gamma = \frac{1}{2}, d = 0.54337, \rho = 0.95, p = \theta = 1, K = 900, L = 600 \) the elasticity is 1.46 if \( \lambda = 0.5 \), but rises to 1.62 if \( \lambda = 0.4 \). The model therefore describes
a mechanism that explains the observation that deeper downturns are associated with a higher elasticity of trade to GDP.

3.3 Trade flows after the shock

In $T + 1$, productivity is restored to $\theta$ and so the wage rate, rental rate and national income are $w$, $r$, and $M$, respectively. The young consumer in period $T + 1$ expects constant income $m$ over his life, so demands goods according to equations (49) and (50). The old consumer, however, has a stock of durables from $T$, given by equation (62), which are lower than in the steady state. With a smaller stock of durables from period 1, the consumer’s period 2 expenditure on durable purchases is higher than in the steady state. To see this, consider that in period $T + 1$ the older generation consumes goods in fixed proportion to wealth,

$$
pC_{X,2}^{T+1} = C_{Y,2}^{T+1} = \frac{1 - \gamma}{2} \left( m + dpC_{X,1}^T + dC_{Y,1}^T \right) \tag{68}
$$

$$
P_NC_{N,2}^{T+1} = \gamma \left( m + dpC_{X,1}^T + dC_{Y,1}^T \right), \tag{69}
$$

where $C_{Y,1}^T = pC_{X,1}^T = g(\gamma, d, \rho, \lambda) \lambda m$. Substituting for $T$ consumption levels and subtracting $dC_{Y,1}^T$ from $pC_{X,2}^{T+1}$ to find durable purchases gives

$$
pD_{X,2}^{T+1} = D_{Y,2}^{T+1} = \left( \frac{1 - \gamma}{2} - \gamma d \lambda g(\gamma, d, \rho, \lambda) \right) m > D_{Y,2} = pD_{X,2} \tag{70}
$$

$$
P_NC_{N,2}^{T+1} = \gamma \left( 1 + 2 \lambda d g(\gamma, d, \rho, \lambda) \right) m < P_NC_{N,2}. \tag{71}
$$

Hence, the old generation spends a larger share of income on durables than in the steady state, while the young generation spends exactly the same share as in the steady. The aggregate effect is that a larger proportion of national income is spent on durables in $T$ than in the steady state,

$$
pC_{X,T+1} = C_{Y,T+1} = \frac{1 - \tilde{\gamma}_{T+1}(\gamma, d, \rho, \lambda)}{2} M > \frac{1 - \tilde{\gamma}(\gamma, d, \rho)}{2} M \tag{72}
$$

$$
P_NC_{N,T+1} = \tilde{\gamma}_{T+1}(\gamma, d, \rho, \lambda) M < \tilde{\gamma}(\gamma, d, \rho) M, \tag{73}
$$
where $\gamma_{T+1} (\gamma, d, \rho, \lambda) = \frac{1+\gamma}{2} - f (\gamma, d, \rho) + \gamma d \lambda g (\gamma, d, \rho, \lambda)$ and is increasing in $\gamma, d, \rho$ and $\lambda$. Furthermore, if $\beta + \alpha - 2\nu < 0$, then the increase to $\gamma_{T+1} (\gamma, d, \rho, \lambda)$ gives rise to a level of trade flows that overshoots the steady state level of trade, and is given by

$$pM_{X}^{T+1} = X_{Y}^{T+1} = \left(\frac{(1-\gamma_{T+1})(1-\beta) + \gamma_{T+1}(1-\nu) + 1 - \gamma_{T+1}}{2}\right) rK$$

$$(74)$$

$$- \left(\frac{(1-\gamma_{T+1})\beta + \gamma_{T+1}\nu}{(\beta - \alpha)} - \frac{1 - \gamma_{T+1}}{2}\right) wL > pM_{X} = X_{Y}$$

Finally, in period $T + 2$, the equilibrium discussed in section 2.2 is restored, since all consumers have income $m$ in both periods of their life. The findings are summarised in Proposition 4.

**Proposition 4** If $\beta + \alpha - 2\nu < 0$ and $d > 0$, then trade flows overshoot the steady state level before returning to it in the two periods following an unanticipated, one period fall in productivity.

**Proof.** This follows from equations (70) to (74) and condition 1. ■

Returning to the example where $\alpha = \frac{1}{3}, \beta = \frac{2}{3}, \nu = \frac{3}{5}, \gamma = \frac{1}{2}, d = 0.54337, \rho = 0.95, p = \theta = 1, K = 900$ and $L = 600$, we can determine $\gamma_{T+1} (\lambda)$ by substituting the parameters into $f (\gamma, d, \rho)$, $g (\gamma, d, \rho, \lambda)$, and equations (70) and (49). For $\lambda = 0.5$, $\gamma_{T+1} = 0.51126$, which is smaller than $\gamma$. Trade flows are computed from equation (74) to be $73.311 \times 2^{\frac{3}{5}}$, larger than steady state trade flows at $67.5 \times 2^{\frac{3}{5}}$. Figure 2 summarises the effects of the shock on GDP and trade flows, for the parameter values we have been using thus far.

4 Conclusions

There is systematic evidence that trade flows are more volatile than GDP, with the trade collapse of 2008 a striking example of this. Moreover, the observed large decline in demand for durable goods has been posited as key to explaining the trade collapse. While durable goods are commonly incorporated into macro models, there is relatively limited analysis of the role of product durability in the theoretical trade literature. Comparative advantage models characterised by homothetic preferences and constant returns to scale technologies have the feature that trade flows change in proportion with uniform productivity shocks. This paper shows that by embedding durability...
of traded goods into an otherwise standard Heckscher-Ohlin framework with two traded and one non-traded, non-durable sector, it is possible to explain the excess trade volatility phenomenon, both in the period of the national income shock and in the recovery phase.

Overlapping generations of consumers who generate future wealth through the purchase of durables are shown to maximise life-time utility by skewing their consumption towards durables when young. In turn, the stock of durables carried from the first year of life lowers demand for durable goods when consumers are old. The aggregate effect is that durability of traded goods endogenously shifts preferences away from traded goods towards non-traded goods in the economy. Provided the non-durable sector is sufficiently capital intensive, embedding durability in the model gives rise to an endogenous increase in the share of domestically produced goods in consumption. The model thus offers an alternative explanation for the home bias phenomenon, as well as for Trefler’s “missing trade”, that does not hinge on the presence of transport costs or increasing returns.

Shocking the equilibrium with a one period, unanticipated uniform decline in productivity induces a re-optimisation away from durables by both young and old in the economy. For the young it is due to a reduced willingness to trade-off utility in youth for utility in later life when period 1 income is shocked. For the old it is the large stock of durables carried forward from youth, which
explains the fall in durable purchases. The aggregate effect is a more than proportional decline in international trade, provided the non-traded sector is sufficiently capital intensive. Furthermore, the elasticity of trade flows with respect to GDP is found to be increasing both in the degree of durability and the size of the shock. Thus the model provides microfoundations for the asymmetric shock to the demand for durable goods observed in recessions and clarifies the link between this endogenous shift in preferences and international trade flows. It also offers an explanation for the observation that deeper downturns are associated with a higher elasticity of trade to GDP.

The model clearly has its limitations. While it offers one mechanism for understanding trade volatility, it does not address other factors thought to have contributed to the trade collapse such as vertical production linkages. Moreover, the emphasis is on demand for consumer durables, and does not consider demand for capital goods. The small economy assumption makes the model tractable, but limits the analysis to the effects of a domestic shock while prices are kept constant. Furthermore, the only intertemporal link in the model is the stock of durable goods that are carried forward; consumers are unable to borrow or lend. Examining how access to capital markets may affect trade volatility is an interesting avenue for future research. Finally, the only determinant of international trade considered is comparative advantage. An examination of trade models based on economies of scale or with heterogeneous firms may provide further mechanisms for understanding the determinants of trade volatility.
References


