FCPNLO 7

Program, titles and abstracts

Wednesday 25 September 2019

19:00-21:00 *Get-together drinks (and dinner)*

Thursday 26 September 2019

9:30-10:00 *Registration and morning coffee*

10:00-10:10 *Welcome of the Head of Department Michael Melgaard*

10:10-11:10 Sergei Fedotov (University of Manchester)

Space-dependent variable order fractional equations and super-diffusive intracellular transport

Abstract: I will discuss the asymptotic representation of the solution of the variable-order fractional diffusion equation with a non-trivial advection term that causes ultraslow spatial aggregation of sub-diffusive particles. I will also discuss the phenomenon of cumulative inertia in intracellular transport involving multiple motor proteins. I will provide a theoretical link between this effect and the classical Lévy walk model.

11:10-11:40 *Morning coffee break*

11:40-12:40 Mark Ainsworth (Brown University)

Fractional Cahn-Hilliard Equation(s): Analysis, Properties and Approximation

Abstract: The classical Cahn-Hilliard equation [1] is a nonlinear, fourth order in space, parabolic partial differential equation which is often used as a diffuse interface model for the phase separation of a binary alloy. Despite the widespread adoption of the model, there are good reasons for preferring models in which fractional spatial derivatives appear [2,3]. We consider two such Fractional Cahn-Hilliard equations (FCHE). The first [4] corresponds to considering a gradient flow of the free energy functional in a negative order Sobolev space H^{α}, $\alpha \in [0,1]$ where the choice $\alpha = 1$ corresponds to the classical Cahn-Hilliard equation whilst the choice $\alpha = 0$ recovers the Allen-Cahn equation. It is shown that the equation preserves mass for all positive values of fractional order and that it indeed reduces the free energy. The well-posedness of the problem is established in the sense that the H_1-norm of the solution remains uniformly bounded. We then turn to the delicate question of the L^{∞} boundedness of the solution and establish an L^{∞}-bound for the FCHE in the case where the non-linearity is a quartic polynomial. As a consequence of the estimates, we are able to show that the Fourier-Galerkin method delivers a spectral rate of convergence for the FCHE in the case of a semi-discrete approximation scheme. Finally, we present results obtained using computational simulation of the FCHE for a variety of choices of fractional order α. We then consider an alternative FCHE [3,5] in which the free energy functional involves a fractional order derivative. (Joint work with Zhiping Mao)

References:

[1] J.W. Cahn and J.E. Hilliard, Free energy of a non-uniform system. I. Interfacial Free Energy, J. Chem. Phys, 28, 258-267 (1958).

[2] L. Caffarelli and E. Valdinoci, A Priori Bounds for solutions of non-local evoluation PDE, Springer, Milan 2013.

[3] G. Palatucci and O. Savin, Local and global minimisers for a variational energy involving a fractional norm, Ann. Mat. Pura Appl., 4, 673-718 (2014).

[4] M. Ainsworth and Z. Mao, Analysis and Approximation of a Fractional Cahn-Hilliard Equation, SIAM J. Numer. Anal. 55 (2017), no. 4, 1689-1718. [5] M. Ainsworth and Z. Mao, Well-posedness of the Cahn-Hilliard Equation with Fractional Free Energy and Its Fourier-Galerkin Discretization, Chaos Solitons Fractals 102 (2017), 264-273.

12:40-14:00 Lunch

14:00-15:00 Elżbieta Motyl (University of Łódź)

Fractionally dissipative stochastic quasi-geostrophic type equations

Abstract: The two-dimensional surface quasi-geostrophic equation

$$\frac{\partial \theta}{\partial t} + (v \cdot \nabla)\, \theta = 0$$

is important is important in modeling geophysical fluid dynamics. Here $\theta : \mathbb{R}^2 \to \mathbb{R}$ represents the temperature and $v : \mathbb{R}^2 \to \mathbb{R}^2$ is the velocity of the fluid. The velocity is expressed in terms of the temperature by the relation $v = R^T\theta = (-R_2\theta, R_1\theta)$, where R_j denotes the j-th Riesz transform ($j = 1, 2$). In addition, various generalizations of this equation are considered. We consider stochastic fractionally dissipative quasi-geostrophic type equation on $\mathbb{R}^d$ with a multiplicative Gaussian noise

$$\begin{cases} du(t) + [(\mathcal{R}u(t) \cdot \nabla)u(t) + \nu(-\Delta)^{\alpha}u(t)]\, dt = f(t)\, dt + G(t, u(t))\, dW(t), \\ u(0) = u_0. \end{cases}$$

Using appropriate approximations and the stochastic compactness method we prove the existence of the weak solutions. In the $2D$ sub-critical case we also prove the pathwise uniqueness of the solutions. This is joint work with Z. Brzeźniak (University of York).

References:

[1] Z. Brzeźniak, E. Motyl, Fractionally dissipative stochastic quasi-geostrophic type equations on $\mathbb{R}^d$, SIAM J. Math. Anal., 51 (3), 2306–2358 (2019).
[2] Z. Brzeźniak, E. Motyl, Existence of a martingale solution to the stochastic Navier-Stokes equations in unbounded $2D$ and $3D$ domains, J. Differential Equations, 254, 1627–1685 (2013).

15:00-15:30 *Afternoon coffee break*

15:30-16:30 Rainer Klages (Institute of Theoretical Physics, Technical University of Berlin and School of Mathematical Sciences, Queen Mary University of London)

Origin of anomalous diffusion in dynamical systems: Three examples

Abstract: Consider equations of motion that generate dispersion of an ensemble of particles. FFor a given dynamical system an interesting problem is not only what type of diffusion is generated by its equations of motion but also whether the resulting diffusive dynamics can be reproduced by some known stochastic model. I will discuss three examples of dynamical systems generating different types of diffusive transport: The first model is fully deterministic but non-chaotic by displaying a whole range of normal and anomalous diffusion under variation of a single control parameter [1]. The second model is a dissipative version of the paradigmatic standard map. Weakly perturbing it by noise generates subdiffusion due to particles hopping between multiple attractors [2]. The third model randomly mixes in time chaotic dynamics generating normal diffusive spreading with non-chaotic motion where all particles localize. Varying a control parameter the mixed system exhibits a transition characterised by subdiffusion [3]. In all three cases I will show successes, failures and pitfalls if one tries to reproduce the resulting diffusive dynamics by using simple stochastic models. Joint work with all authors on the references cited below.

References:

[1] L. Salari, L. Rondoni, C. Giberti, R. Klages, Chaos 25, 073113 (2015).
[2] C.S. Rodrigues, A.V. Chechkin, A.P.S. de Moura, C. Grebogi and R. Klages, Europhys. Lett. 108, 40002

(2014).
[3] Y. Sato, R. Klages, Phys. Rev. Lett. 122, 174101 (2019).

19:00-21:00 Dinner at the Indian Summer

End of Day 1

Friday 27 September 2019

10:00-11:00 Adrian Baule (Queen Mary University of London)

Weak Galilean invariance as a selection principle for coarse-grained diffusive models

Abstract: How does the mathematical description of a system change in different reference frames? Galilei first addressed this fundamental question by formulating the famous principle of Galilean invariance. It prescribes that the equations of motion of closed systems remain the same in different inertial frames related by Galilean transformations, thus imposing strong constraints on the dynamical rules. However, real world systems are often described by coarse-grained models integrating complex internal and external interactions indistinguishably as friction and stochastic forces. Since Galilean invariance is then violated, there is seemingly no alternative principle to assess a priori the physical consistency of a given stochastic model in different inertial frames. Here, starting from the Kac-Zwanzig Hamiltonian model generating Brownian motion, we show how Galilean invariance is broken during the coarse-graining procedure when deriving stochastic equations. Our analysis leads to a set of rules characterizing systems in different inertial frames that have to be satisfied by general stochastic models, which we call *weak Galilean invariance*. Several well-known stochastic processes are invariant in these terms, except the continuous-time random walk for which we derive the correct invariant description. Our results are particularly relevant for the modeling of biological systems, as they provide a theoretical principle to select physically consistent stochastic models before a validation against experimental data.

Reference:

[1] A. Cairoli, R. Klages, and A. Baule, PNAS 115, 5714 (2018).

11:00-11:30 *Morning coffee break*

11:30-12:30 Lehel Banjai (Heriot Watt University)

Efficient numerical solution of time fractional sub-diffusion or diffusion-wave equations

Abstract: In this talk we discuss fast and memory efficient numerical solution of time fractional partial differential equations. The numerical method we describe belongs to the family of fast and oblivious quadratures with significant improvements in both computational costs and the ease of implementation. Furthermore, the new algorithm potentially allows for easier implementation of adaptive schemes needed to deal with often low regularity solutions. We present two approaches to using the described algorithms in solving time fractional PDE. Firstly as a fast and memory efficient discretization of the fractional time derivative combined with classical time-stepping. Secondly as a fast, memory efficient, and parallelizable method to finding the solution of the PDE at a final time. Time fractional subdiffusion or diffusion-wave equations can be approached in both ways. Whereas a fractionally damped wave equation coming from medical ultrasound applications is an example where the former approach is needed. The talk will end with some numerical experiments. This is joint work with Katie Baker (Heriot-Watt), Michael Karkulik (Universidad Técnica Federico Santa María), and Maria Lopez-Fernandez (Sapienza).

12:30-14:00 Lunch

14:00-15:00 Zdzisław Brzeźniak (University of York)

Vortex sheets for 2D stochastic Euler equations

Abstract: In the deterministic case, we have existence of solutions to 2D Euler equations with singular vorticity, in particular nonnegative vortex sheets (initial vorticity concentrated on a line). We consider the

2D stochastic Euler equations, in the vorticity form, with transport noise:

$$\partial_t \xi + u \cdot \nabla \xi + \sum_k \sigma_k \cdot \nabla \xi \circ \dot{W}^k = 0$$

$$\xi = \text{const} + \operatorname{curl} u$$

(σ_k given vector fields, W^k independent Brownian motions). We prove the existence of non-negative H^{-1}-valued solutions. Our result includes positive vortex sheets as initial data. My talk is based on a joint work with Mario Maurelli (Milano).

15:00-15:30 *Afternoon coffee break*

15:30-16:30 Gianni Pagnini (BCAM)

Non-autonomous stochastic differential equations and fractional diffusion

Abstract: The equivalence in distribution is shown between a family of stochastic processes driven by non-autonomous stochastic differential equations, with time-dependent drift and standard white noise, and a family of randomly-scaled Gaussian processes, i.e., processes built by the product of a Gaussian process times a non-negative independent random variable [1]. This result establishes a connection between non-autonomous stochastic differential equations with standard white noise and fractional diffusion. In fact, an example of randomly-scaled Gaussian process is the so-called generalised grey Brownian motion whose density function, for proper values of the parameters, solves a number of fractional diffusion equations [2]. The proof is based on the study of the centre-of-mass like variable of a heterogeneous ensemble of Ornstein?Uhlenbeck processes featuring a population of relaxation times and a population of noise amplitudes. The link between the Ornstein?Uhlenbeck process and the Langevin equation provides a framework for discussing under a physical perspective the application of this approach for modelling anomalous diffusion in biological systems [3, 4].

References:

[1] D'Ovidio, M., Vitali, S., Sposini, V., Sliusarenko, O., Paradisi., P, Castellani, G., Pagnini, G., Centre-of-mass like superposition of Ornstein-Uhlenbeck processes: A pathway to non-autonomous stochastic differential equations and to fractional diffusion. Fract. Calc. Appl. Anal., 21, 2018, 1420-1435.

[2] Molina-García, D., Pham, T. Minh, Paradisi ,P. Manzo, C., Pagnini, G., Fractional kinetics emerging from ergodicity breaking in random media. Phys. Rev. E, 94, 2016, 052147.

[3] Vitali, S., Sposini, V., Sliusarenko, O., Paradisi, P., Castellani, G., Pagnini, G., Langevin equation in complex media and anomalous diffusion. J. R. Soc. Interface, 15, 2018, 20180282.

[4] Sliusarenko, O., Vitali, S., Sposini, V., Paradisi, P., Chechkin, A., Castellani, G., Pagnini, G., Finite energy Lévy-type motion through heterogeneous ensemble of Brownian particles. J. Phys. A: Math. Theor., 52, 2019, 095601.

Discussions and end of Day 2

Saturday 28 September 2019

10:00-11:00 Virginia Kiryakova (Institute of Mathematics and Informatics, Bulgarian Academy of Sciences)

Special functions of fractional calculus and fractional order models

Abstract: The developments in theoretical and applied science require knowledge of the properties of mathematical functions, from elementary trigonometric functions to the multitude of Special Functions (SF). These functions appear whenever natural phenomena are studied, engineering problems are formulated, and numerical simulations are performed. They also crop up in probability theory and statistics, financial models, and economic analysis. This survey talk aims to attract attention to classes of SF that were not so popular (or some of them not introduced) until Fractional Calculus (FC) gained the important role, related to the boom of applications of fractional order models. The so-called Special Functions of Fractional Calculus (SF of FC) are basically Fox H-functions, among them - the generalised Wright hypergeometric functions, in particular the Mittag-Leffler type functions and various extensions from the important class of the multi-index Mittag-Leffler functions. The SF of FC are unavoidable tool in solutions of fractional order ODEs (as fractional relaxation-oscillation equations) and PDEs (as fractional wave-diffusion equations), of stochastic fractional differential equations, in control systems of fractional order, quantum mechanics, etc. They appear also as kernel-functions of the Generalised Fractional Calculus (GFC) operators; and as pdf-, cdf- or expectations in probability. Some of our basic results on SF of FC are briefly discussed, as follows: (i) all of them can be represented as GFC operators of basic elementary functions (among them the pdf functions of some probability distributions like gamma-, beta-, etc.); (ii) classification and basic properties; (iii) evaluation of FC and GFC operators of these SF in the general case; (iv) numerous examples. For their extended list and details, see the references below authored by V. Kyriakova.

References:

[1] Generalized Fractional Calculus and Applications, Longman-J. Wiley, Harlow-N. York, 1994.
[2] All the special functions are fractional differintegrals of elementary functions. J. Phys. A: Math. & General 30 (1997), 5085-5103, doi:10.1088/0305-4470/30/14/019.
[3] Multiple (multi-index) Mittag-Leffler functions and relations to generalized fractional calculus. J. Comput. Appl. Math. 118 (2000), 241-259, 2000, doi:10.1016/S0377-0427(00)00292-2.
[4] The special functions of fractional calculus as generalised fractional calculus operators of some basic functions. Computers and Math. with Appl. 59 (2010), 1128-1141, doi:10.1016/j.camwa.2009.05.014.
[5] The multi-index Mittag-Leffler functions as an important class of special functions of fractional calculus. Computers and Math. with Appl. 59 (2010), 1885-1895, doi:10.1016/j.camwa.2009.08.025.
[6] Fractional calculus operators of special functions-The result is well predictable, Chaos Solitons and Fractals 102 (2017), 2-15, doi: 10.1016/j.chaos.2017.03.006.

11:00-11:30 *Morning coffee break*

11:30-12:30 Conference Closure: Vassili Kolokoltsov (University of Warwick)

Probabilistic interpretation and probabilistic solutions for generalized fractional PDEs

Abstract: We develop a unified approach (or in fact several equivalent ones, with probabilistic or analytic essentials to choose from) for introducing generalised fractional operators and solving the related PDEs. In particular, we show how the main standard classes of operators and equations (Hadamard, Erdeyi-Kober, Caputo-Dzerbashian, etc.) fit to the scheme. The approach leads to many fruitful extensions and applications.

12:30-14:00 **Lunch**

Discussions and end of Day 3