Project: Probabilistic and combinatorial analysis of coalescence

Imagine that you start with n dots on a paper. Each time you pick two dots randomly and connect them. This simple procedure gives rise to a sequence of growing graphs. And if you look at the connected components there (blocs of dots connecting to each other), they will coalesce at a stochastic rate until eventually every dot is absorbed into one big component. It is not difficult to check that the coalescent rate is proportional to $x \cdot y$, where x, y are the respective sizes of the two blocs which have been merged. A system of particles which coalesce at such a rate is called the multiplicative coalescent. And the above random graph process is a combinatorial construction of the multiplicative coalescent. One can consider other merging rules, which will lead to other coalescence process, including the Kingman coalescent, additive coalescent, etc.

The various combinatorial constructions allow us to explore the probabilistic theory of coalescence through its many connections with random graphs, branching processes, or Brownian motions. There is also a rich literature on the applications of the coalescent models. In particular, the Kingman coalescent is ubiquitous in the study of population genetics.

Recommended modules: Probability Models, Random Processes.
Keywords: coalescent, random graph, branching process

References

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