Advances in last passage percolation

Titles and abstracts

Monday 24 June 2019

10:30-11:00 *Registration and morning coffee*
11:00-11:10 *Welcome*

11:15-12:15 Timo Seppäläinen (University of Wisconsin-Madison)

Geometry of the corner growth model

Abstract: The corner growth model is a last-passage percolation model of random growth on the square lattice. It lies at the nexus of several branches of mathematics: probability, statistical physics, queueing theory, combinatorics, and integrable systems. It has been studied intensely for 40 years. We review properties of the geodesics, Busemann functions and competition interfaces of the corner growth model and present new qualitative and quantitative results on the overall geodesic picture and the joint distributions of the Busemann functions. Based on collaborations with Louis Fan (Indiana), Firas Rassoul-Agha and Chris Janjigian (Utah).

12:30-14:00 LUNCH

14:15-15:15 Ofer Busani (University of Bristol)

Transversal Fluctuations in Last Passage Percolation

Abstract: In Last Passage Percolation(LPP) we assign i.i.d Exponential weights on the lattice points of the 1st quadrant of $\mathbb{Z}^2$. We then look for the up-right path going from $(0,0)$ to (n,n) that collects the most weights along the way. One is then often interested in questions regarding (1) the total weight collected along the maximal path, and (2) the behaviour of the maximal path. It is known that this path's fluctuations around the diagonal is of order $n^{2/3}$. The proof, however, is only given in the context of integrable probability theory where one relies on some algebraic properties satisfied by the Exponential Distribution. We give a probabilistic proof for this phenomenon where the main novelty is the probabilistic proof for the lower bound. Joint work with Márton Bálazs and Timo Seppäläinen.

15:20-15:40 *Afternoon coffee break*

15:45-16:45 Janosch Ortmann (UQAM - Université du Québec á Montréal)

The directed landscape

Abstract: The conjectured limit of last passage percolation is a scale-invariant, independent, stationary increment process with respect to metric composition. In joint work with Duncan Dauvergne and Bálint Virág (Toronto), we were able to prove this for Brownian last passage percolation. In particular, it follows that last passage values along any space-like curve converge uniformly to an Airy process. In this talk, I will describe the construction of the limiting object, called the directed landscape, and explain how it arises as the full limit of last passage percolation across independent Brownian motions.

End of Day 1

Tuesday 25 June 2019.

10:30-11:00 *Morning coffee*

11:00-12:00 Nikos Zygouras (University of Warwick)

The two-dimensional KPZ and other marginally relevant disordered systems.

Abstract: In joint works with Francesco Caravenna and Rongfeng Sun we have initiated a program of studying scaling limits of disordered systems, where disorder has a "*marginally relevant*" effect. In the language of stochastic analysis and renormalisation theory this corresponds to studying randomly perturbed models at the "*critical dimension*". One such model is the two-dimensional KPZ equation. A consequence of our previous works has been that the two-dimensional KPZ with the noise mollified in space on scale ε and scaled as $\hat{\beta}/\sqrt{|\log \varepsilon|}$ undergoes a phase transition with an explicit critical point $\hat{\beta}_c = \sqrt{2\pi}$. In a more recent work we show that the solution to the mollified and renormalised equation has a unique limit in the entire subcritical regime $\hat{\beta} \in (0, \hat{\beta}_c)$, which we have identified as the solution to an additive Stochastic Heat Equation, establishing so-called Edwards-Wilkinson fluctuations.

12:15-14:00 LUNCH

14:15-15:15 Elia Bisi (University College Dublin)

Transition between characters of classical groups, decomposition of Gelfand-Tsetlin patterns and last passage percolation.

Abstract: We introduce two families of symmetric polynomials that interpolate between irreducible characters of $\mathrm{Sp}_{2n}(\mathbb{C})$ and $\mathrm{SO}_{2n+1}(\mathbb{C})$ and between irreducible characters of $\mathrm{SO}_{2n}(\mathbb{C})$ and $\mathrm{SO}_{2n+1}(\mathbb{C})$.

We then study the last passage percolation model with various symmetries via a number of identities that involve orthogonal/symplectic characters and our interpolating polynomials, thus going beyond the link with classical Schur polynomials originally found by Baik and Rains. We achieve this by applying the Robinson-Schensted-Knuth correspondence to triangular arrays and using a decomposition procedure for Gelfand-Tsetlin patterns.

As an application, we provide an explanation of why the Tracy-Widom GOE and GSE distributions from random matrix theory admit formulations in terms of both Fredholm determinants and Fredholm Pfaffians. Based on joint work with Nikos Zygouras.

15:20-15:40 *Afternoon coffee break*

15:45-16:45 Promit Ghosal (Columbia University)

KPZ equation correlation in time.

Abstract: The KPZ equation is a fundamental stochastic PDE related to modeling random growth processes, Burgers turbulence, interacting particle system, random polymers etc. It is related to another important SPDE, namely, the stochastic heat equation (SHE). In this talk, we focus on the time correlation of the solution of the KPZ equation. For instance, we investigate the correlation between $\mathcal{H}^{\mathbf{nw}}(t_1, 0)$ and $\mathcal{H}^{\mathbf{nw}}(t_2, 0)$ for $t_1 > t_2 > 0$ where $\mathcal{H}^{\mathbf{nw}}(t, 0)$ denotes the narrow wedge solution of the KPZ equation at time t and the spatial point 0. Our analysis is based on the tail probabilities of the KPZ equation and the Brownian Gibbs property of the KPZ line ensemble.

This talk will be based on a joint work with Prof. Ivan Corwin and Prof. Alan Hammond.

19:00: Conference Dinner at 'Archipelagos' restaurant.

End of Day 2

Wednesday 26 June 2019.

10:30-11:00 *Morning coffee*

11:00-12:00 Márton Bálazs (University of Bristol)

How to initialise a second class particle?

Abstract: This talk will be on interacting particle systems (but time permitting, I'll make some connections to LPP). One of the best known models in the field is the simple exclusion process where every site has 0 or 1 particles. It has long been established that under certain rescaling procedure this process converges to solutions of a deterministic nonlinear PDE (Burgers' equation). Particular types of solutions, called rarefaction fans, arise from decreasing step initial data.

Second class particles are probabilistic objects that come from coupling two interacting particle systems. They are very useful and their behaviour is highly nontrivial.

The beautiful paper of P. A. Ferrari and C. Kipnis connects the above: they proved that the second class particle of simple exclusion chooses a uniform random velocity when started in a rarefaction fan. The extremely elegant proof is based, among other ideas, on the fact that increasing the mean of a Bernoulli distribution can be done by adding or not adding 1 to the random variable.

For a long time simple exclusion was the only model with an established large scale behaviour of the second class particle in its rarefaction fan. I will explain how this is done in the Ferrari-Kipnis paper, then show how to do this for other models that allow more than one particles per site. The main issue is that most families of distributions are not as nice as Bernoulli in terms of increasing their parameter by just adding or not adding 1. To overcome this we use a signed, non-probabilistic coupling measure that nevertheless points out a canonical initial probability distribution for the second class particle. We can then use this initial distribution to greatly generalize the Ferrari-Kipnis argument. I will conclude with an example where the second class particle velocity has a mixed discrete and continuous distribution.

This is joint work with Attila László Nagy.

12:15-14:00 LUNCH

14:15-15:15 Paul Chleboun (University of Warwick)

Kinetically constrained models and growth processes

Abstract: We will discuss the relaxation and out-of-equilibrium dynamics of certain kinetically constrained models (KCMs), and related growth processes. KCMs are particle systems on integer lattices, where each vertex is labeled either 0 or 1, which evolve according to very simple rules: i) with rate one and independently for each vertex, a new value 1/0 is proposed with probability 1-q and q respectively; ii) the proposed value is accepted if and only if the current state in a neighbourhood of this vertex satisfy a certain constraint. The out-of-equilibrium dynamics of KCMs are extremely rich and display many of the key features of real glasses. We will discuss some related growth processes which display features of last passage percolation, and introduce a type of intermediate passage percolation.

This will include joint work with Alessandra Faggionato and Fabio Martinelli.

15:20-15:40 *Afternoon coffee break*

15:45-16:45 Elnur Emrah (Royal Institute of Technology, KTH)

First-order asymptotics for the exactly solvable inhomogeneous exponential corner growth model.

Abstract: We consider the exactly solvable corner growth model with independent exponentially distributed waiting times. The rates of the exponentials are given by an additively separable function of the site

coordinates. We obtain simple explicit variational formulas for the first-order asymptotics of the growth process under a decay condition on the rates. Subject to further mild conditions, we prove the existence of the limit shape, describe it explicitly and observe geometric features such as potential flat segments and spikes near the axes. We connect these results to various other models in the literature, including the TASEP with step initial conditions and disorder in the jump rates of particles and holes. The standard subadditive arguments do not apply in our setting, so our methodology is based instead on concentration bounds and estimating the boundary exit probabilities of the geodesics in the increment-stationary version of the model. Our methods are primarily probabilistic, with the only input from integrable probability being the distributional invariance of the last-passage times under permutations of the parameter sequences. Joint with C. Janjigian and T. Seppäläinen.

End of Day 3