

## Counting and classifying

Understanding the nature of science has been a longstanding problem for human thought, a domain where mainly historians and philosophers (epistemologists) have been active. In a nutshell, science is just a language that we use to describe reality and uses a limited subset of our reasoning capabilities. The solution to the problem is based on two abilities that we share with other species on Earth: counting and classifying. We can create the whole building of theoretical science starting from these two abilities and using probability theory. Contrary to other attempts to build science from first principles, this approach is bottom-up and does not use optimisation principles. It just uses the human abilities of counting and classifying objects, as well as basic propositional logic to describe the number of objects belonging to different classes. We start from the problem of allocating  $n$  objects into  $g$  classes or categories. Specific allocations can be considered as facts (taking place or not) whereas events are propositions (true or not) about facts. Not everything in the world is known and we are often left with a set of possibility. Therefore, probability theory naturally enters in our description of the world. Classes or categories can be also seen as labels. Time is another important label representing subsequent observations of the system under scrutiny. As time passes, objects can change class (move around classes). In general, the exact laws are not known, leading to a description of movement in terms of stochastic processes. Coming back to our allocation of  $n$  objects into  $g$  categories, we can naturally describe the random time dynamics in terms of birth-death Markov chains (if memory is not infinite). In the appropriate limit, these Markov chains can be approximated by diffusions (stochastic differential equations). Finally, if in some regime, fluctuations are small enough, the time evolution of diffusions is well described by ordinary differential equations for expected values. From the problem of allocating  $n$  objects into  $g$  classes, one is led to a probabilistic description encompassing all the models used in science:

- starting from Markovian dynamics for the description of change,
- leading to Markovian diffusions in the continuous limit, and
- justifying the use of deterministic equations when random fluctuations around expected values are negligible.

We want to show that this construction can be repeated in a more general setting, where the Markovian hypothesis is removed and the continuous random description is in terms of random fields. Moreover, we are convinced that the transition from quantum to classical mechanics follows this pattern and this important problem will be addressed as well. This research line will bring probability theory at the forefront of interest in all the fields of science. It will be based on discrete probability, combinatorial stochastic processes, functional limit theorems and on their applications to physics, biology and the social sciences.

This project is deeply influenced by the thought and the results of Johnson, Keynes and Carnap as mediated by Zabell and Costantini. A good starting reference is the book: U. Garibaldi and E. Scalas, *Finitary Probabilistic Methods in Econophysics*, Cambridge University Press 2010.

**Key words:** Probability theory, philosophy of probability, epistemology, Markov chains,

stochastic processes, functional limit theorems, large deviations, fluctuations.