1. Discrete Hammersley process

Consider a collection of independent Bernoulli random variables $\{X_v\}_{v\in\mathbb{Z}^2}$ with $\mathbb{P}(X_v = 1) = p = 1-q$ and interpret the event that $X_v = 1$ as the event of having site v as *marked*. For any rectangle $[m]\times[n] = \{1,2,...,m\}\times\{1,2,...,n\}$ we can define the random variable $L(m,n)$ that denotes the maximum possible number of marked sites that one can collect along a path from $(1,1)$ to (m,n) that is strictly increasing in both coordinates. It is possible that there is more than one optimal path, and any such path is called a 'Bernoulli longest increasing path (BLIP).'

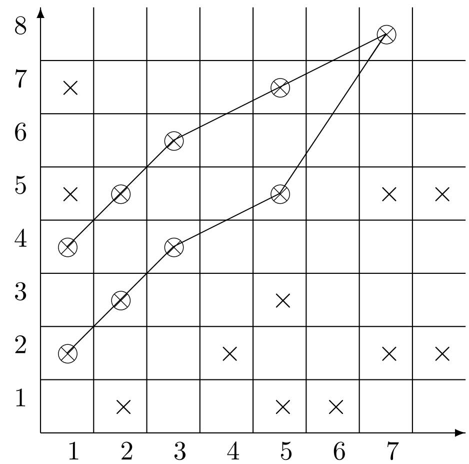

Figure 1. Two possible Bernoulli Longest Increasing paths in the rectangle $[7]\times[8]$. Bernoulli markings are denoted by $\times$. With the notation introduced, we have that $L(7,8) = 5$. A longest increasing path is $\Pi = \{(1,2),(2,3),(3,4),(5,5),(7,8)\}$.

The random variables $-L(m,n)$ satisfy a certain property, called **subadditivity**. By **Kingman's Subadditive Ergodic Theorem** one can prove $n^{-1}L(\lfloor nx\rfloor, \lfloor ny\rfloor) \to \Psi(x,y)$ a.s. and in L^1. Part of the project will be to prove the closed formula for $\Psi(x,y)$ given by

$$\Psi(x,y) = \begin{cases} x, & \text{if } x < py \\ \dfrac{2\sqrt{pxy} - p(x+y)}{q}, & \text{if } p^{-1}y \geq x \geq py \\ y, & \text{if } y < px \end{cases} \tag{1.1}$$

for all $(x,y)\in\mathbb{R}^2_+$. There is a vast literature in statistical physics that studies this model as a simplified alternative to the hard *longest common subsequence* (LCS) model (see below).

Key words: Longest increasing path, Hammersley process, totally asymmetric simple exclusion process, corner growth model, last passage percolation.

In a recent project with Federico Ciech we cast the discrete Hammersley process in the context of an invariant boundary model, and proved several results about the order of the variance of the boundary model. The article is titled " Order of the variance in the discrete Hammersley process with boundary " and it can be found here.