

Working Paper Series

No. 07 -2020

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Kernel density decomposition with an application to the social cost of carbon[☆]

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Abstract

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Keywords: social cost of carbon, kernel density, decomposition, discrete explanatory variables

JEL codes: C14, Q54

1. Introduction

Everything about climate change is uncertain. Many estimates of the social cost of carbon have been published, and kernel densities have been used to visualize the uncertainty about the Pigou tax (Tol, 2018). I here propose a new method to decompose that uncertainty into discrete components, and a new statistical test for whether the components differ from one another.

[☆]The replication package and generic code are on GitHub.

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¹Earlier versions of this paper were presented at the University of Sussex and the University of Surrey.

Kernel densities are a useful tool to describe univariate data (Takezawa, 2005). Simple kernel regression is helpful for specifying the relationship between two variables (Altman, 1992), and kernel quantile regression can be used to show this relationship across the distribution (Yu and Jones, 1998). However, these methods are not suitable if the explanatory variable is categorical. The method proposed here works well for categorical data, and shows both central tendency and spread.

This paper extends previous meta-analyses of the social cost of carbon (Tol, 2005, 2018). The kernel function and bandwidth are the same as in the 2018 paper (see Appendix B), but there are now many more estimates (see Appendix A). My earlier papers used sample splits rather than decomposition and did not conduct statistical tests for differences. Wang et al. (2019) report a meta-analysis of 548 estimates of the social cost of carbon. I have 2786. They focus on the central tendency, while I consider the entire distribution. They assume linearity and normality. I do not.

The paper proceeds as follows. Section 2 presents the decomposition method, and Section 3 the appropriate hypothesis test. Section 4 applies these methods to estimates of the social cost of carbon and its growth rate. Section 5 concludes.

2. Kernel density decomposition

A *kernel density* is defined as

$$f(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - x_i}{h}\right) \quad (1)$$

where x_i are a series of observations, h is the bandwidth, and K is the *kernel function*. The kernel function is conventionally assumed to be a (i) **non-negative** (ii) **symmetric** function that (iii) **integrates to one**, with (iv) **zero mean** and (v) **finite variance** (Takezawa, 2005). That is, any standardized symmetric probability density function can serve as a kernel function, and the Normal density is indeed a common choice.

Conventions are just that. As long as the kernel function is non-negative² and integrates to one, an appropriately weighted sum of kernel functions is non-negative and integrates to one—such a sum is a probability density function. Of course, if the kernel function is **asymmetric**, centralization needs to be carefully considered—is x_i the mean, median or mode of K ?

A kernel density can be seen as a mixture (Makov, 2001; McLachlan and Peel, 2001).³ This reinterpretation opens a route to decomposition. We can construct the kernel density of any subset of x_i . The weighted sum of the kernel densities of the subsets is a kernel density.

²The assumption of non-negativity is relaxed for bias reduction (Jones and Signorini, 1997), a topic unrelated to the current paper.

³Quetelet (1846, 1852) was the first to note that the weighted sum of densities is a density, Pearson (1894) the first to apply this insight.

Indeed, with the right weights and bandwidths, the weighted sum of the kernel densities of subsets of the data is identical to the kernel density of the whole data set. To see this, partition the observations into m subsets of length m_j with $\sum_j m_j = n$, as $x_1, \dots, x_{m_1}, x_{m_1+1}, \dots, x_{m_1+m_2}, x_{m_1+m_2+1}, \dots, x_n$. Then

$$f(x) = \sum_{j=1}^m \frac{m_j}{n} \frac{1}{m_j h} \sum_{i=\sum_{k=1}^{j-1} m_k+1}^{\sum_{k=1}^j m_k} K\left(\frac{x-x_i}{h}\right) =: \sum_{j=1}^m \frac{m_j}{n} f_j(x) \quad (2)$$

This is identical to Equation (1). Moreover, each of the components f_j of the composite kernel density f is itself a kernel density.

I doubt I am the first to notice this, but as far as I know I am the first to write it up.⁴

Note that kernel decomposition works with any set of weights that add to one; and with any kernel function or bandwidth for the subsets:

$$f(x) = \sum_{j=1}^m \frac{w_j}{m_j h_j} \sum_{i=\sum_{k=1}^{j-1} m_k+1}^{\sum_{k=1}^j m_k} K_j\left(\frac{x-x_i}{h_j}\right) =: \sum_{j=1}^m w_j f_j(x; h_j) \quad (3)$$

In this case, the composite kernel density is not be the same as the kernel density fitted to the complete data set. While it may be hard to argue in favour of different kernel functions K_j for different subsets of the data, meaningfully different subsets of the data would have different spreads and hence bandwidths h_j .

The mathematics work for *any* partitioning of the observations, but decomposing a kernel density in this manner makes as much sense as the partitioning. Below, I take a large sample of estimates of the social cost of carbon and decompose its kernel density in three ways: by author, by year of publication, and by discount rate. As the partitioning is intuitive, so is the decomposition.

Three hypotheses are tested: Do different discount rates imply different climate policy? The expected answer is yes. Do estimates change over time? In other words, are we learning? The answer is let's hope so. Do different researchers reach different conclusions? In other words, do subjective judgements affect the estimates? The answer is let's hope not.

The decomposition of the kernel density by author opens another interpretation: Vote-counting (Laplace, 1814). Different experts have published different estimates of the social cost of carbon. These can be seen as votes for a particular Pigou tax. But as the experts are uncertain, they have voted for a central

⁴I searched Scopus for "kernel decomposition", "composite kernel" and "kernel mixture". Cunningham and Williams (1994) show that a spectrogram is the weighted sum of spectrograms. Szymkowiak-Have et al. (2006) decompose kernels by introducing a conditional variable (much like I do) but use this for spectral clustering rather than decomposition. Szafranski et al. (2010) fit alternative kernel densities to the same data, and construct a composite kernel density using model fit as weights; see Kloft et al. (2011) for a discussion of the appropriate weights.

value and a spread. The kernel function is a vote, the kernel density adds those votes.⁵

Different experts have cast different numbers of votes. Below, I count one paper as one vote. One can also argue that it should be one vote per expert, or that papers should be weighted by citations, journal prestige, or author pedigree. Composite kernel densities naturally allow for this, but it is a dangerous route to travel in this case.

3. Inference

Equation (3) holds that the kernel density $f(x)$ is composed of m kernel densities $f_j(x)$ with weight m_j/n . For each interval $\underline{x} < x < \bar{x}$, we can test whether the shares of the component densities equal the overall weights, using the Equality of Proportions test by Pearson (1900). Such a test reveals whether a component density disproportionately contributes to, say, the left tail of the composite density.

If the intervals correspond to p percentiles of the composite distribution, the test statistic is

$$\chi_{(m-1)(p-1)}^2 = n \sum_{k=0}^p \sum_{j=1}^m \frac{\left(\int_{P_k}^{P_{k+1}} f_j(x) dx - \frac{m_j}{n} \right)^2}{\frac{m_j}{n}} \quad (4)$$

Note that the test only works if there are two components or more, $m \geq 2$. If not, there would be nothing to compare. Note also that the distribution needs to be split in two quantiles or more, $p \geq 2$. This is because each component density adds up to its weight m_j/n by construction.

Matlab codes to construct and decompose a kernel and test the decomposition are on GitHub.

4. Application

4.1. The social cost of carbon

The social cost of carbon is the damage done, at the margin, by emitting more carbon dioxide into the atmosphere. If evaluated along the optimal emissions trajectory, the social cost of carbon equals the Pigou (1920) tax that internalizes the externality and restores the Pareto (1906) optimum. Climate change features high on the political and public agenda. Estimating the social cost of carbon is an intellectual challenge, requiring the projection of emissions, concentrations and climates over a long period of time, the estimation the effects of climate change, the valuation of a wide variety of impacts, the aggregation of impacts over people with a huge range of living standards, the consideration

⁵Note the difference with Bayesian updating, which multiplies rather than adds probabilities.

of large uncertainties and ambiguous knowledge, and the discounting of welfare in the distant future and over multiple generations (Tol, 2018).

It is therefore no surprise that there is a large literature on the social cost of carbon spanning four decades, from Nordhaus (1982) to Okullo (2020). I count 2786 estimates in 148 papers. These are estimates for the social cost of carbon of carbon dioxide emitted in the recent past. 54 papers published estimates of the social cost of carbon at two or more points in time, for a total of 648 estimates of the *growth rate* of the social cost of carbon.⁶

4.2. Discount rate

Figure 1 decomposes the kernel density of the social cost of carbon into its components by pure rate of time preference used. Three choices of time preference are most common: 3.0%, 1.5% and 1.0%.⁷ As one would expect, the lower discount rates contribute more to the right tail of the distribution.

Table C.1 shows the contributions of estimates of the social cost of carbon using a particular pure rate of time preference to the overall kernel density (denoted "null") as well as to the five quintiles of that density (denoted Q1-5). The null hypothesis that all shares are equal is firmly rejected; $\chi^2_{24} = 98.6$; $p = 0.000$.

4.3. Author

I split the sample into estimates by those who have published five papers or more (i.e., Christopher W. Hope, William D. Nordhaus, Frederick van der Ploeg, Richard S.J. Tol) and others.

Figure 2 decomposes the kernel density by author. Of the named authors, estimates by van der Ploeg are the narrowest, Tol contributes most to the left tail, and Hope to the right tail.

Table C.2 shows the contributions of estimates of the social cost of carbon published by a particular author to the overall kernel density and its quintiles. Although there are patterns in figure and table, the quintile shares are statistically indistinguishable from the overall shares; $\chi^2_{16} = 19.2$; $p = 0.260$.

4.4. Year of publication

I split the sample into five periods. The key events are the Second Assessment Report of the Intergovernmental Panel on Climate Change, and the first IPCC report to discuss the economic impacts (Pearce et al., 1996), the Third Assessment Report of the IPCC (Smith et al., 2001), the Stern Review (Stern et al., 2006), and the Obama update of the social cost of carbon (on the Social Cost of Carbon, 2013).

⁶The carbon tax should increase over time until emissions are so low that the marginal impacts of climate change start to fall.

⁷"Other" refers to a range of numbers and methods, but mostly constant consumption rates.

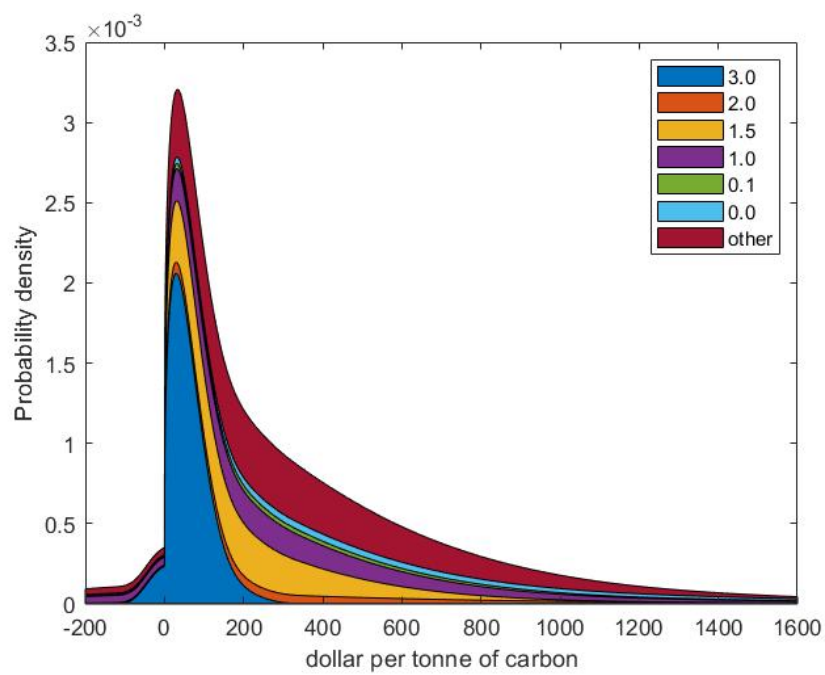


Figure 1: Kernel density of the social cost of carbon and its composition by discount rate.

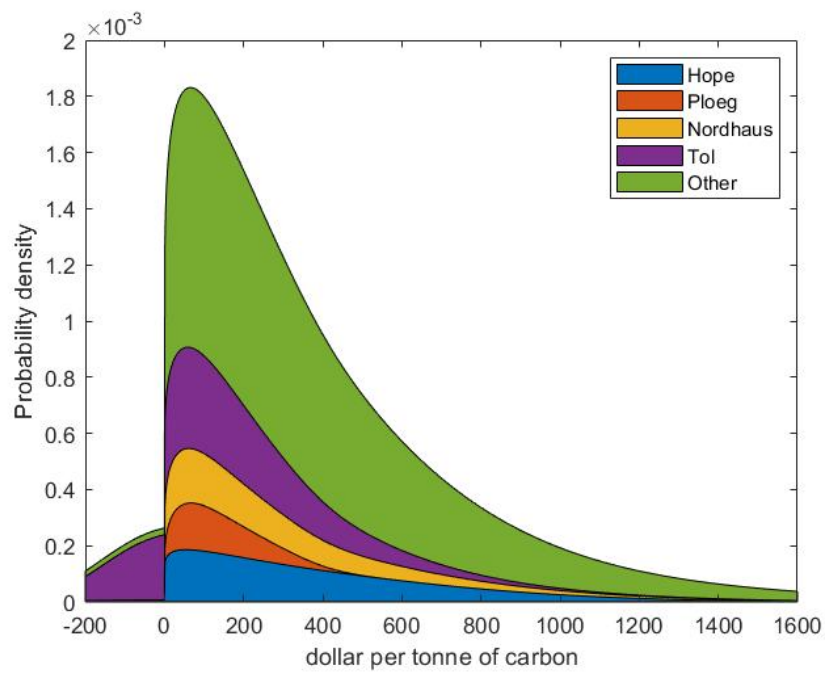


Figure 2: Kernel density of the social cost of carbon and its composition by author.

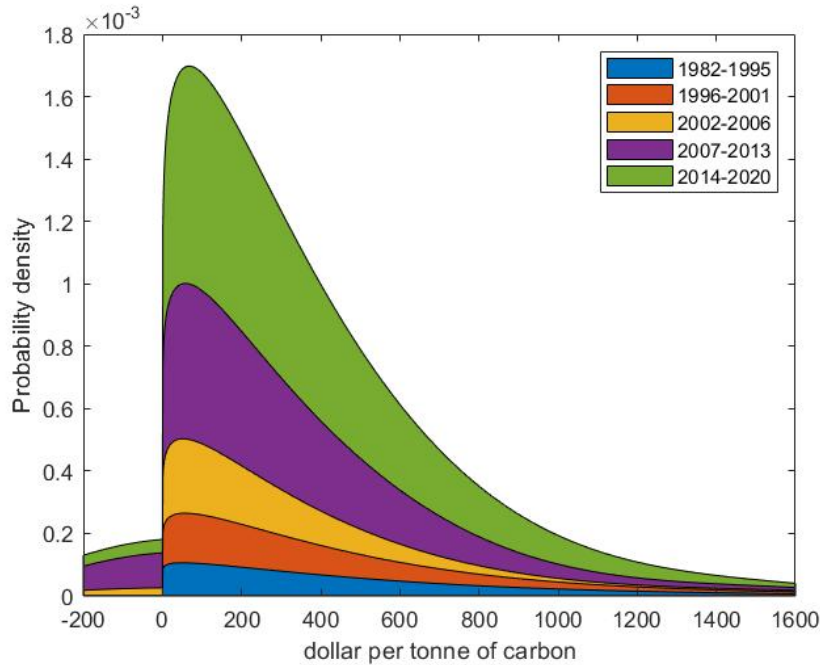


Figure 3: Kernel density of the social cost of carbon and its composition by publication period.

Figure 3 shows the decomposition by publication period. The earlier studies excluded negative estimates, but other patterns are not obvious.

Table C.3 shows the contributions of estimates of the social cost of carbon published in a particular period to the overall kernel density and its quintiles. The quintile shares are statistically indistinguishable from the overall shares; $\chi^2_{16} = 4.14$; $p = 0.999$.

4.5. The growth rate of the social cost of carbon

Figure 4 shows the kernel density of the growth rate of the social cost of carbon, decomposed for the pure rate of time preference. The density is symmetric for a 3.0% utility discount rate. However, for discount rates of 1.5% and 2.0%, little probability mass is added to the left tail, and a lot to the right tail.

Table C.4 shows the shares by quintile of the kernel density. The same pattern is seen as in the graph, but Pearson's test for the equality of proportions does not reject the null hypothesis that the component densities are equal to the composite one; $\chi^2_{24} = 10.6$; $p = 0.992$.

5. Discussion and conclusion

I present a method to decompose kernel densities and statistically test whether the components differ from the composite. I illustrate the proposed method with

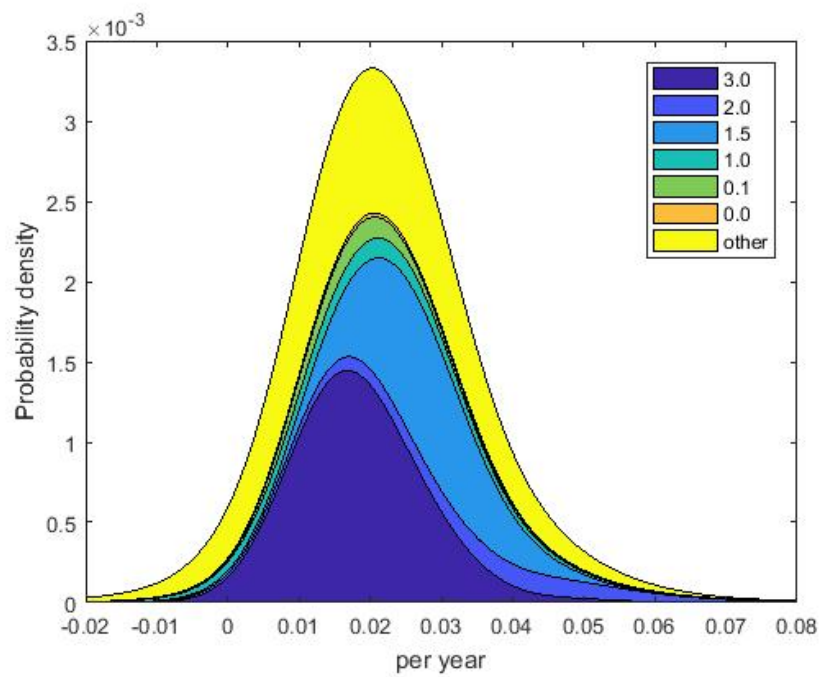


Figure 4: Kernel density of the growth rate of the social cost of carbon and its composition by discount rate.

a meta-analysis of the social cost of carbon and its growth rate. As expected, a lower discount rate implies a statistically significantly higher Pigou tax; its growth rate is not affected by the discount rate. Earlier estimates of the social cost of carbon are not different from later estimates. There appears to be no learning over time. The null hypothesis that different authors publish similar estimates cannot be rejected. Published estimates are not marked by subjectivity.

The proposed method can be applied to any issue with categorical explanatory variables, such as the wages of men and women, energy use of home owners and renters, commuting times by public and private transport, exam scores by ethnicity, or projected corona virus deaths by political affiliation.

The method presented here considers a univariate kernel density and its dependence on a single categorical variable. Generalization to bi- and trivariate kernel densities is immediate, although visualization would be a challenge. If a kernel density can be used as kernel function to form a composite kernel density, then the composite kernel density can of course also be used as a kernel function—and so on. That is, although I present a single nest for a single categorical explanatory variable, deeper nests are possible if so desired.

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Appendix A. New estimates of the social cost of carbon

The previous meta-analysis of the social cost of carbon (Tol, 2018) was extended with estimates reported in Anthoff and Emmerling (2019), Bretschger and Pattakou (2019), Budolfson et al. (2017), Daniel et al. (2019), Dayaratna et al. (2020), Ekholm (2018), Faulwasser et al. (2018), Golub and Brody (2017), Guivarch and Pottier (2018), Hafeez et al. (2017), Hänsel and Quaas (2018), Kotchen (2018), Moore et al. (2017), Nordhaus (2015), Okullo (2020), Ricke et al. (2018), Scovronick et al. (2017), Tol (2019), Yang et al. (2018) and Zhen et al. (2018). The Budolfson and Faulwasser estimates were digitized from graphs.

Glanemann et al. (2020) do not report a carbon tax, Zhen and Tian (2019) report the *relative* social cost of carbon, Paul Kelleher and Wagner (2019) *relative changes* in the social cost of carbon, van der Ploeg and de Zeeuw (2019) the *steady state* social cost of carbon, and Pindyck (2017, 2019) the *average* social cost of carbon.

All data to reproduce this paper are on GitHub.

Appendix B. Bandwidth and kernel function

The choice of kernel function and bandwidth is key to any kernel density estimate, as illustrated in Figure B.5. Kernel and bandwidth should be chosen such to reflect the nature of the data. In this case, the uncertainty about the social cost of carbon is large and right-skewed. Furthermore, the social cost of carbon is, most researchers argue, a cost and not a benefit.

A conventional choice would be to use a Normal kernel function, with a bandwidth according to the Silverman rule, that is, 1.06 times the sample standard deviation divided by the number of observations raised to the power one-fifth. Figure B.5 reveals two problems with this approach: The right tail is thin, and a large probability mass is assigned to negative social costs of carbon. If the bandwidth equals the sample standard deviation to reflect the wide uncertainty, the right tail appropriately thickens but the probability of a Pigou subsidy on greenhouse gas emissions increases too.

Many of the published estimates of the social cost of carbon are based on an impact function that excludes benefits of climate change. Honouring that, I assign a knotted Normal kernel function to these observations, with the mode as its central tendency. The probability of a negative social cost of carbon falls.

The studies that report the possibility of a negative social cost of carbon nonetheless argue in favour of a positive one. A symmetric Normal kernel does not reflect that. I therefore replace it with a Gumbel kernel, which is defined on the real line but right-skewed. Again, I use the mode as its central tendency. This thickens the right tail and thins the left tail.

A knotted Normal kernel is not only peculiar near zero but it also has a thin tail. I therefore replace it with a Weibull kernel, which is defined on the positive real line, near zero near zero, and right-skewed. I use the mode as its central tendency. The right tail of the kernel distribution thickens again. The Weibull-Gumbel kernel distribution is the default used here.

Code and data to reproduce this paper are on GitHub.

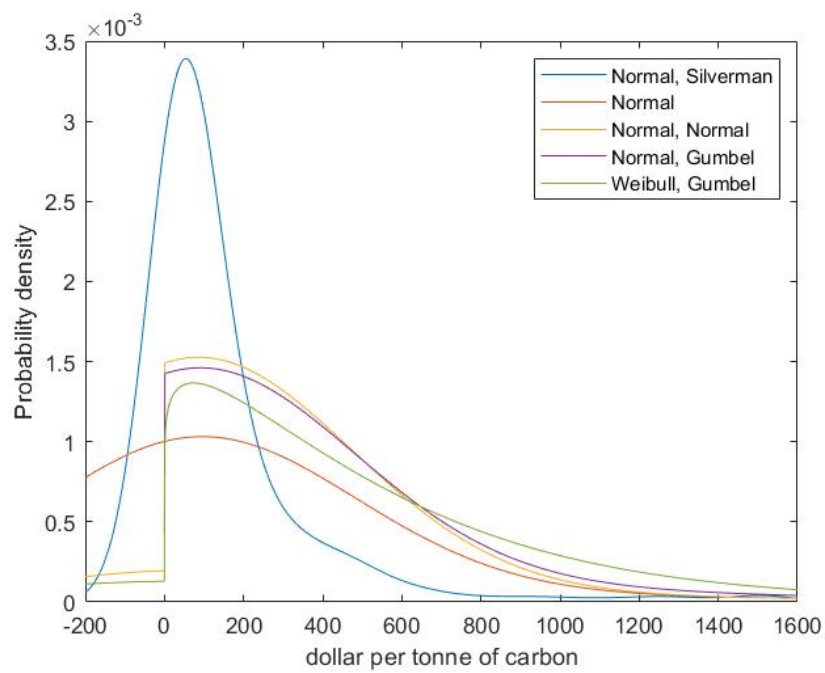


Figure B.5: Kernel density of the social cost of carbon for alternative kernel functions and bandwidths.

Appendix C. Additional results

	3.0	2.0	1.5	1.0	0.1	0.0	other
Q1	0.1677	0.0066	0.0368	0.0354	0.0050	0.0074	0.0544
Q2	0.0520	0.0085	0.0470	0.0271	0.0045	0.0061	0.0567
Q3	0.0028	0.0088	0.0399	0.0284	0.0051	0.0077	0.0614
Q4	0.0000	0.0099	0.0289	0.0295	0.0060	0.0107	0.0680
Q5	0.0000	0.0168	0.0119	0.0347	0.0096	0.0268	0.0779
Null	0.0445	0.0101	0.0329	0.0310	0.0060	0.0117	0.0637

Table C.1: Observed and hypothesized contribution to the kernel density by quintile and pure rate of time preference.

	Hope	Nordhaus	Ploeg	Tol	Other
Q1	0.0184	0.0135	0.0175	0.0722	0.0900
Q2	0.0218	0.0173	0.0217	0.0403	0.1140
Q3	0.0218	0.0092	0.0196	0.0330	0.1172
Q4	0.0233	0.0009	0.0174	0.0225	0.1223
Q5	0.0220	0.0000	0.0146	0.0097	0.1397
Null	0.0214	0.0082	0.0182	0.0355	0.1166

Table C.2: Observed and hypothesized contribution to the kernel density by quintile and author.

	1982-1995	1996-2001	2002-2006	2007-2013	2014-2020
Q1	0.0095	0.0142	0.0284	0.0758	0.0754
Q2	0.0126	0.0190	0.0267	0.0595	0.0862
Q3	0.0130	0.0189	0.0239	0.0586	0.0874
Q4	0.0143	0.0191	0.0204	0.0569	0.0879
Q5	0.0219	0.0229	0.0140	0.0468	0.0867
Null	0.0143	0.0188	0.0227	0.0595	0.0847

Table C.3: Observed and hypothesized contribution to the kernel density by quintile and publication period.

	3.0	2.0	1.5	1.0	0.1	0.0	other
Q1	0.0628	0.0054	0.0056	0.0125	0.0053	0.0029	0.0772
Q2	0.1029	0.0057	0.0227	0.0091	0.0104	0.0019	0.0644
Q3	0.0858	0.0072	0.0527	0.0085	0.0084	0.0017	0.0609
Q4	0.0540	0.0108	0.0726	0.0083	0.0045	0.0017	0.0582
Q5	0.0207	0.0339	0.0484	0.0087	0.0028	0.0028	0.0588
Null	0.0652	0.0126	0.0404	0.0094	0.0063	0.0022	0.0639

Table C.4: Observed and hypothesized contribution to the kernel density of the growth rate of the social cost of carbon by quintile and pure rate of time preference.