Fermions in a Soft Wall Extra Dimension

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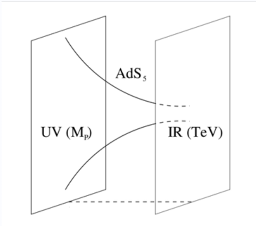
1 Abstract

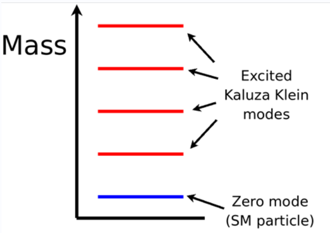
Over the last ten years, there has been a huge increase in the study of warped extra dimensional models which have been used to provide elegant solutions to some of the problems of the Standard Model. For example, allowing fermions to propagate in the extra dimension can give rise to the fermion mass hierarchy. An interesting variation on the original warped extra dimension is the "soft wall" model where the extra dimension has a smooth space-time cutoff. We will study the behaviour of fermions in such a soft wall background, in particular we see if it is possible to produce the fermion mass hierarchy without introducing dangerous flavour changing processes.

2 Hard Wall Extra Dimension

In 1999, Randall and Sundrum came up with a novel solution to the hierarchy problem [1], explaining why the weak force is so much stronger than gravity. Their model proposed the existence of a small warped extra dimension bounded by two "hard wall" branes. The strongly warped anti-de Sitter space means that whilst gravity is weak at the IR brane (where we live) it is strong at the UV brane hence the hierarchy is simply produced by the geometry of the extra dimension.



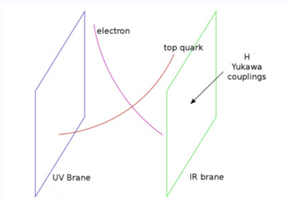
Warped extra dimension – gravity is strong at the UV brane but weakens towards the IR. *Source – arXiv: hep-ph/0601213v1.*



Particles propagating in a compactified extra dimension form a tower of massive modes.

When particles are allowed to propagate in a compactified extra dimension they form a tower of quantized excited states known as Kaluza Klein (KK) modes. From a 4D point of view the excited states will appear as more massive particles. We identify the zero modes with the Standard Model (SM) particles and the excited states as unique signatures of extra dimensions to be looked for at future collider experiments.

It has also been shown [2, 3, 4] that by placing the SM fermions in the extra dimension, the fermion mass hierarchy can be neatly explained. Fermion masses are generated by Yukawa couplings to the Higgs field which is localized near the IR brane. A large hierarchy in the fermion masses can then be produced by altering the location of the fermion wave-functions in the extra dimension, overlapping more or less with the Higgs field.



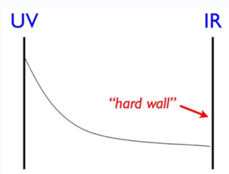
Fermion mass hierarchy explained by the amount of overlap fermion wave functions have with the Higgs.

3 Soft Wall Extra Dimension

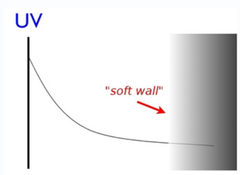
Inspired by work from AdS/CFT, the warped extra dimension models have been extended by removing the IR brane and replacing it with a smooth space-time cutoff, known as a "soft wall" [5]. With *z* being the coordinate in the extra dimension, the soft wall model is realised via the action:

$$S = \int d^5x \sqrt{|g|}\, e^{-\Phi(z)} \mathcal{L},$$

where $\mathcal{L}$ is the matter field Lagrangian, and $\Phi(z)$ is the "dilaton" which mediates the strength of the soft wall.



Hard Wall: in the original set up, the extra dimension is bounded by two "hard wall" branes.



Soft Wall: the IR brane is removed and the extra dimension extends to infinity but is dynamically cut off by a dilaton field.

4 Soft Wall Phenomonology

We study power-law behaviour for the dilaton field

$$\Phi(z) = (\mu z)^{\nu},$$

where μ is a mass scale and ν is a dynamical parameter which alters the effective "strength" of the soft wall. For different values of ν different Kaluza Klein mass spectra are produced, the masses increase with mode number *n* as:

Hard Wall:

$$m_n^2 \sim n^2$$

Soft Wall:

$\nu \leq 1$	*Continuous mass spectra*
$\nu = 2$	$m_n^2 \sim \mu^2 n$
$\nu \to \infty$	$m_n^2 \sim \mu^2 n^2$

For large ν, we recover the hard wall spectrum.

5 Fermions in a Soft Wall Extra Dimension

Whilst analysis of bosons in the soft wall background is relatively straightforward, fermions present a much more difficult problem. The usual method of analysis in hard wall models involves finding the wave-functions of free particles propagating in the extra dimension and then treating gauge and Yukawa couplings as perturbations.

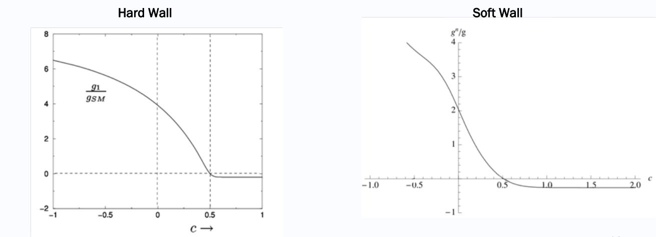
Unfortunately, unlike bosons, in the soft wall background fermions do not directly feel the effect of the dilaton and as free particles form a continuous KK spectrum. It is only through their interaction via Yukawa couplings with the Higgs that we get a discrete, normalisable mode spectrum for fermions. Thus a perturbative approach is not possible.

Due to the complicated nature of the interactions, an analytic solution is not generally possible for multiple generations of fermions, and a numerical approach will need to be developed. However for a simplified single generational model, an analytic solution is possible which reproduces some of the nice features of hard wall models:

- Able to produce a large mass hierarchy
- Electroweak constraints less stringent than for hard wall models

Electroweak constraints:

Due to the presence of extra Kaluza Klein states, gauge couplings between the excited states could produce dangerous flavour changing processes. These processes place important constraints on the model. It has been shown in hard wall models that due to the near universal nature of the gauge couplings in the extra dimension, these processes are quite heavily suppressed and experimental constraints are satisfied with a Kaluza Klein scale of about 10 TeV which still offers a good solution to the hierarchy problem [6].



Z boson gauge coupling of the first KK state relative to its SM value as a function of the fermion location (c < ½ localised near IR brane, c > ½ localised near UV) in both the hard wall (left [6]) and soft wall (right [5]).

The plots above show the gauge coupling of the first excited state of the Z boson as a function of fermion location in the extra dimension. The coupling in the soft wall does not get as strong as in the hard wall providing even less stringent constraints than the hard wall model!

The above analysis has still only been carried out in the soft wall case for a simplified single fermion generation model. It remains to be seen whether the same features will apply for a full three generation model. We are currently developing a numerical solution to this problem.

References

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