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Tail-effect and the Role of Greenhouse Gas Emissions Control

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Abstract: This paper investigates the role of emissions control on reducing the tail-effect of the fat-tailed distribution of the climate sensitivity. Through a simple analysis on temperature distributions and some numerical simulations using the well-known DICE model, we find that the option for emissions control effectively prevents the tail-effect. Climate policy based on HARA utility is less sensitive to fat tails than climate policy based on CRRA utility.

JEL classification: Q54, Q58, H23

Uncertainty is central to climate policy. It implies stricter optimal emissions control as uncertainties are asymmetric (Tol 2003) and consequences are irreversible (Ingham et al. 2007). Deep (or fat-tailed, structural) uncertainty is a further complication (Weitzman 2009). Weitzman's 'Dismal Theorem' – that expected utility maximization cannot be used to guide climate policy – is controversial. See Tol (2003), Hennlock (2009), Horowitz and Lange (2009), Karp (2009), Ackerman et al. (2010), Costello et al. (2010), Nordhaus (2011), Pindyck (2011), Weitzman (2011), Anthoff and Tol (2013), among others.

Anthony Millner recently extended the Dismal Theorem by introducing abatement policy, and alternative welfare functions (Millner 2013). He argues that, when climate policy is explicitly included into the model, whether or not the tail dominates depends on parameter values such as the elasticity of marginal utility. However, Millner's model is too stylized for practical applications and, moreover, it omits the cost of emissions control.

Hwang et al. (2013a), using DICE (Nordhaus 2008), numerically show that the Dismal Theorem does not hold when emission control is introduced. In this paper we combine Millner (2013) and Hwang et al. (2013a), numerically examining the Weitzman-Millner theorem.

The constant relative risk aversion (CRRA) utility function, that is typically used in economics for its analytical tractability, violates the requirements for the use of unbounded utility functions in maximization of expected utility (Arrow 1974). Other functional forms such as the hyperbolic absolute risk aversion (HARA) function are advocated in the literature (e.g., Arrow 2009; Millner 2013), as this functional form prevents marginal utility from diverging as consumption approaches zero. Thus, we use the HARA utility function instead of the CRRA utility function and investigate the effect of using this better suited functional form for the utility function on policy and welfare.[1]

I. Dismal Theorem and the Cost of Emissions Control

A. Extension of the Weitzman-Millner Model

Millner (2013) adds emissions control to the model by Weitzman (2009). We extend the Weitzman-Millner (WM) model by adding the cost of emissions control as follows:

[1] A few papers have investigated the role of utility functions under fat-tailed uncertainty. For instance, Ikefuji et al. (2010) apply the 'Burr' utility into a model of climate change. On the other hand, most literature dealing with fat tails set bounds on consumption (e.g., Newbold and Daigneault 2009; Costello et al. 2010; Dietz 2011) or utility (e.g., Pindyck 2011), while maintaining CRRA.

(1) $\max_{\mu\in[0,1]} U\big(1-\Lambda(\mu)\big)+\beta\mathbb{E}\,U\big(C(\mu;s)\big)=U\big(1-\Lambda(\mu)\big)+\beta\int_{s\in S}U\big(C(\mu;s)\big)\,g(s)ds$

where μ is the rate of emissions control in the first period, U is the utility function, Λ is the current abatement cost (a fraction of utility), β is the discount factor, $\mathbb{E}$ is the expectation operator, C is future consumption, s is an uncertain variable such as the equilibrium climate sensitivity, S is the set of s, g is the probability density function of s. If $\Lambda = 0$, the model is Millner's. If $\mu = 0$, the model is Weitzman's.

This is a simple two-period model including climate policy. The problem of the decision maker is to choose the rate of emissions control in the first period so as to maximize social welfare, defined as the discounted sum of expected utility of consumption. A unit increase in carbon emissions today induces future climate change, and thus reduces expected utility. This is due to the loss of future consumption as a consequence of a higher temperature. Thus the decision maker controls the level of carbon emissions today. Abatement cost is increasing and convex in the emissions control rate. Current consumption is gross output minus the abatement cost and the damage cost. The gross output of the economy today is normalized to 1, and the damage cost today is, without loss of generality, assumed to be zero. The uncertain variable is assumed to have a fat-tailed distribution, which means that its moment generating function is infinite and thus the first moment does not exist (Weitzman 2009).

B. Temperature Distribution

In order to get insight into the role of emissions control we specify the temperature response model. The global mean surface air temperature change is assumed to have a relation with radiative forcing as follows (Wigley and Schlesinger 1985; Gregory and Forster 2008; Baker and Roe 2009).[2]

(2) $$T_{AT}=\lambda RF/RF_{2\times CO_2}$$

where T_{AT} is the future atmospheric temperature change, RF is the radiative forcing change which is a decreasing function of the emissions control rate ($\partial RF/\partial\mu < 0$), λ is the equilibrium climate sensitivity, and $RF_{2\times CO_2}$ is the radiative forcing from a doubling of carbon dioxide.

[2] This equation is intuitive in terms of the definition of the equilibrium climate sensitivity. From this relation, a doubling of CO_2 induces temperature increase of λ. Gregory and Forster (2008) verify this relation with historical data. Furthermore, a successive application of the (time-dependent) temperature response model of DICE or FUND produces a similar relation.

Let us suppose that the climate sensitivity has the following probability density function (PDF) with parameters $\bar{f}$ and σ_f (Roe and Baker 2007).

(3) $$g_\lambda(\lambda) = \left(\frac{1}{\sigma_f\sqrt{2\pi}}\right)\frac{\lambda_0}{\lambda^2} exp\left\{-\frac{1}{2}\left[\frac{\left(1-\bar{f}-\frac{\lambda_0}{\lambda}\right)}{\sigma_f}\right]^2\right\}$$

Equation (3) is derived from the assumption that the equilibrium climate sensitivity is related to the total feedback factors as follows: $\lambda = \lambda_0/(1-f)$, where λ_0 is the reference climate sensitivity in a blackbody planet, f (<1) is the total feedback factor normally distributed with mean $\bar{f}$ and standard deviation σ_f. Transforming the random variables, we derive the density function of temperature as follows. We only present the kernel of the distribution, which is related to the emissions control today, for simplicity.

(4) $$g_T(T_{AT}) = g_\lambda\big(\lambda(T_{AT})\big)\frac{\partial\lambda(T_{AT})}{\partial T_{AT}} \propto \frac{RF}{{T_{AT}}^2} exp\left\{-\frac{1}{2}\left[\frac{1-\bar{f}-\frac{\lambda_0 RF}{T_{AT}}}{\sigma_f}\right]^2\right\}$$

where g_T is the temperature distribution, $\lambda(T_{AT}) = RF_{2\times CO_2}\, T_{AT}/RF$.

Then Equation (5) holds for any a>0.

(5) $$\lim_{T_{AT}\to\infty}\frac{g_T}{exp(-aT_{AT})} \propto \lim_{T_{AT}\to\infty}\frac{RF}{{T_{AT}}^2} exp\left\{-\frac{1}{2}\left[\frac{1-\bar{f}-\frac{\lambda_0 RF}{T_{AT}}}{\sigma_f}\right]^2 + aT_{AT}\right\} = \infty$$

That is, g_T has fat tails in the sense that the tail falls more slowly than exponentially (Weitzman 2013). The mode of the (smooth, unimodal) distribution can be found from the condition: $\partial g_T/\partial T_{AT} = 0$. We find that the mode of the distribution decreases in the emissions control rate. That is, the emissions control reduces the probability of high temperature increases.

C. The Role of Greenhouse Gas Emissions Control

Now that we are equipped with the setting of the model and the temperature distribution, let us return to the problem of the decision maker in the extended WM model. From Equation (1) the optimal climate policy should satisfy the first order condition as follows. For the derivation, we assume a HARA utility function, $U(C) = \zeta\{\eta + C/\alpha\}^{1-\alpha}$ and a polynomial climate impact function, $C = Y/\left(1+\pi T_{AT}^{\gamma}\right)$ where Y is the gross output in the future, α(>0),

$\eta(\geqslant 0)$, $\pi(>0)$, $\zeta(<0)$ and $\gamma(>1)$ are parameters. Note that current consumption is normalized to be 1 and there is no damage from climate change today.

$$(6)\quad \frac{\partial \Lambda}{\partial \mu}\frac{\partial U(1-\Lambda)}{\partial \Lambda} \propto \beta \frac{\partial RF}{\partial \mu}\int_{\{T_{AT}\}} \frac{\pi\gamma T_{AT}}{RF}\frac{YT_{AT}^{\gamma-1}}{\left(1+\pi T_{AT}^{\gamma}\right)^2}\frac{1-\alpha}{\alpha}\zeta\left\{\eta + \frac{Y}{\alpha\left(1+\pi T_{AT}^{\gamma}\right)}\right\}^{-\alpha} g_T(T_{AT})dT_{AT}$$

$$\propto \frac{\partial RF}{\partial \mu}\int_{\underline{T}}^{\infty} T_{AT}^{-\gamma-2}\left\{\eta + \frac{YT_{AT}^{-\gamma}}{\alpha\pi}\right\}^{-\alpha} exp\left\{-\frac{1}{2}\left[\frac{1-\bar{f}-\frac{\lambda_0 RF}{T_{AT}}}{\sigma_f}\right]^2\right\} dT_{AT}$$

where $\{T_{AT}\}$ is the set of temperature increases. For the first line in Equation (6), we apply Equation (2), chain rule ($\frac{\partial U}{\partial \mu} = \frac{\partial RF}{\partial \mu}\frac{\partial T_{AT}}{\partial RF}\frac{\partial C}{\partial T_{AT}}\frac{\partial U}{\partial C}$), and the fact that radiative forcing is independent of the temperature distribution. For the second line, we apply Equation (4) and assume that πT_{AT}^{γ} is far greater than 1 for $T_{AT}>\underline{T}$.[3]

For the CRRA case (η=0), Equation (6) becomes:

$$(7)\quad \frac{\partial \Lambda}{\partial \mu}\frac{\partial U(1-\Lambda)}{\partial \Lambda} \propto \frac{\partial RF}{\partial \mu}\int_{\underline{T}}^{\infty} T_{AT}^{\gamma(\alpha-1)-2}\, exp\left\{-\frac{1}{2}\left[\frac{1-\bar{f}-\frac{\lambda_0 RF}{T_{AT}}}{\sigma_f}\right]^2\right\} dT_{AT}$$

In the limit as the temperature goes to infinity, the exponential term of the right hand side (RHS) becomes constant, and thus the convergence criterion of the expectation depends on both the exponent of the damage cost function (γ) and the elasticity of the marginal utility (α) as follows: $2-\gamma(\alpha-1)>1 \leftrightarrow \gamma(\alpha-1)<1$.[4] This criterion implies that the higher (respectively, lower) the exponent of the damage cost function is and the higher (resp., lower) the elasticity of the marginal utility is, the more likely is the expected utility to diverge (resp., converge).[5]

[3] That is, $1+\pi T_{AT}^{\gamma} \approx \pi T_{AT}^{\gamma}$ for $T_{AT}>\underline{T}$. Note that the domain of integration changes and the other terms which are not related to temperature are dropped.

[4] Note that $\int_{S}^{\infty} S^{-p} dS$ exists for any uncertain variable S if and only if p>1.

[5] Note that this criterion is derived from the various assumptions with the simple two-period model. Thus it is not directly applicable to our numerical simulations in Section 3. In addition, this criterion is sensitive to the climate sensitivity distribution. For instance, if a power function, $g_{\lambda} = \kappa\lambda^{-p}$, where κ and p(>1) are constants, is used then the convergence criterion is $p-\gamma(\alpha-1)>1$. Many fat-tailed distributions usually used in the literature have functional forms similar to the power function (e.g., the Student-t distribution by Weitzman (2009), the Pareto distribution by Nordhaus (2011), Pindyck (2011) and Weitzman (2013), and the Cauchy distribution).

For the HARA case (η>0), $\left\{\eta + YT_{AT}^{-\gamma}/\alpha\pi\right\}^{-\alpha}$ in Equation (6) also becomes constant as the temperature increases arbitrarily high. Consequently the expectation converges if and only if $\gamma + 2 > 1$. By the assumption ($\gamma > 1$) it is clear that the expectation exists for the HARA case.

Finally, let us consider the role played by the cost of emissions control. First of all, as an extreme case, suppose that the cost is zero. Since future consumption depends on future damage and the decision maker can control emissions without loss of current consumption, zero emission is optimal (μ=1). Consequently the changes in radioactive forcing and temperature are all zero and there is no dismal future.[6] For the other extreme case, in which the full reduction of emissions costs the total world output, the full reduction (μ=1) cannot be optimal if RHS of Equations (6) (the expected marginal damage cost) converges, since the left hand side (LHS) (the marginal abatement cost) diverges as μ approaches 1. Even if RHS diverges, it cannot be justifiable to set μ=1 if it would cost all we produce. In usual cases where the cost of emissions control is between the two extreme cases, LHS of the Equations (6) are finite. If RHS also converges the optimal climate policy is determined in a way to balance LHS and RHS. The optimal carbon tax is low (respectively, high) for a low (resp., high) unit cost of emissions control. If the RHS diverges, on the other hand, there is no answer for Equation (6). However, if we are able to constrain the range of the climate sensitivity through temperature observations or climate research (Hwang et al. 2013b), the unit cost of emissions control plays the similar role as the case above: a low unit cost of emissions control effectively reduces the tail-effect.

II. Numerical Model and Methods

The numerical model and methods of the current paper are similar to those of Hwang et al. (2013a), but we here use a Gauss-Hermite quadrature method (Judd 1998). This greatly reduces the computation time without much loss of accuracy.

(8) $$\max_{\mu_t, I_t} \mathbb{E} \sum_{t=0}^{T} \beta^t U(C_t, L_t) = \sum_{i=1}^{N} w_i \sum_{t=0}^{T} \beta^t L_t \zeta \left\{\eta + \frac{(C_{t,i}/L_t)}{\alpha}\right\}^{1-\alpha}$$

where i (1, 2, …, N=10) and t (1, 2, …, T=60) denote the integration node and the time period (number of decades after the year 2005) respectively, w_i is the integration weight,[7] I is

[6] Of course in reality the current carbon stock, even if we stop adding carbons from now on, induces adverse climate impacts. However, it would not be that severe to induce a catastrophe.

[7] The integration nodes and weights are produced from the normal distribution of the total feedback factors.

the gross investment, U is the population-weighted HARA utility function, C is consumption, L is labor force, $\beta = (1+\rho)^{-1}$ is the discount factor, ρ(=0.015) is the pure rate of time preference, α(=2), η and ζ(=-10^{-6}) are parameters.

We use the temperature response model of FUND (Anthoff and Tol 2008) since it has a form similar to Equation (2). We assume that the climate sensitivity has a fat-tailed distribution as in Equation (3) with $\bar{f}$=0.60, which corresponds to the climate sensitivity of 3°C/2xCO_2 and σ_f=0.13 following Roe and Baker (2007). We apply the damage function of Weitzman (2012) as a reference case as follows since this functional form highly magnifies the effect of uncertainty, and thus is appropriate for the purpose of the current paper: $\Omega_t = 1/\left[1 + \pi_1 T_{AT_t} + \pi_2 {T_{AT_t}}^2 + \pi_3 {T_{AT_t}}^{\pi_4}\right]$, where π_1=0, π_2=0.0028388, π_3=0.0000050703, and π_4=6.754. For the DICE damage case in Section 3, π_3 and π_4 are set to zero. The following abatement-cost function of DICE is applied: $\Lambda_t = \theta_{1,t}\mu_t^{\theta_2}$, where θ_1 is the adjusted cost of backstop technology (DICE cost case),[8] θ_2 =2.8 is a parameter. For the zero and high cost case, that we will illustrate in Section 3, θ_1 is set to zero and one, respectively. Unless otherwise noted, we use the same parameter values, initial conditions, and equations as in DICE 2007.[9]

We gradually increase the upper bound of the climate sensitivity distribution from 5°C/2xCO_2, holding the parameters of the distribution unchanged. Then we compare the behaviors of the variables of interest such as the optimal carbon tax as the upper bound changes. In addition, we experiment with the parameter of the HARA utility function in order to investigate its effect on policy and welfare. Specifically we start with η=0.001 and then gradually decrease the value until it becomes zero so that the utility function becomes CRRA.[10]

III. Tail-effect and the Role of Emissions Control

We investigate the distribution of temperature, damage costs, and consumption for the CRRA case. The HARA cases show patterns for the distributions that are qualitatively similar to the CRRA case (results not shown, same for the distributions of damage costs and consumption). We set an upper bound of 25°C/2xCO_2 to the climate sensitivity since the no-

[8] θ_1 is a time-varying exogenous variable in DICE. The initial value is 0.056 (in 2005) and it gradually decreases to 0.004 in 2605.

[9] We set the lower bounds of economic variables such as consumption, capital stock, and the gross world output to be less than 0.001US$ per person per year. In addition we remove the upper bounds of temperature increases.

[10] For η>0.001, the optimal carbon tax generally increases as η increases but this does not affect the main results of this paper.

policy case is not solvable for greater climate sensitivities (which are very unlikely anyway).[11] This bound is practically suitable for investigating the points we argued in Section 1.

Figure 1 shows the temperature distributions for the specifications discussed in section 2. We observe that policy effectively shifts the mode of the distribution and thus reduces the probability of high temperature increases. If the cost is lower (respectively, higher) than the DICE abatement cost, the density of the right tail becomes much thinner (resp., fatter). In addition, the temperature distributions are shifted toward lower temperature increases as time goes by in the *presence* of policy, whereas they are shifted toward higher temperature increases in the *absence* of policy. This confirms that the possibility of emissions control prevents the temperature from increasing.

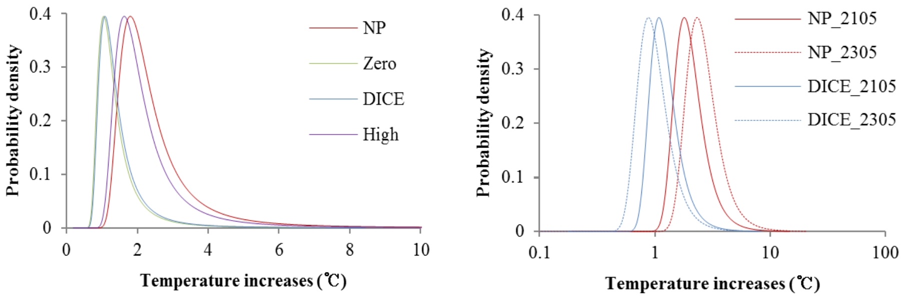

FIGURE 1. TEMPERATURE DISTRIBUTIONS

Notes: (Left panel): The PDFs of the temperature increases in 2105. (Right panel): The evolution of the PDFs over time. NP, Zero, DICE, and High cost refer to the no-policy, zero, DICE, high cost case, respectively. Note that x-axis in the right panel is on log (base 10) scale.

The left panel in Figure 2 shows the distributions of damage costs. The probability of high damage costs is reduced when climate policy is present. These results are consistent with the PDFs of temperature distributions. The distributions of consumption have thin *right* tails whereas the distributions of damage costs have thin *left* tails.[12] As with the PDFs of the damage costs, the PDFs of consumption significantly differ from case to case. Main implications are 1) climate policy greatly reduces the tail-effect and 2) the effects are sensitive to the cost of policy.

[11] By the no-policy we mean that $\mu = 0$ throughout all time periods.

[12] This is because 1) low temperature increases induce almost negligible impacts on the damage costs, and 2) the abatement costs are same across all states of the world.

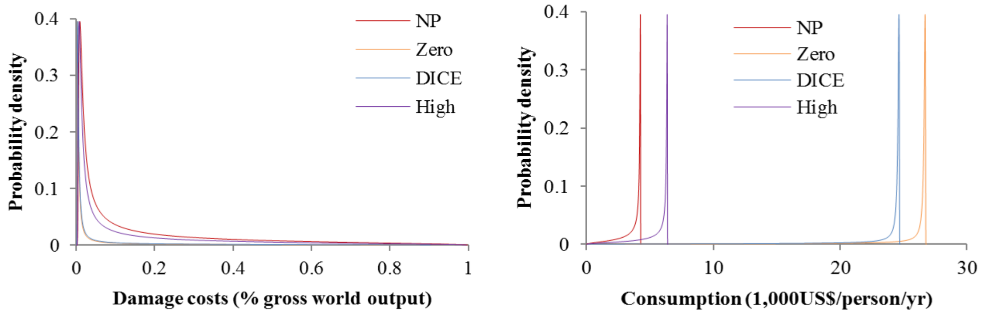

FIGURE 2. THE DISTRIBUTIONS OF DAMAGE COSTS AND CONSUMPTION IN 2105

Notes: (Left panel): Damage costs distributions. (Right panel): Consumption distributions

Figure 3 presents the optimal carbon tax and social welfare of the DICE cost case. The optimal carbon tax arbitrarily increases as the upper bound of climate sensitivity increases for the CRRA (η=0) case. However, for the HARA cases (η>0), such a tail-effect rarely arises. Similarly, there is a sharp fall in social welfare around the higher upper bounds of the climate sensitivity for the CRRA case, whereas such a deep fall is not present for the HARA cases.

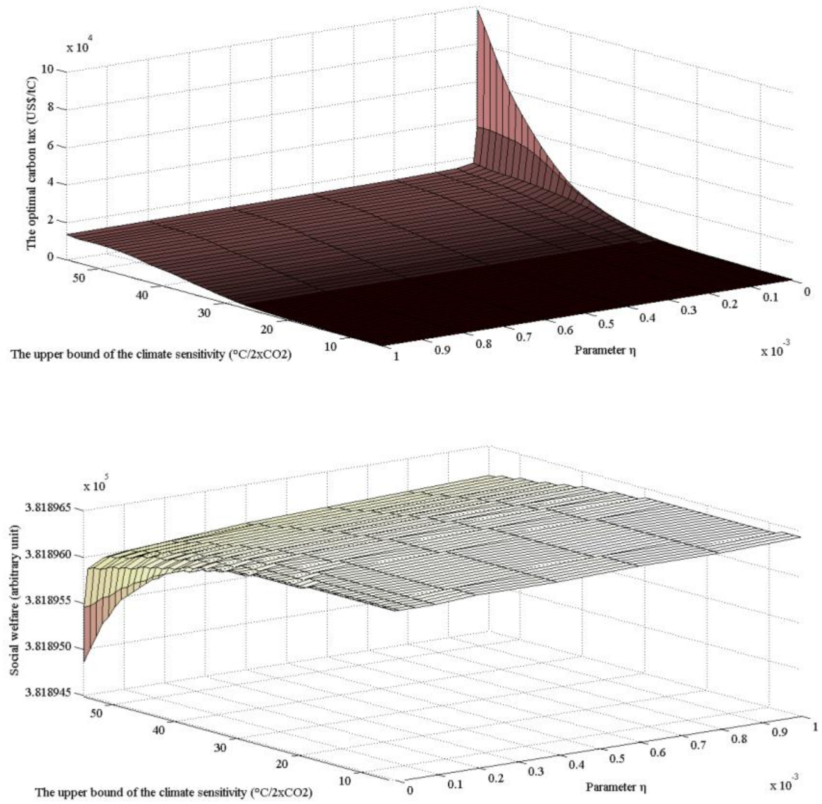

FIGURE 3. THE OPTIMAL CARBON TAX AND WELFARE (DICE COST CASE)

Notes: (Top panel): The optimal carbon tax in 2015. Note that x-axis (for parameter η) is presented in the reversed order. (Bottom panel): Social welfare.

Some sensitivity analyses are presented in Figure 4. We observe that the tail-effect is highly sensitive to the cost of emissions control and whether or not the tail-effect is present depends on the parameters values of the utility function and the damage function. For instance, if the DICE damage function instead of the Weitzman's damage function is applied, the tail-effect is greatly reduced. A low value of α also reduces the tail-effect.

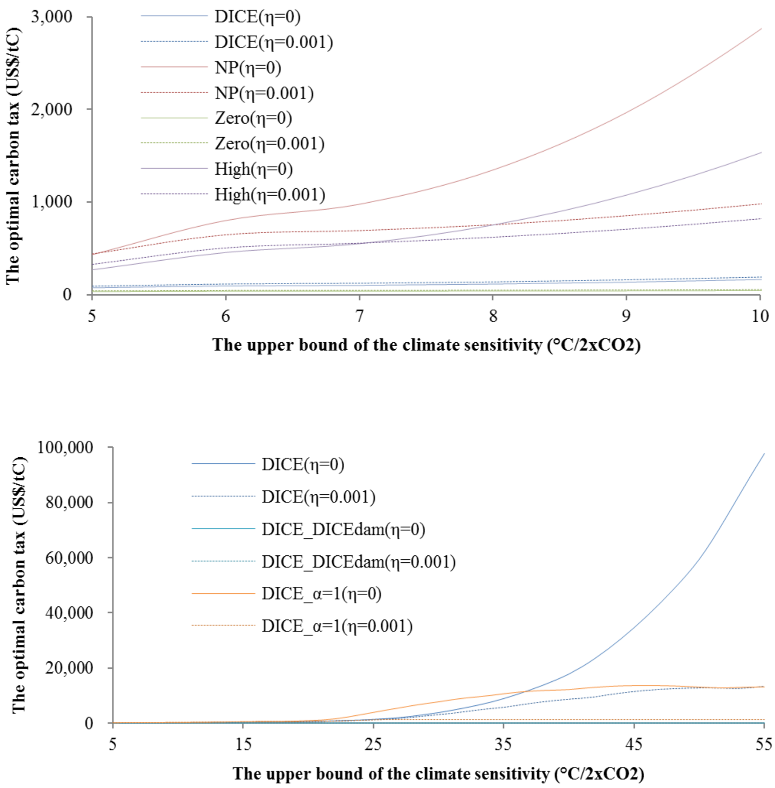

FIGURE 4. THE OPTIMAL CARBON TAX IN 2015 (SENSITIVITY ANALYSIS)

Notes: DICE refers to the reference case, where the Weitzman's damage function and the DICE abatement cost function are applied and α=2. The other cases are the sensitivity analyses on the abatement cost (Zero, High), on the damage function (DICE_DICEdam: the DICE damage function is applied instead of Weitzman (2012)'s damage function), and on the parameter of the HARA utility function (DICE_α=1: α is set to 1 instead of 2). For the NP (no-policy) case the social cost of carbon is presented. Since the NP case is not solvable for higher upper bounds of the climate sensitivity, we only present the results up to 10°C/2x CO_2 in the top panel.

IV. Conclusion

This paper has investigated the role of emissions control on reducing the tail-effect of the fat-tailed distribution of the climate sensitivity. The main results are that 1) the option for emissions control effectively prevents the tail-effect, and that 2) if the HARA utility function is used instead of the CRRA utility function, the tail-effect in the sense of Weitzman's Dismal Theorem does not arise, and that 3) the role of emissions control in reducing the tail-effect is sensitive to the cost of emissions control.

For the derivation of the analytical results and numerical simulations, we applied the climate sensitivity distribution derived from the feedback analysis (Equation 3). Even if other fat-tailed distributions are used the main implications of this paper, above mentioned, would not change since the same arguments can be applied to many other distributions. For other distributions only the convergence criterion should be altered (see Footnote 5). Quantitatively, our results may be sensitive to missing feedback factors. For instance, positive feedbacks such as carbon dioxide or methane emissions from forest dieback or melting permafrost would increase the stringency of climate policy (Torn and Harte 2006). However, since those missing feedbacks can be represented as an addition to the value of the total feedback factors, our arguments would still hold qualitatively even with the introduction of such missing feedback factors.

REFERENCES

Ackerman, Frank, Elizabeth A. Stanton, and Rarnon Bueno. 2010. "Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE." *Ecological Economics* 69 (8): 1657-1665.

Anthoff, David, and Richard S. J. Tol. 2010. "The Climate Framework for Uncertainty, Negotiation and Distribution (FUND), Technical Description, Version 3.3." *http://www.fund-model.org*.

Anthoff, David, and Richard S. J. Tol. 2013. "Climate policy under fat-tailed risk: An application of FUND." *Annals of Operations Research* (in press).

Arrow, Kenneth J. 1974. "The use of unbounded utility functions in expected-utility maximization: Response." *The Quarterly Journal of Economics* 88 (1): 136-138.

Arrow, Kenneth J. 2009. "A note on uncertainty and discounting in models of economic growth." *Journal of Risk and Uncertainty* 38 (2): 87-94.

Baker, Marcia B., and Gerard H. Roe. 2009. "The shape of things to come: Why is climate change so predictable?" *Journal of Climate* 22 (17): 4574-4589.

Costello, Christopher J., Michael G. Neubert, Stephen A. Polasky, and Andrew R. Solow. 2010. "Bounded uncertainty and climate change economics." *Proceedings of the National Academy of Sciences* 107 (18): 8108-8110.

Dietz, Simon. 2011. "High impact, low probability? An empirical analysis of risk in the economics of climate change." *Climatic Change* 108 (3): 519-541.

Gregory, Jonathan M., and Piers M. Foster. 2008. "Transient climate response estimated from radiative forcing and observed temperature change." *Journal of Geophysical Research* 113: D23105.

Hansen, James, Andrew Lacis, David Rind, Gary Russell, Peter Stone, Inez Fung, Reto Ruedy, and J. Lerner. 1984. “Climate sensitivity: Analysis of feedback mechanisms.” *Geophysical Monograph Series* 29: 130-163.

Hennlock, Magnus. 2009. “Robust control in global warming management: An analytical dynamic integrated assessment.” Resource for the Future Discussion Paper RF 09-19.

Horowitz, John, and Andreas Lange. 2008. “What's wrong with infinity - a note on Weitzman's Dismal Theorem.” http://faculty.arec.umd.edu/jhorowitz/weitzman_final.pdf.

Hwang, In Chang, Fredric Reynes, and Richard S. J. Tol. 2013a. “Climate policy under fat-tailed risk: An application of DICE.” *Environmental and Resource Economics*: 56(3), 415-436.

Hwang, In Chang, Richard S. J. Tol, and Marjan W. Hofkes. 2013b. “Active learning about climate change.” University of Sussex Economics Department Working Paper Series No 65-2013.

Ikefuji, Masako, Roger J.A. Laeven, Chris Muris, and Jan R. Magnus. 2010. “Expected utility and catastrophic risk in a stochastic economy-climate model.” Tilberg University CentER Discussion Paper No. 2010-122.

Ingham, Alan, Jie Ma, and Alistair Ulph. 2007. “Climate change, mitigation and adaptation with uncertainty and learning.” *Energy Policy* 35: 5354-5369.

Judd, Kenneth L. 1998. *Numerical methods in economics*. Cambridge, MA: MIT press.

Karp, Larry. 2009. “Sacrifice, discounting and climate policy: five questions.” CESifo Working Paper No. 2761.

Millner, Antony. 2013. “On welfare frameworks and catastrophic climate risks.” *Journal of Environmental Economics and Management* 65: 310-325.

Newbold, Stephen C., and Adam Daigneault. 2009. “Climate response uncertainty and the benefits of greenhouse gas emissions reductions.” *Environmental and Resource Economics* 44 (3): 351-377.

Nordhaus, William D. 2008. *A question of balance: Weighing the options on global warming policies*. New Haven and London: Yale University Press.

Nordhaus, William D. 2011. “The economics of tail events with an application to climate change.” *Review of Environmental Economics and Policy* 5 (2): 240-257.

Pindyck, Robert S. 2011. “Fat tails, thin tails, and climate change policy.” *Review of Environmental Economics and Policy* 5 (2): 258-274.

Roe, Gerard H., and Marcia B. Baker. 2007. “Why is climate sensitivity so unpredictable?” *Science* 318 (5850): 629-632.

Tol, Richard S. J. 2003. “Is the uncertainty about climate change too large for expected cost-benefit analysis?” *Climatic Change* 56 (3): 265-289.

Weitzman, Martin L. 2009. "On modeling and interpreting the economics of catastrophic climate change." *The Review of Economics and Statistics* 91 (1): 1-19.

Weitzman, Martin L. 2011. "Fat-tailed uncertainty in the economics of catastrophic climate change." *Review of Environmental Economics and Policy* 5 (2): 275-292.

Weitzman, Martin L. 2012. "GHG targets as insurance against catastrophic climate damages." *Journal of Public Economic Theory* 14 (2): 221-244.

Weitzman, Martin L. 2013. "A Precautionary Tale of Uncertain Tail Fattening." *Environmental and Resource Economics* 55: 159-173.

Wigley, Tom M. L, and Michael E. Schlesinger. 2013. "Analytical solution for the effect of increasing CO_2 on global mean temperature." *Nature* 315(20): 649-652.