Gauge-Yukawa theories at large N_F

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Outline

- Motivation
- Large N_F expansion in a nutshell
- Calculations of the resummed Yukawa beta function
- Large N_F beta functions - state of the art
- Asymptotically Safe model building with large N_F
- Conclusions

Perturbative asymptotic safety

D.F.Litim, F. Sannino, JHEP 1412, 178 (2014)

Gauge-Yukawa theory with $G = SU(N_C)$, N_F fermions and N_F^2 scalars

$$\alpha_{g,y} = \frac{g(y)^2 N_C}{(4\pi)^2}$$

$$\beta_g = \frac{d\alpha_g}{d\ln\mu} = \alpha_g^2(-B + C\alpha_g - D\alpha_y)$$

$$\beta_y = \frac{d\alpha_y}{d\ln\mu} = \alpha_y(E\alpha_y - F\alpha_g)$$

UV interacting fixed point

$$\text{parameter } \epsilon = \frac{N_F}{N_C} - \frac{11}{2} \quad \longrightarrow \quad B = -\frac{4}{3}\epsilon$$

$$\text{Veneziano limit: } N_F, N_C \to \infty, \ \frac{N_F}{N_C} \to \text{const.} \quad \longrightarrow \quad \epsilon \ll 1$$

UV GY fixed point is perturbatively stable

Scenarios with large N_F

To employ this idea as a model building tool we need finite N_C

large N_F required

- $SU(3) \times SU(2)$ — *A.Bond, G.Hiller, KK, D.Litim, JHEP 1708, 004 (2017)*
- $U(1)$ — *A.Bond, G.Hiller, KK, D.Litim, PoS EPS-HEP2017 (2017) 542*

perturbative stability in question — *D.Barducci, M. Fabbrichesi, C.M.Nieto, R.Percacci, V.Skrinjar, arXiv:1807.05584*

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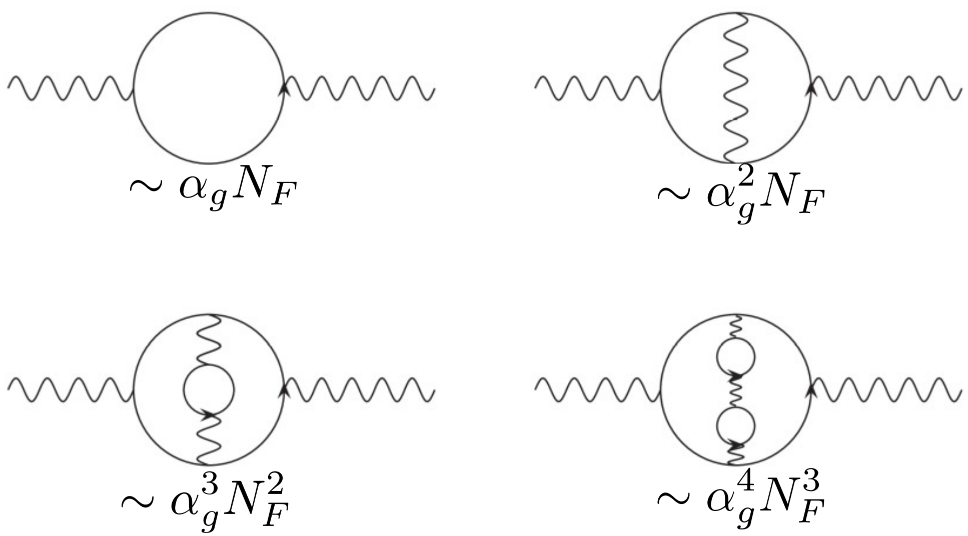


$$\alpha_g N_F \sim 1$$

↓

perturbative expansion in α_g breaks down

$$\beta_g = \alpha_g^2 \left(B N_F + C \alpha_g N_F + D \alpha_g^2 N_F^2 + \cdots \right)$$

Large N_F expansion in a nutshell

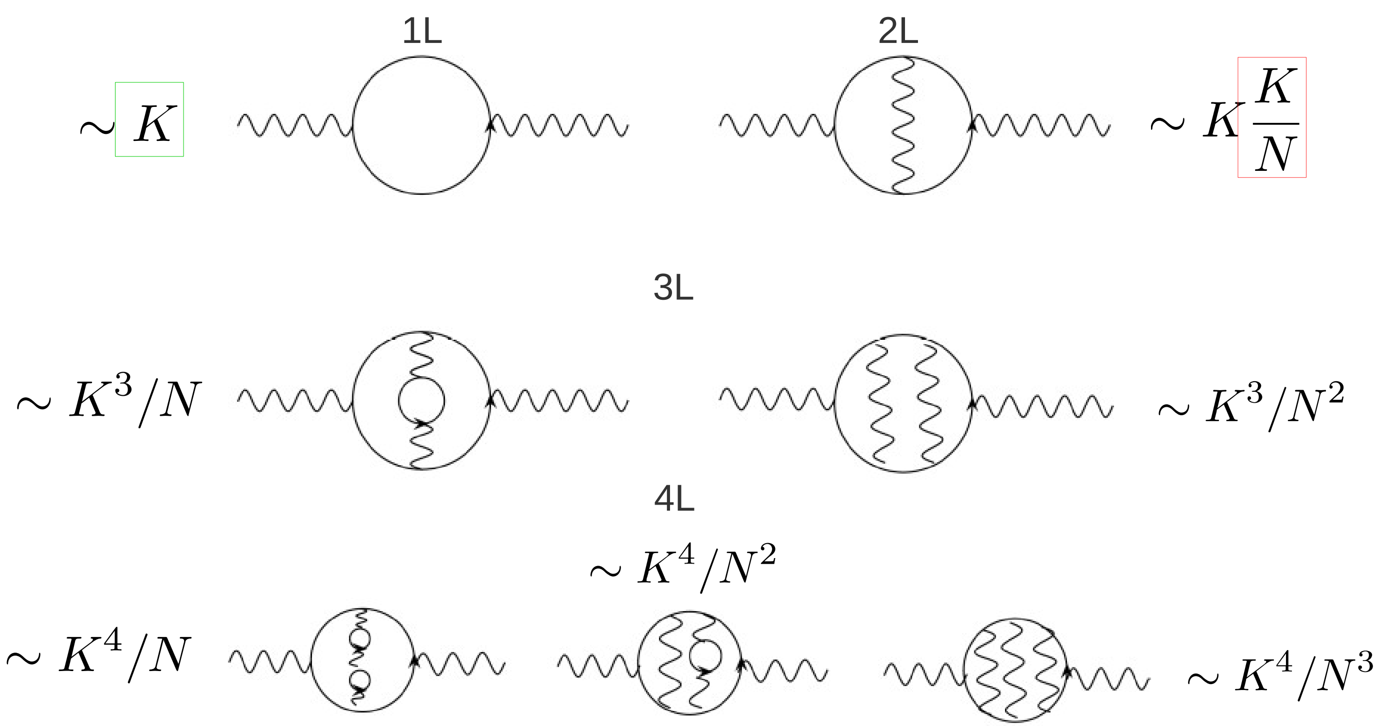
Let us reorganize the perturbation series in $1/N_F$

QED: A. Palanques-Mestre and P. Pascual, *Commun. Math. Phys.* 95 (1984) 277
QCD: J.A.Gracey, *Phys. Lett. B737 (1996) 178-184*
B.Holdom, Phys. Lett. B694 (2011) 74-79

t'Hooft coupling: $K = \alpha_g N_F/\pi$

$$\beta(K) \equiv \frac{d\log K}{d\log Q} = \frac{2}{3}K\left(1 + \sum_{i=1}^{\infty}\frac{F_i(K)}{N_F^i}\right)$$

Bubble counting:



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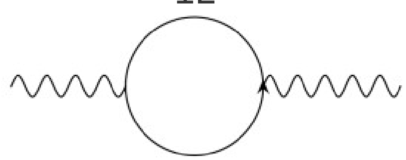
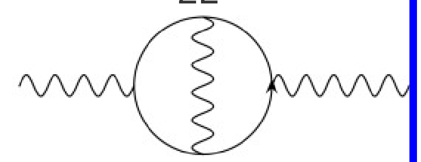
Bubble counting:

1L

$\sim K$



2L

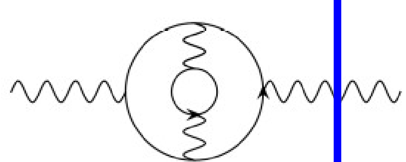


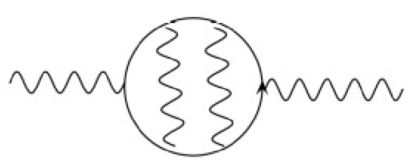
$\sim K\frac{K}{N}$

For large N_F $F_1(K)$ dominates

3L

$\sim K^3/N$

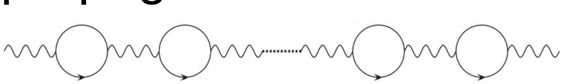




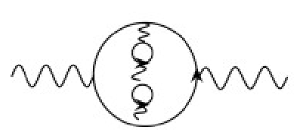
$\sim K^3/N^2$

4L

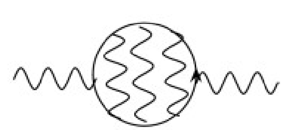
Replace gauge propagator with



$\sim K^4/N$

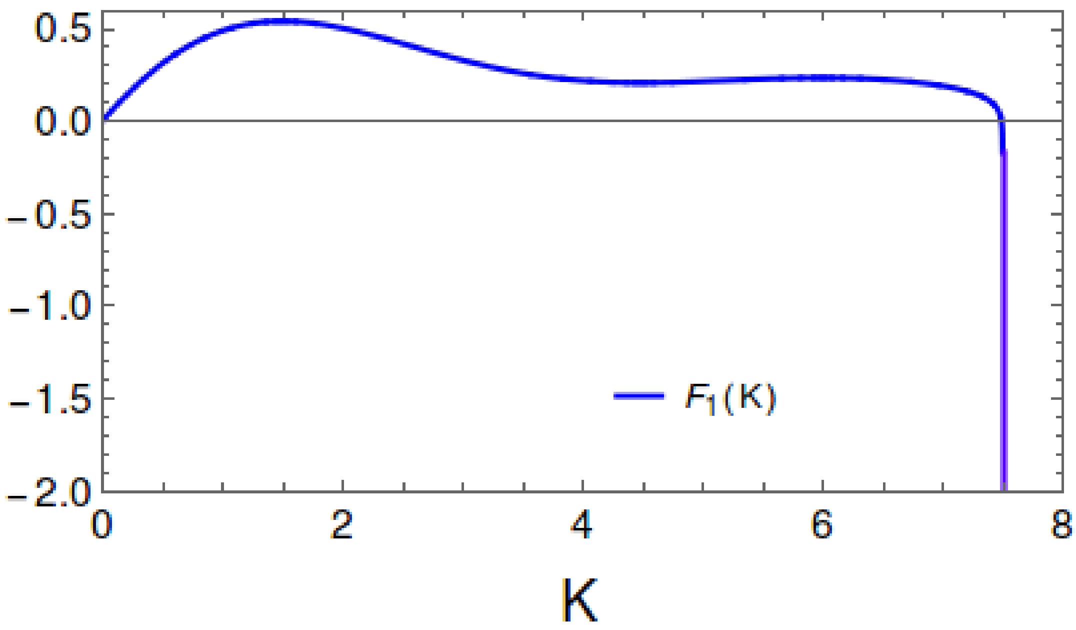


$\sim K^4/N^2$



$\sim K^4/N^3$

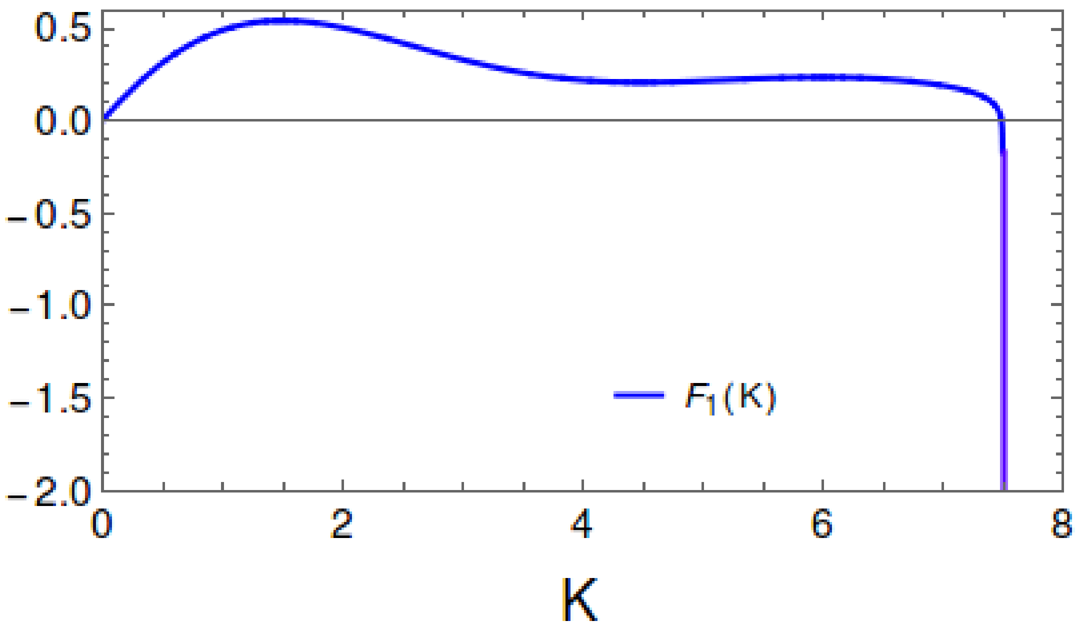
Large N_F expansion in a nutshell

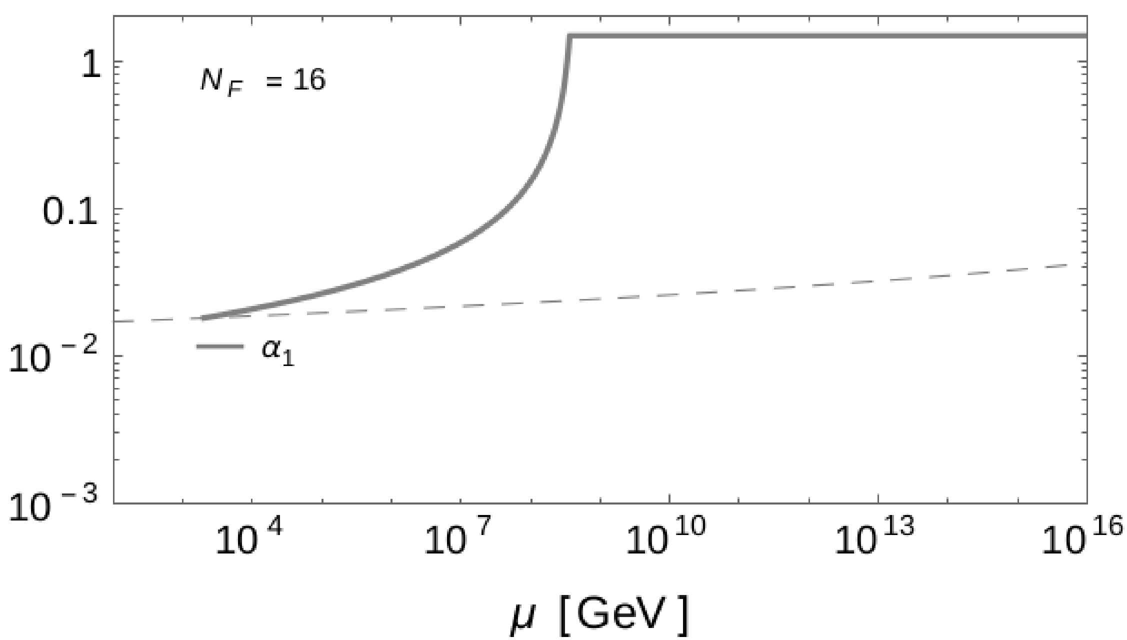


- there exists a **final analytical limit** of the infinite series
- $F_1(K)$ has a pole (in QED)

$$K = \frac{15}{2}$$

Large N_F expansion in a nutshell





- there exists a **final analytical limit** of the infinite series
- $F_1(K)$ has a pole (in QED)

$$K = \frac{15}{2}$$

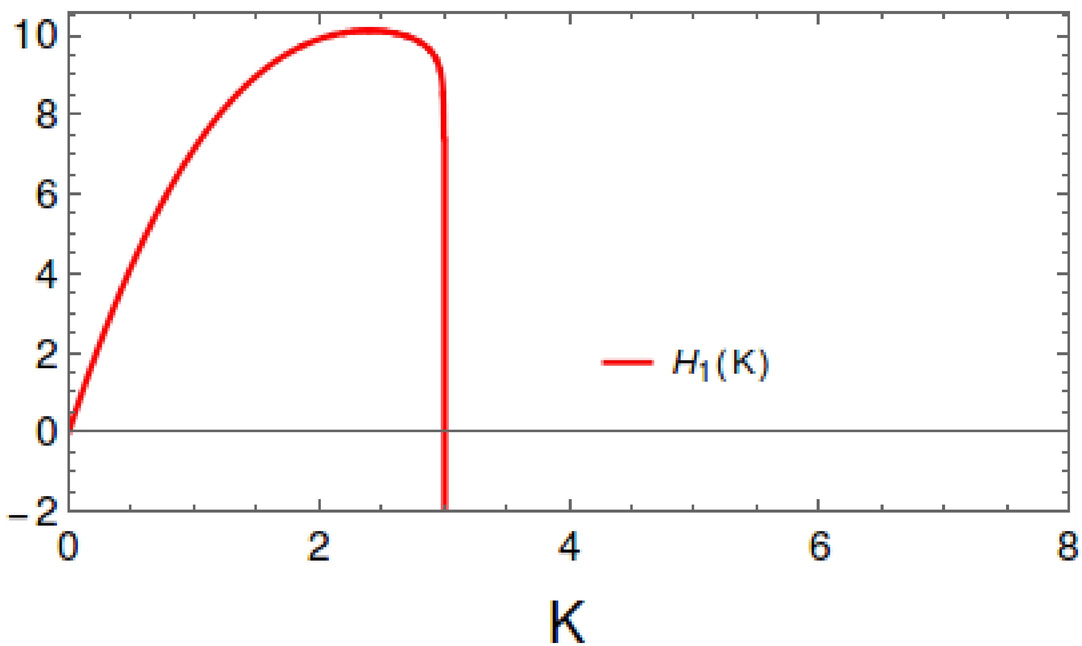
$$\beta(K) = \frac{2}{3}K\left(1 + \frac{F_1(K)}{N_F}\right)$$

Holdom, 2010
Mann et all. 2017

- gauge coupling develops an **interacting UV fixed-point**

$$\alpha_g^* = \frac{15\pi}{2N_F}$$

Large N_F expansion in a nutshell



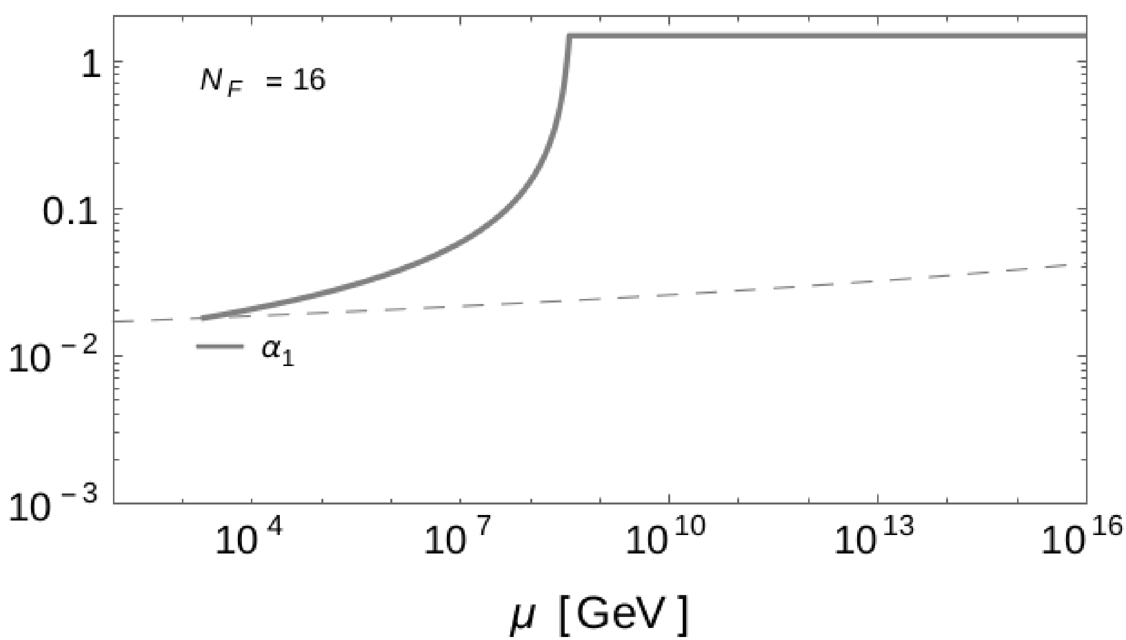
- there exists a **final analytical limit** of the infinite series
- $F_1(K)$ has a pole (in QED)

$$K = \frac{15}{2}$$

***K* = 3 for QCD**

$$\beta(K) = \frac{2}{3}K\left(1 + \frac{F_1(K)}{N_F}\right)$$

Holdom, 2010
Mann et all. 2017

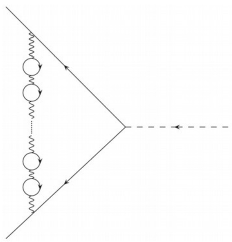


- gauge coupling develops an **interacting UV fixed-point**

$$\alpha_g^* = \frac{15\pi}{2N_F}$$

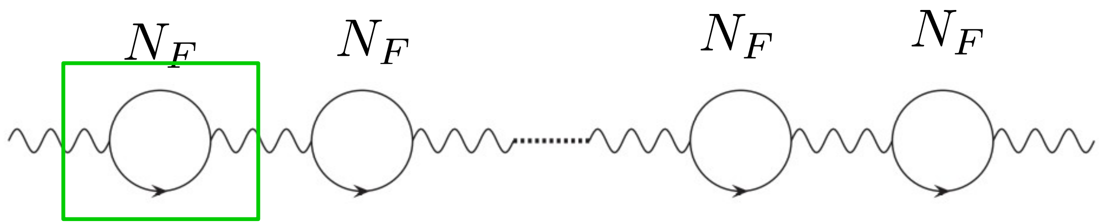
Resummation for the Yukawa coupling

KK, E.M. Sessolo,
JHEP 1804 (2018) 027



$$\beta_y(K) \equiv \frac{d\ln\alpha_y}{d\ln\mu} = \sum_{i=1}^{\infty} \frac{Y_i(K)}{N_F^i}$$

Step 1: Resum the gauge propagator: $\Delta_{B\,\mu\nu} = \frac{g_{\mu\nu}}{k^2 - i\epsilon}$



$$i\Pi^{\rho\sigma}(k) = i\left(g^{\rho\sigma}k^2 - k^\rho k^\sigma\right)\Pi(k^2)$$

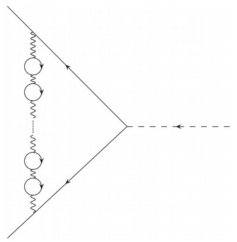
$$\Pi(k^2) = -2K\left(-\frac{4\pi\mu^2}{k^2}\right)^{\frac{\epsilon}{2}} \frac{\Gamma\left(\frac{\epsilon}{2}\right)\Gamma\left(2-\frac{\epsilon}{2}\right)^2}{\Gamma\left(4-\epsilon\right)}$$

$$\Delta'_{\mu\nu} = \Delta_{B\,\mu\nu} + \Delta_{B\,\mu\rho}\Pi^{\rho\sigma}(k)\Delta_{B\,\sigma\nu} + \ldots$$

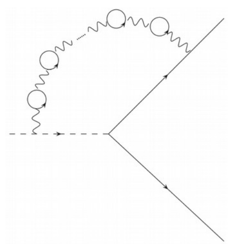
$$\Delta'_{\mu\nu} = \frac{g_{\mu\nu}}{(k^2 - i\epsilon)\left[1 - \Pi(k^2)\right]} = \frac{g_{\mu\nu}}{k^2}\sum_{n=0}^{\infty}\Pi^n(k^2)$$

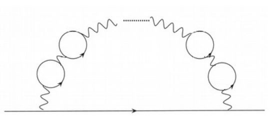
Resummation for the Yukawa coupling

Step 2: Calculate resummed renormalization constants

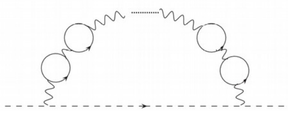


$$Z_1 - 1 = -\Lambda_Y(p, p')\Big|_{\text{poles in } \epsilon}$$





$$Z_{2\,L,R} - 1 = \frac{d}{dp}\Sigma_2(p)_{L,R}\Big|_{\text{poles in } \epsilon}$$



$$Z_\phi - 1 = \frac{d}{dp^2}S(p^2)\Big|_{\text{poles in } \epsilon}$$

Resummation for the Yukawa coupling

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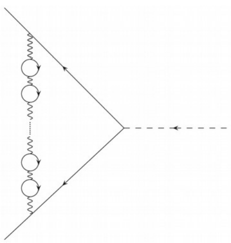
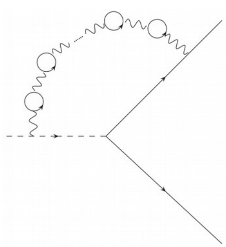


$$Z_1 - 1 = -\Lambda_Y(p,p')\Big|_{\text{poles in }\epsilon}$$

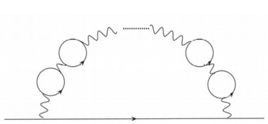
$$\Lambda_Y(p,p') = \sum_{n=0}^{\infty} \Lambda_Y^{(n)}(p,p')$$

$$\Lambda_Y^{(n)}(p,p') = (-ig_{1,0})^2 q_L q_R \mu^\epsilon \boxed{\Pi^n(0)} \int \frac{d^dk}{(2\pi)^d}\gamma_\nu \frac{i(p-k+m_0)}{(p-k)^2-m_0^2} y \frac{i(p'-k+m_0)}{(p'-k)^2-m_0^2}\gamma^\nu \boxed{\frac{-i\left(\mu^2\right)^{\frac{n\epsilon}{2}}}{\left(k^2\right)^{1+\frac{n\epsilon}{2}}}}$$

insert resummed propagator
into 1-loop diagram



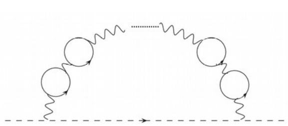
$$+q_S^2(-ig_{1,0})^2 \mu^\epsilon \Pi^n(0) \int \frac{d^dk}{(2\pi)^d}\gamma_\nu \frac{i(p'-k+m_0)}{(p'-k)^2-m_0^2} y \frac{i(2l-k)^\nu}{(l-k)^2-M_0^2} \frac{-i\left(\mu^2\right)^{\frac{n\epsilon}{2}}}{\left(k^2\right)^{1+\frac{n\epsilon}{2}}}$$



$$Z_{2\,L,R} - 1 = \frac{d}{dp}\Sigma_2(p)_{L,R}\Big|_{\text{poles in }\epsilon}$$

$$-i\Sigma_2(p)_{L,R} = \sum_{n=0}^{\infty}\left[-i\Sigma_2^{(n)}(p)_{L,R}\right]$$

$$-i\Sigma_2^{(n)}(p)_{L,R} = (-ig_{1,0}\,q_{L,R})^2 \mu^\epsilon \Pi^n(0) \int \frac{d^dk}{(2\pi)^d}\gamma_\mu \frac{i(p-k+m_0)}{(p-k)^2-m_0^2}\gamma^\mu \frac{-i\left(\mu^2\right)^{\frac{n\epsilon}{2}}}{\left(k^2\right)^{1+\frac{n\epsilon}{2}}}$$



$$Z_\phi - 1 = \frac{d}{dp^2}S(p^2)\Big|_{\text{poles in }\epsilon}$$

$$-iS(p^2) = \sum_{n=0}^{\infty}\left[-iS^{(n)}(p^2)\right]$$

$$-iS^{(n)}(p^2) = (-ig_{1,0}\,q_S)^2 \mu^\epsilon \Pi^n(0) \int \frac{d^dk}{(2\pi)^d} \frac{i(2p-k)_\mu(2p-k)^\mu}{(p-k)^2-M_0^2} \frac{-i\left(\mu^2\right)^{\frac{n\epsilon}{2}}}{\left(k^2\right)^{1+\frac{n\epsilon}{2}}}$$

Resummation for the Yukawa coupling

Step 3: Use resummation formula

$$\frac{d\Sigma_2(p)_{L,R}}{dp} = \frac{3}{4N_F}\frac{q^2_{L,R}}{q^2}\sum_{n=1}^{\infty}\left(-\frac{2}{3}K_0\right)^n \frac{1}{n\epsilon^n} H_2(n,\epsilon)$$

pole extraction

$$K_0^n = Z_3^{-n}K^n = \left(1-\frac{2}{3}\frac{K}{\epsilon}\right)^{-n} K^n = K^n \sum_{i=0}^{\infty}\binom{-n}{i}\left(-\frac{2}{3}\frac{K}{\epsilon}\right)^i$$

renormalization of the gauge coupling

$$H_2(n,\epsilon) = \sum_{j=0}^{\infty} H_2^{(j)}(\epsilon)(n\epsilon)^j\,, \qquad H_2^{(0)}(\epsilon) = \sum_{i=0}^{\infty}\widetilde{H}_2^{(i)}\epsilon^i\,, \qquad \widetilde{H}_2^{(0)} = 1$$

$$\frac{d\Sigma_2(p)_{L,R}}{dp} = \frac{3}{4N_F}\frac{q^2_{L,R}}{q^2}\sum_{n=1}^{\infty}(-1)^n\left(\frac{2K}{3}\right)^n \sum_{j=0}^{n-1}\frac{H_2^{(j)}(\epsilon)}{\epsilon^{n-j}} \sum_{i=0}^{n-1}\binom{n-1}{i}(-1)^i(n-i)^{j-1}$$

$$\sum_{i=0}^{n-1}\binom{n-1}{i}(-1)^i(n-i)^{j-1} = \begin{cases} 0, & j > 0 \\ -(-1)^n\, 1/n, & j = 0 \end{cases}$$

resummation formula (Palanques-Mestre, Pascual)

$$\frac{d\Sigma_2(p)_{L,R}}{dp} = -\frac{3}{4N_F}\frac{q^2_{L,R}}{q^2}\sum_{n=1}^{\infty}\left(\frac{2K}{3}\right)^n\frac{H_2^{(0)}(\epsilon)}{n\epsilon^n} \stackrel{\frac{1}{\epsilon}}{=} -\frac{3}{4N_F}\frac{q^2_{L,R}}{q^2}\sum_{n=1}^{\infty}\left(\frac{2K}{3}\right)^n\frac{\widetilde{H}_2^{(n-1)}}{n\epsilon}$$

Resummation for the Yukawa coupling

Step 4: Derive the beta function

$$H_2^{(0)}(\epsilon) = \sum_{i=0}^{\infty} \widetilde{H}_2^{(i)} \epsilon^i$$

$$\epsilon \frac{\partial Z_{2\,L,R}}{\partial K} = -\frac{1}{2N_F} \frac{q_{L,R}^2}{q^2} \boxed{\sum_{n=0}^{\infty} \widetilde{H}_2^{(n)} \cdot \left(\frac{2K}{3}\right)^n} = -\frac{1}{2N_F} \frac{q_{L,R}^2}{q^2} \boxed{H_2^{(0)} \left(\frac{2K}{3}\right)}$$

...and similar for other renormalization constants

$$Y_1(K) K^{-1} = -4 \frac{q_L q_R}{q^2} Y_1^{(0)} \left(\frac{2}{3} K\right) - \frac{q_S^2}{q^2} Y_2^{(0)} \left(\frac{2}{3} K\right)$$

$$\frac{1}{2} \frac{q_L^2 + q_R^2}{q^2} H_2^{(0)} \left(\frac{2}{3} K\right) - \frac{q_S^2}{q^2} H_\phi^{(0)} \left(\frac{2}{3} K\right)$$

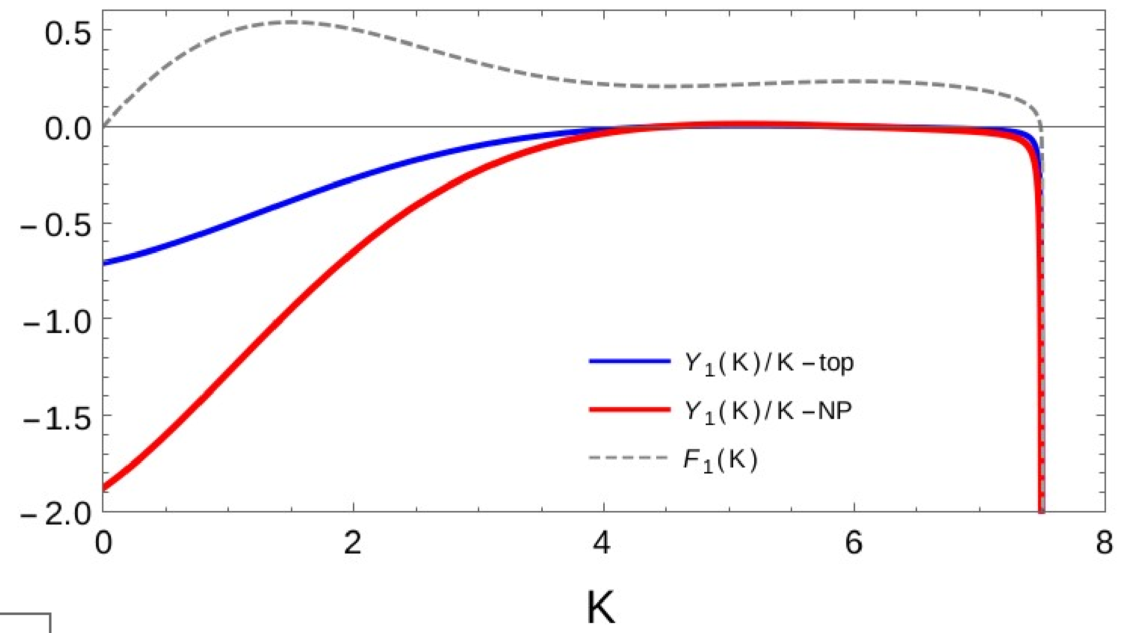
$$\beta_y(K) \equiv \frac{d \ln \alpha_y}{d \ln \mu} = \frac{Y_1(K)}{N_F}$$

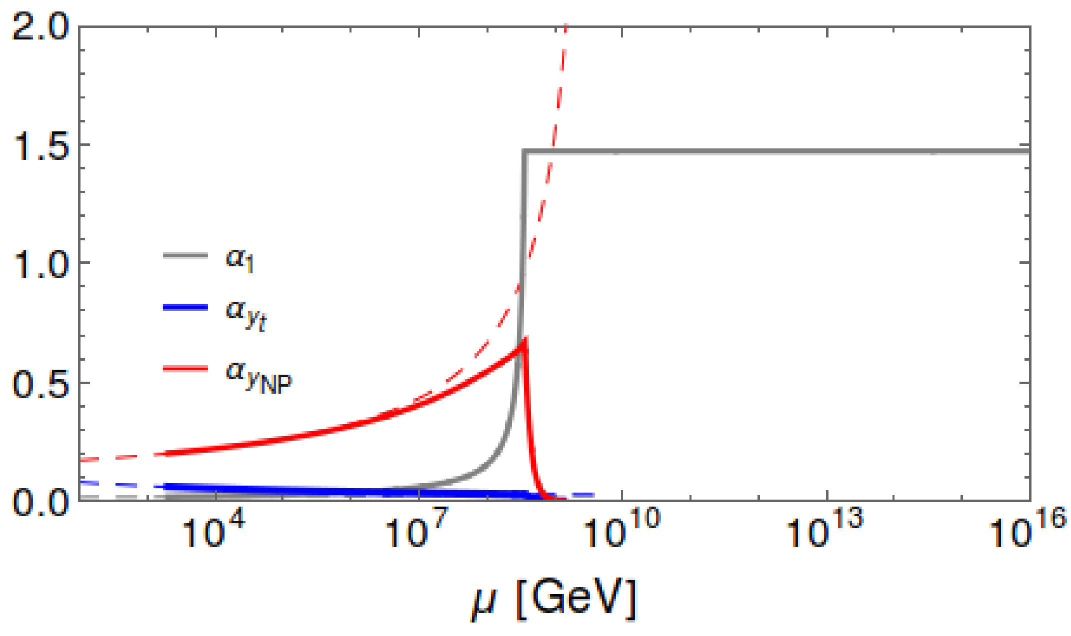
Yukawa coupling at large N_F

- there exists a **final analytical limit** of the infinite series

- $Y_1(K)/K$ has a pole

$$K = \frac{15}{2}$$



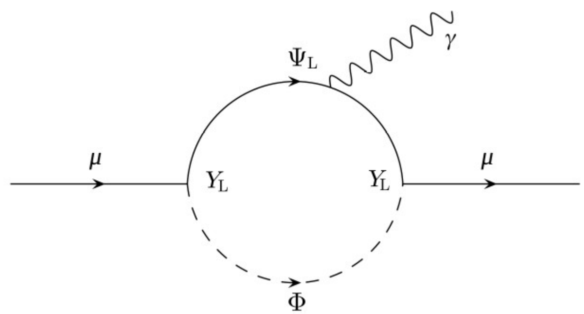


- models with large Yukawa couplings might(?) become UV complete

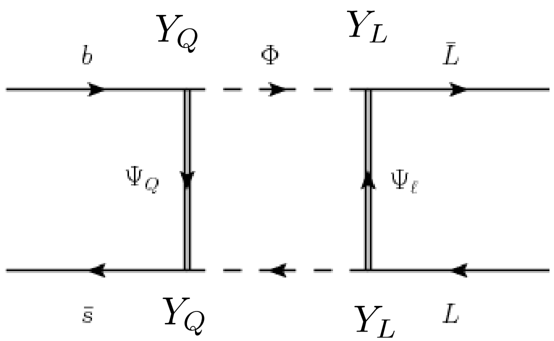
Interlude - pheno applications

Many BSM scenarios can explain anomalies in *g-2* and R_K through 1-loop contributions, ex.

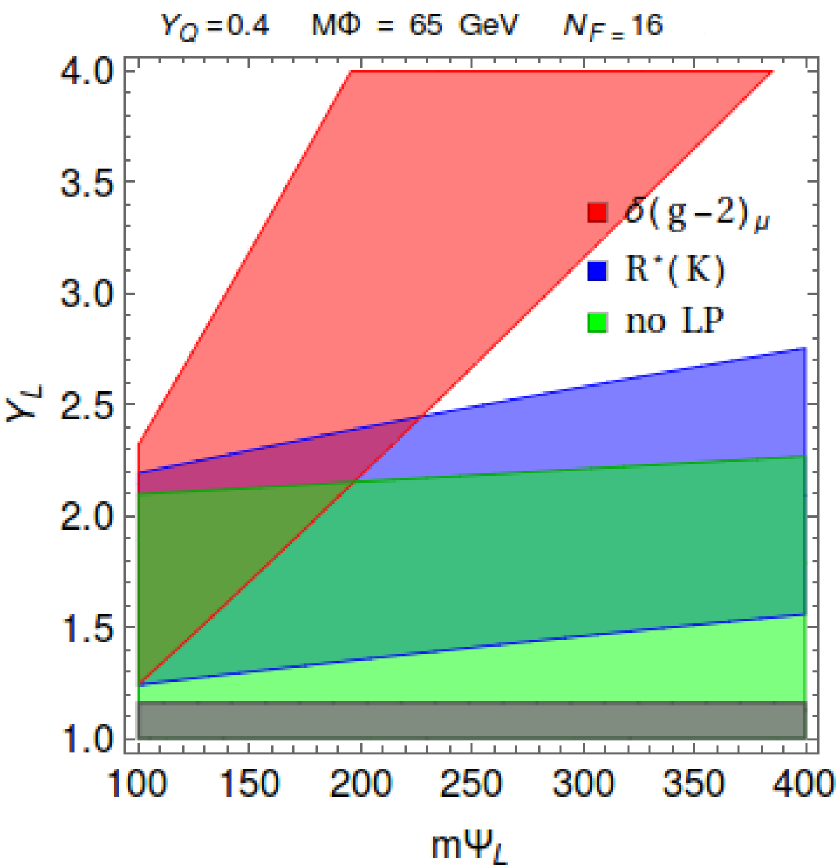
$$\psi_L(\mathbf{1},\mathbf{1},-1) \qquad \psi_Q(\mathbf{3},\mathbf{1},-1/3) \qquad \phi(\mathbf{1},\mathbf{2},-1/2)$$



KK, E.M.Sessolo,
JHEP 1709 (2017) 112



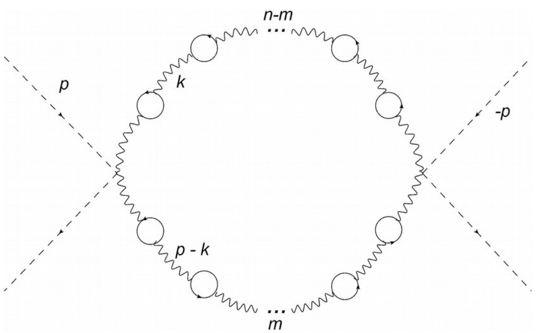
P.Arnan, L.Hofer, F.Mescia, A.Crivellin
JHEP 1704 (2014) 043



RGE in the limit of large N_F

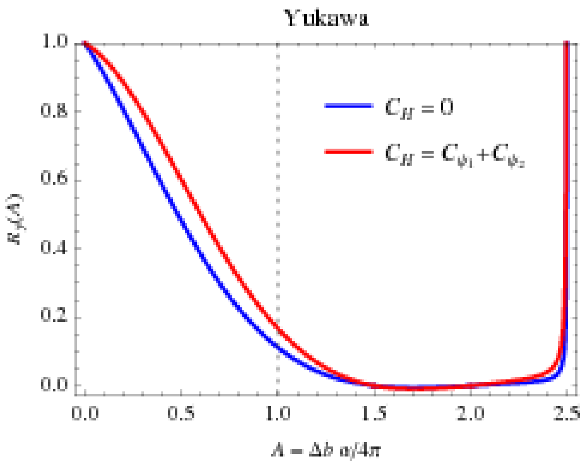
1-loop RGE for the Yukawa coupling

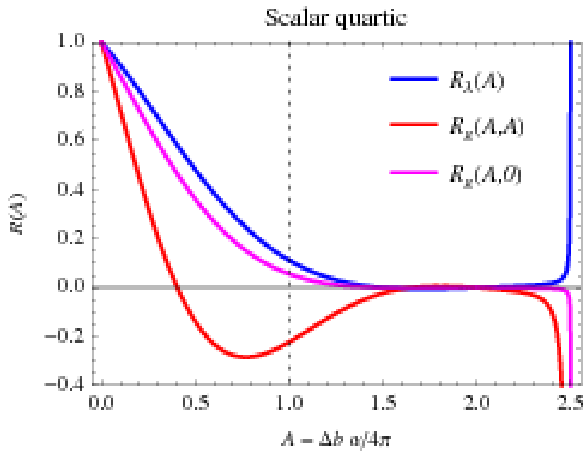
Scalar quartic coupling at large N_F



Pelaggi, Plasencia, Salvio, Sannino, Smirnov, Strumia, Phys.Rev. D97 (2018)

$$(\beta_\lambda)_{1/N} = -c_1 \lambda K R_\lambda(K) + \boxed{d_1 K^2 R_g(K)}$$





R_λ, R_g have poles at

$$\boxed{K = \frac{15}{2}}$$

Quartic coupling stays perturbative only for non-abelian gauge groups

Large N_F - state of the art

	β_g	β_y	β_λ
U(1)	$ K = 7.5	# K = 7.5	& K = 7.5
SU(N)	% K = 3	# K = 7.5	& K = 7.5
U(1) x SU(N)	K = 7.5	K = 7.5	K = 7.5
SU(N) x SU(M)	K = 3	K = 7.5	K = 7.5
Yukawa	Y = 3	Y = 5	
U(1) + Yukawa	K =7.5, Y = 3	K=7.5, Y=5	

$ *Palanques-Mestre, Pascual*, 1984;

% Gracey, 1996, *Holdom*, 2010;

Kowalska, Sessolo, *JHEP 1804 (2018) 027*;

& *Pelaggi, Plasencia, Salvio, Sannino, Smirnov, Strumia*, *Phys.Rev. D97 (2018)*

Large N_F - state of the art

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U(1)	$ K = 7.5	# K = 7.5	& K = 7.5
SU(N)	% K = 3	# K = 7.5	& K = 7.5
U(1) x SU(N)	! K = 7.5	! K = 7.5	! K = 7.5
SU(N) x SU(M)	! K = 3	! K = 7.5	! K = 7.5
Yukawa	Y = 3	Y = 5	
U(1) + Yukawa	K =7.5, Y = 3	K=7.5, Y=5	

$ *Palanques-Mestre, Pascual*, 1984;

% Gracey, 1996, *Holdom*, 2010;

Kowalska, Sessolo, *JHEP 1804 (2018) 027*;

& *Pelaggi, Plasencia, Salvio, Sannino, Smirnov, Strumia*, *Phys.Rev. D97 (2018)*

! *Antipin, Dondi, Sannino, Thomsen, Wang*, Phys.Rev. D98 (2018);

Large N_F - state of the art

	β_g	β_y	β_λ
U(1)	$ K = 7.5	# K = 7.5	& K = 7.5
SU(N)	% K = 3	# K = 7.5	& K = 7.5
U(1) x SU(N)	! K = 7.5	! K = 7.5	! K = 7.5
SU(N) x SU(M)	! K = 3	! K = 7.5	! K = 7.5
Yukawa	+ Y = 3	@ Y = 5	
U(1) + Yukawa	+ K =7.5, Y = 3	+ K=7.5, Y=5	

$ *Palanques-Mestre, Pascual*, 1984;

% Gracey, 1996, *Holdom*, 2010;

Kowalska, Sessolo, *JHEP 1804 (2018) 027*;

& *Pelaggi, Plasencia, Salvio, Sannino, Smirnov, Strumia*, *Phys.Rev. D97 (2018)*

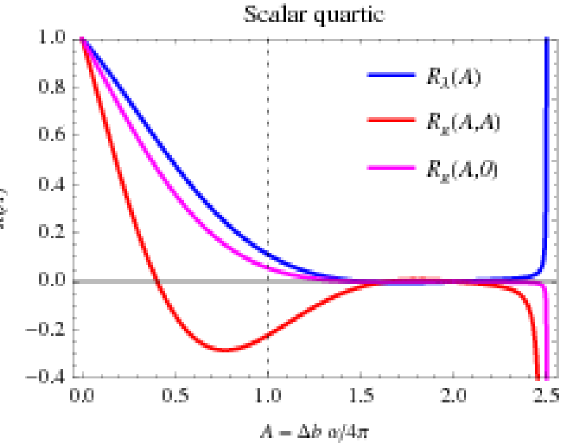
! *Antipin, Dondi, Sannino, Thomsen, Wang*, Phys.Rev. D98 (2018);

@ *Alanne, Blasi*, 1806.06954;

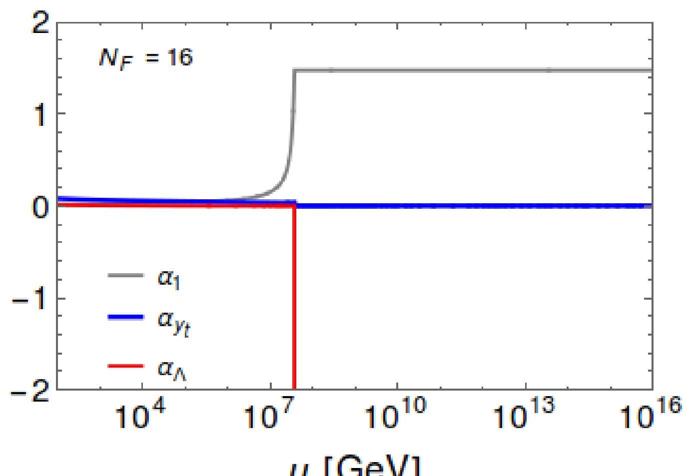
+ *Alanne, Blasi*, 1808.03252;

AS Standard Model (gauge)

	β_g	β_y	β_λ	
U(1)	K = 7.5	K = 7.5	K = 7.5	X
SU(N)	K = 3	K = 7.5	K = 7.5	
U(1) x SU(N)	K = 7.5	K = 7.5	K = 7.5	X
SU(N) x SU(M)	K = 3	K = 7.5	K = 7.5	
Yukawa	Y = 3	Y = 5		
U(1) + Yukawa	K =7.5, Y = 3	K=7.5, Y=5		



G.Pelaggi, A.Plasencia, A.Salvio, F.Sannino, J.Smirnov, A.Strumia
arXiv:1708.00437v2



No!

AS GUT extensions of the SM

	β_g	β_y	β_λ	
U(1)	K = 7.5	K = 7.5	K = 7.5	X
SU(N)	K = 3	K = 7.5	K = 7.5	v
U(1) x SU(N)	K = 7.5	K = 7.5	K = 7.5	X
SU(N) x SU(M)	K = 3	K = 7.5	K = 7.5	v
Yukawa	Y = 3	Y = 5		
U(1) + Yukawa	K =7.5, Y = 3	K=7.5, Y=5		

For example:

Pati-Salam: *SU(4) x SU(2) x SU(2)*

Molinaro, Sannino, Wang
arXiv: 1807.03669

Trinification: $SU(3)_C \times SU(3)_R \times SU(3)_L$

Wang, Al Balushi, Mann, Jiang
arXiv: 1812.11085

AS Standard Model (Yukawa)?

	β_g	β_y	β_λ	
U(1)	K = 7.5	K = 7.5	K = 7.5	X
SU(N)	K = 3	K = 7.5	K = 7.5	V
U(1) x SU(N)	K = 7.5	K = 7.5	K = 7.5	X
SU(N) x SU(M)	K = 3	K = 7.5	K = 7.5	V
Yukawa	Y = 3	Y = 5		
U(1) + Yukawa	K =7.5, Y = 3	K=7.5, Y=5		?

1-loop Higgs quartic:

$$16\pi^2\beta(\lambda) = 24\lambda^2 + \lambda(12y_t^2 - 9g_2^2 - 3g_1^2) + \boxed{\frac{3}{8}g_1^4 + \frac{9}{8}g_2^4 + \frac{3}{4}g_1^2g_2^2 - 6y_t^4}$$

The first pole must has K > 7.5

Conclusions

- Gauge and Yukawa couplings can remain asymptotically safe/free in the large-N_F limit.
- In abelian theories the scalar quartic couplings run into a pole.
- At the moment no viable AS extension of the SM in the large-N_F limit.
- More calculations needed.