What is a Complex Innovation System?

J. Sylvan Katz
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C.A.McLeish@sussex.ac.uk

Contact
T.Ciarli@sussex.ac.uk
D.Rotolo@sussex.ac.uk

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J. Sylvan Katz

SPRU, Jubilee Building
University of Sussex
Falmer, Brighton, BN1 9SL, UK

Johnson-Shoyama Graduate School of Public Policy,
University of Saskatchewan Campus,
Diefenbaker Building, 101 Diefenbaker Place,
Saskatoon, SK, Canada, S7N 5B8

Science Metrix
1335, Mont-Royal E.
Montreal, Quebec
Canada H2J 1Y6

j.s.katz@sussex.ac.uk
Abstract

Innovation systems are frequently referred to as complex systems, something that is intuitively understood but poorly defined. A complex system dynamically evolves in non-linear ways giving it unique properties that distinguish it from other systems. In particular, a common signature of complex systems is scale-invariant emergent properties. A scale-invariant property can be identified because it is solely described by a power law function, \( f(x) = kx^\alpha \) where the exponent, \( \alpha \), is a measure of the scale-invariance. The purpose of this paper is to describe and illustrate that innovation systems have properties of a complex adaptive system and in particular scale-invariant emergent properties indicative of their complex nature. These properties can be quantified and used to inform public policy.

The global research system is an example of an innovation system. Peer-reviewed publications containing knowledge are a characteristic output. And citations or references to these articles are an indirect measure of the impact the knowledge has on the research community. These measures are used to illustrate how scale-invariant properties can be identified and quantified.

Peer-reviewed papers indexed in Scopus and in the Web of Science were used as the data sources to produce measures of sizes and impact. Papers indexed in Scopus were classified into fields using the Scopus, NSF and MAPS schemes. The evolution of the overall and field level impact distributions were examined to see if they had a reasonable likelihood of being scale-invariant as they aged. Also, correlations between impact and size were explored to see if they were scale-invariant too.

The findings show that the distribution of impact has a reasonable likelihood of being scale-invariant with scaling exponents that tended toward a value of less than 3.0 with the passage of time. Scale-invariant correlations are shown to exist between the evolution of impact and size with time and between field impact and sizes at points in time. However, it was found that care must be exercised making these measures as the method of classification may hide emerging properties.

The confirmation that an innovation has common characteristics of an adaptive complex system and scale-invariant emergent properties allows us to confidently say that the global research system has a reasonable likelihood of being a complex innovation system. And using the recursive nature of scale-invariance it is reasonable to assume that regional, national, local and sectoral level systems contained within it are complex with scale-invariant properties too. Measures and models based on the scale-invariant properties of complex innovation systems can provide new and novel insights about an innovation system useful for informing public policy.

Keywords: innovation system, scale-invariant, scale-free, complex system, complex adaptive system, power law, innovation policy, bibliometric, scientometric
What is a complex innovation system?

1. Introduction

First, let’s ask another question. “Why should we care if an innovation system is or is not complex?” Complex systems have unique properties that are distinctly different from those of other systems. Most, if not all, real-world complex systems possess a particular property - *scale-invariance* - that can be measured and quantified. Conventional measures often used as indicators of performance of an innovation system are incapable of quantifying this property (Katz, 2012). If an innovation system is not complex then there is no need to worry and the current indicator will suffice. However, if an innovation system is complex it will be shown that *scale-independent* measures are needed to fully inform innovation policy. This paper focuses on the identification and quantification of scale-invariant emergent properties of an innovation system to illustrate that it is a complex system.

Some people might ask if we can answer the question “Is a given innovation or research system complex?” By definition the only answer is yes. The logic is simple. An innovation system is a social construct created by a biological system of humans. By definition biological systems, particularly humans are complex adaptive systems. Innovation or research is an emergent property illustrative of their adaptive capabilities. However, this simple answer doesn’t provide any guidance as to how to identify and use naturally occurring scale-invariant characteristics of a complex innovation system to inform public policy. We will focus on the more general question “What is a complex innovation system?” requiring descriptions and illustrations of how to identify, measure and use the unique property of scale-invariance.

The design of effective innovation policy to benefit society and the economy is partially predicated on the notion that decision makers have reliable evidence-based measures to inform their decisions (Husbands, Lane, Marburger, & Shipp, 2011; Lane, 2010). This is an area of intense investigation as witnessed by recent articles on Science Metrics in Nature and the National Science Foundation’s (NSF) The Science of Science & Innovation Policy (SciSIP) program (Abbot et al., 2010; Lane, 2009).

An innovation system is a network of organisations within an economic system involved in the creation, diffusion and use of scientific and technological knowledge as well as the organisations responsible for the coordination and support of these processes (Dosi, 1988). The concept of innovation refers to the development, adaptation, imitation and adoption of knowledge and technologies that are new to a given context. Innovation systems can found at many levels of the economy such as global, regional, national, local and sectoral levels. National systems of innovation have been a particular area of strong interest with many peer-reviewed papers on the topic (Filippetti & Archibugi, 2011; Freeman, 1995; Lundvall, Johnson, Andersen, & Dalum, 2002).

Innovation is an interactive process between many actors, including companies, universities and research institutes (Leydesdorff & Etzkowitz, 1996). Innovation does not follow a linear
path that begins with research and then moves through development, design, engineering and production resulting with the introduction of new products and processes. Innovation is a non-linear process with feedback between the different stages of development.

The character of an innovation system emerges from the interactions between its members and the members of other systems. Some of the interactions are more “rule-like” than others because they are governed by laws, regulations, treaties, etc. Other interactions are more random because they are governed by personal, social, political and economic forces. Economic emergence in an innovation system, that is the appearance of economic structures that cannot be explained by examining their components, requires that their analysis be fully embedded in complex economic system theory (Foster & Metcalfe, 2012; Harper & Endres, 2012). Intuitively we know that innovation and the systems in which it is embedded must be complex but how can it be shown?

As mentioned earlier innovation is a non-linear process often seen as an emergent property of a complex adaptive social system (Cooksey, 2011; Dougherty & Dunne, 2011; Katz, 2000; Kuhlmann et al., 1999; Lemay & Sá, 2012; Pavitt, 2003; Rose-Anderssen, Allen, Tsinopoulos, & McCarthy, 2005). Non-linear effects from feedback occur throughout the innovation process; non-linear processes are known generator of scale-invariant properties. And, it has been demonstrated that conventional measures used as indicators of properties of an innovation system are deficient in their ability to characterise scale-invariant emergent properties (Katz, 2006; Katz & Cothey, 2006).

The remaining sections of this paper describe and illustrate how complex systems theory and techniques used to identify and quantify scale-invariant emergent properties can be applied to innovation systems to better inform public policy. First an overview of complex systems theory and techniques for identifying and quantifying scale-invariance is provided. Scale-invariant properties of innovation systems that have already been identified are reviewed. And the principles discussed in the preceding sections are applied to size and impact data to illustrate how scale-invariant properties can be identified and measured. The objective is to provide a clearer understanding of what a complex innovation system is.

2. Complex Systems and Scale-Invariance

Over the past few decades an extensive literature has been published on the study of complex physical, biological and social systems. Complex systems differ from complicated systems. Generally speaking a complicated system is understood through structural decomposition while a complex system is understood through a functional analysis (Poli, 2013). Complicated systems tend to be distinctive and specialized occurring relatively rarely while complex systems tend to be generic and pervasive. Complex systems have some generally accepted properties (Baranger, 2001; Marković & Gros, 2014; Vicsek, 2002). Their structure spans several scales. Their constituents are interdependent and interact in nonlinear ways. These interactions give rise to novel and emergent dynamics. The combination of structure and emergence is viewed as self-organization.

There are a number of types of complexity such as algorithmic, computational, mathematical, physical and symbolic complexity. We will focus on physical and symbolic
complexity exemplified by biological systems and human languages. These are two types of systems that most people intuitively agree are complex (Stephens, 2012).

Humans generate speech and written words from a need to communicate meaning in a given world or social context; their utterances obey a complex system of syntactic, lexical, and semantic regularity (Piantadosi, 2014). Symbolic complexity in human language is partially exemplified by two scale-invariant properties: the Zipf distribution of words in documents, irrespective of language, and the Lotka distribution of documents published by researchers and others, irrespective of language and culture. Lotka distributions are used to model scale-invariant properties of human information production processes such as the published output of a research system (Egghe, 2009).

Physical complexity often involves the interplay between chaos and non-chaos producing critical points where self-organization is most likely to occur (Schroeder, 1991; Whitfield, 2005). A complex systems is neither too ‘ordered’ nor too ‘disorder’ but finely balanced between the two (Stephens, 2012). In order for a complex biological system to survive and evolve there must be interplay between competition and co-operation at different scales. For example, this is a fundamental characteristic of insect & animal colonies and human activities. Complex biological/social systems are called adaptive systems because they can adapt to a changing environment. A small subset of adaptive complex systems are self-reproducing and experience birth, growth and death.

Emergent properties are the most often observed real world phenomenon in a complex system. Emergent properties are patterns and regularities arising through interactions among smaller or simpler entities in a system that themselves do not exhibit such properties. In biological systems interactions at lower levels emerge as objects expressing their properties at a higher level (Cohen & Harel, 2007). Emergent properties tend to arise as new objects from one scale to another. Emergent properties are a key generic property of complex adaptive economic system; it is what makes economies become complex (Harper & Endres, 2012).

An emergent property is identified by the scaling behavior of variables describing a structural feature or a dynamical characteristic of the system (Marković & Gros, 2014). In summary scaling behaviour occurs when an identical or statistically similar property occurs at many levels of observation. This property is frequently referred to as a scale-invariant property because it appears to be statistically similar irrespective of scale. It is commonly associated with things like the self-similar structure of geometrical and natural fractals (Mandelbrot, 1967).

Scale-invariance can be perfect, as in the case of a deterministically defined geometrical fractal, or it can be statistical, as in the case of jaggedness of an island shoreline or the billowness of clouds in the sky (Madhushani & Sonnadara, 2012; Mandelbrot, 1967). Scaling properties can be measured and used to characterise attributes of a complex system. Measures based on scale-invariant properties are called scale-independent or scale-adjusted measures (Katz, 2006). Scale-Adjusted Metropolitan Indicators (SAMIs) have been used to make comparisons between cities with scaling properties and provide meaningful rankings.
of urban systems (Bettencourt, Lobo, Strumsky, & West, 2010). Unlike conventional per capita indicators scale-independent measures are dimensionless and independent of size.

Scale-invariance is mathematically defined as \( p(bx) = g(b)p(x) \) for any \( b \) (Newman, 2005). In other words, if we increase the scale or units by which we measure \( x \) by a factor of \( b \), the shape of the distribution \( p(x) \) is unchanged, except for an overall multiplicative constant. The only two mathematic functions\(^2\) that possess this property are shown in Figure 1: (a) power law probability distributions defined by \( p(x) = kx^{-\alpha} \) for \( x \geq x_{\text{min}} \), the value of \( x \) at which the power law tail of the distribution begins, and (b) power law correlations, defined by \( f(x) = cx^n \), where \( k \) & \( c \) are constants. Power law functions are characterized by their linear appearance on a log-log scale. The exponents \( n \) and \( \alpha \), also called scaling factors, are measures of the scale-invariance of properties. From this point forward the symbol \( \alpha \) will be used to denote the exponent of either a scaling distribution or correlation

![Figure 1 – Power law (a) probability distribution and (b) correlation](image)

There is a degenerate form of a power law distribution called a power law with exponential cut-off given by \( f(x) = kx^{-\alpha}e^{-\lambda x} \). In these cases some entities in the far right hand tail of the distribution do not occur with as high a probability as would be expected of a pure power law distribution as seen in Figure 2. For a pure power law scale-invariance is found for all \( x > x_{\text{min}} \). Scale-invariance for a power law with exponential cut-off is limited to the region \( x_{\text{max}} > x > x_{\text{min}} \) where \( x_{\text{max}} \) is the value of \( x \) where the exponential cut-off starts to dominate the power law. The region of scale-invariance between \( x_{\text{min}} \) and \( x_{\text{max}} \) can be several orders of magnitude in size. Some people think that the exponential cut-off of the power law is due to finite size of the data set, but recently it has been shown that it might also be an effect of finite observation time (Kazuko et al., 2006). And models also show that the probability distribution tends to evolve from exponential to a power law with exponential cut-off to a pure power law given enough time.

\(^2\) While \( f(x) = p(x) \) when \( n = \alpha, x_{\text{min}} = 1 \) & \( \alpha > 0 \) for illustrative purposes distributions and correlations are discussed separately
Random and natural populations drawn from a scale-invariant distribution are scale-invariant too. A natural population is one that preserves its clustering, ‘community’ or small world structure (Girvan & Newman, 2002; Palla, Derenyi, Farkas, & Vicsek, 2005). Hence a scale-invariant property likely to occur in large complex innovation system likely will occur for any natural smaller innovation system within it making it complex too. It gets more difficult to statistically confirm the property as system size decreases. We will use this property later.

A number of physical processes are known to generate power law distributions. For example, they can be generated by combinations of exponential distributions (Naranan, 1970; Newman, 2005; Reed & Hughes, 2003). They can be generated using the same multiplicative process that generates lognormal distributions by imposing lower bounds (Mitzenmacher, 2003). This is one reason it is difficult to distinguish a power law from lognormal distribution. Other power law generating mechanisms include specific Yule processes such as preferential attachment and critical phenomena that occur with continuous phase transitions (Barabási & Albert, 1999; Newman, 2005). History-dependent processes are ubiquitous in nature and social systems and frequently exhibit scale-invariant properties (Corominas-Murtra, Hanel, & Thurner, 2015). History-dependent stochastic process associated with complex systems become more constrained as they unfold and their state space or set of possible outcomes decreases as they age. This reduction of state space over time leads to the emergence of power laws. The universal nature of power law distributions may be partially explained by the diversity of mechanisms that can produce them.

The existence of scale-invariant properties maybe necessary but it is not sufficient to define a complex innovation system. Scale-invariance is associated with simple and complex physical systems as well as complex biological/social systems (Stephens, 2012). In addition to scale-invariant properties a complex biological/social system needs to be distinguishable from a physical system.
A physical system only has to do one thing – “be” – it doesn’t make choices and it is constrained by physical laws to find a state of least energy or action. A biological/social system has to make choices consistent with the restrictions imposed by the laws of physics. A complex physical system evolves solely in state space while a complex biological/social system evolves in spaces of state and strategy allowing it to adapt. By definition an innovation system is a biological/social system that makes choices adapting itself and its environment to changing knowledge, practices and technologies and as well as to changing economic, social and political forces in which it is embedded.

A premise of this paper is that since an innovation system has the necessary characteristics to distinguish it from a complex physical system and if it can be shown to have scale-invariant emergent properties then it can be claimed with reasonable certainty that it is a complex innovation system.

3. Innovation Systems and Scaling Properties

Many scaling properties have been observed for innovation systems. For example, the distribution of stock price fluctuations for more than 16,000 US companies was found to be a power law with an exponent ≈ 3.0 (Plerou, Gopikrishnan, Nunes Amaral, Meyer, & Stanley, 1999). The growth dynamic of business and university research activities are scale-invariant (Matia, Nunes Amaral, Luwel, Moed, & Stanley, 2005; Plerou, Amaral, Gopikrishnan, Meyer, & Stanley, 1999). The dynamics are independent of size which the authors suggest is indicative of a universal mechanism involved in the growth dynamics of complex organizations. And a recent study of the European aerospace research area shows a variety of scale-free topologies in joint venture networks (Biggiiero & Angelini, 2015). The researchers explored eight properties of the joint venture activities in 192 FP6 Aerospace projects involving 1165 organizations from 47 countries. They used the Clauset et.al. (2009) methodology that will be explained later and they found that 6 of the 8 properties had a good likelihood of have scale-invariant characteristics.

Cities are society’s predominant engine of innovation and wealth creation (Bettencourt, Lobo, Helbing, Kühnert, & West, 2007). Scaling correlations occur between city sizes and such thing as new patents issued, numbers of inventors, GDP, number of R&D establishments, private sector R&D employment and overall R&D employment. These scale-invariant properties of cities are related to economic productivity and creative output; they are superlinear with scaling exponents > 1.0 (Arbesman, Kleinberg, & Strogatz, 2009; Bettencourt, Lobo, & Strumsky, 2007). Generally speaking, the properties of most socioeconomic systems such as innovation and wealth creation are strongly predicted by scaling laws that are non-linear functions of population size (Bettencourt et al., 2010).

Scale-invariant models composed of a small number of power law correlations have been constructed of the evolution of the European, Canada and Chinese innovation systems. They were built using the scaling correlations between GERD & GDP and GDP & population (Gao, Guo, Katz, & Guan, 2010; Katz, 2006). These models will be discussed in detail later.

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3 The unique case of learning computer programs such as genetic algorithms which do make choices is discussed in Stephens’ paper where he argues that adaption cannot arise from fitness functions that are specified apriori.
Patent based indicators have been used and accepted as measures of innovation for a considerable period of time. A scaling correlation has been shown between firm R&D expenditures and number of patents issued (Bound, Cummins, Griliches, Hall, & Jaffe, 1984). And recently, the distribution of patents among applicants within OECD countries was shown to be scale-invariant using the Clauset et.al (2009) methodology (O’Neale & Hendy, 2012). Recent unpublished analysis\(^4\) using 2013 patent data that included more countries found that at the aggregate level there was a power law distribution with \(\alpha = 1.92\pm0.01\) with \(p = 0.40\)\(^5\). And using the data from the last two columns of Table 1 in the paper it can be seen there is a scaling correlation across the OECD countries between numbers of patents and patent applicants in each country. It has a scaling exponent of 1.37\(\pm0.07\) \((R^2 = 0.95)\) suggesting that patent applications increased 2.6 times for a doubling in number of applicants in a country. Also, a patent-patent citation network constructed from 1963 to 1997 US patents has been shown to be scale-free (Brantle & Fallah, 2007). Patent measures have a variety of scale-invariant properties.

As illustrated above a variety of measures can be used to investigate the scale-invariant properties of an innovation system. However some measures are frequently self-reported, statistically sampled and incomplete. Sometimes they are reported in different units requiring conversion before they can be used for comparative purposes. These data tend to be noisy and inaccurate making them difficult to analyse for comparative purposes. And the quality of these data makes distributional analysis particularly difficult especially when the data are disaggregated into smaller groups.

On the other hand, some properties such as impact and size measured using numbers of citations to peer-reviewed publications and numbers of papers published by a group are accurate and relatively noise free. And a variety of well tested schemes are available to agglomerate publications into things like research fields and subfields of investigation to examine effects of scale. Large datasets covering decades of published research are available making them ideal for illustrating scale-invariant emergent properties of an evolutionary system. Datasets containing measures of other properties are frequently orders of magnitude smaller making distributional analysis at smaller scales problematic.

Peer-reviewed publications often containing new knowledge are a common output of the global research system based in institutions and organizations like universities, hospitals, government research labs, private sector labs, etc. References or citations to these articles are used as an imperfect but quantifiable indirect measure of the impact this knowledge has on the community. Publication and citation data were derived from two widely accepted data sources: the Web of Science (WoS) and Scopus. Peer-reviewed publications and citations to them are used in this paper as examples of clean, reliable and reproducible measures of size and impact to explore scale-invariant properties of the global research system as an exemplar innovation system. The same principles can be applied to measures such as numbers of patents and patent applicants as discussed earlier.

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\(^4\) Personal communication with Dr. O’Neale 06/07/2015

\(^5\) \(p>1.0\) is significant – see Clauset et.al. for an explanation
An interesting property of scale-invariance is its recursive nature. For example, if we know that a property of the global research system is likely to be scale-invariant then any regional, national, local, sectoral, etc research system within it will likely have that scale-invariant property too. And if the global research system is a complex innovation system then smaller research systems contained within it are complex too.

3.1 Scaling Distributions

Citation distributions have been the object of intense investigation for fifty years. Price reported the right skewed nature of these distributions and later proposed a ‘cumulative advantage’ mechanism to explain the power law nature of these heavy tailed distributions (de Solla Price, 1965, 1976). However, until recently a robust methodology for determining the likelihood of a real-world distribution having a power law tail had not been generally accepted.

In the past a standard method for determining if a distribution was a power law was to use linear regression methods on log transformed data. However, Rousseau and others had suggested that the maximum likelihood estimation technique was better (Rousseau & Rousseau, 2000). In 2005 an in-depth article reviewed the theories and empirical evidence for the existence of power-laws as well as generating mechanisms proposed to explain them (Newman, 2005). And in 2009 an excellent paper appeared in SIAM that detailed a comprehensive methodology for determining the existence of a power law distribution is gaining wide acceptance (Clauset, Shalizi, & Newman, 2009). The article examined the theory, practice and difficulties for determining the best fit model to real-world empirical data that have power law probability distributions. It convincingly showed that the technique of using a linear regression of log transformed data is flawed and that the maximum likelihood estimation technique could be used to draw meaningful conclusions about real-world data. Also the authors supplied the R:, C and Matlab routines required for the analysis for use by others.6

A few papers have appeared recently that used the Clauset et.al. techniques to explore citation distributions. Two papers looked at field level citation distributions for 1998-2002 WoS and Scopus data (Albarrán, Crespo, Ortuño, & Ruiz-Castillo, 2011; Brzezinski, 2015). Albarrán and colleagues used WoS data and Brzezinski used Scopus data. Both groups used a 5 year fixed citation window to create the citation distributions and both encountered a similar difficulty. The WoS and Scopus field/subfield assignments allow a journal, hence an article, to be assigned to more than one field/subfield. The result is there is a good chance that highly cited articles will appear in more than one field/subfield. This has the potential of impacting the shape of the tail of the citation distributions.

Albarrán et.al. reported that the distribution for 17 of 22 (77%) fields and 140 of 219 (64%) subfields had reasonable likelihood of being power law distributions. The authors did not check to see if any other heavy tailed distributions might fit the data. Brzezinski reported the citation distributions of 14 of 27 (52%) Scopus fields had a reasonable likelihood of being power law distributions. He computed the log-likelihood ratio as describe in Clauset et.al. to

6 Available at http://tuvalu.santafe.edu/~aaronc/powerlaws/
see if a given distribution was best fit by a power law or another heavy tailed distributions. Both groups combined multiple years of publications and they combined the citation counts to these publications. It is unclear what effect mixing citation distributions for multiple years has on the overall shape of the distribution compared to looking at the citation distributions for individual years.

Using the same methodology a longitudinal study examined the evolution of the citation distributions to peer-reviewed papers indexed in the WoS between 1984 and 2002 and cited from the year of publication to 2009 (Katz, 2012). This approach provided observation windows ranging from 6 to 25 years. Also, articles were uniquely assigned to one of 13 fields using the National Science Foundation (NSF) journal classification scheme\(^7\) so that citation distributions at the field level could be explored. The scaling exponents, \(\alpha\) that had significant p-values\(^8\) for annual citations to all papers declined over time as the distribution evolved and it approached a value near 3.0, a value first reported by Redner (Redner, 1998). In addition, the citation distributions for 7 of 13 (54\%) fields and 124 of 276 (45\%) subfields had scaling exponents with significant p-values and \(\alpha<3.0\).

Power law distributions with \(\alpha<3.0\) have infinite variance (Newman, 2005). They don’t reside in the domain of attraction of Gaussian distributions\(^9\); hence, the Central Limit Theorem no longer applies and population averages cannot be used to characterise them. The population average is only significant when the variance is finite. However, real data are finite and have a finite sample maximum but given any growing population with a maximum \(x\) value at a point in time there is a non-negligible chance at some later time the maximum value will be exceeded. If one calculates the means of random samples drawn from a power law distribution with \(\alpha<3.0\) the values of the means will have a power law distribution and vary over orders of magnitudes. And the variance grows as \(n^{(3-\alpha)/(\alpha-1)}\) where \(n\) is the size of the tail at a point in time.

Population averages cannot accurately characterize real-world distributions with \(\alpha<3.0\). Many traditional measures including things like the mean-normalized citation score\(^10\) (MNCS) which are used for comparative and evaluative purposes are valid only for Gaussian population distributions and power law distributions with \(\alpha\geq3.0\) (Waltman, van Eck, van Leeuwen, Visser, & van Raan, 2011). Such measures are poor indicators of the emerging properties of real-world complex systems that usually have scaling exponents in the range \(2<\alpha<3\) (Clauset et al., 2009; Katz, 2006; Newman, 2005).

Many networks have small world properties where the diameter\(^11\) of the network \(d = \log N\) and \(N\) is the number of nodes in the network (Newman, 2001). Complex networks like citation networks tend to have scale-invariant degree distributions. As mentioned previously real-world complex networks usually have scaling exponents with \(2<\alpha<3\). When the magnitude of the exponent is in this range the average diameter of the network shrinks

\(^7\) Updated by Science-Metrix, Montreal, Canada
\(^8\) \(P \geq 0.10\) as suggested by Clauset et. al. is considered significant
\(^9\) Newman (2011) SIGMETRICS posting
\(^10\) MNCS is the observed citation impact of a paper is divided by the expected citation impact, i.e. the average citation impact of papers from the same publication year and subject.
\(^11\) The diameter of a network is the average distance between nodes in the network
from logN to loglogN. So if N=10^{10} the mean distance between nodes shrinks from 10 when \( \alpha>3 \) to 1 when \( \alpha<3 \). The network becomes an ultra-small world network (Cohen & Havlin, 2003).

Until recently it was difficult to examine the evolution of large citation distributions. Usually a snapshot was taken at a point in time of the distribution of citations to papers published in 1 or more contiguous years using a fixed size citation window. These data were analyzed and conclusions drawn about the likelihood that these distributions were power law distributions or other heavy-tailed distributions (e.g. log-normal, Poisson, stretched exponential). It is difficult to do longitudinal studies even using modern high performance computing facilities available on most university campuses. It can take more than 24 hours to run the simulation routine used to determine the p-value for the exponent of a single power law distribution.

An innovation system is a dynamic system and its attributes change with time. Perhaps snapshots of citation distributions are not giving us a full picture. Using a longitudinal approach looking at the evolutionary trends it will be shown that these attributes are not scale-invariant all of the time but they have a reasonable likelihood of being scale-invariant much of the time.

### 3.2 Scaling Correlations

Scale-invariant correlations exist across groups within an innovation system at points in time (Katz, 1999). For example, using 1981-1996 ISI (now WoS) data a scaling correlation with \( \alpha=1.27\pm0.03 \) was found between impact of subject areas measured using citations and the sizes of subject areas measured by numbers of peer-reviewed papers published in the area. The magnitude of the scaling exponent indicates that on average every time the size doubles impact increases \( 2^{1.27} \) or 2.4 times.

The scaling correlation between impact and size was determined across 13 fields and 138 subfields using 1984-2002 WoS data (Katz, 2012). Impact was measured by counting citations to papers using a fixed 6-years citation window. The data were summed over the time interval. It was found that the scaling exponents were the same at the field and subfield levels with \( \alpha=1.28\pm0.09 \) and \( \alpha=1.27\pm0.03 \), respectively.

Scaling correlations with \( \alpha>1 \) have been found between the denominators and numerators of ratios within collections of conventional measures of performance such as citations/paper, GDP/capita and GERD/GDP. Invariably, the numerator is a measure of group size (papers, population & GDP) and used as a normalizing parameter. If these indicators were truly normalized for size we would expect that the scaling correlation between the denominators and numerators would be linear (i.e. \( \alpha=1 \)) indicating effects of size have been removed. Instead they usually have \( \alpha>1 \) indicating the denominators increases non-linearly with size. Many indicators used as comparative measures of the performance of innovation systems are biased by size (Gao et al., 2010; Katz, 2006).

Other scaling correlations are known such as the ones found between numbers of in-links and sizes of University web sites and numbers of collaborative papers and total numbers of
papers published by countries (Archambault, Beauchesne, Côté, & Roberge, 2011; Katz & Cothey, 2006). Impact has been shown to scaling with size from the level of research group and to the level of European Universities (van Raan, 2008a, 2008b). And the sizes of universities have been shown to scale like city sizes and the published scientific output from urban centers scales with the population of the centers (Nomaler, Frenken, & Heimeriks, 2014; van Raan, 2013).

Recently a scaling correlation was reported between impact and numbers of internationally collaborative papers published in Management journals (Ronda-Pupo & Katz, 2015). The scaling correlation between impact and numbers of collaborative papers in each journal was 1.89±0.08 where as the scaling correlation between impact and number of non-collaborative papers was lower with a value of 1.35±0.08. In the field of Management the impact is about 45% greater ($2^{1.89}/2^{1.25}=1.45$) when international collaboration is involved than when it isn’t. This technique can be used to explore the impact of many types of collaboration not just international collaboration across various grouping of entities that publish peer-reviewed research.

Scaling correlations with $\alpha\neq1$ are indicative of emergent structure or self-organization. When the exponent is superlinear, i.e. $\alpha>1.0$, the system exhibits a cumulative advantage and when the exponent is sublinear, i.e. $\alpha<1.0$, it exhibits a cumulative disadvantage. And when $\alpha=1.0$ only linear effects are at play giving no indication of self-organization within the system.

The measured scaling factor for the correlation between citations and papers of $\alpha=1.27 \pm 0.03$ mentioned earlier is a scale-independent measure of the average citedness of peer-reviewed papers produced by fields of different sizes in the global research system. The term citedness is a fuzzy term describing a scale-invariant property akin to terms like the jaggedness of islands and billowness of clouds measured by their fractal dimension - the exponent of a power law relationship.

Any two measures of properties of a system that exhibit exponential growths over time will exhibit a scaling correlation$^{12}$ between those two variables (Katz, 2005). The scaling exponent gives us a measure of the relative growth of the two properties. For example, it has been shown that European and Canadian Gross Expenditures on R&D (GERD) and GDP grew exponential from 1981 to 2000. The scaling exponents for the correlation between these parameters was found to be 1.03 and 1.42 respectively (Katz, 2006). Between 1995 and 2005 the Chinese innovation system had a scaling exponent of 1.67 for the same parameters (Gao et al., 2010). The scaling exponents tell us that the relative growth of GERD for the EU was essentially linear with respect to GDP while the Canadian and Chinese GERD tended to grow 2.67 ($2^{1.45}$) and 3.18 ($2^{1.67}$) times for a doubling of GDP. This indicator can be used to compare the relative growth of impact of fields of vastly different sizes.

---

12 Assume we are given any two exponential processes $x = am^{pt}$ and $y = bm^{qt}$. Let $y = sx^{a}$, then $bm^{qt} = s(am^{pt})^{a}$ or $b/s(a)^{a} = m^{(p-a)qt}$. Because $m^{(p-a)qt}$ is a time-dependent variable and it cannot be equal to $b/s(a)^{a}$, a constant, unless $p-a = q = 0$, therefore, $\alpha = q/p$ and $s = b/a^{q/p}$. This relationship holds even if the two processes are delayed in time with respect to each other or if they have different starting values at $t = 0$. 

13
Using 1984-2002 WoS data the exponent of the scaling correlation for the relative growth of impact with size for 13 fields ranged from $\alpha=1.13$ in physics to $\alpha=2.92$ in biology with an average of $\alpha=1.74$ (Katz, 2012). Field sizes ranged from 40 thousand to 3.2 million documents. When fields were ranked by this measure and compared to the rank determined using average number of citations per paper over the time interval it was clear that the conventional indicator was not a predictor of the growth of a field’s impact.

Scaling correlations also exist across groups within an innovation system at points in time. Scale-invariant models were constructed for the European, Canadian and Chinese\textsuperscript{13} innovation systems using the scaling correlation between the growth of GERD with GDP and the scaling correlation between these parameters across groups within the system at points in time (Katz, 2006). The evolution of these innovation systems were uniquely described by a small number of scale-invariant functions.

The 1980 to 2002 model of the European system clearly showed that shortly after the enactment of the Single European Act in 1986 the scaling correlation between GERD and GDP across EU countries dropped from approximately 1.25 to nearly 1.0. In other words, at the beginning of the period R&D investments increased nearly 2.5 times for each doubling in national size and by the end of the period R&D investment it was increasing linearly with size. R&D investment became more equitable across EU countries with integration into a single market. During the same period the scaling correlation across Canadian provinces moved up a bit from about 1.1 to 1.14 with a brief peak at around 1.2 between 1996 and 1998. On the other hand scaling correlation across Chinese provinces and municipalities rose from about 0.86 in 1995 to around 1.25 in 2002 as R&D investment became more concentrated in the larger ones. These models can be used to examine how the systems might have evolved given the existing scaling trends or if they were changed under different policy regimes. This kind of policy relevant information cannot be conveyed by conventional measures and models.

By definition the exponent, $\alpha$, of the exponent of a power law function is a constant. The exponent remains constant irrespective of changes in the magnitude of the independent variable; it is a scale-independent measure of a scale-invariant property under consideration. Monitoring how the scaling exponent changes can provide insight into how the system is evolving (Katz, 2006). For example, the evolution of the scaling correlation between Gross Expenditure on R&D (GERD) and GDP across countries in the European innovation system between 1980 and 2000 changed from being very superlinear with an exponent of 1.30 to being essentially linear by 2000. This illustrates an effect that European integration had on R&D intensity. In comparison over the same time frame the scaling exponent across provinces in the Canadian innovation system was superlinear with an exponent of around 1.10 rising briefly to 1.20.

Scale-invariance can be used to construct dimensionless relative measures adjusted for size that are useful for comparative purposes. For example, the expected impact of an entity of a given size, $P$, in the system having a scaling factor, $\alpha$, is given by $C_e \approx P^\alpha$. The relative impact of any entity in the system is given by the ratio of its observed impact, $C_o$, to its

\textsuperscript{13} The model for the Chinese system has not been published. It was prepared for a specifically for a presentation
expected impact $C_e$. The magnitudes of the relative impacts of entities of vastly different sizes in the system can be compared with the confidence that effect of size have been removed.

4. Data and Methodology

Two data sets were used to examine the evolution of citation distributions and examine the correlations between impact and size. One data set consisted of publications indexed in Scopus in 1997 and 1998 with annual citation counts to each paper from the year of publication to 2013. These data were supplied by Elsevier\textsuperscript{14}. They were used to study the evolution of citation distributions over 16 and 17 year time spans and consisted of more than 800,000 peer-reviewed documents each year that were cited more than 20 million times. The second data set consisted of 10.9 million peer-reviewed source documents indexed in the WoS between 1984 and 2002 with annual citation counts for each document from the year of publication to 2009 totalling about 120 million citations (Katz, 2012). These data were prepared by Science Metrix\textsuperscript{15}.

A field level analysis of the evolution of citation distributions was done using three journal schemes to classify documents: Scopus, UCSD Map of Science (MAPS) and the National Science Foundation (NSF) journal classification schemes. Earlier it was mentioned that the Scopus scheme allows a journal and the articles it contains to be assigned to one or more of 27 research fields. This raised a question. Does this impact the field level citation distributions?

This question is examined by comparing the findings using the Scopus scheme to the non-overlapping MAPs and NSF schemes. The MAPS scheme was designed for visualizing maps of science (Börner et al., 2012). It used algebraic and clustering techniques to assign journals to one of 554 unique journal clusters which were aggregated into 14 unique science areas. Journals that covered multiple fields were not used in analysis reducing the size of the analyzed data by 10% from its original size. The NSF\textsuperscript{16} scheme assigns each journal hence each article to one of 13 unique fields based on citation patterns and expert opinion. It has not been updated for a while so articles in newer journals are not classified reducing the analyzed data set by approximately 7% from the original size.

The methodology described by Clauset et al. and the Matlab, R: and C software routines they used were used in this analysis (Clauset et al., 2009; Gillespie, 2014). In particular, the maximum likelihood estimation method (MLE) was used to determine the scaling exponents of the cumulative probability distributions. A technique devised by Clauset and colleagues (Aaron Clauset, Maxwell, & Kristian Skrede, 2007) was used to estimate the lower bounds (i.e. $x_{\text{min}}$) the point at which the power law tail begins. And the p-values for exponents were determined using Monte Carlo simulations. These simulations were computationally

\textsuperscript{14} These data were awarded as part of Project #7 of the 2013 Elsevier Bibliometric Research Program grants http://ebrp.elsevier.com/grantedProposals2013.asp

\textsuperscript{15} Web of Science data was provided through a Research Fellowship at Science Metrix in Montreal Canada http://www.science-metrix.com/

\textsuperscript{16} An exclusive classification used in the Science & Engineering Indicators since the 1970s. It was originally designed by CHI Research.
intensive taking up to 25 hours to determine one p-value running on an 8 node, 6 processors per node, Sun Microsystems, cluster using parallel Matlab. In addition likelihood ratio tests were done comparing the best-fit power law model to other heavy tailed distributions: Poisson, log-normal, exponential, stretched exponential and power with exponential cut-off. It is important to note that in Clauset et al. the authors state that for real-world data it is extremely difficult to tell the difference between log-normal and power-law behavior.

The distributions for each year in the evolution of the overall citation distribution were assigned one of four likelihoods of being best modeled by a power law (none, moderate, good or power law with exponential cut-off) as described in the Clauset paper. The assignments are based on the amount of statistical support there is for a distribution being modeled by a power law distribution. “None” indicates the data is probably not power-law distributed; “moderate” indicates that the power law is a good fit but that there are other plausible alternatives; “good” indicates that the power law is a good fit and that none of the alternatives considered is plausible. In some cases, it is noted as “with cut-off,” meaning a power law with exponential cut-off is favored over a pure power law.

5. Results and Discussion

The first part of this section presents an analysis of the evolution of the impact of the global research system by exploring the evolution of citation distributions to peer-reviewed papers indexed in Scopus and WoS overall and at field levels. The second section presents an analysis of the scaling correlations between the growth of the research system’s impact and size over time and between field impact and field sizes measured across fields at points in time.

5.1 Impact Probability Distributions

Table 1 gives the results of the tests of the fit of a power law model to the evolution of the distribution of citations to 1997 and 1998 peer-reviewed documents indexed in Scopus. One of four likelihood categories was assigned to each distribution annually. Table 2 gives similar results of the tests of the fit of a power law model to the evolution of the distribution of citations to 1984 peer-reviewed documents indexed in the WoS database. This table is an example of the analysis done for each of five years of WoS data from 1984 to 1988 that had the longest evolution times. Both tables give the magnitude of the scaling exponent, α, its p-value and the log-likelihood ratios test (LR) for alternative heavy tailed distributions and their respective p-values. Positive values of LR indicate that the power-law model is favored over the alternative (Clauset et al., 2009). The final column of the table summarizes the statistical support for the power-law fit for each year in the evolution of the distributions.

[SEE TABLES 1 and 2 at end of document]

The values in Table 3(a) summarize the results from Table 1. It gives the number of times that each type of support occurred in the evolution of the 1997 & 1998 citation distributions. Except for one instance the LR values for the Poisson, exponential and stretched exponential models were positive ruling them out as potential candidates. The LR values for the log-normal model were negative except for three instances and because they
had insignificant p-values they could not be ruled out as a possibility. In 98% of the instances the distributions had a moderate likelihood of being modeled a power law or a power law with an exponential cut-off. And the scaling exponents for the distributions decreased in magnitude over the evolution time interval from 3.36 and 3.21 to 3.06 and 2.99 in 1997 and 1998, respectively.

Table 3(b) summarizes the findings for five years of WoS data for which example data were given for 1984 in Table 2. In 93% of the instances the distributions had at least a moderate chance of being a power law or power law with exponential cut-off. And in 28% of cases of the WoS distributions were assigned a likelihood support of ‘good’.

And as we move from observation times of 17 & 16 years using Scopus to 25 to 21 years using the WoS there is a greater chance the distributions will have a good fit to a power law or power law with exponential cut-off. These data support the notion that the distribution of the impact of knowledge measured using citations to peer-reviewed documents published by the global research system has a reasonable likelihood of having scale-invariant properties.

Table 3 – Support for a power law distribution (a) Scopus and (b) Web of Science

<table>
<thead>
<tr>
<th>(a) Scopus</th>
<th>1997</th>
<th>1998</th>
<th>Total</th>
<th>% Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3%</td>
</tr>
<tr>
<td>Moderate</td>
<td>14</td>
<td>10</td>
<td>24</td>
<td>71%</td>
</tr>
<tr>
<td>Good</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>9%</td>
</tr>
<tr>
<td>Power law w/cut-off</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>18%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Web of Science</th>
<th>1984</th>
<th>1985</th>
<th>1986</th>
<th>1987</th>
<th>1988</th>
<th>Total</th>
<th>% Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>8</td>
<td>7%</td>
</tr>
<tr>
<td>Moderate</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>22</td>
<td>18%</td>
</tr>
<tr>
<td>Good</td>
<td>11</td>
<td>6</td>
<td>0</td>
<td>9</td>
<td>12</td>
<td>38</td>
<td>32%</td>
</tr>
<tr>
<td>Power law w/cut-off</td>
<td>13</td>
<td>12</td>
<td>19</td>
<td>3</td>
<td>5</td>
<td>52</td>
<td>43%</td>
</tr>
</tbody>
</table>

Table 4 examines the field level distributions of citations to peer-reviewed papers indexed in the Scopus database in 1997 & 1998. Journals were assigned to one or more of 27 overlapping Scopus fields, one of 13 non-overlapping NSF fields and one of 13 non-overlapping MAPS fields. The data show that in 55-70% of the cases there is a ‘good’ likelihood that a field level citation distribution could be modeled by a power law or a power law with exponential cut-off.

These data illustrate that the impact of field level knowledge measured using citations to peer-reviewed publications from the global research system has a reasonable likelihood of having scale-invariant probability distributions too. Furthermore the result seems to be independent of whether papers are assigned to overlapping or non-overlapping fields.
Table 4 – Support for power law distribution – field level analyses

<table>
<thead>
<tr>
<th></th>
<th>Scopus</th>
<th>NSF</th>
<th>MAPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Likelihood</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>none</td>
<td>39</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>moderate</td>
<td>142</td>
<td>98</td>
<td>52</td>
</tr>
<tr>
<td>good</td>
<td>25</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>cut-off</td>
<td>253</td>
<td>123</td>
<td>137</td>
</tr>
<tr>
<td>none</td>
<td>26</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>moderate</td>
<td>172</td>
<td>85</td>
<td>64</td>
</tr>
<tr>
<td>good</td>
<td>59</td>
<td>38</td>
<td>21</td>
</tr>
<tr>
<td>cut-off</td>
<td>175</td>
<td>85</td>
<td>109</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>none</td>
<td>65</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>moderate</td>
<td>314</td>
<td>183</td>
<td>116</td>
</tr>
<tr>
<td>good</td>
<td>84</td>
<td>48</td>
<td>42</td>
</tr>
<tr>
<td>cut-off</td>
<td>428</td>
<td>210</td>
<td>246</td>
</tr>
</tbody>
</table>

The scaling exponents with significant p-values for the field level distributions were examined at the end of the observation timeframe to determine how many fields had $\alpha < 3.0$. Using the Scopus journal classification scheme 24 of the 54 (44%) field level distributions had $\alpha < 3.0$. Also, 8 of the 24 (30%) fields had distributions with $\alpha < 3.0$ both years. Using the NSF scheme 14 of 26 (54%) distributions had $\alpha < 3.0$ and 4 of 13 (31%) fields had distributions with $\alpha < 3.0$ both years. And using the MAPS scheme 10 of 28 (36%) of the distributions had $\alpha < 3.0$ and 5 of 14 (36%) fields had distributions with $\alpha < 3.0$ both years. Irrespective of the method used to assign papers to fields a significant number of distributions have $\alpha < 3.0$ by the end of the observation time frame.

5.2 Impact Scaling Correlations

The simplest scaling correlation that any system will exhibit occurs between parameters that grow exponentially at the same time. Let’s examine how the increasing size of the global research system correlates with its impact. This is illustrated using WoS data because it covers a larger time frame. Figure 3a depicts the exponential growth over time on a log-linear scale of peer-reviewed papers and citations to these papers counted using a fixed 6 year time window. Figure 3b depicts the scaling correlation between citations and papers on a log-log scale.

The ratio of the exponential growth exponents, $0.023/0.013 = 1.77$, is within the error limit range for the measured scaling exponent of $1.79 \pm 0.04$. The scaling exponents tells us that on average for every doubling in peer-reviewed published output the impact is expected to increase by $2^{1.79}$ or 3.5 times. The scaling exponent, 1.79, is a scale-independent measure of the scale-invariant relative growth of the impact of the global research system over a 21 year time frame.
As discussed earlier scaling correlations have been found between impact and group size in the global research system at a point time. For example consider the scaling correlation between the impacts of research fields and their sizes based on Scopus data classified using the three journal classification schemes and done at two points in time. Table 4 gives the magnitudes of the scaling correlation across fields in 1997 and 1998 between number of citations and field sizes determined using the Scopus, NSF and MAPS journal classification methods.

Table 4 – Scaling exponents for scaling correlation between impact and field sizes

<table>
<thead>
<tr>
<th>Scheme</th>
<th>No. Fields</th>
<th>1997</th>
<th></th>
<th>1998</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>α</td>
<td>R²</td>
<td>α</td>
<td>R²</td>
</tr>
<tr>
<td>Scopus</td>
<td>27</td>
<td>0.96 ± 0.09</td>
<td>0.83</td>
<td>0.96 ± 0.09</td>
<td>0.83</td>
</tr>
<tr>
<td>NSF</td>
<td>13</td>
<td>1.21 ± 0.08</td>
<td>0.91</td>
<td>1.19 ± 0.07</td>
<td>0.92</td>
</tr>
<tr>
<td>MAPS</td>
<td>13</td>
<td>1.27 ± 0.12</td>
<td>0.96</td>
<td>1.26 ± 0.11</td>
<td>0.96</td>
</tr>
</tbody>
</table>

The data show that the non-overlapping NSF and MAPS field assignments have \( \alpha > 1.0 \) indicative of a self-organization. On the other hand the overlapping Scopus field assignment has \( \alpha = 1.0 \) not indicative of self-organized structure. The reason for the difference is that in the NSF & MAPS instances highly cited papers, those most likely to be found in the heavy tail, are assigned to only one field while in the Scopus case there is a high likelihood that their citation counts will be assigned to more than one field. The methods used to classify entities in an innovation system can affect our view of its self-organizing structure.

6. Conclusions

The global research system was used as an example of an innovation system. Peer-reviewed publications and citations were used as measures of size and impact, respectively. The
distribution of impact and the correlation between impact and size over time and at points in time were examined to see if scale-invariant properties could be identified. Many other parameters have been shown to have scale-invariant characteristics. They can be used for similar investigations but some of them have limitations as described earlier. Citations and papers were used to illustrate the concepts because the Web of Science and Scopus datasets are relatively clean, long time series are readily available and these measures have long history of use in the study of innovation systems.

The global research system has the general characteristics of a complex system. Its adaptive nature distinguishes it from a complex physical system. At different levels of observation the evolution of the impact tends to be scale-invariant. The scaling exponent tended toward < 3.0 as the distributions evolved. However, in some field became it was < 3.0 early in their evolution. Scale-invariant correlations exist between the growth of impact & size over time and between impact and size at points in time. The scaling exponent of the later correlation is a systemic measure of the ‘average impact’ of the all fields in the system. It can be used as a reference function to calculate a scale-independent measure of how much impact a field is having relative to the average system impact.

A scale-invariant property has a unique characteristic. It is solely characterized by a power law $f(x)=kx^\alpha$ where $\alpha$ is a constant that quantifies the scale-invariant property. Due to its recursive characteristics any natural community drawn from a population exhibiting a scale-invariant property will display that scale-invariant property too. Since the global research system is likely to be a complex innovation system with scale-invariant characteristics then any regional, national, local or sectoral research system within the global system is likely to be a complex system with scale-invariant properties too.

What are the implications of the findings for policy makers? Perhaps the most important take away is scale-invariance is natural and is frequently found as an emergent property of a complex system. Many current measures used to inform public policy are based on population averages that cannot quantify scale-invariant properties except under the very rare and unusual instance when the scale-invariance is linear. Rankings based on current measures are distorted because they are size dependent rankings. Furthermore, population based averages are only useful when the scaling exponent of a scale-invariant distribution is $\geq 3.0$. Generally speaking real-world scale-invariant properties tends to become $< 3.0$ as the system evolves. At this point the variance is infinite or at least very large making the usefulness of such measures questionable. On the other hand scale-independent measures based on natural scale-invariant emergent properties of a complex system are useful because they are dimensionless, independent of size and comparable over the lifetime of the system irrespective of the magnitude of the variance.

Only a small number of scale-invariant functions are needed to create a scale-independent model that can be run forward under constant or changing policy frameworks (Katz, 2006, 2012). Scale-independent evidence based measures are the only measures that capture
naturally occurring but rarely quantified scale-invariant emergent properties of a dynamically evolving complex innovation system. Scale-independent measures and models would be useful tools to include in any basket of measures used to inform public innovation policy.

The focus of this article was to answer the question “What is a complex innovation system?” The premise proposed was that if an innovation system can be shown to have the characteristics of a complex adaptive system and if it has scale-invariant properties indicative of self-organizing process then the system is likely a complex system. From the evidence in this article there is a reasonable likelihood that many, if not all, innovation systems are complex with scale-invariant characteristics.
Acknowledgements

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References


Table 1 - Test of evolution of power law distributions for citations to peer-reviewed 1997 and 1998 documents indexed in the Scopus database. Statistically significant p-values are denoted in **bold**. For each year in the evolution of the distribution p-values for the fit to the power-law model and likelihood (LR) ratios with p-values are given for alternatives distributions.

<table>
<thead>
<tr>
<th>Scopus 1997 N = 801,501</th>
<th>Log-normal</th>
<th>Possion</th>
<th>Exponential</th>
<th>Stretch Exp</th>
<th>Power law + cut-off</th>
<th>Support for power law</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time years</strong></td>
<td>α</td>
<td>p</td>
<td>LR</td>
<td>p</td>
<td>LR</td>
<td>LR</td>
</tr>
<tr>
<td>1</td>
<td>3.36</td>
<td>0.44</td>
<td>-0.92 0.36</td>
<td>7.12 0.00</td>
<td>5.49 0.00</td>
<td>0.50 0.62</td>
</tr>
<tr>
<td>2</td>
<td>3.10</td>
<td>0.00</td>
<td>-2.14 0.03</td>
<td>13.11 0.00</td>
<td>9.52 0.00</td>
<td>-0.46 0.65</td>
</tr>
<tr>
<td>3</td>
<td>3.23</td>
<td>0.83</td>
<td>-0.94 0.35</td>
<td>8.60 0.00</td>
<td>5.75 0.00</td>
<td>0.19 0.85</td>
</tr>
<tr>
<td>4</td>
<td>3.24</td>
<td>0.69</td>
<td>-0.68 0.50</td>
<td>6.91 0.00</td>
<td>5.10 0.00</td>
<td>0.63 0.53</td>
</tr>
<tr>
<td>5</td>
<td>3.25</td>
<td>0.52</td>
<td>-0.43 0.67</td>
<td>5.90 0.00</td>
<td>4.68 0.00</td>
<td>0.55 0.49</td>
</tr>
<tr>
<td>6</td>
<td>3.23</td>
<td>0.43</td>
<td>-0.44 0.66</td>
<td>5.46 0.00</td>
<td>4.48 0.00</td>
<td>0.98 0.33</td>
</tr>
<tr>
<td>7</td>
<td>3.23</td>
<td>0.54</td>
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Table 2 - Test of evolution of power law distributions for citations to peer-reviewed 1984 documents indexed in the Web of Science database. Statistically significant p-values are denoted in **bold**. For
each year in the evolution of the distribution p-values for the fit to the power-law model and likelihood (LR) ratios with p-values are given for alternatives distributions.

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