SPRU
Science Policy Research Unit

Working Paper Series

SWPS 2015-35 (December)

Social Network and Private Provision of Public Goods

Bulat Sanditov,
Saurabh Arora

University of Sussex

SPRU Working Paper Series (ISSN 2057-6668)

The SPRU Working Paper Series aims to accelerate the public availability of the research undertaken by SPRU-associated people of all categories, and exceptionally, other research that is of considerable interest within SPRU. It presents research results that in whole or part are suitable for submission to a refereed journal, to a sponsor, to a major conference or to the editor of a book. Our intention is to provide access to early copies of SPRU research.

Guidelines for authors

Papers shall be submitted in pdf or Word format. They should contain a title, an abstract, and keywords. Papers should be submitted to one of the Editors, who will process them and send them to the appropriate Associate Editor. Two members of SPRU will be asked to provide a short written review within three weeks. The revised versions of the paper, together with a reply to the reviewers, should be sent to the Associate Editor, who will propose to the Editors its publication on the series. When submitting the authors should indicate if the paper has already undergone peer-reviewing, in which case the Associate Editors may decide to skip internal review process.

Websites

SWPS: www.sussex.ac.uk/spru/research/swps

IDEAS: ideas.repec.org/s/sru/ssewps.html

Research Gate: www.researchgate.net/journal/2057-6668_SPRU_Working_Paper_Series

Social network and private provision of public goods

Bulat Sanditov[1,*] and Saurabh Arora[2]

[1]TELECOM Ecole de Management, Institut Mines-Télécom, France

[2]Science Policy Research Unit, University of Sussex, UK

*Corresponding author: 9 rue Charles Fourier, 91011 Evry Cedex, France. We are grateful to two anonymous referees and the guest editor, Zakaria Babutsidze, for valuable comments. Previous versions have benefitted from generous input provided by Francesco Lissoni, Uwe Cantner, Mauro Napoletano and the participants of EMAEE'13, WEHIA'13, GCW'13 conferences, the GREThA seminar at University of Bordeaux and Jena Economic Research Seminar at Friedrich Schiller University. The usual disclaimer applies.

Abstract

Using a simple model with interdependent utilities, we study how social networks influence individual voluntary contributions to the provision of a public good. Departing from the standard model of public good provision, we assume that an agent's utility has two terms: (a) 'ego'-utility derived from the agent's consumption of public and private goods, and (b) a social utility which is the sum of utility spillovers from other agents with whom the agent has social relationships. We establish conditions for the existence of a unique interior Nash equilibrium and describe the equilibrium in terms of network characteristics. We show that social network always has a positive effect on the provision of the public good. We also find that, in networks with "small world"-like modular structures, 'bridging' ties connecting distant parts of social network play an important role inducing the agent's contribution to public good. Assumptions and results of the model are discussed in relation to the role of social capital in community-level development projects and to the effect of innovation networks on firms' R&D investments.

1 Introduction

Our interest in studying the relationship between structure of social networks and private provision of a public good is motivated by two economic contexts.

First, in **development studies**, social capital is believed to be an important factor in enabling action by individual members of a community. In community-driven development, an increasing number of field studies have found social capital to be a statistically significant predictor of provision/absence of community-level public services. For example, Pargal et al. (2002) studied the establishment of voluntary solid waste management systems in Bangladesh and found that it is more likely to be undertaken in neighborhoods with higher levels of social capital. (Note that the level of social capital is a function of the structure of the local social network.) Similar effects of social capital were reported by Isham and Kähkönen (2002) on community-based water projects in Indonesia and by Motiram and Osberg (2010) on household access to drinking water in India. This widespread evidence suggests that different levels of social capital, based on different network structures, may afford different potentials with respect to action for establishing a community public service.

The other context motivating our interest in the effect of network structure on production of public goods comes from the **economics of innovation**. It has long been recognized that knowledge has characteristics of a public good and, therefore, the argument goes, when production of knowledge relies on private voluntary contributions (for example firms' investments in R&D), knowledge is under-produced as compared to the social optimum (Arrow, 1962). However, specific features of knowledge make the problem of knowledge production, in some key aspects, different from the standard problem with public goods.

First, the important part of firms' knowledge generated through their own R&D activities remains tacit and held by the knowledge employees. Unlike explicit or codified parts of knowledge, this tacit knowledge, often tied in with the firm's own technological activities, does not flow freely into the public domain. Instead, it may spillover to other firms through the network of interfirm interactions such as a network of R&D alliances, a network of co-inventors, or a buyer-supplier network. As highlighted by the extensive literature on proximity and innovation (e.g. Boschma, 2010; Balland, 2012), spillovers of tacit knowledge are most effective between firms that are cognitively proximate (in their knowledge bases). Such cognitive proximity may be developed by tapping into and utilizing the same pool of generally-available public knowledge.

Further, this 'global' public knowledge (that may be more readily available as codified) may be complementary to firms' own knowledge generated through R&D (a large part of which may be 'sticky' or tacit and thereby remaining within the firm and its local network). Such complementarity rests on the observation that firms' abilities to assimilate public knowledge and put it to use depend on firms' absorptive capacities, which can only be accumulated by carrying out their own R&D activities (Cohen and Levinthal, 1989). In addition, this complementarity between public and private knowledge renders pure free-riding strategy impractical. The complementarity stimulates firms' own investment in R&D, and therefore mitigates the problem of underproduction of public knowledge. The absorptive capacity argument suggests that the stock of public knowledge, and the desire to absorb it within the firm, is an important factor in driving firms' willingness to invest in R&D.

This interplay between the global-public and local-private types of knowledge implies that a firm may face a trade-off between investing in the production of public knowledge and in the production of private knowledge. Negotiation of this trade-off may require that a firm has to take into account of how its investment in global-public knowledge, in addition to the direct effect on its own R&D capabilities, stimulates other firms in an industry or a region to conduct research in technological domains that may be complementary (or related) to its own. Knowledge generated through research by others may later spill over back to the focal firm through the global interfirm network of the entire industry or region. On the other hand, the focal firm's R&D investment in the development of its own specific technological domain is more likely to have an effect on complementary R&D activities of the firms with which it has direct connections. The latter local knowledge is then likely to be of immediate relevance to the focal firm and its network of directly connected firms. In contrast, the global-public knowledge is likely to have general relevance, akin to a general-purpose technology (David, 1990; Gambardella and McGahan, 2010), which can be accessed by other firms in the entire network for further development, thereby generating global spillovers. The balance in this trade-off between investing in general/global and specific/local knowledge depends on the stock of public knowledge, the structure of the global knowledge network and the firm's own position in this network.

In both contexts (social capital in community development and knowledge spillovers for innovation), network structure is likely to play an important role in determining individual decisions to contribute to the provision of a public good. In this paper, we employ a formal model to examine this influence of the network structure. Our model differs from other models of public goods on networks in several respects.

First, our model is a model of interrelated utilities. The payoff to an agent may 'spill over' to another agent if the two are engaged in a social relationship. Therefore, one's motivation to invest in a public good, in addition to the direct effect on one's own utility, lies in the increase of the well-being of other agents for whom one cares. Second, public good in our model is global, i.e. available to all. Consistent with the two economic contexts mentioned above, the public good is provided on the level of the whole population. Third, besides the global public good there is a private good which, as we shall see, due to utility spillovers is akin to a local public good and the global and local goods are complementary. Thus our model can also be seen as a network game with interplay between global and local public goods.

The paper is organized as follows. Section 2 reviews recent literature on models with public goods on networks. Section 3 introduce the model and presents private equilibrium allocations of the public good. Section 4 discusses the relationship with earlier works. Section 5 concludes the paper.

2 Literature

Our model is related to two streams in economic literature. First, and directly related to our model, is the literature on network games. The recent decade has seen a surge of interest in network games, including network games with public goods. Given the large and ever increasing number of studies in this field, we limit ourselves to reviewing only a few seminal works. For a comprehensive survey of recent advances on network games, we refer the reader to Jackson and Zenou (2014).

Ballester et al. (2006) analyzed a general network game with linear-quadratic payoffs such that the marginal utility of agent's own action, while diminishing in her own action, linearly depends on the actions of her neighbors. They show that this game has a unique interior Nash equilibrium in which agents' actions are proportional to their Bonacich centrality scores.[1] This striking result is significant because it bridges the gap between emerging economic research on network games and the established theory of social networks in the social sciences.

[1] *Bonacich centrality* is a measure used in social network analysis to describe the power and importance of an actor in a social network.

Bloch and Zenginobuz (2007) proposed a model of local public goods with spillovers. In their model, agents are distributed across a fixed number of separate jurisdictions. Agents divide their income between consumption of a private good and contribution to the provision of a local public good. The utility of an agent depends on her consumption of the private good and the total amount of the public good, which is equal to the sum of the local public good produced in the jurisdiction and spillovers of public goods produced in other jurisdictions. (All local public goods are perfect substitutes.) The matrix of spillover intensities describes the structure of spillovers between jurisdictions. Bloch and Zenginobuz (2007) found that when the intensity of spillovers is relatively low (all row sums of spillovers matrix are less than one), the equilibrium level of the public good is unique and the increasing intensity of spillovers results in multiple equilibria. They also found that a jurisdiction that is more central (and therefore benefits more from public goods produced in other jurisdictions) contributes less of the public good.

Bramoullé and Kranton (2007) examined a different public good game on a network. In their model, agents have to make a decision about costly investment in a public good. This public good is local and an agent's payoff is a function of the sum of an agent's own contribution and the contributions of her direct neighbors. This game has two types of equilibria: specialized and hybrid. In a specialized equilibrium, some agents invest in public goods while others choose not to invest but free-ride on contributions of their neighbors. Such specialization is socially optimal in a configuration in which contributors are linked to many non-contributors. Interestingly, a new link may reduce social welfare because it may reduce the agent's incentive to contribute.

Bramoullé et al. (2014) unified and extended the results of Bramoullé and Kranton (2007) and Ballester et al. (2006). They developed a general approach to analyze games with strategic substitutes with linear best reply functions based on the theory of potential games and therefore encompass earlier works on network games mentioned above. They show that, in games with linear best reply functions, it is the lowest eigenvalue of the interaction matrix that defines certain properties of the Nash equilibria such as uniqueness, the number of contributors and stability. Under the specifications of the model that we study in this paper, just as in Bramoullé et al. (2014) model, agents' best reply functions are linear and therefore the results obtained by Bramoullé et al. must also hold.

The second stream of literature on which we base our model concerns systems of altruistic utility functions (Becker, 1976; Bergstrom, 1999). In this literature, connections between agents are defined as interdependence between agents' utilities. In network games such as those discussed above, a social network is essentially the network of effects of one agent's action on payoffs of the agents with whom she is connected. In the literature on interdependent utilities, the network accounts for the effects of one agent's payoff on the payoffs of her direct neighbors. Most research in this tradition is concerned with some general properties of systems or models in which the network structure of interactions plays no significant role, such as systems of two individuals or systems with a special pattern of interactions.[2] Notwithstanding the lack of interest in structure of interrelated utilities in this literature, some results on systems with general patterns of interactions are also useful, as in the model we develop in the next section.

Bergstrom (1999) studied systems of altruistic individuals and has shown that under certain conditions, a system of interdependent utility functions can be disentangled into a system of independent utility functions that define individual preferences over the set of possible allocations of individual consumption bundles. Ley (1997) studied the optimal provision of public goods in the context of a additively separable altruistic utility functions. In his model, an agent receives spillovers from the utilities of other individuals with whom she has social ties. Perhaps surprisingly, he found that the Samuelson condition for optimal allocation in a model without interactions also holds in a model with interdependent utility functions.[3]

Finally, in a recent study, Bourlès and Bramoullé (2013) analyzed a transfer game on a network defined in terms of interdependencies between agents' utility functions. They find that, for any network and any utility function, there is a unique profile of equilibrium incomes. However, equilibria on a generic network are multiple, except on trees.

Our model, motivated by the two economic contexts discussed in the introduction, is closely related to both streams of the economic literature discussed above. Following studies on systems of altruistic utilities, we analzse a population of agents woven into a network of social relationships where the utility of an agent depends on utilities of the other agents with whom she has direct relationships. Similar to network games, we are interested in equilibrium outcomes and the relationship between network structure and equilibrium level of a public good.

Our model is different from other models of public goods on networks in several respects. First, in contrast to other network games with public goods, our networks are networks of interrelated utilities. Our interpretation of social relationships is similar to the traditional notion of close relationships between individuals adopted in the social sciences (Wellman, 1979), which refers to feelings and sentiment based on reciprocity and mutual care generally found in social networks of kinship, friendship and social affinity. It may also be useful in the case of networks other than social

[2]Examples of such systems include systems of utilities in which individuals care equally for all other individuals and systems with a fixed pattern of interactions. For instance, Bergstrom (1999) examines a model where agents are located on a line and each agent is connected to exactly one agent on the right and one agent on the left. (This system of utility functions can be used in the framework of overlapping generations to account for intergenerational altruism.)

[3]Also see Bergstrom (2006).

networks, such as knowledge networks, where it is reasonable to assume that the interactions take the form of payoff spillovers from *alter* to *ego* rather than a direct effect of *alter*'s actions one *ego*'s payoff.

Second, in our model, agents make contributions to a public good that are *global*, i.e. provided on the level of the whole population. In this respect, it is different from most other models where public goods are *local*, i.e. accessible only by an agent's direct neighbors. This makes it consistent with the two economic contexts mentioned in the introduction, because both community-level public services and public knowledge are, in principle, accessible by all members of a community.

Third, because of payoff spillovers, the private good, complementary to the global public good in our model, is in fact a kind of a local public good (Ley, 1997). Thus our model can be seen as a model with global and local public goods with some level of complementarity between the two. In the context of R&D networks, it may correspond to a firm's investment in building its own absorptive capacities, which increases the value of public knowledge to the firm.

3 Model

There are n agents in the economy. Each agent indexed by $i \in \{1, .., n\}$ has endowment $w_i = 1$, which she divides between consumption of private good, x_i, and a contribution to public good, y_i. The marginal rate of transformation between private and public good is 1, and the amount of public good provision is simply a sum of contributions of all agents, $Y = y_1 + .. + y_n$.

Agent i's utility, V_i, is sum of two terms. The *ego* part U_i, henceforth referred as ego-utility, is a function of the consumption of the private good, x_i, and the consumption of the public good, Y. Throughout the paper we use *ego* utility functions

$$U_i(x_i, Y) = v(Y)x_i, \tag{1}$$

where $v(\cdot)$ is the Constant Relative Risk Aversion (CRRA) or isoelastic utility function. These functions belong to a special class of quasi-concave utility functions $U_i = v(Y)x + u_i(Y)$ that generate economies with transferable private utilities (Bergstrom and Cornes, 1983) and has been used earlier in models with systems of altruistic preferences (Ley, 1997; Bergstrom, 2006).[4] It is also important for us that, with this specification, we obtain best-reply functions similar to ones in other network games discussed in the previous section.

The *alter* part of V_i is sum of utility "spillovers" from the agents with whom agent

[4]Ley (1997) has shown, that in a system of interrelated utilities such as (2), Pareto efficiency of an allocation is independent of the distribution of private consumption. However, this result concerns only the flat part of the utility possibility frontiers, because his analysis excludes corner solutions.

i has direct connections

$$V_i = \underbrace{U_i(x_i, Y)}_{ego} + \underbrace{\sum_{j \neq i} \alpha_{ij} V_j}_{alter} . \tag{2}$$

Social relationships channelling utility spillovers constitute a weighted undirected network g with adjacency matrix $\mathbf{A} = \|\alpha_{ij}\|$. We assume that utility spillovers are non-negative ($\alpha_{ij} \geq 0$). Equation (2) can be rewritten in matrix form as

$$\mathbf{v} = \mathbf{u} + \mathbf{A}\mathbf{v}, \text{ or } \mathbf{u} = (\mathbf{I} - \mathbf{A})\mathbf{v}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{v} = (V_1, V_2, .., V_n)$ and $\mathbf{u} = (U_1, U_2, .., U_n)$ are the vectors of agents' overall utilities and ego-utilities, respectively.

3.1 Systems of interrelated utility functions

Systems of additively separable utility functions have been studied by Bergstrom (1999) and Ley (1997). Bergstrom (1999) has shown that a system of interdependent utility functions (2) can be disentangled to induce a unique and well-behaved, or "normally benevolent", system of independent utility functions when matrix $(\mathbf{I} - \mathbf{A})$ is dominant diagonal: a matrix of form $(\mathbf{I} - \mathbf{A})$, where $\mathbf{A} \geq 0$, is said to be dominant diagonal if its row sums are all positive, i.e.

$$\sum_{j=1}^{n} \alpha_{ij} < 1, \text{ for any } i \in \{1, .., n\}. \tag{3}$$

A dominant diagonal matrix is invertible and the elements of the inverse matrix are non-negative. When the adjacency matrix $\mathbf{A}$ satisfies (3), by inverting matrix equation (2) we can obtain

$$\mathbf{v} = \mathbf{B}\mathbf{u}, \text{ where } \mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}. \tag{4}$$

A system of utility function is said to be normally benevolent when an agent is never worse off because of any other agent is being better off (Bergstrom, 1999). When matrix $(\mathbf{I} - \mathbf{A})$ is dominant diagonal, the system of utility functions (4) is normally benevolent, because elements of $\mathbf{B}$ are non-negative.

Matrix $\mathbf{B}$ has particular meaning in social network theory.[5] Its elements can be interpreted as a measure of influence that one individual exerts on another. Notice

[5] Matrices of form $(\mathbf{I} - \mathbf{A})$ and their inverse are well-known to economists in the context of input-output analysis. Matrix $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$, where $\mathbf{A}$ is a matrix of technical coefficients, is a Leontief inverse that describes the relationship between vector of sector outputs and final demands. When a network defined by adjacency matrix $\mathbf{A}$ is strongly connected, a diagonally dominant matrix $(\mathbf{I} - \mathbf{A})$ is an M-matrix (Horn and Johnson, 1994, p.131)

that $(\mathbf{I}-\mathbf{A})^{-1}$ can be decomposed into a series of matrix powers of $\mathbf{A}$: $\mathbf{B} = \sum_{t=0}^{\infty} \mathbf{A}^t$. The element ij of matrix $\mathbf{A}^t$ counts the number of walks from i to j of length t discounted by the product of the edge weights along the walk. Thus B_{ij} sums up all walks that start at i and end at j with appropriate weights (e.g. Ballester et al. 2006; Elliott and Golub 2013). The vector of row sums of matrix $\mathbf{B}$, known as the vector of *Bonacich centralities*, characterizes agents' influence or power over the network.

We may interpret B_{ij} in terms of utility flows: the well-being of agent i spills over to i's neighbors (the intensity of spillover is proportional to the strength of social relationships) and then to neighbors of neighbors and so on. The coefficient B_{ij} counts for total utility flow passing all walks connecting i to j and therefore traces the total effect of an increase in utility of agent i on the utility of j channelled through the social network.

The diagonal element B_{ii} corresponds to the part of the 'well-being flow' that returns to i: an increase in i's utility (e.g. due to increase in private consumption) spills over to i's neighbors increasing their utilities, but increasing their utilities, in turn, spills over back to i. The B_{ii} sums up the direct effect (equal to 1), the effect of neighbors described above and all effects of higher order due to neighbors of neighbors, and so on. In general, the magnitude of B_{ii} is larger when there are many paths starting and ending in i, which depends on the size of i's network and its density. Hence, B_{ii} is a measure of i's embeddedness in the network.

3.2 Best-reply functions

The system (4) defines agents' preferences over different allocations $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. Agent i maximizes her utility V_i by choosing x_i given the vector of choices of all other agents $\mathbf{x}_{-i}$. Her optimization problem is

$$\begin{aligned} \max_{x_i} V_i(x_i, Y) \quad &= \quad \sum_{j=1}^{n} B_{ij} U_j(x_j, Y) \qquad (5) \\ \text{subject to} \quad & \quad Y = W - (x_i + x_{-i}), \\ & \quad 0 \le x_i \le 1. \end{aligned}$$

The first order condition for this optimization problem is

$$\frac{dV_i}{dx_i} = \frac{\partial V_i}{\partial x_i} - \frac{\partial V_i}{\partial Y} = B_{ii}\frac{\partial U_i(x_i, Y)}{\partial x_i} - \sum_{j=1}^{n} B_{ij}\frac{\partial U_j}{\partial Y} = 0$$

or

$$B_{ii}\frac{\partial U_i(x_i, Y)}{\partial x_i} = \sum_{j=1}^{n} B_{ij}\frac{\partial U_j(x_j, Y)}{\partial Y} \qquad (6)$$

when $x_i \in (0, 1)$. In addition, an agent's budget constraint defines two possible corner solutions

$$B_{ii}\frac{\partial U_i(x_i,Y)}{\partial x_i} > \sum_{j=1}^{n} B_{ij}\frac{\partial U_j(x_j,Y)}{\partial Y}, \qquad x_i = 1;$$

$$B_{ii}\frac{\partial U_i(x_i,Y)}{\partial x_i} < \sum_{j=1}^{n} B_{ij}\frac{\partial U_j(x_j,Y)}{\partial Y}, \qquad x_i = 0.$$

The LHS of equation (6) and the inequalities above represent the marginal utility of consuming the private good. The factor B_{ii} accounts for the sum of direct effect of private consumption on ego-utility U_i and the return ('circular') utility spillovers. Similarly, factors B_{ij} in the RHS of equation (6) describe the effect of an increase in j's utility on the utility of agent i. The RHS sums up utility flows from all other agents and stands for the marginal effect of i's investment in public good on her utility.

Assuming that ego-utility functions are quasi-concave in the form of (1), the solution to the optimization problem (6) can be written in a closed form. The first order condition (6) is

$$v(Y)B_{ii} = \sum_{j=1}^{n} B_{ij}x_j v'(Y), \tag{7}$$

for $x_i \in (0,1)$, or

$$\frac{Y}{\epsilon_y} = \sum_{j=1}^{n} \frac{B_{ij}}{B_{ii}} x_j.$$

Considering that the elasticity of CRRA function $v(Y)$ is a constant, $\epsilon_y(Y) = \epsilon$, and $Y = W - (x_1 + \cdots + x_i)$, where $W = n$ is the sum of agents' endowments, we obtain

$$(1+\epsilon)x_i = W - \sum_{j\neq i}\left(1+\epsilon\frac{B_{ij}}{B_{ii}}\right)x_j$$

Finally, the best reply function of agent i is

$$BR_i(\mathbf{x}_{-i}) = \begin{cases} 0, \text{ if } W < \sum_{j\neq i}\left(1+\epsilon\frac{B_{ij}}{B_{ii}}\right)x_j; \\ 1, \text{ if } W > \sum_{j\neq i}\left(1+\epsilon\frac{B_{ij}}{B_{ii}}\right)x_j + (1+\epsilon); \\ \frac{1}{1+\epsilon}\left(W - \sum_{j\neq i}\left(1+\epsilon\frac{B_{ij}}{B_{ii}}\right)x_j\right), \text{ otherwise.} \end{cases} \tag{8}$$

Notice that agents' best reply functions are linear in the actions of the others. For properly defined $\tilde{x}$ and $\tilde{\mathbf{B}}$, we can re-write BR_i as

$$BR_i(\mathbf{x}) = \min\{1, \max\{0, \tilde{x} - \tilde{\mathbf{B}}\mathbf{x}\}\}.$$

Therefore, our model belongs to the class of the network games studied in Bramoullé et al. (2014) and their results concerning existence and stability of Nash equilibria must hold for our model.[6] However, following the approach of Bramoullé et al. (2014), we would have to re-formulate our model in terms of the network induced by matrix $\tilde{\mathbf{B}}$, which lacks a straightforward intuitive interpretation with respect to the original social network. The criteria for existence and stability of equilibria in the game would also be formulated in terms of eigenvalues of $\tilde{\mathbf{B}}$. Thus, instead of relying on the general results of Bramoullé et al. (2014), we establish conditions and prove the existence and uniqueness of internal equilibrium in our model by solving the system of first order conditions (7). In this way, we obtain the equilibrium allocation $\mathbf{x}$ in a closed form and can describe the equilibrium in terms of the original network (matrices $\mathbf{A}$ and $\mathbf{B}$).

3.3 Equilibrium

We focus on the interior equilibrium in our model i.e. the equilibrium where agents' budget constraints ($0 \leq x_i \leq 1$) are not binding and agents' first order conditions are as given by equation (7). Define vector $\boldsymbol{\chi}$ as

$$\chi_i = B_{ii} - \sum_{i=1}^{n} A_{ij} B_{jj}$$

or in matrix form

$$\boldsymbol{\chi} = (\mathbf{I} - \mathbf{A})\mathbf{b},$$

where $\mathbf{b} = (B_{11}, B_{22}, \dots B_{nn},)$, and let $\bar{\chi}$ be the average of χ_i over $i = 1, \dots, n$. The following proposition establishes conditions for existence of the interior equilibrium and describes how agents' actions in equilibrium depend on their positions in the network.

Proposition 1. *If for any $i \in \{1, \dots, n\}$: $\chi_i \geq 0$ and $(\chi_i - \bar{\chi}) \leq \frac{\epsilon}{n}$, then there is a unique interior equilibrium $\mathbf{x}^* = \{x_1^*, \dots, x_n^*\}$:*

$$x_i^* = \frac{\chi_i}{\bar{\chi} + \epsilon/n}$$

and $0 \leq x_i^* \leq 0$ for any $i \in \{1, \dots n\}$.

Proof. Assuming that agents' budget constraints are not binding, the system of first order conditions (7) defining the internal equilibrium can be written in a matrix form as

$$\mathbf{b} = \mathbf{B}\mathbf{z}, \tag{9}$$

where $\mathbf{b} = \text{diag}(\mathbf{B})$ is the vector of diagonal elements of matrix $\mathbf{B}$ and $\mathbf{z}$ is the vector of agents' marginal rates of substitution: $z_i = \frac{v'(Y)x_i}{v(Y)}$. Inversion of this equation ($\mathbf{B}$

[6] The general version of their model with heterogeneous payoff impacts and upper bound on agents' actions (Bramoullé et al., 2014, p.919).

is non-singular) gives us

$$\mathbf{z} = (\mathbf{I} - \mathbf{A})\mathbf{b} \equiv \boldsymbol{\chi},$$

and using definition of z_i we have

$$x_i = \frac{v(Y)}{v'(Y)}\chi_i,$$

i.e. in equilibrium, consumption of private good x_i is proportional to χ_i.

To find the equilibrium level of x_i we need only to determine the level of public good in equilibrium, Y^*. Summing up x_i we obtain

$$X^* = \frac{v(Y)}{v'(Y)} \sum_{i,j} \chi_i$$

where $X^* = (x_1^* + \cdots + x_n^*)$ is the total consumption of private good. Taking into account the balance condition: $Y = (W - X)$, where $W = n$ is the sum of agents endowments, and the fact that, for CRRA utility function $v(Y)$: $v(Y)/v'(Y) = Y/\epsilon$, this equation can be written as

$$n - Y^* = \frac{Y^*}{\epsilon} \sum_{i,j} \chi_i,$$

solving for which we find the equilibrium level of public good

$$Y^* = \frac{1}{\bar{\chi}/\epsilon + 1/n}, \tag{10}$$

and, finally,

$$x_i^* = \frac{Y^*}{\epsilon}\chi_i = \frac{\chi_i}{\bar{\chi} + \epsilon/n}.$$

It is straightforward to verify that the conditions of the proposition ensure that the equilibrium consumption of private good defined by the expression above respects the budget constraint $0 \leq x_i^* \leq 1$. Therefore, $\mathbf{x}^*$ defines an interior equilibrium. By its very construction, this equilibrium is unique because the system of linear equations $\mathbf{b} = \mathbf{B}\mathbf{z}$ has a unique solution. ∎

Notice that the structural properties of the social network enter into agents' actions through $\boldsymbol{\chi}$. Vector $\boldsymbol{\chi}$ plays an important role in describing the equilibrium in our model: the equilibrium vector of agents' private consumption $\mathbf{x}^*$ is a multiple of $\boldsymbol{\chi}$, while average $\bar{\chi}$ determines the equilibrium level of the public good.

4 Discussion

There are two opposite effects of a social network on an agent's incentives to contribute to a public good, as can be inferred from equation (6). On the one hand, the social network lets the agent internalize part of the social benefit due to her investment in the public good. Contribution to the public good raises the well-being of every individual and the network channels the increases in other agents' utilities to the contributing agent. As a result, the marginal utility of the public good on an agent's utility ($\partial V_i/\partial Y$) is larger than the marginal effect of the public good on the agent's ego-utility ($\partial U_i/\partial Y$). The magnitude of this effect depends on the coefficients B_{ij} that measure how well is agent i connected to the others, according to the RHS of equation (6).

On the other hand, social network also amplifies the effect of consumption of the private good. An increment to an agent's ego-utility due to more consumption of the private good spills over to the agent's direct neighbors, increasing their utility. This, in turn, enhances utilities of the neighbors of the agent's neighbors and the process goes on. Thus, the private good in the model is, in fact, a kind of *local public good*. Part of the utility flow initiated by an agent's consumption of the private good returns to the agent. This accrues to the marginal effect of private consumption, and therefore we have factor B_{ii} in the LHS of (6) which stands for the intensity of returning utility spillovers (or 'self-effect').

In this section, we examine properties of the equilibrium in our model. First, we show that, for any agent, the 'global' positive effect of the social network on the provision of the public good always prevails over the negative 'local' effect, i.e. the social network encourages the agent's investment in the public good. We also show that, despite the positive effect of the social network in equilibrium, the public good is under-produced. Second, we examine the impact of network structure on provision of the public good. We focus on small-world modular networks and, using some numerical examples, we show the importance of social ties spanning across groups of densely connected agents.

4.1 Equilibrium and social optimum

Below we analyze the Pareto efficiency of the equilibrium defined by Proposition 1 (section 3.3). We start with a comparison against the standard model of the public good, where there is no social network ($A = 0$) and therefore no utility spillovers ($B = I$), which may serve as a useful benchmark. Our interest in such settings, which we refer to as an *egoistic* society, owes to the result of Ley (1997) on the relationship between Pareto efficient allocations in an egoistic society and socially optimal allocations in a model with interrelated utilities.

Social optimum in egoistic society In a standard public good model, a socially optimal allocation of private consumption, $\hat{\mathbf{x}}^e$, must satisfy Samuelson's condition:

$$\sum_{j=1}^{n} z_j(\hat{x}_i^e, \hat{Y}^e) = 1,$$

where z_j is agent j's marginal rate of substitution between public and private goods.

However, if the production of a public good relies on agents' voluntary contributions, then the level of the public good in the economy is lower than the socially optimal because agents are only willing to contribute $y_i^e = (w_i - x_i^e)$ such that an agent's marginal rate of substitution between the consumption of private and public goods is equal to 1.[7] Under this condition, an agent's first order condition is

$$z_i(x_i^e, Y^e) = 1.$$

Consequently, agents' marginal rates of substitution sum to n rather than 1, as is required by Samuelson's condition. Thus, in an egoistic society, where provision of the public good depends on voluntary contributions of agents, the public good is always underproduced: $Y^e < \hat{Y}^e$.

Egoistic society vs. social network In our model, the social network allows an agent to internalize the positive externality resulting from her investment in the public good and, therefore, the agent's contribution to the public good is larger than in an *egoistic* society. In the equilibrium defined by (9), the marginal rate of substitution of agent i is

$$z_i = \left(1 - \sum_{j \neq i} \frac{B_{ij}}{B_{ii}} z_j\right) < 1.$$

i.e. the social network lowers agents' marginal rates of substitution. Since the rate of marginal substitution of public good Y for agent's consumption of private good x_i is diminishing with Y (for any reasonable utility functions including the chosen quasi-concave utilities), it implies that

Lemma 1. *Networks of social relationships encourage investment in the public good. The agents contribute to the provision of the public good more than they would do if there were no social network.*

However, agents do not internalize the positive externalities of their investment in the public good fully and therefore the sum of agents' contributions in equilibrium will never reach the optimum level $\hat{Y}^e$ defined by Samuelson's condition.

[7] By assumption, the marginal rate of transformation between private and public goods is equal to 1.

Lemma 2. *The equilibrium level of public good, Y^*, does not reach the socially optimal level of public good in the egoistic society, $\hat{Y}^e$: $Y^* < \hat{Y}^e$*

Proof. Indeed, if we divide (7) by its LHS, we notice that

$$1 = \sum_{j=1}^{n} \frac{B_{ij}}{B_{ii}} z_j < \sum_{j=1}^{n} z_j \tag{11}$$

where we use the fact that for strictly diagonally dominant A, the diagonal elements of its inverse are always larger than its off-diagonal elements i.e. $B_{ii} > B_{ij}$ for any $j \neq i$ (Horn and Johnson, 1994, Theorem 2.5.12). Since the sum of agents' marginal rates of substitution is larger than 1 (required by Samuelson's condition), in the private equilibrium the level of the public good (Y^*) is always below the socially optimal $\hat{Y}^e$. ■

Social efficiency of equilibrium allocation Ley (1997) has shown that a Pareto efficient allocation in a model with interrelated utilities is also a Pareto efficient allocation in egoistic society. Furthermore, he has found that, for quasi-concave ego-utility functions, all Pareto efficient allocations have the same level of the public good. Given his result, analysis of (in)efficiency of the equilibrium in our model could have been reduced to testing whether our equilibrium satisfies Samuelson's condition of the standard model (and according to Lemma 2, it does not). However, Ley's result does not apply to all Pareto efficient allocations, but concerns only allocations where agents' budget constraints are not binding i.e., $x_i < 1$ for any $i \in \{1, .., n\}$. We proceed as follows. First, we find a Pareto efficient allocation that is *comparable* to the private equilibrium. Then we show that, in this socially efficient allocation, the level of public good is higher than in the equilibrium, when agents' budget constraints are not binding.

The following lemma characterizes the equilibrium allocation in terms of its location in the space of utilities.

Lemma 3. *In the equilibrium, the vector of agents' utilities $\mathbf{v}^* = (V_1^*, \ldots, V_n^*)$ is a multiple of the diagonal of matrix B:*

$$\mathbf{v}^* = \gamma \mathbf{b}$$

where $\mathbf{b} = diag(B)$ and constant $\gamma > 0$.

Proof. From the first order condition (7) we know that

$$\sum_{j=1}^{n} B_{ij} x_j = B_{ii} \frac{v(Y)}{v'(Y)}.$$

Substituting that expression into (4), we get

$$V_i(x_i, Y) = \sum_{j=1}^{n} B_{ij} v(Y) x_j = B_{ii} \frac{v(Y)^2}{v'(Y)}.$$

Factor $v(Y)^2/v'(Y)$ depends only on the Y and is the same for all agents. Thus, the vector of utilities $v^* = (V_1^*, \ldots, V_n^*)$ is a multiple of the vector of the diagonal elements of matrix B. ■

According to Corollary 3, the equilibrium vector of agents' utilities $\mathbf{v}^*$ is a multiple of the vector of matrix $\mathbf{B}$'s diagonal $\mathbf{b}$. First, let us find all allocations $\mathbf{x}$ such that the vector of agent utilities $\mathbf{v}(\mathbf{x})$ is parallel to $\mathbf{v}^*$:

$$\mathbf{v} \equiv v(Y)\mathbf{B}\mathbf{x} = \gamma \mathbf{b}.$$

Solving this system of linear equations with respect to $\mathbf{x}$, we get

$$x_i = \frac{\gamma}{v(Y)} \chi_i.$$

Now, assuming that an agent's budget constraint is not binding ($x_i \geq 1$), we find a condition on any allocation $\mathbf{x}$ such that $\mathbf{v} = \gamma \mathbf{b}$. Summation over i gives us $\gamma(Y) = v(Y)(W - Y)/\sum \chi_j$. Using it in the expression above, we have

$$x_i = \frac{W - Y}{\sum \chi_j} \chi_i.$$

The allocations $\mathbf{x}$ we obtain by changing $(W - Y)$ from 0 to W correspond to points in the space of utilities belonging to a ray starting at the origin passing through $\mathbf{v}^*$. Among them there is a Pareto efficient allocation $\hat{\mathbf{x}}$ such that the corresponding $\mathbf{v}(\hat{\mathbf{x}})$ is the point where the ray intersects with the utility-possibility frontier. This allocation must correspond to $\mathbf{x}$ with maximum $\gamma(Y)$. Hence, for such allocation

$$\frac{d\gamma}{dY} = v'(Y)(W - Y) - v(Y) = 0,$$

from which we find the socially optimal level of public good in our model[8]

$$\hat{Y} = \frac{1}{1/(n\epsilon) + 1/n}, \tag{12}$$

and the corresponding $\hat{\mathbf{x}}$ is

$$\hat{x}_i = \frac{W - \hat{Y}}{\sum \chi_j} \chi_i.$$

Comparing the equilibrium allocation $\hat{x}^*$ with $\hat{x}_i$ allows us to state our

[8]The Samuelson condition for the egoistic society would result in the same expression for $\hat{Y}$.

Proposition 2. *The equilibrium allocation $\hat{x}^*$ is not socially efficient: the level of public good in equilibrium is lower than the socially optimal level of public good.*

Proof. Assume that for any i, allocation $\hat{x}_i$ defined above is such that $0 < \hat{x}_i < 1$. Notice that the socially optimal level of public good, $\hat{Y}$, is higher than the equilibrium provision of public good Y^*

$$\hat{Y} = \frac{1}{1/(n\epsilon) + 1/n} > Y^* = \frac{1}{\bar{\chi}/\epsilon + 1/n}$$

where we use the fact that $\bar{\chi} > 1/n$, because in equilibrium an agent's marginal rates of substitution is equal to χ_i and, according to Lemma 2, the sum of the marginal rates of substitution is larger than 1. The lower the value of $\bar{\chi}$, the closer the equilibrium allocation $\mathbf{x}^*$ to the utility-possibility frontier.

Let us verify that, in this Pareto efficient allocation, $\hat{\mathbf{x}}$, the agents' budget constraints are not binding. Notice that for any agent i, $\hat{x}_i < x_i^*$:

$$\hat{x}_i = \frac{W - \hat{Y}}{\sum \chi_j}\chi_i < x_i^* = \frac{W - Y^*}{\sum \chi_j}\chi_i < 1,$$

because $\hat{Y} > Y^*$ and under the condition of Proposition 1, the budget constraints are not binding $(0 < x_i^* < 1)$. This justifies our assumption that $\hat{x}_i < 1$ made earlier, and therefore equation (12) correctly defines the socially optimal level of public good. ■

Summing up the results concerning the efficiency of the private equilibrium, we may state that social network encourages agents' investment in public good, yet the equilibrium level of public good falls short of the socially optimal level.

4.2 Network structure and public good: importance of bridging ties

Although for any agent the 'global' positive effect of social network on the provision of the public good always dominates the negative 'local' effect, the balance between the two depends on the position that the agent occupies in the network. The equilibrium vector of consumption of private good $\mathbf{x}^*$ depends on the diagonal elements of matrix $\mathbf{B}$. The intensity of return spillovers B_{ii} depends on the size of i's extended network (number of neighbors, neighbors of neighbors, and so on, discounted by their distance from i), and on the density of connections within this network. The larger and denser is i's network, the more possibilities there are for 'circular' utility flows and therefore the larger is B_{ii}. The larger is B_{ii}, the more i spends on private consumption, and the less is her contribution to the public good.

Many real world networks have "small-world" modular structure: they consist of tightly-knit clusters (communities) of nodes with only a few connections running between the clusters.[9] Within a densely connected cluster there are multiple short paths connecting its members, while there are only a few paths running across clusters.

This suggests that the intensity of spillovers within a cluster, which include return or 'circular' spillovers, is relatively high in comparison with the intensity of spillovers between clusters. Thus, other things being equal, for an individual in a cluster whose connections are local, i.e. within the cluster, the trade-off between the global public good and the local public good is biased towards the latter. This is because she is not internalizing positive externalities of her investment in the public good from outside her cluster. By contrary, an individual whose connections span many clusters may get more benefits from investing in the global public good, because she is exposed to inter-cluster utility flows.

(a) Social network (b) Matrix **B**

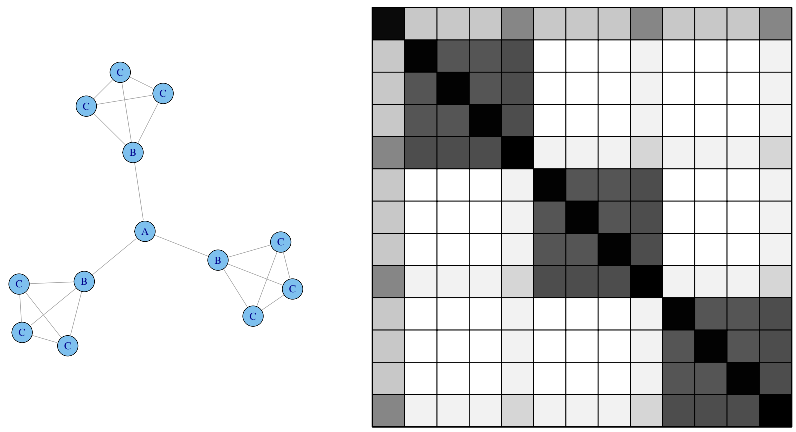


Figure 1: An example of a modular social network. All links have the same strength $\alpha = 0.23$. **Left:** Social network, **Right:** Matrix $B = (I - A)^{-1}$. Shading corresponds to the magnitude of B_{ij}. Nodes sorted by clusters (individual A is in the first row/column, individuals B are in rows/columns 2,6, and 13.

To illustrate this point, consider a network depicted in Figure 1a. The network consists of three fully connected clusters and one individual (type A) connected to each of the clusters through one member of the cluster (type B). All links have the same strength and high intensity of spilllovers $\alpha = 0.23$. The value of the parameter ϵ in the CRRA ego-utility function is set at 0.8, so that the conditions of the Proposition 1 are satisfied.[10] The values of χ_i, diagonal elements B_{ii}, and the agent's contribution to public good y_i for the three types of nodes in this network are reported in Table 1.

[9]In social networks, those clusters may correspond to members of the same family, same neighborhood, close circles of colleagues, or same leader's constituency, etc. For example, Arora and Sanditov (2015) studied social networks of farmers in a village in Southern India and found that clusters of farmers in this network are formed around cluster leaders, most of whom are important persons in the village.

[10]The lower the value of α, the shorter the effective distance across which utility spillovers flow and, consequently, the lower the importance of agent A bridging the three clusters.

According to the typology of brokerage roles in social network analysis (Gould and Fernandez, 1989), the agent in the center (type A) is a 'liaison broker' because she bridges otherwise disconnected agents (type B) belonging to different clusters. An agent of type B is a 'gatekeeper' because she connects other members of her group (type C) to the outside world, across a structural hole (Burt, 2000). Agents of type C do not fill any structural hole and play no brokerage roles in this network.

Table 1: Values of χ_i, B_{ii}, and contributions to public good, y_i, for the three types of agents in a network shown in Figure 1.

	χ_i	B_{ii}	y_i	Brokerage role
A	0.21	1.29	0.49	*Liason*
B	0.26	1.55	0.39	*Gatekeeper*
C	0.42	1.44	0.02	–

An individual of type C has no direct inter-cluster ties, and hence investment in the global public good for her is less attractive than investment in the private (local public) good, as she receives only limited benefits from the related increase in payoffs of individuals outside her cluster (low values of B_{ij} outside blocks in Figure 1b). Therefore, in comparison with the others, individuals of type C contribute the least to the global public good compared to the other individuals (Table 1).

Consider the liaison broker at the center of the network (type A). Her ego-network has the same size as networks of individuals of type C (each having three direct neighbors), but A's network is extremely sparse – there are no connections running between the three of her neighbors. Although A is positioned outside of any cluster, her ego-network spans all of them. In contrast to individuals of type C, her balance is in favor of investing more in the global public good, because the intensity of spillovers from all other agents B_{ij} is not negligible (first row of matrix $\mathbf{B}$ as shown in Figure 1b), while the intensity of return or 'circular' spillovers B_{ii} is relatively low.

An individual of type B belongs to a cluster but has one external tie (with agent A). This tie exposes her to utility flows from the 'outside world' and gives her more incentive to invest in the global public good *vis-à-vis* individuals C. At the same time, being a part of a tightly connected cluster, an individual B also enjoys higher intensity of return spillovers (unlike A who has the lowest B_{ii}). In addition, B's ego-network is larger than the networks of any individual of another type. As a result, with respect to the global public good, an individual of type B contributes more to the production of public good than individuals C, but less than individual A.

With respect to overall production of the global public good (therefore social efficiency), having agents breeding inter-cluster ties, like individual A in the social network shown in Figure 1, is important for two reasons. First, as discussed above, such agents tend to contribute more to the (global) public good than agents with a similar number of connections but whose connections are only local (agents of type

C). Second, by bridging otherwise disconnected clusters, they let utility flows spill over cluster boundaries, allowing other agents to internalize part of the positive externalities from their investment in the global public good and therefore encouraging the production of the public good.

The literature on social capital distinguishes between 'bridging' and 'bonding' social capital. Bonding social capital is associated with relationships in close-knit networks, i.e. in networks where individuals know each other, such as families, close circles of colleagues, etc. By contrast, 'bridging' capital involves 'cross-cutting ties' linking together groups of individuals otherwise distant or disconnected (Burt, 2000). In the context of our discussion, a *bonding link* refers to a social relationship within a tightly knit group of individuals, while a *bridging link* is a social tie that cuts across such groups. In the social network shown in Figure 1a, ties between agents of type C and ties between agents of type B and C are bonding ties, while the ties involving agent A are bridging ties.

The results of our model suggest that, other things being equal, bridging ties stimulate investment in the (global) public good, while bonding ties tend to encourage private consumption (local public good). In a generic network, however, the effect of adding a tie largely depends on various characteristics of the positions of agents connected by the tie. As a consequence, singling out the effect of bridging/bonding ties on the global public good is no trivial task. Nevertheless, the effect becomes salient on locally-dense regular network structures. As an example, consider the network shown in Figure 2a. In this network, agents are placed on a circle at locations indexed $1, 2, \dots, n$. An agent is connected to k closest neighbors on each side and all connections are of the same strength α. Hence, the degree of a node in this network is equal to $2k$.

Let us place a tie between two agents who are not connected (e.g. shown by a dashed line at Figure 2a).[11] A tie placed between agents located close to each other on the circle corresponds to a bonding tie because the network is locally dense and has many alternative paths running between closely located agents (as is likely in cases where agents belong to the same tightly-knit community). On the other hand, a tie between agents located at the opposite sides of the circle is more of a bridging tie – such agents belong to different communities that are distant from each other. Thus, varying the distance between agents connected by the newly added tie, we obtain a spectrum of bonding/bridging ties.

Before proceeding to the effect of bonding/bridging social capital, notice that, in accordance with Lemma 1, the larger the value of α, the stronger must be the effect of the social network on public good provision for both bridging and bonding ties. Figure 2b reports on the change in the provision of the (global) public good due to the addition of a new link, ΔY, as a function of the strength of social relationships

[11]The value of α must be lower than $1/(2k+1)$ for matrix $(\mathbf{I} - \mathbf{A})$ to be dominant diagonal.

α for the ties placed between agents at the distance d =3, 5 and 7. As expected, ΔY is an increasing function of α.

The panel chart in Figure 2c shows the effect of the shortcut distance, d, on the change in provision of the global public good corresponding to the addition of a new tie, ΔY. As one can see, the longer the shortcut distance, the more resources are spent on production of the public good. This result suggests that, other things being equal, a bridging tie that brings together agents from distant communities has a stronger effect on private provision of the (global) public good than a bonding tie does and this, in fact, holds for any value of $\alpha \in (0, 1/(2k+1))$.

Notice that 1-D lattice networks such as the one shown in Figure 2a are specific instances of "small-world" networks of the Watts-Strogatz model (Watts and Strogatz, 1998). In this model, each link in a regular lattice is 'rewired' with a given probability p: if the value of p is small, then the resulting network is similar to the original regular network, and hence its local structure is rich in bonding ties. In contrast, if the probability of rewiring p is high, the local structure disappears and the resulting network structure becomes similar to an Erdös-Renyi random graph where most connections are bridging ties. In this family of networks ranging from a regular network to a random graph, p serves as an index of randomness (see Figure 3a). Using Monte-Carlo simulations, we can examine how the effect of the shortcut distance on provision of the public good depends on the randomness (hence local structure) of the network.

Figure 3b presents a panel chart with the results of numerical experiment for two values of $\alpha = 0.01$ and 0.05 and three values of rewiring probability $p = 0$, 0.1 and 1.[12] The left-most column of the panel corresponds to $p = 0$, which is the regular 1-D lattice and the same as the diagrams in the top row of Figure 2c. In locally dense regular networks saturated with bonding links, a bridging social tie has a larger effect than a bonding tie. This also holds true for moderately rewired networks ($p = 0.1$). However, as the local network structure becomes less pronounced, the effect of the shortcut distance vanishes (right column, $p = 1$).

5 Conclusions

What does our model say with respect to the two economic contexts mentioned in the introduction of this paper? The first and obvious conclusion is that networks of mutual care and reciprocity may help to mitigate the classical problem of under-provision of the public good. An early author on social capital wrote: "The individual is helpless socially, if left entirely to himself. [...] If he may come into contact

[12]From the population of random small-world networks, we sample only networks satisfying two sets of conditions: they produce dominant diagonal matrices $(\mathbf{I} - \mathbf{A})$ and they meet the conditions of Proposition 1. With larger values of α, Monte-Carlo simulations become impractical because most randomly generated networks fail those conditions.

with his neighbor, and they with other neighbors, there will be an accumulation of social capital, which may immediately satisfy his social needs and which may bear a social potentiality sufficient to the substantial improvement of living conditions in the whole community." (Hanifan, 1916, p.130) In our model, it is the network of social relationships that helps an individual to appreciate the fact that a personal contribution to the provision of a public good can make not only a direct difference for herself, but also her action has an effect on the whole community which pays back in terms of increased well-being of the others for whom she cares.

The model, however, is overly simple to account in full for the complex dynamic relationships between individual action and social capital. While social capital is often used in the implementation of community-level development projects, it is also simultaneously built as it is used, not least through the establishment of community public services (cf. Falk and Kilpatrick, 2000). In addition to being an attribute of a community as a whole (for example, in the form of generalized trust), social capital has also been argued to be an attribute of individuals (Bourdieu, 1986; Portes, 2000; Paldam, 2005). The latter is observable as the "ability of people to cooperate voluntarily" (Portes, 2000, p.2), or as the "size" or structure of an individual's network ties (Burt, 2000; Bourdieu, 1986, p.249). It is then reasonable to believe that the two types of social capital (at the level of a community as a whole and at the individual level) are complementary and mutually reinforcing. For instance, by cooperating voluntarily with each other, individuals may build generalized trust in their community, which may, in turn, nurture voluntary cooperation between individual community members. Thus, the effect of social network is likely to be much stronger than what our model may have predicted.

In the context of innovation networks, it is widely accepted that R&D collaboration networks between organizations are an important mechanism by which new technological knowledge diffuses through modern economies. This belief is also largely shared by policy makers; for instance, European Framework Programmes for research and technological development have devoted significant attention to policy instruments for encouraging and strengthening R&D networks within the European Research Area. The results of our model suggest that R&D networks do not only serve as mechanisms for knowledge diffusion, but may also create additional incentives (e.g. through 'circular' spillovers) to contribute to the production of 'global' knowledge relevant to all R&D players as opposed to producing 'local' knowledge that is relevant only to a focal firm and its closest neighbors.

Another result from the model with respect to the overall level of investment in public goods is that not only does having social ties between agents matters, but also important is the knowledge of who these ties actually connect. Our discussion of modular small-world type networks suggests that a 'cohesive' network structure rich in social ties spanning across families, neighborhoods, and circles of close friends can do better than a ‘segmented’ network structure where links are local to homogeneous groups. In the latter segmented network structure, an individual finds that investment in a local public good, which benefits the individual and members of her close social circle, brings larger returns than investment in a global public good that benefits all individuals in the entire network indiscriminately. In contrast, in a ‘cohesive’ network structure, an individual would be more willing to invest in a global public good because a social network spanning different clusters allows her to observe that investment in the global public good can make a difference for other individuals in her wider network beyond the circle of family and close friends (without producing any detrimental effects). This result seems to be in line with some empirical evidence on community-based development projects, which finds difference in the effect of ‘bridging’ and ‘bonding’ social capital in the establishment of public services (e.g. Motiram and Osberg, 2010).

With respect to R&D networks, this result implies that the impact of policies to encourage R&D collaborations will depend on how policy-induced connections fit into the pre-existing structure of knowledge networks. Much work in the economic and management literature on innovation networks emphasizes the importance of long-distance ties, particularly for more substantial non-incremental innovations (e.g. Burt, 2000; Cowan and Jonard, 2003; Uzzi and Spiro, 2005; Cowan and Jonard, 2007). By bridging otherwise distant parts of social and innovation networks, inter-cluster connections may foster information exchange, maintain diversity of knowledge and enhance individual creativity. Our model offers a different (and complementary) rationale for higher efficiency of network structures with inter-cluster bridging links: in a network with diverse ties, agents (individuals, firms, regions) have more incentive to invest in a ‘global’ public good because the network allows them to internalize larger part of positive externalities resulting from such investment. Thus, policies to encourage collaborations within densely connected clusters (e.g. between firms within the same region) might end up stimulating an agent to invest in a ‘local’ type of knowledge beneficial to the agent and her closest network neighbors. Such a sub-optimal outcome may be countered by promoting the formation and strengthening of inter-cluster connections that stimulate agents’ investments in the production of less specific, general knowledge. Alternatively, to encourage the production of such ‘global’ knowledge, policy makers may be better off promoting innovation in existing R&D networks that are sparsely connected.

References

Arora, Saurabh and Bulat Sanditov, “Cultures of Caste and Rural Development in the Social Network of a South Indian Village,” *SAGE Open*, 2015, *5* (3).

Arrow, Kenneth, “Economic Welfare and the Allocation of Resources for Invention," in R.R. Nelson, ed., *The Rate and Direction of Inventive Activity: Economic and Social Factors*, National Bureau of Economic Research, September 1962, pp. 609–626.

Balland, Pierre-Alexandre, "Proximity and the Evolution of Collaboration Networks: Evidence from Research and Development Projects within the Global Navigation Satellite System (GNSS) Industry," *Regional Studies*, 2012, *46* (6), 741–756.

Ballester, Coralio, Antoni Calvó-Armengol, and Yves Zenou, "Who's Who in Networks. Wanted: The Key Player," *Econometrica*, September 2006, *74* (5), 1403–1417.

Becker, Gary, "A theory of social interactions," *Journal of Political Economy*, 1976, *82*, 1063–1094.

Bergstrom, Theodore C., "Systems of Benevolent Utility Functions," *Journal of Public Economic Theory*, 1999, *1* (1), 71–100.

_ , "Benefit-Cost in a Benevolent Society," *American Economic Review*, March 2006, *96* (1), 339–351.

_ **and Richard C. Cornes**, "Independence of Allocative Efficiency from Distribution in the Theory of Public Goods," *Econometrica*, November 1983, *51* (6), 1753–65.

Bloch, Francis and Ünal Zenginobuz, "The effect of spillovers on the provision of local public goods," *Review of Economic Design*, November 2007, *11* (3), 199–216.

Boschma, Ron, "Proximity and Innovation: A Critical Assessment," *Regional Studies*, 2010, *39* (1), 61–74.

Bourdieu, Pierre, "The Forms of Capital," in J. Richardson, ed., *Handbook of Theory and Research for the Sociology of Education*, Greenwood, 1986.

Bourlès, Renaud and Yann Bramoullé, "Altruism in Networks," 2013. mimeo.

Bramoullé, Yann and Rachel Kranton, "Public goods in networks," *Journal of Economic Theory*, July 2007, *135* (1), 478–494.

_ **,** _ **, and Martin D'Amours**, "Strategic Interaction and Networks," *American Economic Review*, 2014, *104* (3), 898–930.

Burt, Ronald, "The Network Structure of Social Capital," *Research in Organizational Behaviour*, 2000, *22*, 345–423.

Cohen, Wesley M. and Daniel A. Levinthal, "Innovation and Learning: The Two Faces of R&D," *Economic Journal*, September 1989, *99* (397), 569–96.

Cowan, Robin and Nicolas Jonard, "The dynamics of collective invention," *Journal of Economic Behavior & Organization*, December 2003, *52* (4), 513–532.

_ **and** _ , "Structural Holes, Innovation and the Distribution of Ideas," *Journal of Economic Interaction and Coordination*, 2007, *2* (2), 93–110.

David, Paul A., "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox," *AEA Papers and Proceedings*, 1990, *80* (2), 335–361.

Elliott, Matthew and Benjamin Golub, "A Network Approach to Public Goods," in "Proceedings of the Fourteenth ACM Conference on Electronic Commerce" EC '13 ACM New York, NY, USA 2013, pp. 377–378.

Falk, I. and S. Kilpatrick, "What is social capital? A study of interaction in a rural community," *Sociologica ruralis*, 2000, *40* (1), 87–110.

Gambardella, Alfonso and Anita M. McGahan, "Business Model Innovation: General Purpose Technologies and their Implications for Industry Structure," *Long Range Planning*, 2010, *43*, 262–271.

Gould, R.V. and R.M. Fernandez, "Structures of Mediation: A Formal Approach to Brokerage in Transaction Networks," *Sociological Methodology*, 1989, *19*, 89–126.

Hanifan, L. J., "The Rural School Community Center," *Annals of the American Academy of Political and Social Science*, 1916, *67*, pp. 130–138.

Horn, Roger A. and Charles R. Johnson, *Topics in matrix analysis*, Cambridge University Press, 1994.

Isham, Jonathan and Satu Kähkönen, "How do participation and social capital affect community-based water projects? Evidence from Central Java, Indonesia," in Christiaan Grootaert and Thierry van Bastelaer, eds., *The Role of Social Capital in Development: An Empirical Assessment*, Cambridge University Press, 2002.

Jackson, Matthew O. and Yves Zenou, "Games on Networks," in Peyton Young and Shmuel Zamir, eds., *Handbook of Game Theory, Vol. 4*, Elsevier Science, 2014.

Ley, Eduardo, "Optimal provision of public goods with altruistic individuals," *Economics Letters*, January 1997, *54* (1), 23–27.

Motiram, Sripad and Lars Osberg, "Social Capital and Basic Goods: The Cautionary Tale of Drinking Water in India," *Economic Development and Cultural Change*, 2010, *59* (1), 63–94.

Paldam, Martin, "Social Capital and Social Policy," in "Paper presented at New Frontiers of Social Policy Conference" 2005.

Pargal, Sheoli, Daniel Gilligan, and Mainul Huq, "Does social capital increase participation in voluntary solid waste management? Evidence from Dhaka, Bangladesh," in Christiaan Grootaert and Thierry van Bastelaer, eds., *The Role of Social Capital in Development: An Empirical Assessment*, Cambridge University Press, 2002.

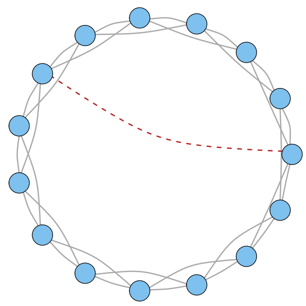
Portes, Alejandro, "The Two Meanings of Social Capital," *Sociological Forum*, 2000, *15* (1), 1–12.

Uzzi, Brian and Jarrett Spiro, "Collaboration and Creativity: The Small World Problem," *American Journal of Sociology*, 2005, *111* (2), 447–504.

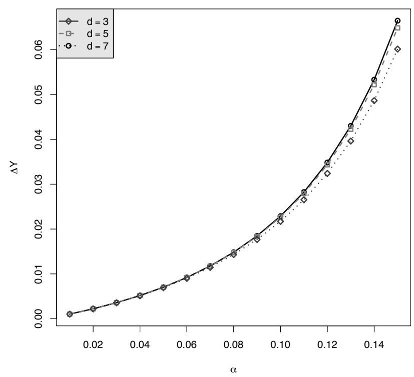
Watts, Duncan J. and Steven H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, 1998, *393* (6684), 440–442.

Wellman, Barry, "The Community Question: The Intimate Networks of East Yorkers," *American Journal of Sociology*, 1979, *84* (5), 1201–1231.

(a) Regular lattice with a shortcut



(b) Effect of the strength of social ties



(c) Effect of shortcut distance on the provision of the public good on a regular lattice

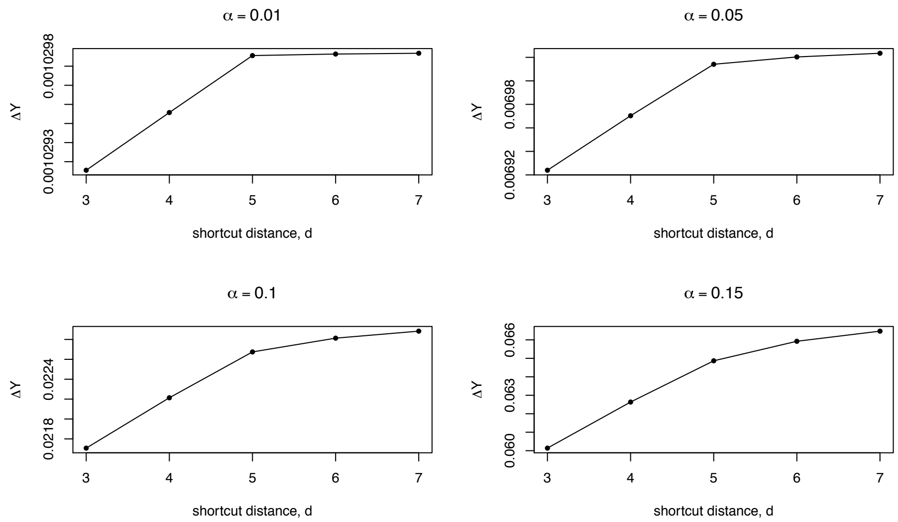
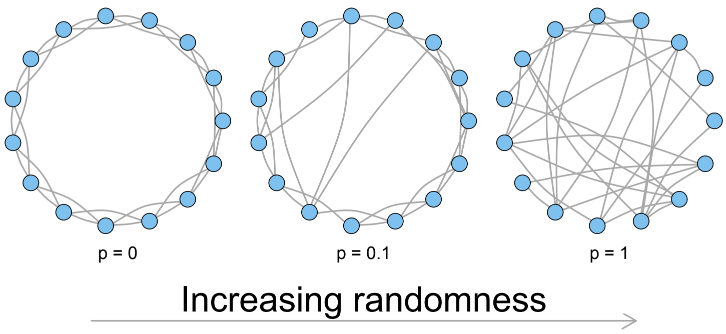


Figure 2: The effect of a bridging tie ($\epsilon = 0.8$). **a:** Regular lattice of 15 agents and 30 links with a shortcut placed between agent 1 and agent 7 (dash line). All links have the same strength, α. **b:** The change in the provision of the public good due to a shortcut ΔY as a function of the strength of social relationships α for shortcuts at distance 3,5, and 7. **c:** The change in the provision of the public good due to a shortcut ΔY as a function of shortcut distance d for several values of α

(a) Watts-Strogatz “small-world” networks



(b) Effect of shortcut distance on the provision of the public good in “small-world” networks

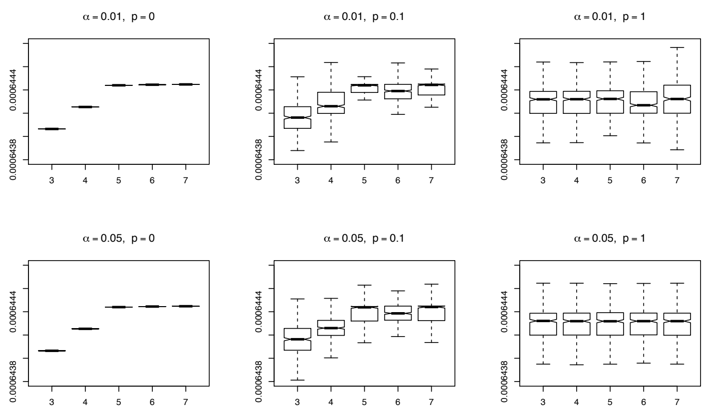


Figure 3: The effect of a bridging tie in Watts-Strogatz “small-world” networks for p =0, 0.1 and 1 and α =0.01 and 0.05. **Top:** Watts-Strogatz “small-world” networks. **Bottom:** Box-and-whisker plot of ΔY as a function of shortcut distance d in “small-world” networks (500 simulation runs for each α,p, and d)

Recent papers in the SPRU Working Paper Series:

SWPS 2015-24. Lee Stapleton. August 2015. Do Academics Doubt Their Own Research?

SWPS 2015-25. Mariana Mazzucato. September 2015. From Market Fixing to Market-Creating: A New Framework for Economic Policy.

SWPS 2015-26. Caetano Penna, Mariana Mazzucato. September 2015. The Rise of Mission-Oriented State Investment Banks: The Cases of Germany's KfW and Brazil's BNDES.

SWPS 2015-27. Aldo Geuna, Matteo Piolatto. October 2015. Research Assessment in the UK and Italy: Costly and Difficult, but Probably Worth (at Least for a While).

SWPS 2015-28. Mariana Mazzucato. October 2015. The Green Entrepreneurial State.

SWPS 2015-29. Lorenz Gollwitzer, David Ockwell, Adrian Ely. November 2015. Institutional Innovation in the Management of Pro-Poor Energy Access in East Africa.

SWPS 2015-30. Ben Martin. November 2015. Twenty Challenges for Innovation Studies.

SWPS 2015-31. Colin Nolden, Steve Sorrell, Friedemann Polzin. November 2015. Innovative Procurement Frameworks for Energy Performance Contracting in the UK Public Sector.

SWPS 2015-32. Gisela Di Meglio, Jorge Gallego, Andrés Maroto, Maria Savona. November 2015. Services in Developing Economies: A new chance for catching-up?

SWPS 2015-33. Gabriele Pellegrino. November 2015. Barriers to Innovation: Can Firm Age Help Lower Them?

SWPS 2015-34. Ohid Yaqub. December 2015. Explaining Variation in Medical Innovation: The Case of Vaccines, and the HIV AIDS effort.

Suggested citation:
Bulat Sanditov, Saurabh Arora (2015). Social Network and Private Provision of Public Goods. SPRU Working Paper Series (SWPS), 2015-35: 1-27. ISSN 2057-6668.
Available at www.sussex.ac.uk/spru/swps2015-35

SPRU – Science Policy Research Unit
University of Sussex
Falmer, Brighton, BN1 9SL,United Kingdom
www.sussex.ac.uk/spru
Twitter: @SPRU
SWPS: www.sussex.ac.uk/spru/research/swps