Artificial Viscosity in Smoothed Particle Hydrodynamics

University of Sussex

Benedict Marrable, MSc Astronomy **Supervisor: Peter Thomas**

1 Abstract

The project is an investigation into the underlying effect of artificial viscosity in a smoothed hydrodynamics code. The purpose is to find the most effective and accurate way of applying artificial viscosity.

2 Smoothed Particle Hydrodymanics

Smoothed Particle hydrodynamics is a method of modeling fluid flow, it was first developed more than three decades ago by Lucy (1977) and Gingold & Monaghan (1977). Since then it has become one of the standard and widely used computational techniques for modeling astronomical fluids.

It is a particle based model, where each particle has its own mass m_i and specific energy&$_i$. All other properties are inferred from neighbouring particles through a weighted average.

For a continuous distribution, the average value of some quantity A at the location i would be

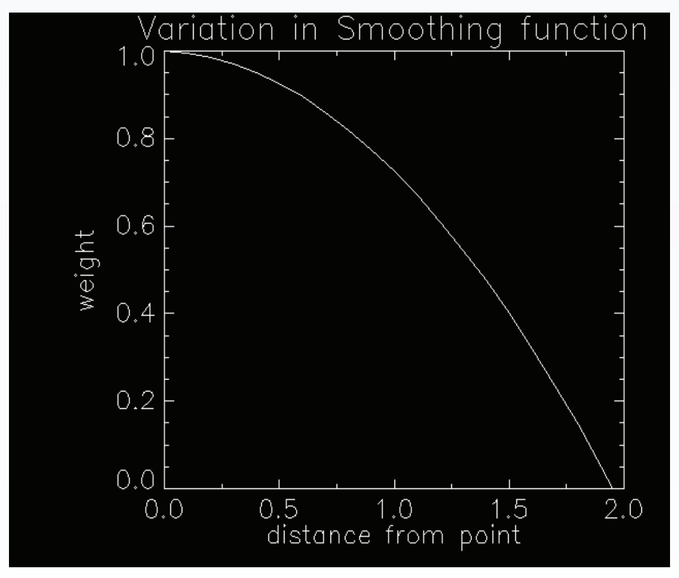
$$\langle A_i \rangle = \int A(r) W(r - r_i) dV$$

where W is the smoothing kernel, in SPH the integral is replaced by a sum,

$$\langle A_i \rangle = \sum_j \frac{m_j}{\rho_j} A_j W_{ij}$$

which extends over all particles, j, within the smoothing sphere. Where ($_j$ is the density of particle j.

The smoothing kernel is monotonically decreasing with distance, this infers the property of weight, the closer the neighbouring particle the more influence it has over the particles properties.



The Smoothing Kernel, which depicts the weight of a neighbour

The SPH code which I use is that developed by Thomas & Couchman (1992), which is called Hydra, named after the mythical beast of the same name.

One of the largest problems associated with SPH when in the presence of shocks, is stopping gases from passing through each other and intermingling. In SPH the standard way of preventing this from happening is by introducing an artificial viscosity term.

3 Artificial Viscosity

The artificial viscosity works by introducing an additional pressure to prevent flows from interpenetrating. This pressure is applied when calculating the properties of the particles, it takes convergent flows slows them down and dissipates the energy into heat, to equate for conservation of energy.

In Hydra, the artificial viscosity is added by replacing the pressure force f with a pressure force g, where

$$g = \frac{2\varepsilon_i}{3}\left(\alpha M_{ij} + \beta M_{ij}^2\right)$$

and &$_i$is the particle's specific energy, &$_j$is the neighbouring particles' properties, (is the mean density across all neighbouring particles, α and β are arbitrary constants of artificial viscosity. M_{ij} is the Mach number given by

$$M_{ij} = \begin{cases} 0, & r_{ij} \cdot v_{ij} > 0, \\ \dfrac{h_i \left| r_{ij} \cdot v_{ij} \right|}{c_{ij}\left(r_{ij}^2 + 0.01 h_i^2\right)}! & r_{ij} \cdot v_{ij} < 0, \end{cases}$$

the sound speed c_{ij}

$$c_{ij} = \frac{c_i + c_j}{2}$$

the particle separation,

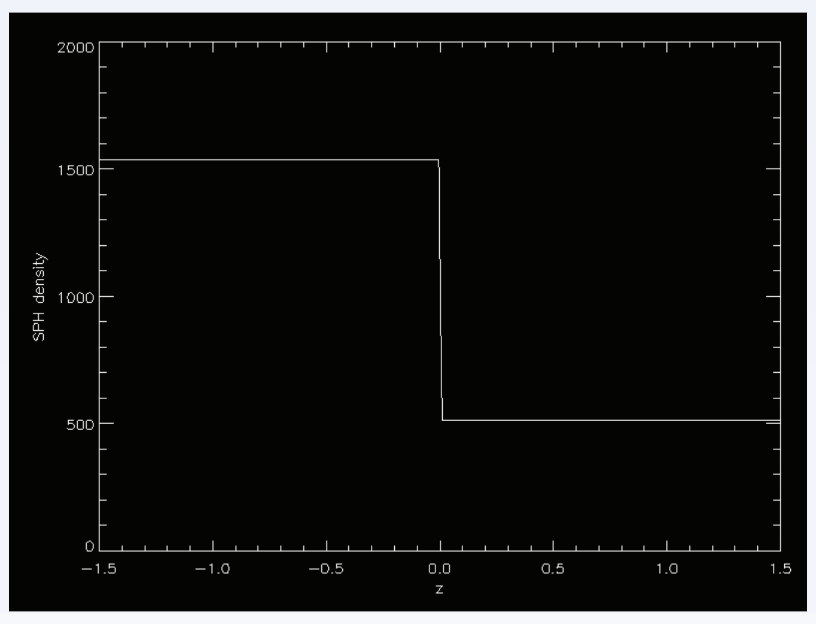
$$r_{ij} = \left| r_{ij} \right| = \left| r_j - r_i \right|$$

We test different values of g, to find the most appropriate value for the artificial viscosity, for several different shock tests.

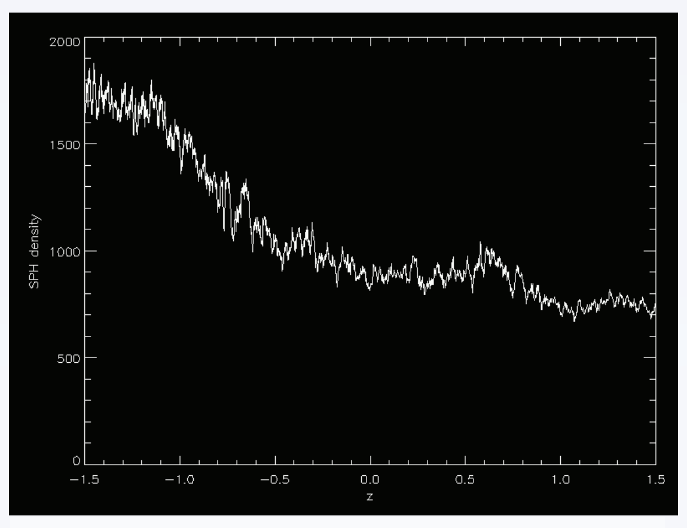
4 Testing

We set up 4 different shocks ranging from no shock to a shock jump of 4 in density. Testing with different values of α and β. The diagram on the right shows a typical density jump of 3.

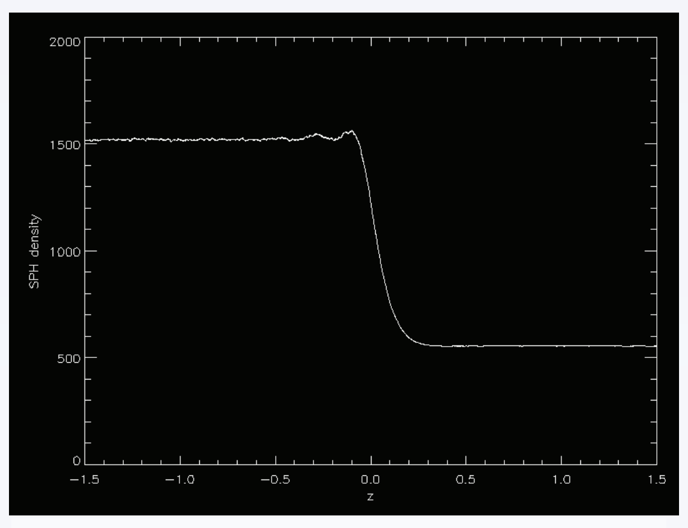
Running these on for a standard period of 2 time units, and investing the accuracy of the shock produced. The below diagrams show the change after a time 2 with different values for the artificial viscosity constants αand β.



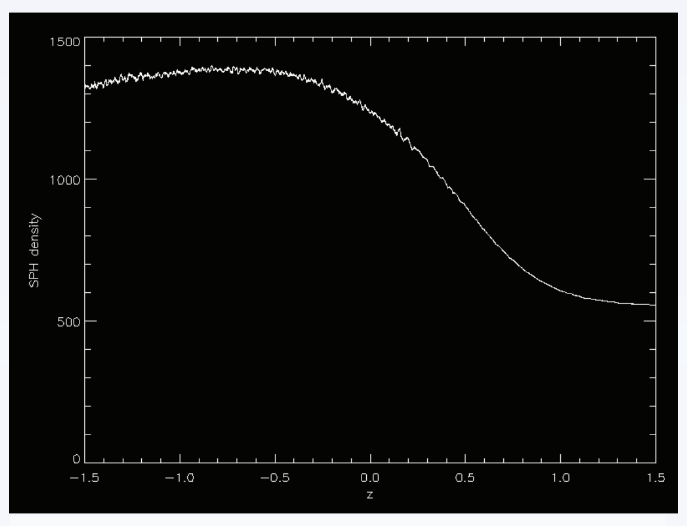
A graph showing the density for a jump of factor 3 at time zero, before Hydra has been run



Shock of density jump 3, after a time 2, using α= 0, β= 0



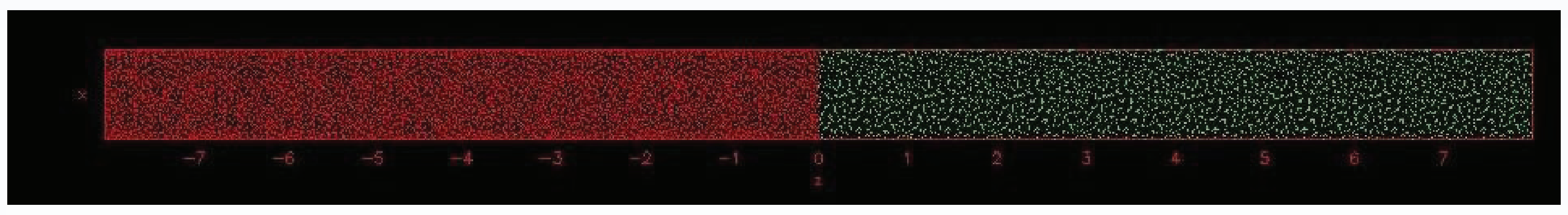
Shock of density jump 3, after a time 2, using α= 1, β= 2



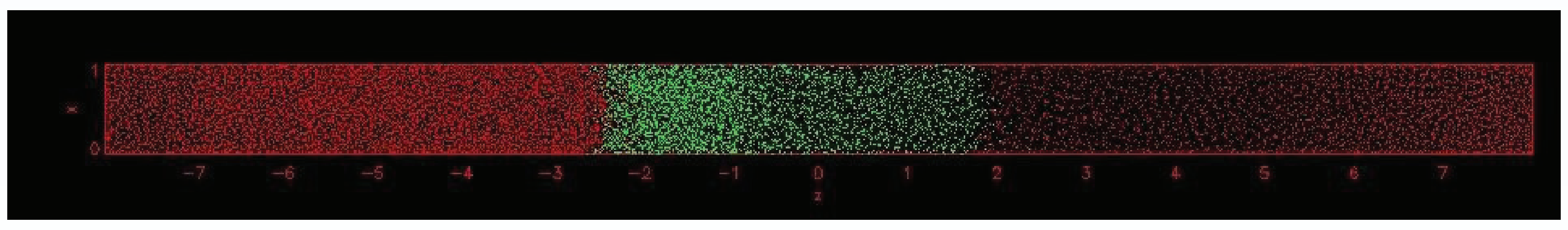
Shock of density jump 3, after a time 2, using α= 10, β= 20

Here you can clearly see that there is a resounding difference which occurs with the different values of the constants α and β. In the case where there is no artificial viscosity (α=0 and β=0) we see that the shock is very badly modelled, as there is no shock front left whatsoever and huge fluctuations in the data points. In the case where there is large artificial viscosity (α=10 and β=20), the shock front has been too heavily smoothed. The best representation of the shock comes from the middle diagram (where α=1 and β=2). Here we see a steep broad shock front, with little fluctuations in the data points.

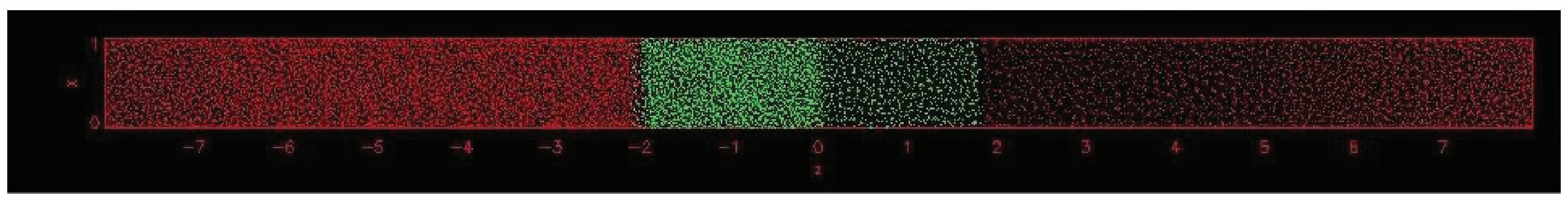
The only thing that remains left to test is for interpenetration, to do that we require a form of distinguishing where the particles were initially placed. This is done by numbering particles. In the below diagrams we see the location of all the particles in the y-plane.



The locations of all the particles, in the initial setup file. From the viewpoint of looking down through the y-plane. This is for a density jump of 3, and it can clearly be seen there are more red particles than green.



With no artificial viscosity, after time 2, we can clearly see there is some interpenetration.



With the standard α= 1, β= 2, after time 2, we can clearly see there is an excellent separation between the green and the red particles, this means very little interpenetration has occurred.

5 Conclusions and Future Work

From the results, it shows that there is a middle ground in which to place the artificial viscosity. Too much artificial viscosity and the shock dampens out, too little and the shock is not accurately modelled, and oscillations occur. For many different shocks a standard form of α=1 and β=2 give a good model of the shock.

Future tests and experiments, include changing the force g, to find a more appropriate way of equating the artificial viscosity for a range of shocks.

We will also look at an experiment, whereby two different gases pass by each other without any kind of shock occurring. Since particles in this situation would be converging at times, there would be as it stands an artificial viscosity applied. However this is not what is required as it will push the two gases apart and create a gap.

References

[1] B. W. Ritchie, P. A. Thomas: R. Astron. Soc., **323**, 743-756 (2001)
[2] M. Nejad-Asgher, A. R. Khesali, J. Soltani: Astrophys. Space. Sci., **313**, 425-430 (2008)
[3] Monaghan: Annu. Rev. Astron. Astrophys., **30**, 543 (1992)
[4] Monaghan, Gingold: J. Comp. Phys., **52**, 374 (1983)
[5] Lucy,: Astron. J., **82**, 1 (1989)