University of Sussex
Business, Management & Economics

Economics Department Working Paper Series

No. 24-2011

A Framework to Analyze the Impact of Exchange Rate Uncertainty on Output Decisions

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Abstract: Southern Cone economies exhibit a high record of exchange rate volatility. In this context, firms tend to contract dollar debt, irrespective of their trade orientation, and without available hedging instruments. This exposes them to bankruptcy risk, in the event of large exchange rate movements. This paper provides a framework to analyze the output effect of exchange rate uncertainty in that context, by focusing on the channel uncertainty-output that operates through the financial strategy of the firm. We find that increases in exchange rate uncertainty increase the probability of bankruptcy, thus increasing expected marginal bankruptcy costs, and reducing optimal output of a risk-neutral firm. Furthermore, we find that firms with higher than average liquidity balances will face lower marginal bankruptcy costs, thus producing more than the average firm. The model displays persistence, as any shock to current profits affects future liquidity balances, and so, future output. This framework can easily be extended to explain the response of other firms' decisions to exchange rate uncertainty, such as investment.

JEL Classification: F31, G33, D81

Key Words: Exchange rates, Bankruptcy Costs, Production Under Uncertainty

1 Introduction

Developing countries have experienced higher exchange rate volatility than developed countries. Southern Cone economies, where exchange rate risk hedging instruments have been unavailable until recently, show particularly high volatility records, which generate high exchange rate uncertainty. In this context, firms contract dollar-debt, which leads to currency mismatches in their balance sheets, and expose them to exchange rate risk.

Given these characteristics, in this paper we investigate the output response to a rise in the degree of exchange rate uncertainty faced by competitive firms, and consider as separate cases firms producing non-tradable and firms producing tradable goods. We focus on the effects of increases in uncertainty that increase the probability of extreme exchange rate outcomes. Most of the literature on the link between uncertainty and economic activity overlooks the effects of the financial strategy of the firm, relies on exogenously determined attitudes to risk and on distributional assumptions that mainly reduce uncertainty to the variance of the random variable. This paper focuses on the channel linking uncertainty and output that operates through the financial strategy of the firm. In our model, firms rely on credit markets to finance working capital, and contract dollar-debt. The firm faces bankruptcy in the event of an exchange rate outcome that makes the debt repayment larger than the output proceeds. Therefore, it internalizes the expected bankruptcy costs when making the optimal output decision, thus showing aversion to bankruptcy risk. To identify the pure-risk effect on output we focus on changes in perceptions that lead to a higher probability of extreme exchange rates, keeping a constant mean. Our analysis does not rely on distributional assumptions of the exchange rate.

Our main results are the following. First, when firms exhibit currency mismatches, obtaining revenue in pesos and repaying dollar-debt, an increase in exchange rate uncertainty reduces optimal output, as the marginal bankruptcy costs increase. The anticipation of generalized government bailouts in the event of drastic exchange rate movements exerts the opposite effect on output. However, our contention is that in the event of high uncertainty, firms will not take bailouts for granted and will act cautiously, hoarding liquidity and disengaging from risky activities. Second, firms' liquidity balances matter for the choice of output: firms with high liquidity balances will face low bankruptcy risks, which leads them to produce more than the average firm. This introduces persistence in the output effects of shocks to profits because any shock to profits, such as a reduction in output prices, or a depreciated exchange rate, will decrease firm's next period liquidity balances, and so, next period's output.

The remainder of the paper is structured as follows. Section 2 presents different timing scenarios associated with production, describing the sources of exchange rate uncertainty in each of them. Section 3 discusses two limitations of the standard theory of production under uncertainty: the oversimplification of the concept of uncertainty, and the absence of discussion on the financial strategy of the firm. Section 4 presents a production model for a competitive risk-neutral firm producing non-tradables. We first discuss the effects of increases in expected depreciation. Then, we incorporate the possibility of bankruptcy and analyze the output effects of increases in uncertainty. Section 5 analyzes the case of a firm producing tradable goods, the possibility of government bailouts, and firm reactions to irreducible uncertainty. Section ?? concludes.

2 Exchange Rate Uncertainty in the Production Decision

Uncertainty arises when the consequence of a decision is not a single sure outcome, but a number of possible outcomes. Figure 1 shows three production timing scenarios to visualise the sources of uncertainty in the production decision.

(Sc1) **Certainty**: The firm produces in period one, when inputs are transformed into output instantaneously, with known output and input prices. The proceeds of the output are used to pay inputs. Everything is known with certainty.

(Sc2) **Two Periods, No Working Capital Needs**: In period one the firm makes a commitment to produce and fixes input prices. In period two, inputs are transformed into output instantaneously. Output prices are revealed, output is sold and the proceeds are used to pay for the inputs. Firms do not need working capital. When making the production decision they only face uncertainty with respect to output prices.

(Sc3) **Two Periods, Working Capital Needs**: Here, production takes time. The production process starts in period one, when inputs are purchased, but it is only in period two when output can be sold. Working capital needs arise. Firms contract dollar-debt in period one at the risk-free interest rate (we assume no default risk). In period two, output prices and exchange rates are revealed. When making the production decision firms face uncertainty with respect to output prices and the peso-cost of the dollar debt. Under these conditions, output decisions are risky investment decisions.

Sc(2) and Sc(3) seem to better describe reality than Sc(1). Forward contracts are a way to reduce exposure to real exchange rate risk. However, these have not always been available, and if they have been, costs were prohibitive.[1]

3 The Limitations of the Standard Approach

Two limitations of the standard approach for the analysis of production under uncertainty are discussed here.[2] In Section 3.1 we focus on the validity of a

[1]This assertion is based on an informal discussion with the Secretariat of the Uruguayan Union of Exporters. In addition, as argued in Borensztein et al. (2008) this seems to apply to Argentina as well. In Brazil, only large firms use forward contracts, and still it is a recent trend. It is argued that because these countries have had fixed or predetermined exchange rate regimes for long periods, firms frequently have not internalized exchange rate risk as an additional cost, given that the Central Bank was implicitly offering an insurance against variations. That could help explaining absence of a better developed forward market for the exchange rate.

[2]For "standard approach" we understand the work of Sandmo (1971), Hawawini (1978), Leland (1972), and what is found in most Microeconomics textbooks. E.g.: Gravelle and Rees (1992).

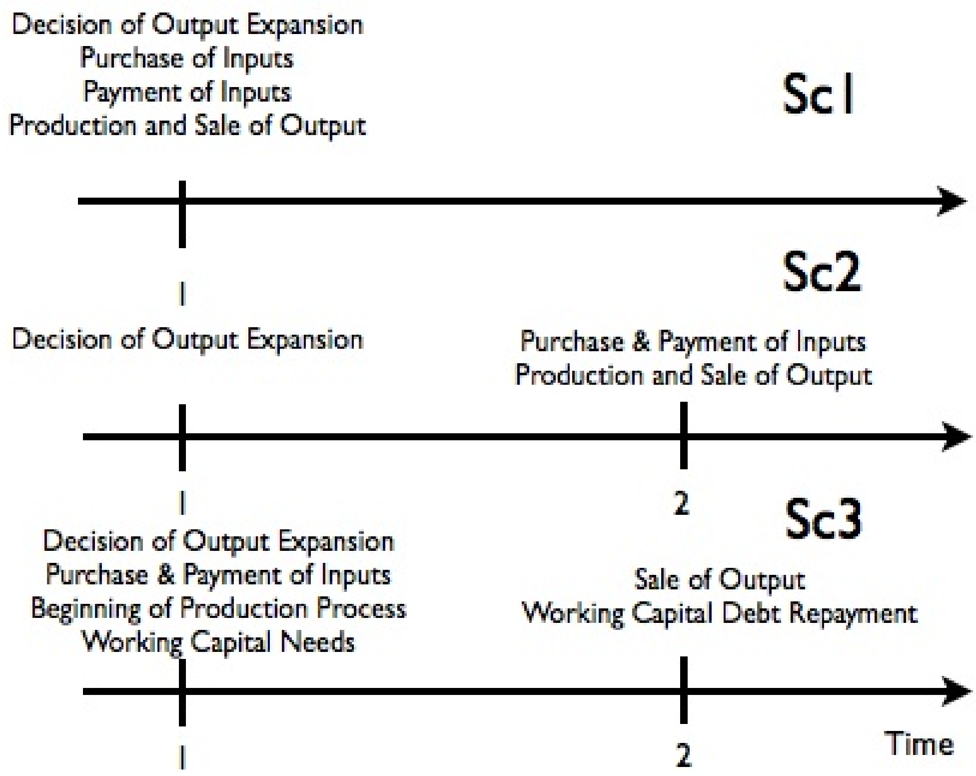

Figure 1: Timing of the Production Decision

mean-variance analysis (M-V), and in Section 3.2 on the implicit assumptions the standard production theory makes on the capital structure.

3.1 Is the Variance a Good indicator of Uncertainty?

The analysis of the impact of uncertainty on output frequently reduces the concept of uncertainty to the variance of the random variable. That implies that either the utility function of the decision maker is quadratic, or that the probability distribution of the random variable can be fully characterized by parameters of location and scale (with all other parameters being definite functions of these) (See Gravelle and Rees (1992)). How reasonable are these restrictions?

> *"In economics, the relevant probability distributions are not nearly Gaussian, and quadratic utility in the large leads to well-known absurdities".* Samuelson (1970)

To illustrate the problems associated with the assumption on the utility function, take a quadratic function as in equation (1):

$$U(\pi) = a + b\pi + c\pi^2 \tag{1}$$

(where $a > 0$, $b > 0$, $c < 0$).

(1) Agents display negative marginal utility of profits when these exceed $-b/2c$.

(2) Agents display increasing absolute risk aversion (IARA). Pratt (1964) defines absolute risk aversion (ARA) as $r(\pi) = -u''(\pi)/u'(\pi)$. If $r'(\pi) > 0$, agents display IARA. For a function as in (1), $r(\pi) = -2c/b + 2c\pi$, and $r'(\pi) = 4c^2(b + 2cx)^{-2} > 0$.

Instead, what seems to be widely accepted is that agents typically display decreasing absolute risk aversion (DARA). Pratt (1964), for example, suggests that "people might generally pay less for insurance against a given risk the greater their assets" [pp. 122-3], implying DARA. The common findings in the empirical literature support the hypothesis of DARA.

Let us turn the attention to the implications of the assumptions on the distribution. Distributions that differ only by location or scale will have shape parameters that are functions of location and scale (or constants). An example is the Normal, in which the parameters of location (μ) and scale (σ) fully characterize the distribution, and measures of shape, such as skewness and kurtosis are constants ($= 0$, and $= 3$ respectively). In the context of financial studies, it is generally claimed that probability distributions of asset returns cannot be fully parameterized by measures of location and scale as they may also differ in measures of shape.[3]

An example helps to illustrate why reducing the analysis of uncertainty to that of mean and variance of the random variable is inadequate if distributions also differ in shape.

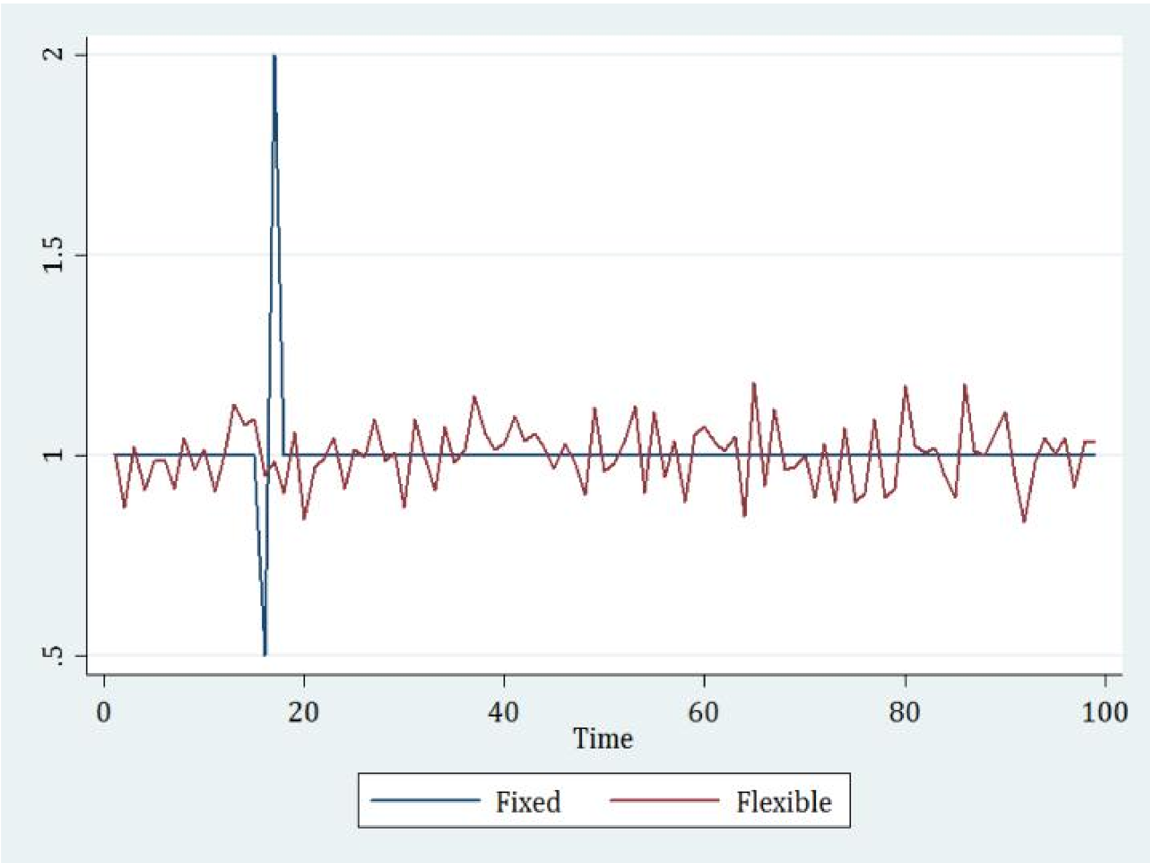

Figure 2: Volatility that "breaks" versus volatility that "bends"

[3]In fact, an unpublished manuscript of Samuelson suggests "Full-Scale Optimization" (FSO), a methodology in which return distributions are used in their entirety (Hagstromer et al. (2007)). The computational burden of this method is significant, and won't be considered here. For more on FSO, see Cremers et al. (2005).

Figure 2 shows the behaviour of two series: "Fixed" and "Flexible". Fixed exhibits an extreme episode in periods $t = 23$ and $t = 24$, and it takes a constant value equal to one for the rest of the period. Flexible shows permanent and mild variation. Would agents perceive the same degree of uncertainty with respect to the value that "Fixed" will take tomorrow and with respect to the value that "Flexible" will take tomorrow? Probably not. Assume now that "Fixed" and "Flexible" are prices of two goods. The behaviour of Fixed has more scope for large effects on, say, accumulated profits than that of Flexible. The producer facing Flexible faces mild or "bounded randomness" whereas that facing Fixed faces "substantial randomness". Uncertainty associated with the former is somehow controllable by averaging while that associated with the latter is less so. Using a M-V approach, we would consider both series as equivalent ($E[u(Fixed)] = E[u(Flexible)]$), as "Flexible" has been constructed by drawing random numbers from a normally distributed population, such that the obtained series approximates to a normal distribution and $E(Flexible) = E(Fixed)$, and $V(Flexible) = V(Fixed)$ ($\mu = 1.005$ and $\sigma = 0.08$). It is possible to have a change in which location and scale don't alter, but shape does (kurtosis in our example). Then, a M-V analysis will give the wrong answer. When distributions differ by location, scale **and** shape, the decision maker will not only look at mean and variance, but also at higher moments of the distribution. (In this case, for example, the kurtosis of "Fixed" is 66, while that of "Flexible" is 2.6.)

Take two countries, with different real exchange rate generating processes. In one, the process is consistent with low-probability extreme events (such as the series "Fixed" in Figure (2)), in the other, is consistent with a low, constant variance (such as the series "Flexible" in the same figure). If uncertainty affects the behaviour of producers, then it is reasonable to expect a larger effect, and therefore a more cautious behaviour of producers in the former country than in the latter, even if during a particular time period, the series shows to be stable.[4]

We cast doubt on the validity of this distributional assumption for the random variable being the real exchange rates (RER) in Argentina, Brazil and Uruguay, as these countries have had fixed nominal exchange rate regimes, with mild variations in RERs, and then, collapses of these regimes with extreme RER movements in several occasions during the period of analysis. In Table (1) we present the first four central sample moments of the distribution of the growth rate of the RER over four different time periods.[5] The variations in the estimates of the parameters of shape (skewness and kurtosis) are evident. For illustrative purposes, we take two subperiods for Uruguay (last three rows of Table (1)) in which mean and variance of RER growth are similar, but skewness and kurtosis

[4]Daniel Heymann has coined a distinction between the volatility that "breaks" and the volatility that "bends". This distinction came up in the context of an informal discussion on exchange rate uncertainty generated by fixed versus flexible exchange rate regimes in Southern Cone countries, where currency crises are relatively frequent. Then, Heymann claimed that, frequently, fixed exchange rate regimes experience "underlying volatility". This is unobservable until it is so large that "breaks" the regime and generates an extreme episode. Underlying volatility in the context of flexible exchange rate regimes, on the other hand, manifests immediately, no matter how large it is, through movements in the exchange rate. Thus, it is less likely to accumulate and generate an extreme episode.

[5]We have arbitrarily chosen the time periods to illustrate the likely inadequacy of this distributional assumption.

change significantly, suggesting that these distributions do exhibit changing patterns in their shape.[6] Thus, the M-V approach seems inappropriate to deal with the randomness of the RER. To capture uncertainty, we will need to take into account higher moments of the distribution of the relevant random variable, besides the looking at mean and variance.

Country	Period	Mean	Variance	Skeweness	Kurtosis
Argentina	1970m1-1978m12	0.006	0.021	4.428	25.334
	1979m1-1991m12	0.013	0.048	7.230	68.257
	1992m1-1998m12	-0.001	0.000	-0.743	3.646
	1999m1-2004m12	0.012	0.005	3.866	19.327
Brazil	1970m1-1978m12	0.001	0.000	-0.895	5.346
	1979m1-1991m12	0.003	0.002	1.315	7.152
	1992m1-1998m12	-0.001	0.000	0.335	5.845
	1999m1-2004m12	0.007	0.003	1.616	8.200
Uruguay	1970m1-1978m12	0.003	0.009	7.791	74.843
	1979m1-1991m12	0.002	0.007	7.475	78.328
	1992m1-1998m12	-0.004	0.000	0.949	7.816
	1999m1-2004m12	0.005	0.001	2.012	14.909
	1996m1-2002m12	0.003	0.001	2.998	23.099
	2003m1-2004m12	0.003	0.001	-0.596	4.881
H_o:**Same** μ, σ	p-value	0.4413	0.3196	0.000	0.000

Table 1: Sample moments for the Growth Rate of the RER

3.2 Are the firm's financing decisions innocuous?

All references to the capital structure of the firm were ignored in our initial exposition of the effects of RER uncertainty on output, which relied on tools of the standard theory of production. When the firm operates in Sc(3), there is a lag between the moment in which the production decision is taken and inputs are purchased, and the moment in which the revenue from the sale of output is obtained, working capital needs will arise. The financial strategy of the firm may add extra uncertainty, and will not be innocuous for optimal output, as we will show in Section 4. Here we discuss some of the options the firm may face in terms of its capital structure, and some of their implications.

Firms could finance working capital needs in excess of the available internally generated funds by issuing equity, or borrowing. When they choose the first alternative, the firm diversifies risk. However, as argued by Myers and Majluf (1984), firms generally do not issue equity to finance working capital needs. The authors find that the announcement of equity offerings reduces stock prices in a significant manner. They argue that informational imperfections are a plausible explanation for this finding. The manager of a firm is asymmetrically informed about the value of the firm. For this reason, if the manager decides to issue stock at a given market price, the investors are only going to be willing to buy it at a lower price, exerting downward pressure on stock prices. That reduction in firm value would be a substantial "cost to false signalling", to be avoided if firms can

[6] We test equality in means, and equality in variances and report the p-values in the last row of the table.

rely on debt-financing.[7] In this case, creditors will not interfere with managerial decisions, but risk will not be diversified, and if the firm cannot meet its financial obligations due to adverse market conditions or poor management, debt will be unforgiving, and the firm will face bankruptcy. Bankruptcy is to be avoided by managers because if it happens, they will suffer a stigma as it is difficult to distinguish whether the financial distress is due to poor management or due to adverse market conditions. This is why, as Greenwald and Stiglitz (1993) argue, managers are averse to bankruptcies and will internalize its expected costs when making the production decision.

Now, if firms tend to use debt as a source of financing, the currency structure of that debt needs to be considered, for in the countries under analysis and over a large portion of the period considered, dollar-debt contracts have been a common phenomenon, largely independent of the trade orientation of the firm.[8] The existence of dollar-debt contracts brings us to Sc(3) presented in Section 2, in which there is another channel through which RER uncertainty will affect production decisions.

The reasons behind debt-dollar contracts are debatable. In general, banking lending rate differentials between domestic and foreign currency were significantly above devaluation expectations - at least those devaluation expectations that explained the banking borrowing rates' differentials (Licandro and Licandro (2003)). A myopic financial manager would then choose to borrow in foreign currency, as it is the apparent cheaper option. However, even if the agent is forward looking and foresees a large exchange rate depreciation, it may be tempted to borrow in foreign currency. This is because if all other agents are doing the same (and he's aware of that), a large depreciation would generate chained bankruptcies, and a collapse in the payment system of the economy. Because the social costs of that outcome are socially undesirable, then a debtor bailout could be ex-post optimal for the government. This induces firms not to internalize the exchange rate risk, and instead, rely on an implicit "free insurance" provided by a lender of last resort (the government). (See Burnside et al. (2001)).[9] Another view argues that dollar debt was deliberately fostered by the governments in these countries in order to show a commitment to the fixed exchange rate regimes. By making the costs of a devaluation extremely high, the government tried to gain credibility (see Levy-Yeyati (2006)) . In some cases,and mainly for long terms, peso debt was just unavailable. Whatever the reasons behind this phenomenon, the existing data reveals that it was widespread in Argentina and Uruguay, and though less prevalent, still significant in Brazil. Kamil (2004) offers some indicators in this respect. His database provides unique information on the currency and maturity structure of firms' liabilities for 10 Latin American countries.[10] The debt-dollarization ratio, calculated as dollar-linked debt as a percentage of total liabilities, is for Argentina during the 1990s, well above 50%, for Brazil, it was lower but still significant and in the range 11-20% during the decade, while for Uruguay the same ratio was in the range 74-84%. It is also possible to see that

[7] Another explanation for the reduction in stock prices after equity issues is that there is a downward sloping demand for a firm's shares. (See Myers and Majluf (1984))

[8] For example, Galiani et al. (2003) argue that in Argentina, debt dollarization was "the rule rather than the exception", and found no relationship between the production mix, or the probability of a sudden nominal devaluation and the degree of debt dollarization.

[9] The optimal output implications of this are discussed in Section 5.2

[10] The dataset covers a wide variety of firms, of different size and trade orientation.

dollar-debt seems to be longer term than non-dollar debt. In the case of Argentina, for example, the ratio of long-term dollar liabilities to total dollar liabilities is in the range of 30-55%, while the ratio of long-term non-dollar liabilities to total non-dollar liabilities is in the range of 11-21%. For Uruguay, the former ratio is in the range 23-54%, while the latter in the range 3-17%.[11]

A framework for the analysis of output decisions under real exchange rate uncertainty should incorporate these elements into account. This is what we do in next section.

4 An Alternative Approach: A Production Model with Bankruptcy Costs and Dollar-Debt

Here we present a simple production model with two key characteristics: production takes time, and firms finance working capital using dollar-debt contracts. The firm operates in Sc(3). We start by considering the output effects of increases in expected depreciation, then we introduce bankruptcy costs, and attempt to answer what is the output effect of mean-preserving increases in exchange rate risk. We start by looking at the case of a firm producing non-tradable goods, and then discuss the implications for one producing tradable goods. Our model owes most to Greenwald and Stiglitz (1993).

4.1 The Model

The firm is neutral to risk and produces a non-tradable good whose price is determined domestically. The firm operates in Sc(3). A single input is used to produce. It is bought at the beginning of the period in a perfectly competitive market. Output is only sold in the period after production, in a perfectly competitive market. For these reasons, the firm needs working capital to buy inputs. Working capital can only be borrowed in foreign currency from financial markets. The exchange rate is expressed as dollars per peso, and it is a random variable.

At the beginning of the period, the firm inherits liquidity balances of size a. These balances are generated by last period's difference between the revenue from the sale of output, and debt repayment.

The price of the only input equals w and is determined in a perfectly competitive input market.

This leads to a level of foreign-currency denominated debt B, along with a nominal interest rate to be paid to the lender, R, which is inherited at the beginning of next period. The peso-value of the debt repayment is unknown today, and will depend on the exchange rate prevailing in the next period.

The following assumptions are made:

(A1) **Production Technology**: Firms produce using only one input, and the production process exhibits diminishing returns. The input requirement function is $\phi(q)$, with $\phi'(q) > 0$, and $\phi''(q) \geq 0$. Note that the input requirement function is the inverse of a production function. Let A be the efficiency parameter of this production function. For simplicity, we assume A=1.

[11] There is no data available for Brazil in this respect.

(A2) **The Source of Randomness**: The exchange rate, measured as dollars per peso, is a random variable $\tilde{e}$. The expected depreciation over the period, $\delta^e = e_0/\tilde{e}_1 - 1 = 0$. $\tilde{e}$ is distributed with distribution function $F_{\tilde{e}}(.)$ and density function $f_{\tilde{e}}(.)$

(A3) **Dollar debt**: The level of debt is determined by the difference between the value of the input bill, and the liquidity balances of the firm at the beginning of the period. Debt can only be contracted in dollars. The peso-value of the repayment to the creditor in the next period is random, as so is the exchange rate.

(A4) **Prices**: There is only one good produced in the economy. To be able to focus on exchange rate uncertainty only, we exclude price uncertainty, and assume a fixed price. The fact that exchange rate volatility tends to be substantially greater than price volatility supports this assumption.

(A5) **Bankruptcy**: Default risk is zero. Firms don't go bankrupt.

(A6) **Creditors' market**: Creditors are risk neutral and perfectly informed.

Expressed in pesos, firms borrowing needs, B, are given by today's difference between the input bill and the inherited liquidity balances:

$$B = (w\phi(q) - a) \tag{2}$$

In foreign currency, $B^{\$} = B \times e_0$. What is to be paid back to the creditor in pesos, is known once the exchange rate is revealed next period, and equals: $B^{\$} = B \times \dfrac{e_0}{\tilde{e}_1} = B \times (1 + \delta)$.

To choose the level of output to be produced, managers will maximize an expected end-of-the-period wealth function which can be re-expressed as in (3):

$$max_q E\Big\{pq - (1+\delta)(1+R)(w\phi(q) - a)\Big\} \tag{3}$$

The first order condition is:

$$p - (1+R)(1+\delta^e)w\phi'(q) = 0 \tag{4}$$

Profits are a concave function of q when $\phi''(q) > 0$. The second order condition is:

$$-(1+R)(1+\delta^e)w\phi''(q) < 0 \quad \text{if} \quad \phi''(q) > 0 \tag{5}$$

4.2 Increases in Expected Depreciation

Proposition 1. *Optimal output is a decreasing and convex function of depreciation expectations, the dollar-return, and the input price if $\phi'(q) > 0$ and $\phi''(q) > 0$.*

Proof.

$$\frac{dq^*}{d\delta^e} = -\frac{\phi'(q)}{(1+\delta^e)\phi''(q)} < 0 \tag{6}$$

$$\frac{d^2q^*}{d(\delta^e)^2} = \frac{\phi'(q)\phi''(q)}{[(1+\delta^e)\phi''(q)]^2} > 0 \tag{7}$$

$$\frac{dq^*}{d(1+R)} = -\frac{\phi'(q)}{(1+R)\phi''(q)} < 0 \tag{8}$$

$$\frac{d^2q^*}{d(1+R)^2} = -\frac{\phi'(q)}{(1+R)^2\phi''(q)} > 0 \tag{9}$$

$$\frac{dq^*}{dw} = -\frac{\phi'(q)}{w\phi''(q)} < 0 \tag{10}$$

$$\frac{d^2q^*}{d(w)^2} = \frac{\phi'(q)\phi''(q)}{[w\phi''(q)]^2} > 0 \tag{11}$$

□

Higher depreciation expectations increase the expected peso-cost of working capital, and induce firms to decrease output to produce at lower marginal costs. The relationship is shown in Figure (3).[12]

An increase in efficiency, A, increases optimal output (the rate at which q increases depends on the structure of ϕ'):

$$\frac{dq^*}{dA} = -\frac{d\phi'(q)/dA}{\phi''(q)} > 0 \tag{12}$$

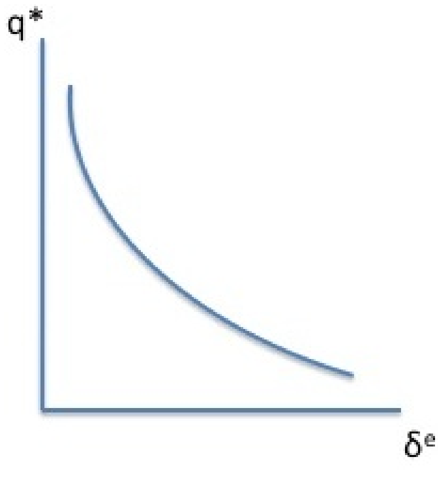

Figure 3: Optimal Output - Expected Depreciation

[12]The figures for input price and dollar-cost of borrowing are analogous.

4.3 Bankruptcies

We now relax A5. When firms' capital structure relies on debt, as argued here, the risk of bankruptcy emerges. Bad states of the world may prevent the firm from meeting its financial obligation due to what Baxter (1967) calls a state of "financial embarrassment". Bankruptcy is costly for firms, and thus to be avoided. We divide costs into direct and indirect costs:

(1) Direct (administrative and restructuring costs): these entail trustees' fees, legal fees, referees' fees as well as the time spent by the managers in litigation, plus, if the firm is forced into receivership by creditors, then costs associated with production disruptions, etc. These are likely to be increasing in firm's size.

(2) Indirect (opportunity cost of lost managerial energies): "financial embarrassment" may affect the stream of operating earnings because of difficulties in obtaining trade credit, disruption in customer relationships, etc. In addition, as argued by Greenwald and Stiglitz (1993), bankruptcies will affect the future prospects by managers, and that effect is likely to be increasing in output. This is because the choice of output levels is a significant role of managers. Bankruptcy with high levels of output should reflect unfavourably on their ability to do this. It implies bad judgement by managers and may thus be unusually costly to their future prospects.

Firms will take the probability of bankruptcy into account when making operating decisions, if the costs associated with that outcome are of sizable magnitudes. Warner (1977), for example, argues that bankruptcy costs are insignificant. He uses data on railroad firms in the US and calculates the ratio of bankruptcy costs to the market value of the firm, and finds this to be at around 1% when the firm's value is considered seven years before bankruptcy, and rising to about 5% when the firm's value is considered one year before bankruptcy. However, his focus is mainly on direct costs. In an attempt to quantify both direct and indirect costs, Altman (1984) compares predicted profits (using data corresponding to three years before bankruptcy) with actual profits and obtains an estimate of bankruptcy costs for industrial firms close to 17.4% of their value three years before bankruptcy. Opler and Titman (1994) reported that during downturns, highly leveraged firms facing financial distress tend to lose substantially more market share than their more conservatively financed competitors — this points to a significant indirect cost of financial distress.

The specification of bankruptcy costs is a moot point, but it seems reasonable to think they are increasing in output, because a) direct costs depend on firm's size, and b) indirect costs — and mainly those related to the manager's reputation, are likely to increase as output increases. In addition, as argued by Greenwald and Stiglitz (1993), for the possibility of bankruptcy not to be ever ignored, bankruptcy costs must increase in output. Otherwise, if, say, they are a fixed cost, profits (increasing in output) may grow so large relative to bankruptcy costs that these are eventually ignored.

In what follows we assume that the associated costs are increasing in output in the form described in equation (13), for the reasons outlined here:

$$Bankruptcy \quad Costs = cq \tag{13}$$

4.4 Solvency Exchange Rate and Ouptut

Because the firm borrows in foreign currency to finance working capital, there is a bankruptcy risk associated with exchange rate levels that would make the debt repayment higher than the output proceeds (the firm operates now in a modified Sc3, with default risk). Lenders can invest their wealth at the risk-free dollar-rate, r, so their expected return from lending to the firms must be, in dollars, at least r. The contracted interest rate at which firms borrow in the debt-market is R. The bankruptcy condition is:

$$\frac{(1+R)Be_0}{\tilde{e}_1} \geq pq$$

$$\frac{(1+R)(w\phi(q)-a)e_0}{\tilde{e}_1} \geq pq \tag{14}$$

A critical value for the exchange rate in period 1, $\bar{e}_1$, that leaves the firm just solvent, assuming $e_0 = 1$, for simplicity can be obtained from (14):

$$\bar{e}_1 = \frac{(1+R)(w\phi(q)-a)}{pq} \tag{15}$$

If in period 1 the exchange rate turns out to be lower (more depreciated) than $\bar{e}_1$, the firm goes bankrupt, while if it is higher (less depreciated) than $\bar{e}$ the firm remains solvent. Notice that the solvency exchange rate, $\bar{e}_1$, is the promised repayment per unit of revenue.

Firms in better than average financial shape (i.e. those with above average inherited liquidity balances (a)) will survive relatively more depreciated exchange rates than the average firm before becoming bankrupt. The lower the portion of working capital that is financed through debt, the larger the depreciation needed to make the firm bankrupt: $\bar{e}_1$ is decreasing in a.

$$\frac{d\bar{e}_1}{da} = -\frac{(1+R)}{pq} < 0 \tag{16}$$

The relationship between $\bar{e}_1$ and output is explained by whether production technology exhibits diminishing, constant or increasing returns. We discuss the cases of diminishing and constant returns (both consistent with competitive equilibrium, and the former, with A1). We don't consider the case of increasing returns as that case would lead to a monopolistic outcome, with endogenous prices, which is out of the scope of this model. Note that $\bar{e}_1$ is a ratio between costs and revenue. Look first the case of $a = 0$, in which firms do not inherit liquidity balances. All input purchases are financed through debt. If as output increases, costs increase at a slower rate than revenue, then, a more depreciated exchange rate is needed to bring the firm to bankruptcy (and conversely).

For any production technology the relationship between the solvency exchange rate and output is given by:

$$\frac{d\bar{e}_1}{dq} = \frac{(1+R)wp\big[\phi'(q)q-\phi(q)\big]+pa}{(pq)^2} \tag{17}$$

To be able to sign equation (17), we will assume a Cobb-Douglas production technology. That implies $\phi(q) = q^{1/\alpha}$. We focus on situations in which firms borrow, i.e.: $w\phi(q) - a > 0$, and $a \geq 0$. Let us consider two cases:

(1) If $\alpha < 1 \Rightarrow$ Diminishing Returns $\Rightarrow \phi'(q)q > \phi(q) \Rightarrow de/dq > 0$. As output increases, working capital requirements increase at an increasing rate $\phi'(q)$ ($\phi''(q) > 0$), while revenues increase at a rate p. This implies that increases in output increase exposure to bankruptcy because they increase the size of debt relative to that of expected revenue. It follows that less depreciated exchange rate realizations will bring firms to bankruptcy, as output increases.

(2) If $\alpha = 1 \Rightarrow$ Constant Returns $\Rightarrow \phi'(q)q = \phi(q) \Rightarrow de/dq \geq 0$. If $a = 0$, then, $\bar{e}$ is constant. Both costs and revenue increase at a constant rate (w and p respectively), and the ratio remains constant (red line in Figure (4)).

Figure (4) shows the relationship between $\bar{e}_1$ and output for the two cases discussed. In each case, we consider three different levels of liquidity balances (zero, "low", "high").

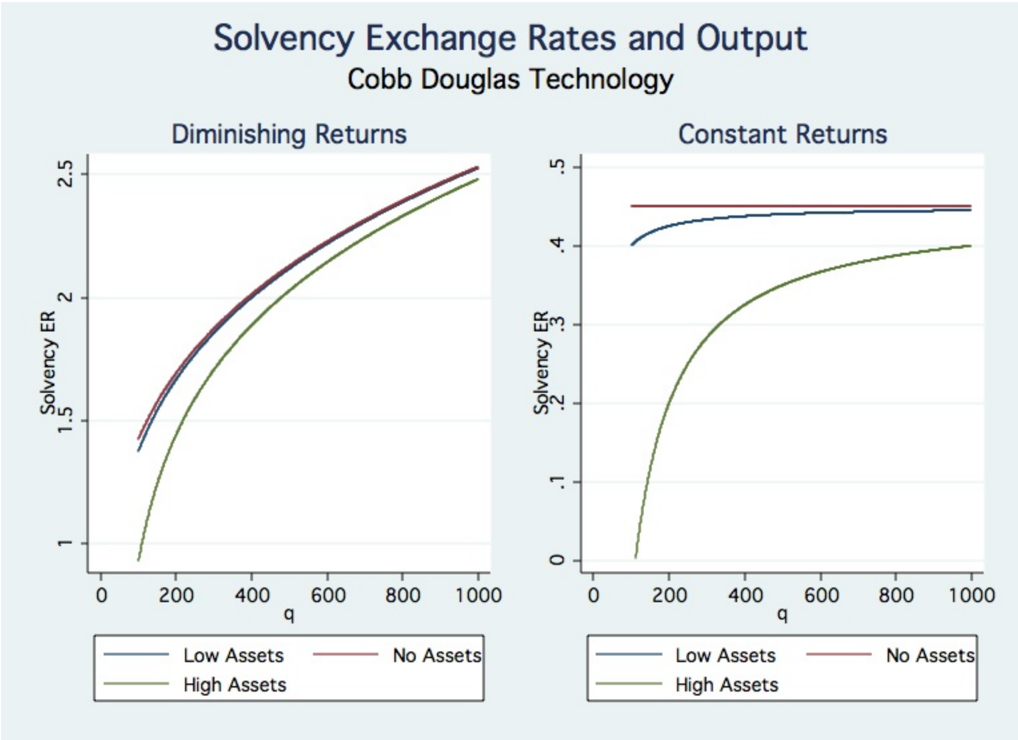

Figure 4: Solvency-Exchange Rate and Output

4.4.1 Lenders' Return and Equilibrium Solvency Exchange Rate

The solvency exchange rate defined in equation (15) can be used to define the dollar-return of lenders, $(1 + \tilde{r})$, as function of the random variable $\tilde{e}_1$, as in equation (18):

$$(1 + \tilde{r}) = \begin{cases} (1 + R) & \text{if } \tilde{e}_1 > \bar{e}_1 \quad (Solvency) \\ \dfrac{pq}{(w\phi(q) - a)}\tilde{e}_1 & \text{if } \tilde{e}_1 \leq \bar{e}_1 \quad (Bankruptcy) \end{cases} \tag{18}$$

Its expected value, $(1 + r)$ equals the sum of the promised return times the probability of solvency plus the output proceeds relative to the debt, valued at the expected exchange rate conditional on bankruptcy (which equals $\int_{-\infty}^{\bar{e}_1} xdF(x)$, where x is the exchange rate and $F(x)$ its distribution function). This is presented in equation (19):

$$
\begin{aligned}
(1+r) &= (1+R)(1-F(\bar{e}_1)) + \frac{pq}{(w\phi(q)-a)}\int_{-\infty}^{\bar{e}_1} xdF(x) \\
(1+r)\frac{(w\phi(q)-a)}{pq} &= \frac{(w\phi(q)-a)}{pq}(1+R)F(\bar{e}_1) + \int_{-\infty}^{\bar{e}_1} xdF(x) \qquad (19)
\end{aligned}
$$

We re-arrange equation (19), and consider the case of constant returns to scale, with $\phi(q) = q$. This yields equation (20), which shows, on the left-hand side, $(h(q))$, the expected repayment per unit of revenue as a function of output, and on the right-hand side, $(z(\bar{e}_1))$, the expected repayment per unit of revenue as a function of the solvency exchange rate, which in turn is equal to the promised repayment per unit of revenue. Thus, (20) expresses the implicit relationship between the level of output produced by the firm and the corresponding exchange rate that leaves the firm just solvent.

$$
h(q) = \left\{(1+r)\frac{(wq-a)}{pq}\right\} = \left\{\bar{e}_1(1-F(\bar{e}_1)) + \int_{-\infty}^{\bar{e}_1} xdF(x)\right\} = z(\bar{e}_1) \qquad (20)
$$

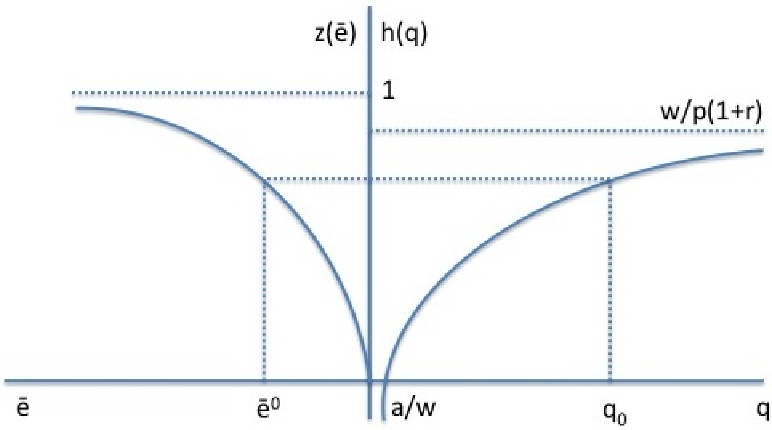

Figure 5: Output-Solvency Exchange Rate

A plot of that implicit relationship between output and the solvency exchange rate is useful, as it allows a more intuitive understanding of the link. This is what we do in Figure (5). The left quadrant of the figure maps the relationship between the firm's promised repayment per unit of revenue (the solvency exchange rate), and the *expected* repayment per unit of revenue. The right quadrant shows the relationship between the level of output and the expected repayment per unit of revenue. Now, consider an increase in the level of output. That will increase the expected repayment per unit of revenue. Consequently, the promised repayment per unit of revenue must increase as well. And that promised repayment equals the solvency exchange rate, $\bar{e}_1^*$. Therefore, $\bar{e}_1^*$ increases in q. Given constant returns, an increasing production makes the firm vulnerable to less depreciated exchange rate outcomes. Notice that as q increases towards infinity, h tends to

$(1+r)w/p$ (which must be lower than 1, if output is positive), while $\bar{e}_1$ approaches a finite limit $\bar{e}_1^0$, which solves equation (21).

$$\frac{w}{p}(1+r) = \left\{ \bar{e}_1^0(1 - F(\bar{e}_1^0)) + \int_0^{\bar{e}_1} x dF(x) \right\} \tag{21}$$

At the same time, the promised repayment per unit of revenue must be 1 when the firm is insolvent for any realization of the exchange rate: as $\bar{e}_1^0$ tends to infinity, $z(\bar{e}_1)$ tends to $E(\tilde{e}_1) = 1$. This means that as q tends to infinity, the probability of bankruptcy tends to a finite limit, $F(\bar{e}_1^0)$. For a profit maximum to exist, then at equilibrium levels of w and r, $p - (1+r)w - cF_0 < 0$, otherwise, q could be increased without bound. This condition holds for a sufficiently large value of c.

Proposition 2. *The solvency exchange rate of equilibrium is an increasing function of q, convex and increasing of w, and a convex decreasing function of p and a.*

Proof. To find the equilibrium response of $\bar{e}_1$ to changes in q, w, p or a we use the implicit function differentiation rule, which expresses that (take the case of $d\bar{e}_1/dq$):

$$\frac{d\bar{e}_1^*}{dq} = \frac{1}{z'} \times \frac{dh}{dq} \tag{22}$$

Using equation (22) we confirm that in equilibrium, the sign of the responses of $\bar{e}_1$ to changes in a and in q are consistent with those showed in equations (16) and (17) respectively. In equilibrium, the more the firm produces, the less depreciated exchange rates needed to bring it to bankruptcy (equation (23)). Increases in output increase their exposure to bankruptcy, because the rate of change of debt is higher than the rate of change of expected revenue as output increases. Second, that the higher the firm's liquidity balances, the more depreciated exchange rate is needed to bring the firm to bankruptcy (equation (24)). And $\bar{e}_1^*$ decreases at an increasing rate with a, because higher values of a not only decrease the size of the debt, but they also decrease the required promised interest payment to lenders, as they reduce the probability of bankruptcy (and conversely for q):

$$\frac{d\bar{e}_1^*}{dq} = \frac{(1+r)a}{(1 - F(\bar{e}_1^*))pq^2} > 0 \tag{23}$$

$$\frac{d\bar{e}_1^*}{da} = -\frac{(1+r)}{(1 - F(\bar{e}_1^*))pq} < 0 \tag{24}$$

$$\frac{d^2\bar{e}_1^*}{da^2} = -\frac{(1+r)f(\bar{e}_1^*)d\bar{e}_1^*/da}{[(1 - F(\bar{e}_1^*))pq]^2} > 0 \tag{25}$$

Analogously we show that a higher output price takes more depreciated exchange rates for the firm to go bankrupt, while the converse applies for a higher input price in equations (26) and (28).

$$\frac{d\bar{e}_1^*}{dp} = -\frac{(1+r)(wq - a)}{(1 - F(\bar{e}_1^*))q} < 0 \tag{26}$$

$$\frac{d^2\bar{e}_1^*}{dp^2} = -\frac{f(\bar{e}_1^*)d\bar{e}_1^*/da(1+r)(wq-a)}{[(1-F(\bar{e}_1^*))q]^2} > 0 \tag{27}$$

$$\frac{d\bar{e}_1^*}{dw} = \frac{(1+r)q}{(1-F(\bar{e}_1^*))} > 0 \tag{28}$$

$$\frac{d^2\bar{e}_1^*}{dw^2} = \frac{f(\bar{e}_1^*)d\bar{e}_1^*/da(1+r)q}{(1-F(\bar{e}_1^*))^2} > 0 \tag{29}$$

□

The probability of bankruptcy, illustrated in Figure (6) can be expressed by substituting the solution value of $\bar{e}_1$, $\bar{e}_1^*$ into $F(\bar{e}_1)$.

$$P(Bankruptcy) = 1 - F(\bar{e}_1^*(.)) \tag{30}$$

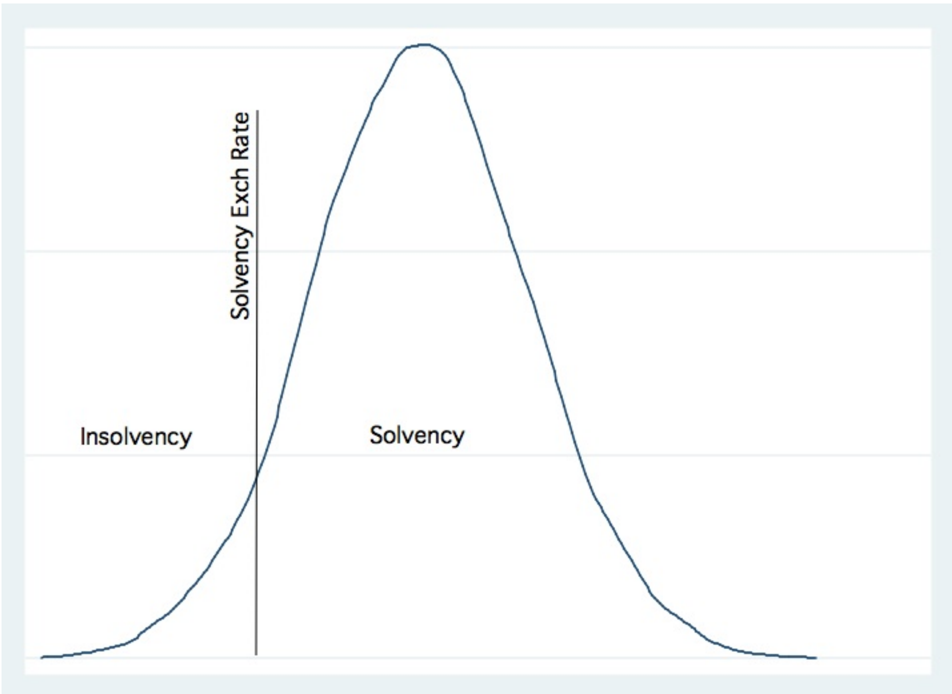

Figure 6: Exchange Rate Distribution: Solvency and Insolvency Zones

4.4.2 Optimal Output Decision

To choose the level of output to be produced, managers will maximize expected wealth at the end of the period (akin to an expected profit function) as presented in equation (31) that incorporates the standard components (i.e.: expected revenue from output sale minus expected repayments to lenders) minus an expected cost of bankruptcy, subject to equation (20). Bankruptcy costs are as in equation (13) for the reasons argued in Section (4.3).

$$max_q E\left\{pq - (1+R)(wq-a)\frac{e_0}{\tilde{e}_1} - cq\right\} \tag{31}$$

Equation (31) can be re-expressed as in equation (32), given that expected depreciation is zero (so, $E(e_0/\tilde{e}_1) = (1 + \delta^e) = 1$)

$$max_q \left\{ pq - (1+r)(wq - a) - cqF(\bar{e}_1) \right\} \tag{32}$$

The first order condition can be expressed as:

$$p = (1+r)w + MBC \tag{33}$$

where MBC is the marginal bankruptcy cost:

$$\begin{aligned} MBC = \frac{dE(BC)}{dq} = \frac{dcq(F(\bar{e}_1^*))}{dq} &= cF(\bar{e}_1^*) + cqf(\bar{e}_1^*)\frac{d\bar{e}_1^*}{dq} \\ &= cF(\bar{e}_1^*) + cf(\bar{e}_1^*)\frac{a(1+r)}{(1-F(\bar{e}_1^*))pq} \end{aligned} \tag{34}$$

Increases in output increase expected bankruptcy costs for two reasons: first, for a given probability of bankruptcy, higher output means higher bankruptcy costs, at a rate c. Second, increases in output increase the probability of bankruptcy. This is because increases in output increase the critical exchange rate at which firms are just solvent. Note that if the production technology exhibited increasing returns, then the sign of MBC would be *a priori* ambiguous. An increase in output would increase the costs of bankruptcy for a given probability, but the effect on the probability of bankruptcy would be ambiguous.

The optimal level of output, q^* can be found just by plugging in equation (34) in (33) and solving for q. This gives:

$$q^* = \frac{cf(\bar{e}_1^*)a(1+r)}{p(1-F(\bar{e}_1^*))(p-(1+r)w - cF(\bar{e}_1^*))} \tag{35}$$

The second order condition equals the opposite of $dMBC/dq$. q^* consistent with equation (35) corresponds to a profit maximum if and only if $dMBC/dq > 0$, which is true under certain conditions, and shown in equation (36).

$$\begin{aligned} \frac{dMBC}{dq} &= c\left[f\frac{de}{dq} + \frac{a(1+r)}{p}(f'\frac{de}{dq}(1-F)^{-1})q^{-1} + f(1-F)^{-2}f\frac{de}{dq}q^{-1} - f(1-F)^{-1}q^{-2})\right] \\ &= c\left[\frac{a(1+r)}{p}f'\frac{a(1+r)}{p}(1-F)^{-2}q^{-3} + f^2(1-F)^{-3}\frac{[a(1+r)]^2}{p^2}q^{-3}\right] \\ &= c\left[\frac{[a(1+r)]^2}{p^2}\frac{1}{(1-F)^2q^3}f' + \frac{a^2}{p^2}\frac{f^2(1-F)^{-1}}{(1-F)^{-2}q^3}\right] \\ &= c\frac{[a(1+r)]^2}{p^2(1-F)^2q^3}\left[f' + \frac{f^2}{(1-F)}\right] \end{aligned} \tag{36}$$

The sign of equation (36) depends on the sign of $[f' + f^2/(1-F)]$. Given that bankruptcies are rare, it is reasonable to think that solvency exchange rates tend to be on the lower tail of the exchange rate distribution (as illustrated in Figure (6)). We discuss this further in Section 4.5.1 and here we make that assumption. If the distribution is unimodal, then f' is positive at relevant levels of output. That implies $dMBC/dq > 0$. Marginal bankruptcy costs increase in output, and the second order condition of a maximum holds. Figure (7) shows the equilibrium where the MBC curve cuts from below the net marginal revenue curve (net marginal revenue= $p - (1 + r)w$).

Irrespective of where in the distribution are the firm's solvency exchange rates, if the hazard function of the exchange rate distribution is monotonically increasing, then $dMBC/dq > 0$. This is because, as shown in equation (37), an increasing hazard function implies $f' + f^2/(1 - F) > 0$. The hazard rate can be thought as the probability of a value of the exchange rate, e_1, occuring, given that a set of values $\{e\} < e_1$ have not occurred. An increasing hazard function means that the likelihood of a realization of the exchange rate e_1, conditional on no lower values having occurred is increasing in e.

$$\begin{aligned} \frac{d}{de}[\frac{f}{(1-F)}] &= \frac{f'}{(1-F)} + \frac{f^2}{(1-F)^2} > 0 \quad \text{which implies} \\ (1-F)\frac{d}{de}[\frac{f}{(1-F)}] &= f' + \frac{f^2}{(1-F)} > 0 \end{aligned} \tag{37}$$

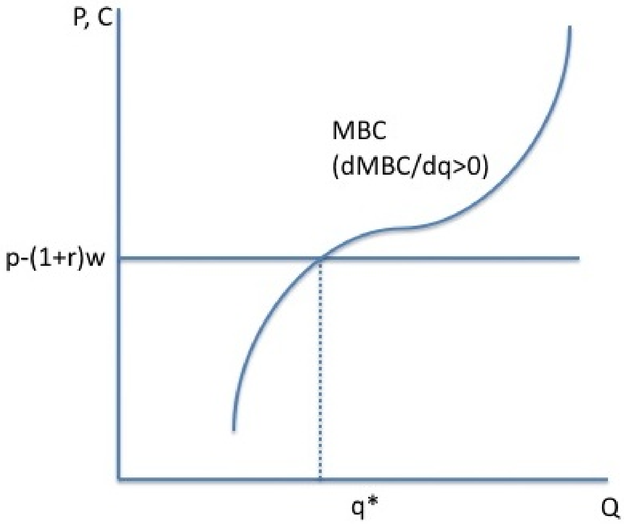

Figure 7: Firm maximising behaviour with costs of bankruptcy

Proposition 3. *Optimal output increases with prices and with liquidity balances if the firm operates with solvency exchange rates in the lower portion of a unimodal exchange rate distribution.*

Proof. Proposition (2) expresses that $\bar{e}_1^*$ is decreasing in output prices, which implies that also MBC are decreasing in output prices:

$$\begin{aligned} \frac{dMBC}{dp} &= cf(e)\frac{de}{dp} + cf'(e)\frac{de}{dp}\frac{a}{pq(1-F)} - \frac{cf(e)}{(1-F(e)}\frac{aq}{[pq(1-F)]^2} \\ &+ cf(e)\frac{a}{pq}\frac{f(e)de/dp}{[pq(1-F(e)]^2} < 0 \end{aligned} \tag{38}$$

The output effect of an increase in output prices, can be seen in Figure 7. It increases net marginal revenue at the same time as it decreases the MBC. Both effects lead to an increase in optimal output. Analytically, we find the optimal output response to changes in the price by totally differentiating the first order condition, which yields:

$$\frac{dq}{dp} = \frac{1 - \dfrac{dMBC}{dp}}{\dfrac{dMBC}{dq}} > 0 \tag{39}$$

Optimal output is increasing in a. By inspection of equation 33, it is possible to see that dq/da is the quotient of $-dMBC/da$ and $dMBC/dq$. Because higher liquidity balances reduce bankruptcy costs, it follows that output increases in a. (See Figure 7)

$$\begin{aligned} \frac{dMBC}{da} &= \frac{d\bar{e}_1}{da}\left[cf(\bar{e}_1) + \frac{cf'(\bar{e}_1)a}{pq(1-F(\bar{e}_1))} + \frac{cf^2(\bar{e}_1)a}{pq(1-F(\bar{e}_1))^2}\right] + \frac{cf(\bar{e}_1)}{pq(1-F(\bar{e}_1))} \\ \frac{dMBC}{da} &= \frac{d\bar{e}_1}{da}\left[\frac{cf'(\bar{e}_1)a}{pq(1-F(\bar{e}_1))} + \frac{cf^2(\bar{e}_1)a}{pq(1-F(\bar{e}_1))^2}\right] < 0 \end{aligned} \tag{40}$$

$$\frac{dq^*}{da} = -\frac{dMBC}{da} \Big/ \frac{dMBC}{dq} > 0 \tag{41}$$

□

The last result is relevant because it implies **persistence**. Any shock that affects today's profits will have an effect on tomorrow's liquidity balances. Firms with higher liquidity balances will produce a higher level of output. So, shocks to today's profits will affect tomorrow's output.

4.5 Increases in Uncertainty

Our purpose is to identify a pure uncertainty effect on output. We explore the output effects of a change in firms' perception about the distribution of the exchange rate, from $\tilde{e}$ to $\tilde{e}'$. As illustrated in Figure (8), the change considered, henceforth "increase in tail-risk", implies that the probability mass in the tails increases, and that in the centre of the distribution falls, while $E(\tilde{e}) = E(\tilde{e}')$. This makes extreme exchange rate outcomes more probable, while keeping the mean value constant.

Proposition 4. *Increases in exchange rate uncertainty lead to a decrease in the level of output produced.*

Proof. The output effect of an increase in uncertainty depends on what happens to the MBC. If the MBC increases after an increase in uncertainty, then output will fall. To observe that, see Figure (7): an increase in MBC leads to a decrease in output produced as the curve MBC shifts upwards (and conversely for a decrease in MBC).

The effect of increases in exchange rate uncertainty on the MBC at the optimum can be calculated by looking at a change in uncertainty that preserves the density function, f, but shifts the distribution F, and then looking at another change that preserves F, and alter f. We examine these changes in equations (42) and (43)[13]:

[13] If we denote the change in the distribution by dy, then we can write the total effect on MBC as $dMBC/dy = cdF/dy + (p - (1+r)w - cF)dln[f/(1-F)/dy]$

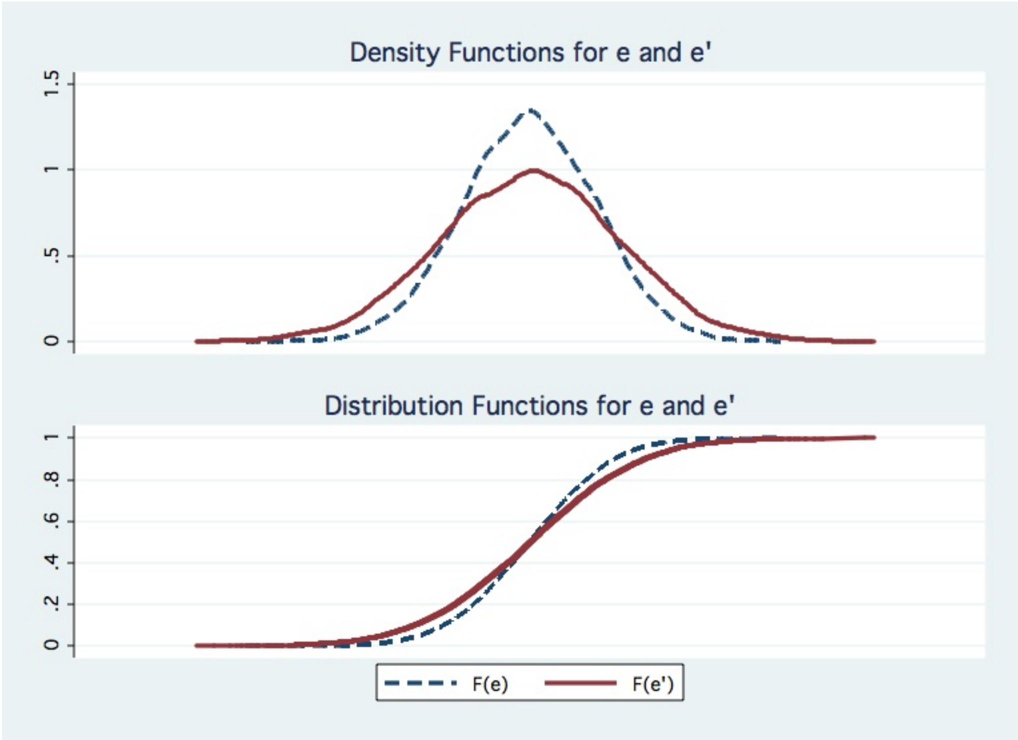

Figure 8: An Increase in Exchange Rate Uncertainty, from e to e'

$$\begin{aligned}\frac{dMBC}{dF} &= c\frac{dF}{dF} + \frac{ca}{pq}\frac{d}{dF}[\frac{f}{(1-F)}] \\ &= c + [p-(1+r)w-cF]\frac{d}{dF}ln[\frac{f}{(1-F)}] \\ &= c + [p-(1+r)w-cF]\frac{1}{(1-F)} > 0 \quad \text{at the optimum.} \end{aligned} \tag{42}$$

$$\begin{aligned}\frac{dMBC}{df} &= c\frac{dF}{df} + \frac{ca}{pq}\frac{d}{df}[\frac{f}{(1-F)}] \\ &= [p-(1+r)w-cF]\frac{d}{df}ln[\frac{f}{(1-F)}] \\ &= [p-(1+r)w-cF]\frac{1}{f} > 0 \quad \text{at the optimum.} \end{aligned} \tag{43}$$

Both types of changes lead to an increase of the MBC, which implies that output falls after an increase in exchange rate uncertainty. □

4.5.1 Are Firms in Zone I or II?

We have argued that there is an ambiguity in the sign of $dMBC/dq$. If the hazard function of the exchange rate distribution is monotonically increasing or if the firms' solvency exchange rates are in the lower portion of a unimodal distribution, then $dMBC/dq > 0$. While the former condition is not easily interpretable, the likelihood of the latter condition holding can be scrutinized. As

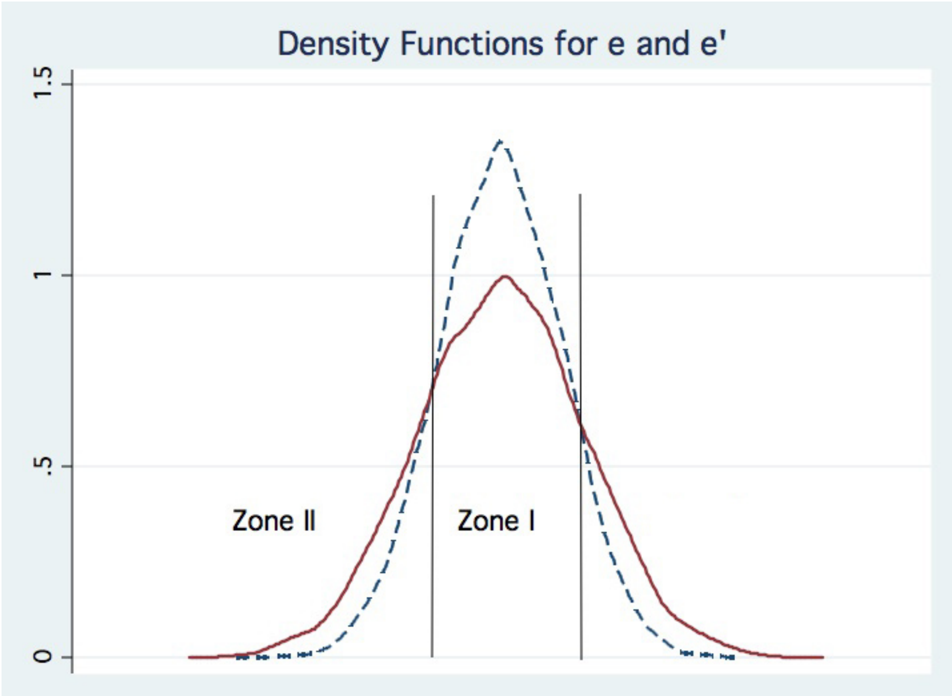

Figure 9: Two Zones: I $f'(\bar{e}_1^*) \leq 0$, II $f'(\bar{e}_1^*) \geq 0$

already argued, bankruptcies are not frequent in practice, and it seems reasonable to assume that solvency exchange rates are on the left tail of the distribution. De Brun et al. (2008) gives us a more accurate indication, though only for Uruguay. The authors performed stress tests to a sample of manufacturing firms in Uruguay. They defined a firm as financially stressed "whenever an exchange-rate depreciation made it unable to meet its amortization and interest payments falling due and or whenever it pushed the firm into a negative equity position" (De Brun et al., 2008, p.229). In Figure (10) we combine the distribution of annual exchange rate changes (solid line, density measured in the axis on the left), with their data on the distribution of firms' solvency exchange rates (% of firms that would be financially distressed if the exchange rate was to change by x%, measured on the axis on the right). It is possible to see that for the great majority of firms, the solvency exchange rates are well to the right of the distribution, probably corresponding to zone II, in our analysis.

5 Extensions

5.1 Firms Producing Tradable Goods

If the risk-neutral firm produces a tradable good 'T', with $p^T = p^{T*}/e$, where p^{T*} is the international price of the good 'T', expressed in foreign currency, then, the value of the exchange rate (and the riskiness of its distribution) is irrelevant for the probability of bankruptcy, as long as this firm has assets (revenue) and liabilities matched in terms of currency denomination. This can be seen by re-examining the bankruptcy condition, expressed by (14) in the light of the new

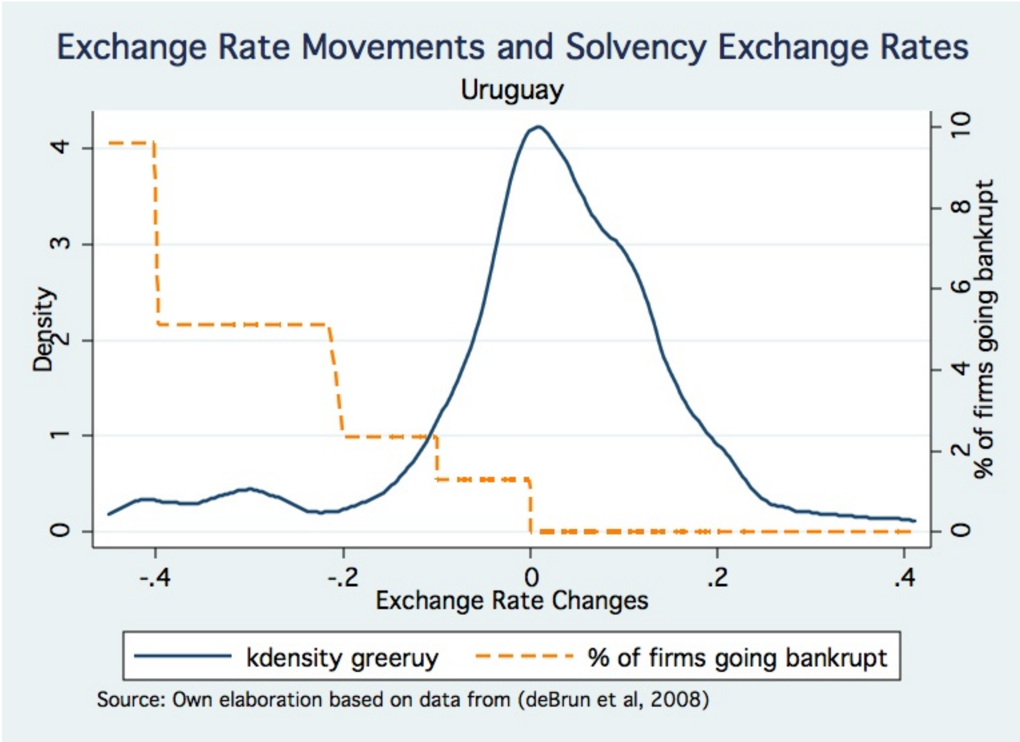

Figure 10: Distribution of actual and solvency exchange rates in Uruguay

setting, and assuming p^*=1, and $e_0 = 1$ for the sake of simplicity.

$$\begin{aligned}(1+R)B^{\$}e_0/\tilde{e}_1 &\geq pq \\ (1+R)(w\phi(q)-a)\times e_0/\tilde{e}_1 &\geq q/\tilde{e}_1 \\ (1+R)(w\phi(q)-a) &\geq q \qquad (44)\end{aligned}$$

5.2 Soft Budget Constraints

If we examine the episodes of sharp depreciations in the countries under analysis, the line of argument proposed by this model, that relies on firms internalizing the costs of potential balance-sheet effects could be contested. This is because, as argued in Section 3.2, it may be ex-post optimal for the government to bailout debtors that are financially distressed as a consequence of the extreme exchange rate. In fact, governments have often acted in support of firms in order to relax the financial constraint they faced. For example, in Argentina, after a sharp real devaluation in 1982 that implied the collapse of a crawling peg system against the dollar, foreign currency denominated corporate debt was converted into domestic currency at the pre-devaluation exchange rate, at the expense of the Argentinean Central Bank. Something similar happened after the abandonment of the currency board regime in 2002, when the government enforced a compulsory conversion of dollar denominated liabilities up to 100,000 dollars into peso denominated liabilities at the one-to-one exchange rate, what is known as the "pesification" of debt. Any potential balance sheet effects were then eliminated (Galiani et al., 2003, p.344). In Uruguay, after the sharp devaluation in 2002, the state-owned bank, which is the main creditor to the manufacturing sector, called for a renegotiation of corporate debt in milder terms. This implied, for example, accepting government bonds as a means of debt-repayments at face value when their market price had plunged to about 60%.[14] Thus, sufficiently liquid firms faced a substantial reduction of their debt. At the same time, the government encouraged the private banking sector to offer debt renegotiation alternatives to the non-banking corporate sector. In Brazil, the "Banco do Brasil" provided some exchange rate risk hedging opportunities some days before the large devaluation of the currency in January 1999.[15]

These type of arrangements are akin to the notion of "soft budget constraints". According to Kornai et al. (2003), a firm faces a soft budget constraint if there is a support organization ready to alleviate part or all of the debt, and the firm managers or owners are aware of this possibility, and internalize it, when making decisions. In the context of our model, let us assume that the government is the support organization, and that it acts by exerting pressure on the banking sector for them to roll over corporate debt. That would mean that firms may only have to pay a portion λ of the debt they face at the beginning of next period.[16] Figure 11 shows the optimal output response to the possibility of a softening of the budget constraint. If firms only pay a portion λ of the debt, that increases the expected net marginal revenue, and at the same time reduces the MBC, as the soft budget constraint reduces the firm's solvency exchange rate. Both effects induce an increase in optimal output. The first order condition of the maximization problem of the firm is:

$$p = (1 + r)w\phi'(q)\lambda + MBC' \tag{45}$$

where $MBC' < MBC$.

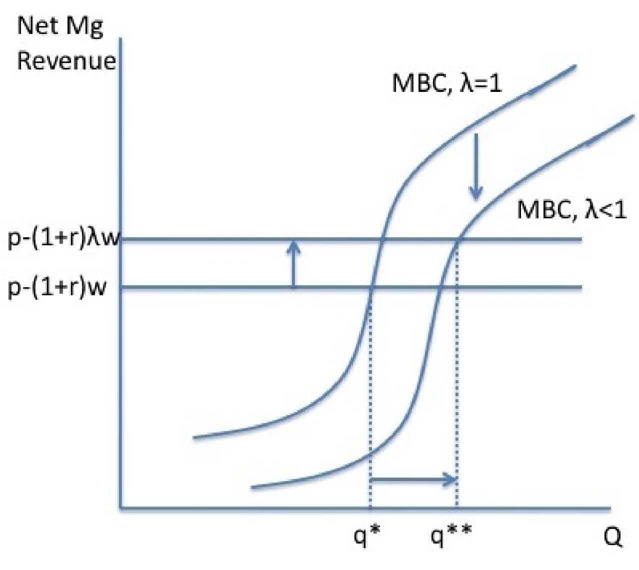

Figure 11: Firm maximising behaviour with costs of bankruptcy and soft budget constraints

[14]Source: Montevideo Stock Exchange.

[15]Personal communication with Pedro Bonomo, from Fundacao Getulio Vargas, Sao Paulo, Brazil.

[16]$0 < \lambda < 1$

If the government only intervenes when there is substantial exchange rate uncertainty, and so, important risks of economic and social disruptions, then, an increase in exchange rate risk will, on the one hand, increase the expected costs of bankruptcy, thus inducing a more "cautious" behaviour of firms (i.e.: reduce output), but on the other hand, will decrease λ, thus inducing the opposite effect on output. The final effect on output is therefore, ambiguous, depending on the relationship between the size of λ and the change in uncertainty.

5.3 High Uncertainty: The Unknown Unknowns

> "*…When, as today, the unknown unknowns dominate, and the economic environment is so complex as to appear nearly incomprehensible, the result is extreme prudence, if not outright paralysis, on the part of investors, consumers and firms. And this behaviour, in turn, feeds the crisis.*" Blanchard (2009)[17]

While Blanchard's quotation refers to the business environment in the financial crisis of 2008/9, it depicts well that of Southern Cone countries in episodes of high exchange rate uncertainty. Caballero and Krishnamurthy (2008) seems to give analytical support to Blanchard. While their focus is on financial risk management decisions instead of production decisions, their insight is relevant for our purposes. The authors show that in the presence of fundamental (or 'Knightian') uncertainty about the economic environment that leads agents to question their world-views and challenge their models used for decision-making, there will be an excessive demand for safety on the part of businesspeople, which leads to disengagements with risky activities and liquidity hoarding.

We argue that in the context of the countries under analysis, high exchange rate uncertainty implies "structural" uncertainty. The concept of "structural" uncertainty is borrowed from Arza (2006) and refers to uncertainty about "the whole set of parameters that defines an economic system at a given time."[18] This set of parameters subject to uncertainty include the new exchange rate regime, if the current one actually collapses, but also other relevant parameters for the firm, such as the government reaction in terms of enforcement of property rights, the level of demand that the firm will face after the severe wealth effects of a possible depreciation, the access to credit, the access to marketing channels the firm may use in the event of possible chained bankruptcies, etc..[19]

Some anecdotal information on the communicational strategy of the government around (before and immediately after) drastic exchange rate movements illustrates how their actions actually tended to increase the perception of uncertainty among decision-makers, in the context of property rights enforcement. The

[17] The concept of the "Unknown unknowns" has been "coined" by former US Defense Secretary, Donald Rumsfeld, on February 12, 2002, referring to the unstable situation in post-invasion Afghanistan.

[18] (Arza, 2006, p. 6). This concept is better suited for our purposes than that of "Knightian uncertainty" used by Caballero and Krishnamurthy (2008). While latter one is related to immeasurable risk over a particular variable, the former is related to a *set* of parameters that are relevant for the decision-maker.

[19] It is worth mentioning that most of the episodes of extreme RER movements in the countries under analysis have had associated output contractions that were large enough to be called "rare disasters", by Barro (2006) in his analysis of rare disasters and asset markets. In particular, he identifies as disasters, the episodes in Argentina in 1979-1985 and in 1999-2002 and those of Uruguay in 1981-1984, and 1998-2002

regularity is that there is no discussion on the "reconstruction" agenda for the period after the large depreciation has actually taken place. In an attempt to grant credibility to the about-to-collapse system, governments tend to rule out the possibility of a depreciation in the first place, even when the rest of the actors in the economy (and the economic fundamentals) argue differently. This is likely to increase the degree of uncertainty faced by all agents in the economy, and paralyze those that make production decisions. In November 1982, journalists asked the General who was acting as de-facto president in Uruguay if there was going to be a devaluation. His answer was: "No, not even if Martians land here". Two days after, the dollar tripled its value against the Uruguayan peso. Then, in 2001, months before the Uruguayan peso was to be devalued again, the President was encouraging people to borrow in foreign currency, arguing that they were not going to devalue. Something similar happened at the time in Argentina, where the President persistently claimed they were not going to devalue their currency. Just a few months later, the currency board was to be abandoned and the value of the dollar drastically increased. Uncertainty is also likely to increase if the strategies designed to deal with the potential balance-sheet problems lack credibility. On this respect, more anecdotal evidence can be presented: in January 2002, after a significant exchange rate depreciation and an ongoing banking crisis, the Argentinean government announced a plan according to which bank depositors were going to recover their deposits in the currency in which those had been originally denominated. At the same time, they announced that debtors would have their dollar-debts converted automatically into peso-debt (Clarin, 2002). The apparent incompatibility of both announcements, in a period in which the Argentinean government did not have access to credit markets to finance the cost of such a combination of proposals, and a depleted stock of foreign exchange reserves, is likely to have increased the degree of uncertainty perceived by agents in the economy.

How reasonable is it to assume that in this context, firms will internalize the behaviour that governments have had in the past and act as if they faced a soft budget constraint, deciding to increase their exposure to exchange rate risk?

Our conjecture is that firms do not take the soft budget constraint for granted. Instead, they will react to high uncertainty with conservatism and caution - or in Blanchard's words: "outright paralysis". Our contention is that the effects of high uncertainty on output decisions will be dominant over any possible effect of perceptions of soft budget constraints. Thus, firms will tend to defer production decisions in a context of high uncertainty.

6 Concluding Remarks

In this chapter we proposed a framework to analyze the impact of exchange rate uncertainty on output decisions when production takes time, and firms finance their working capital needs with dollar debt contracts. The framework seems to be well suited to understand the output implications of currency mismatches in the context of economies with undeveloped exchange rate risk hedging instruments, as it explains a stylized fact: output fluctuates significantly with exchange rate uncertainty.

The main results of this chapter are the following:

First, shocks to profits have persistent effects on output through their effects on liquidity balances. The financial "health" of the firm, understood as the size of the liquidity balances it holds at the beginning of the period matters for the output choice. Firms with higher liquidity balances face lower marginal bankruptcy risks. Thus, any shock that affects today's profits, will affect tomorrow's liquidity balances, and so, firms' output.

Second, when firms face bankruptcy risks, increases in exchange rate uncertainty will increase marginal bankruptcy costs if the firm produces non-tradable goods (i.e.: exhibits currency mismatches), thus inducing firms to reduce output. By internalizing the bankruptcy cost, the firm acts as averse to bankruptcy costs. Because firms are aware that most others are borrowing in dollars, it is reasonable to think that they may anticipate a government bailout in the event of drastic exchange rate depreciations. This is because it may be ex-post optimal for the government to intervene, so as to avoid chained bankruptcies and disruptions in the payment system. Under those circumstances, the effect of increases in uncertainty on output are analytically ambiguous, as the higher the likelihood of extreme exchange rate outcomes, the higher the probability of benefitting from a government bailout. However, our contention is that firms do not take bailouts for granted in the event of high exchange rate uncertainty, for those periods are characterized by lack of information with respect to possible reconstruction agendas, and a generalized state of "irreducible" uncertainty that leads firms to disengage with risky activities and hoard liquidity.

This framework can be extended either to explain other firms' decisions in response to exchange rate uncertainty (e.g investment) or to analyse other uncertainty-output channels such as that operating through input or technology prices (i.e. imported inputs are not easily substitutable and firms may have signed contractual arrangements on foreign technology licenses).

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