

Determining Masses and Spins of New Particles (with missing energy)

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Outline

- Mass determination
 - M_{T2} and related variables
 - Jet contamination
 - Solving decay chains
 - ‘Inclusive’ observables
- Spin determination
 - Decay chains
 - Dileptons
 - Three-body decays
 - Cross sections

Mass Determination with ...

- M_{T2} variable
- Jet contamination
- Solving decay chains
- ‘Inclusive’ observables

See also: review by Barr & Lester,
J.Phys.G 37(2010)123001

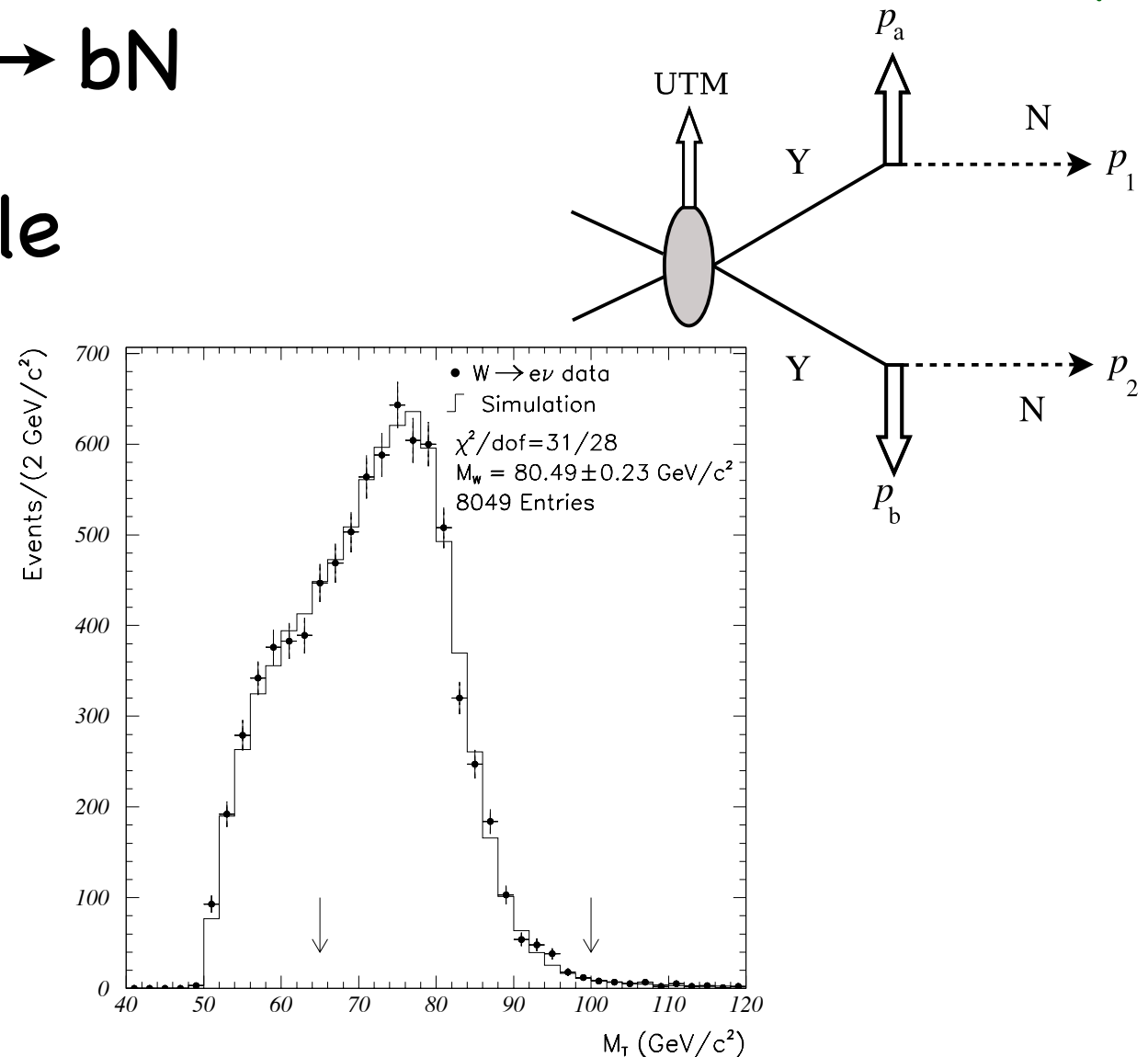
Mass determination with M_{T2}

M_{T2} variable

Lester & Summers, PLB 463(1999)99

- $pp \rightarrow YYX, Y \rightarrow aN, Y \rightarrow bN$
- a, b visible, N invisible
- Transverse mass:

$$m_T^2(\mathbf{p}_T^1, \mathbf{p}_T^a; \mu_N) = \mu_N^2 + m_a^2 + 2(E_T^1 E_T^a - \mathbf{p}_T^1 \cdot \mathbf{p}_T^a)$$



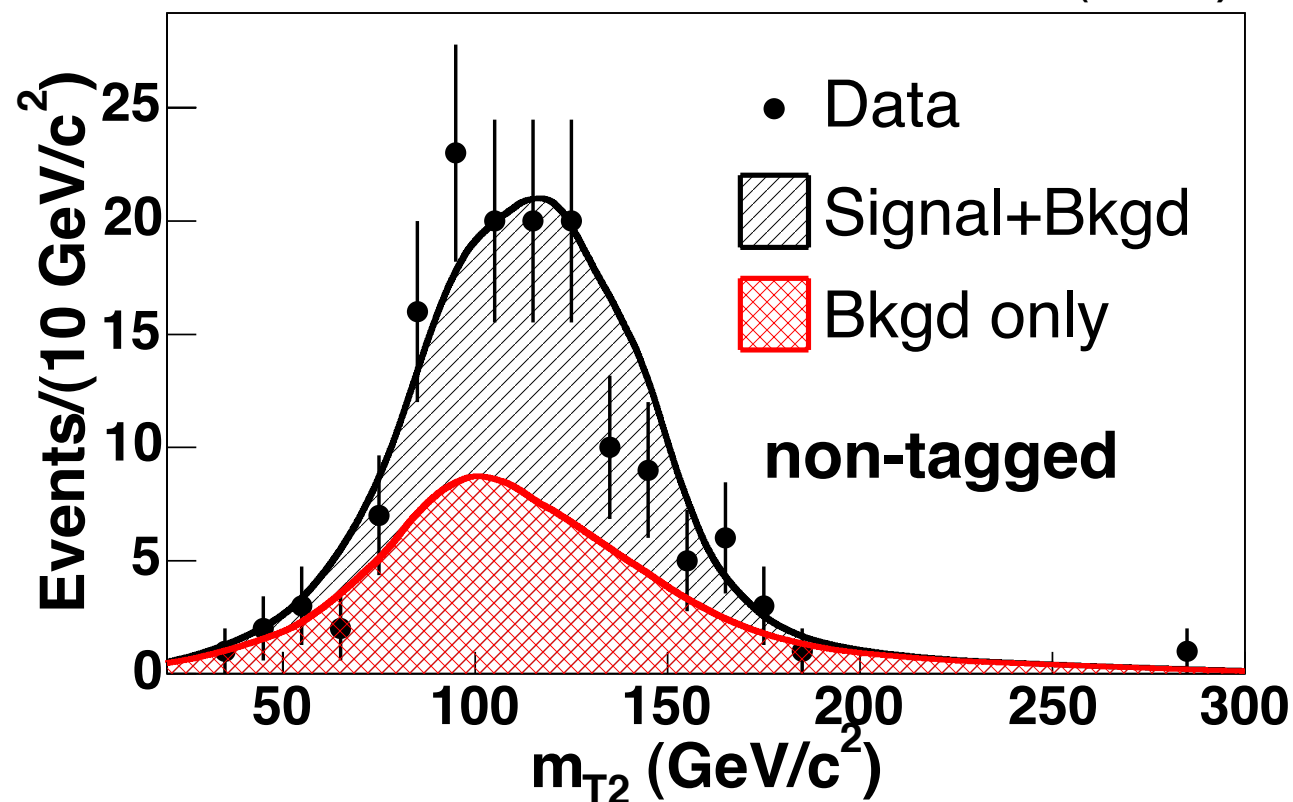
$$m_{T2}^2(\mu_N) \equiv \min_{\mathbf{p}_T^1 + \mathbf{p}_T^2 = \cancel{\mathbf{p}}_T} [\max\{m_T^2(\mathbf{p}_T^1, \mathbf{p}_T^a; \mu_N), m_T^2(\mathbf{p}_T^2, \mathbf{p}_T^b; \mu_N)\}]$$

$$\leq m_Y^2 \text{ when } \mu_N = m_N$$

CDF top mass from M_{T2}

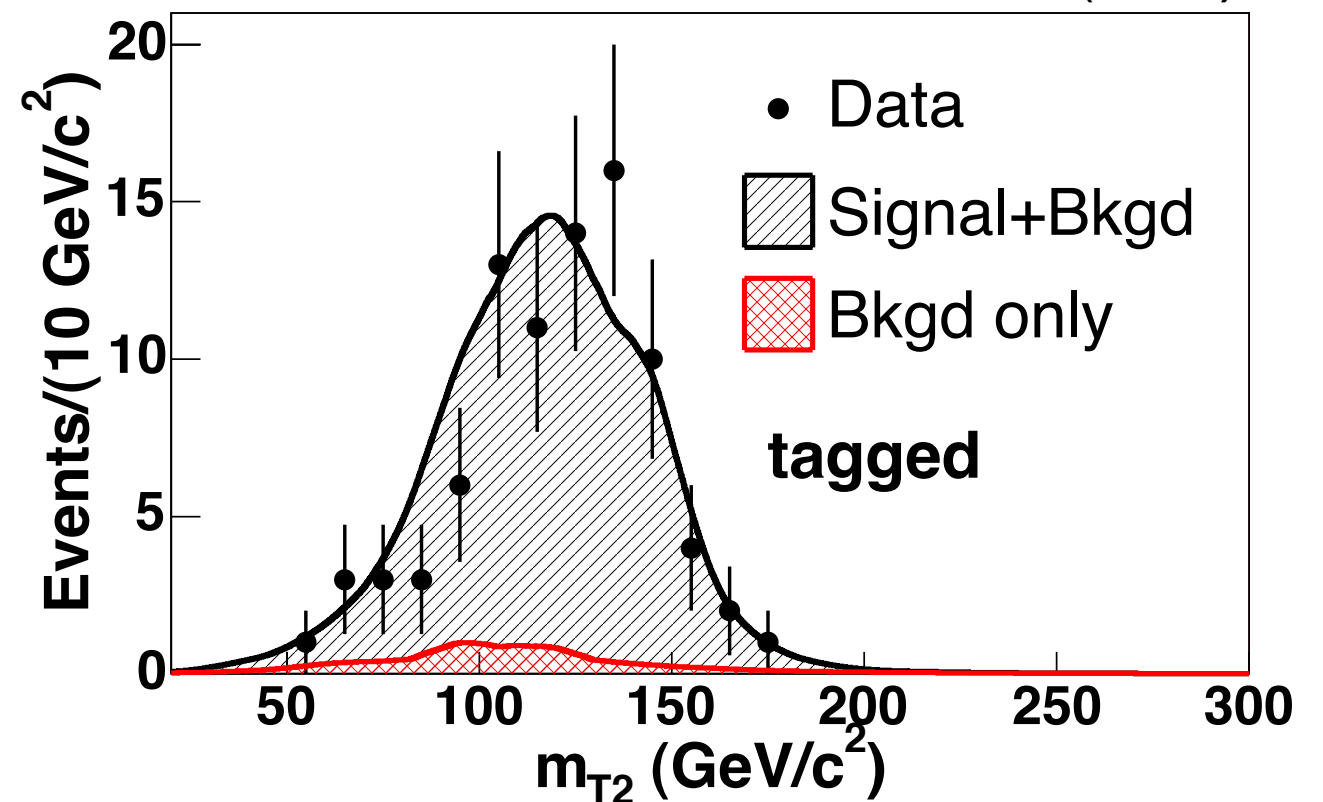
non-tagged

CDF II (3.4 fb⁻¹)



b-tagged

CDF II (3.4 fb⁻¹)

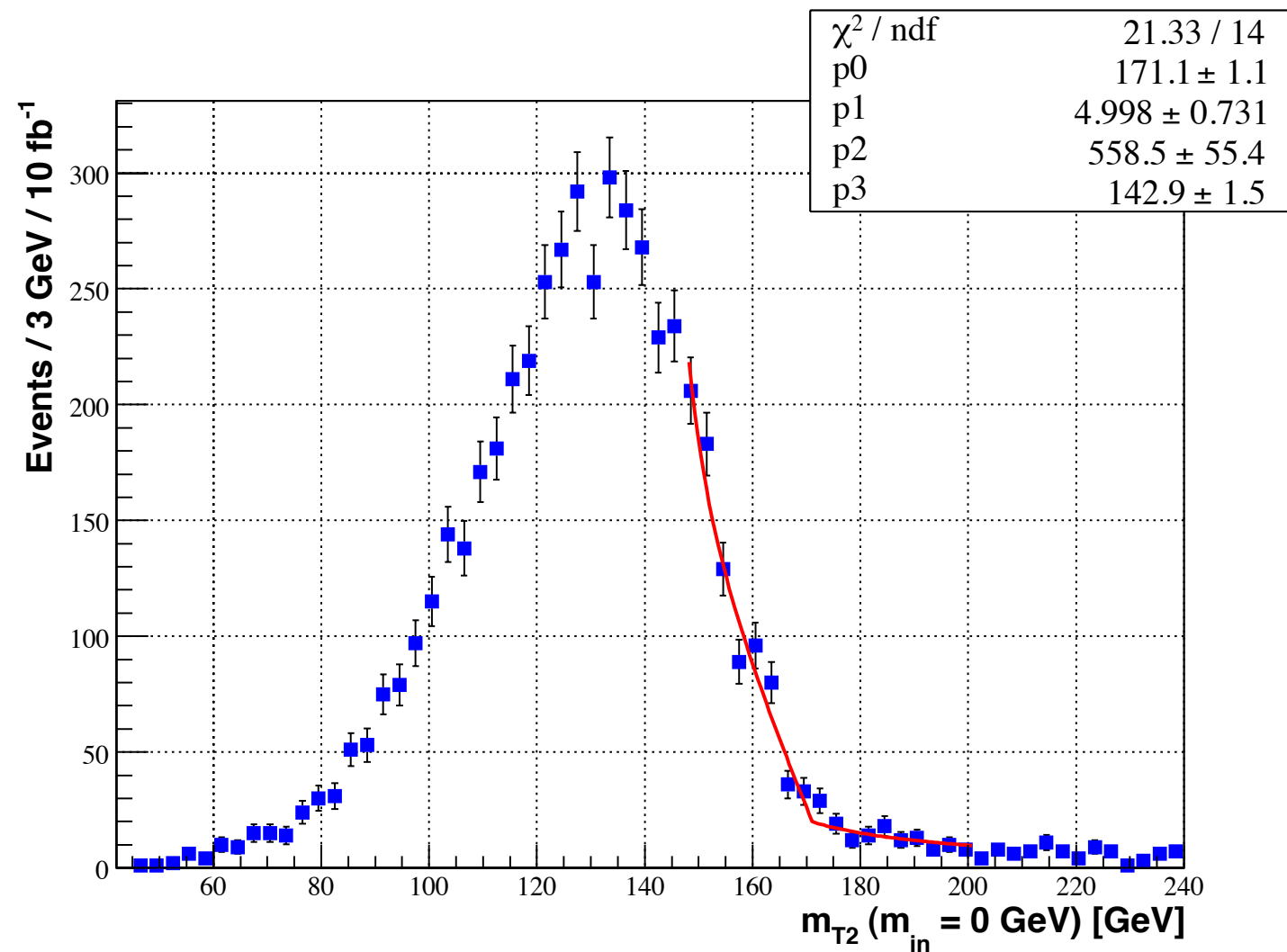


CDF, PRD 81(2010) 031102

● 3.4 fb⁻¹ ⇒ $m_t = 168.0 +5.6/-5.0$ GeV (M_{T2} alone)

Top mass from M_{T2} at LHC?

Cho, Choi, Kim & Park, PRD 78(2008)034019

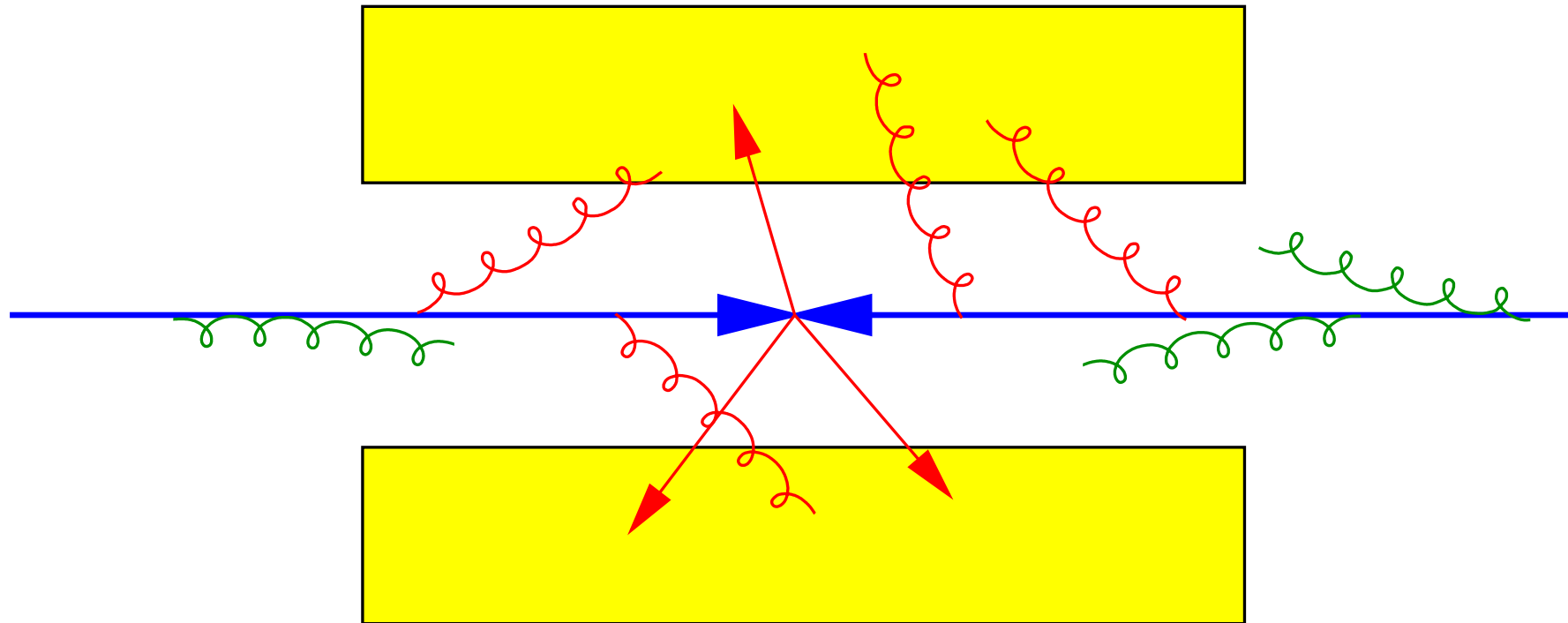


● Input mass 170.9 GeV; PYTHIA+PGS; b-tagging 50%

● 10 fb⁻¹ @ LHC (14 TeV) $\Rightarrow m_t = 171.1 \pm 1.1 \text{ GeV}$

Jet contamination

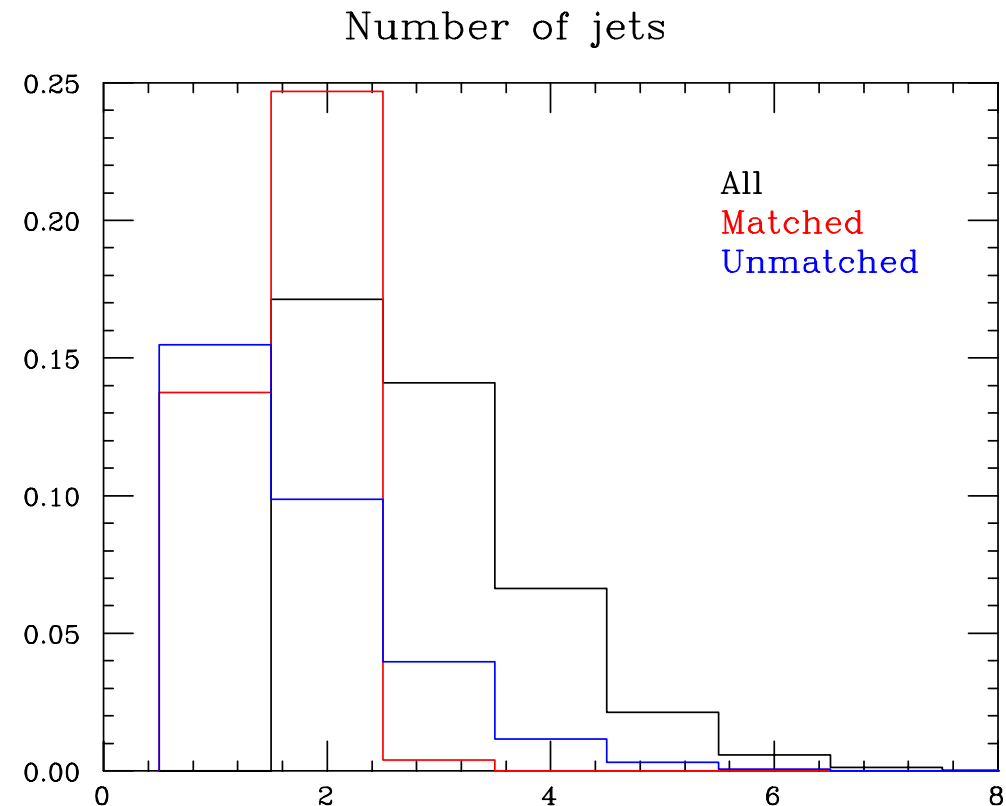
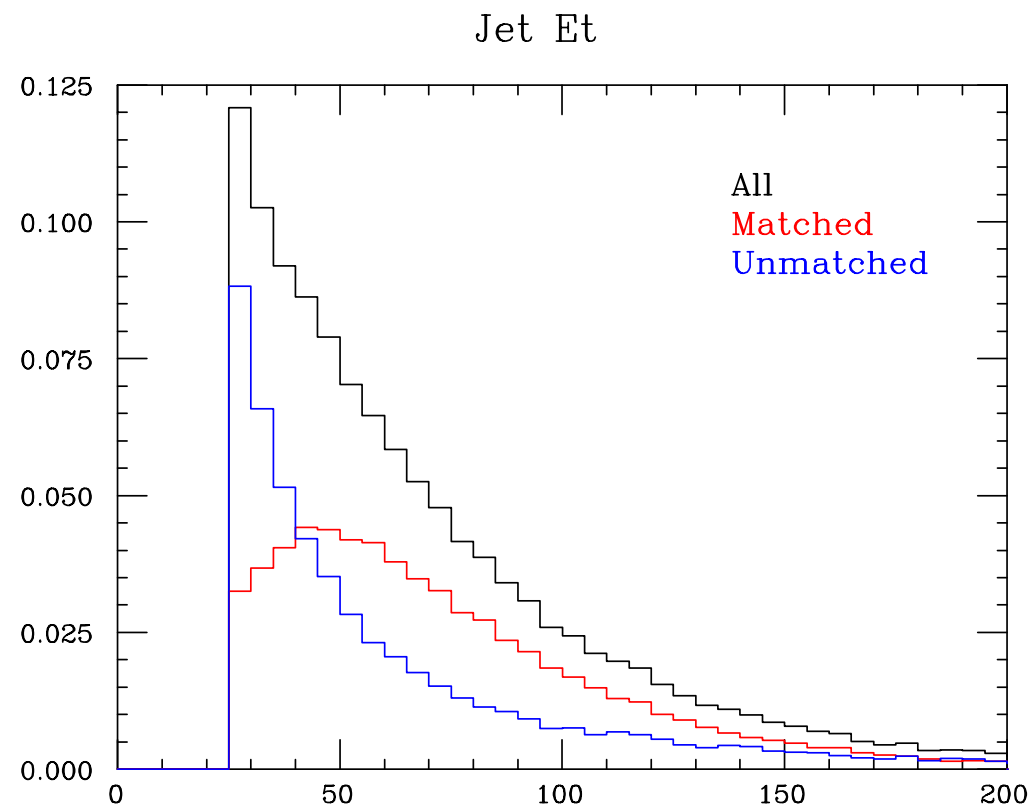
Initial-state QCD radiation



- Irreducible source of “jet contamination”
 - ➔ Misidentification of processes
 - ➔ Combinatorial ambiguities

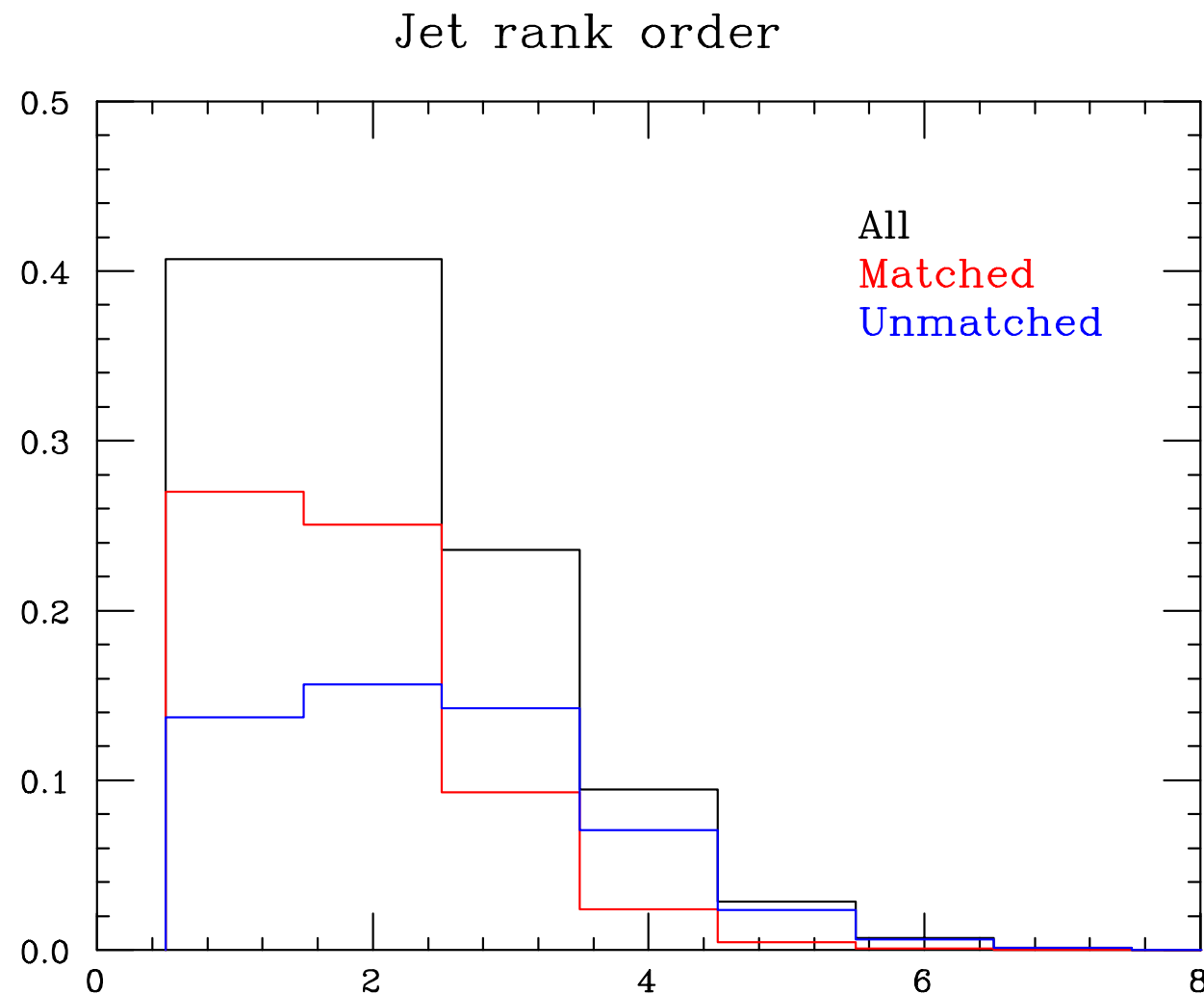
Jet contamination in $t\bar{t}$

- Fully leptonic $t\bar{t}$: 2 jets (+2 leptons + MET)
- Matched = top decay parton within $\Delta R=0.5$ and $\Delta E/E=0.3$
- Generated with MC@NLO (no underlying event)



➔ Half of events have an extra jet

E_T ordering of jets



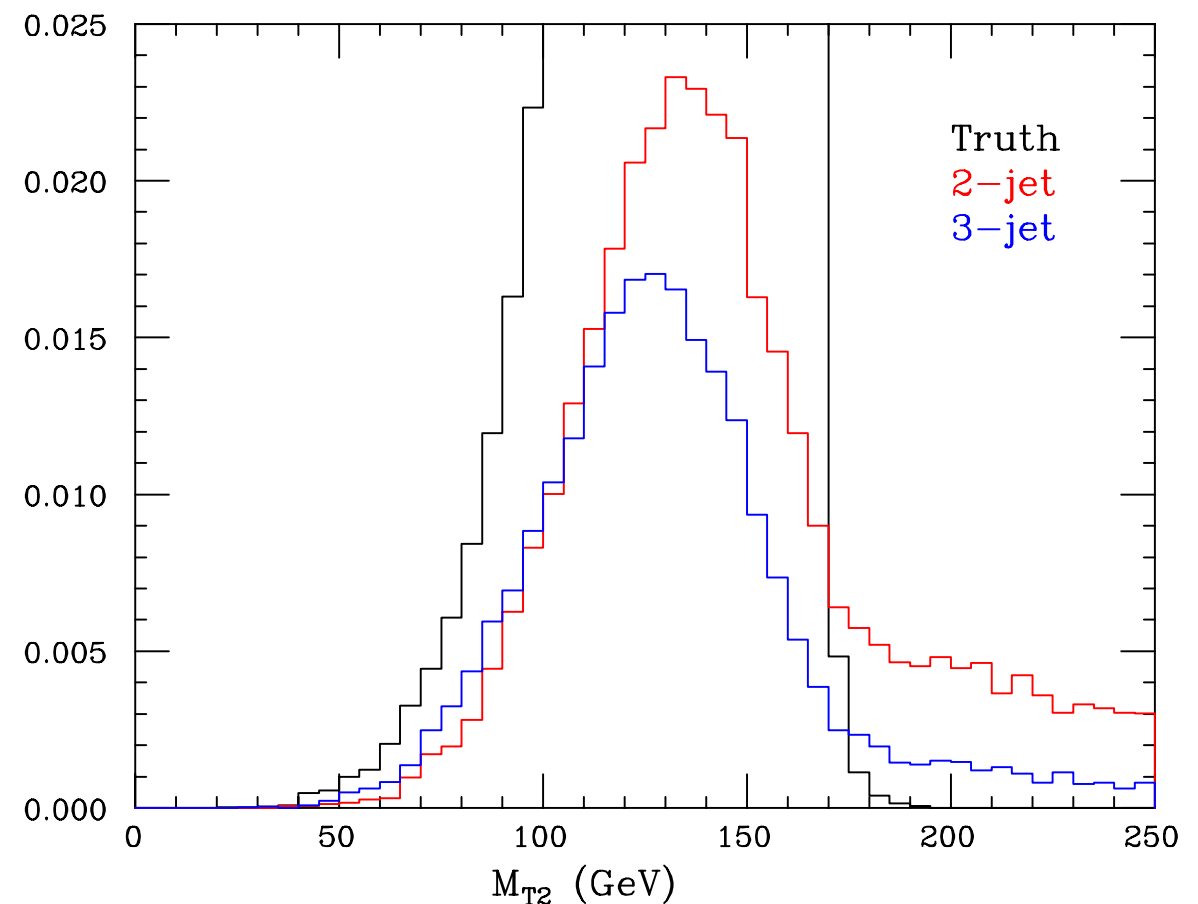
- $P(1 \text{ or both leading jets unmatched}) > 50\%$

Reducing jet contamination in $t\bar{t}$

- New idea: demand more jets, select lowest M_{T2}
As long as one is correct, this cannot raise edge

Alwall, Hiramatsu, Nojiri & Shimizu, PRL103(2009)151802

- 7 fb^{-1} MC@NLO, no b-tagging
- > 50% events have extra jets
- Hardest 2 jets (red) => ISR contaminates edge
- Smallest M_{T2} from 3 hardest (blue) => less contamination

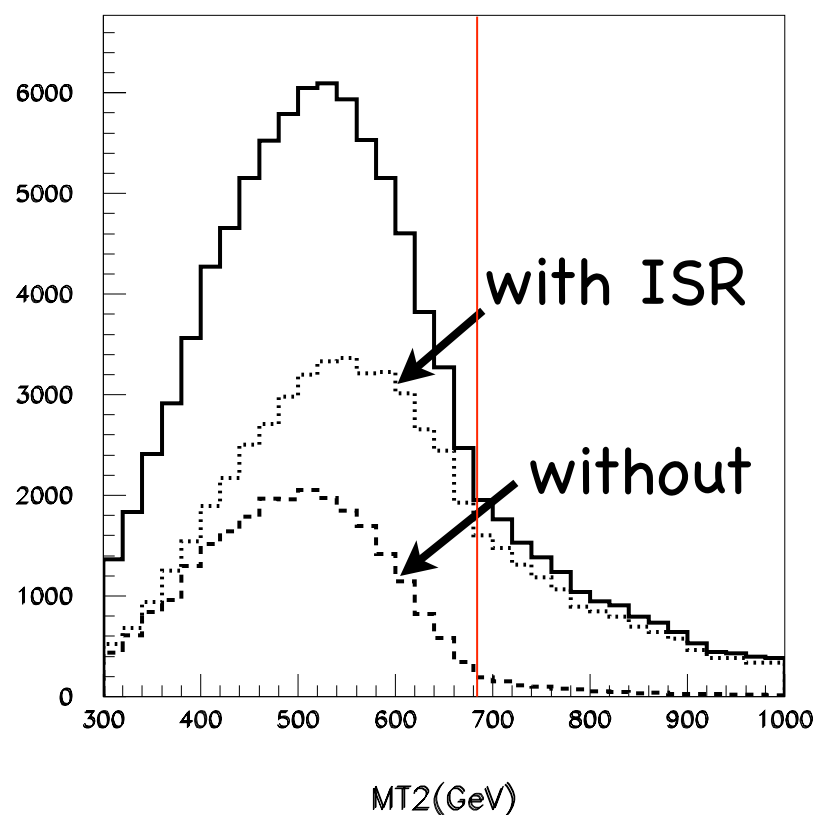


Reducing jet contamination in SUSY

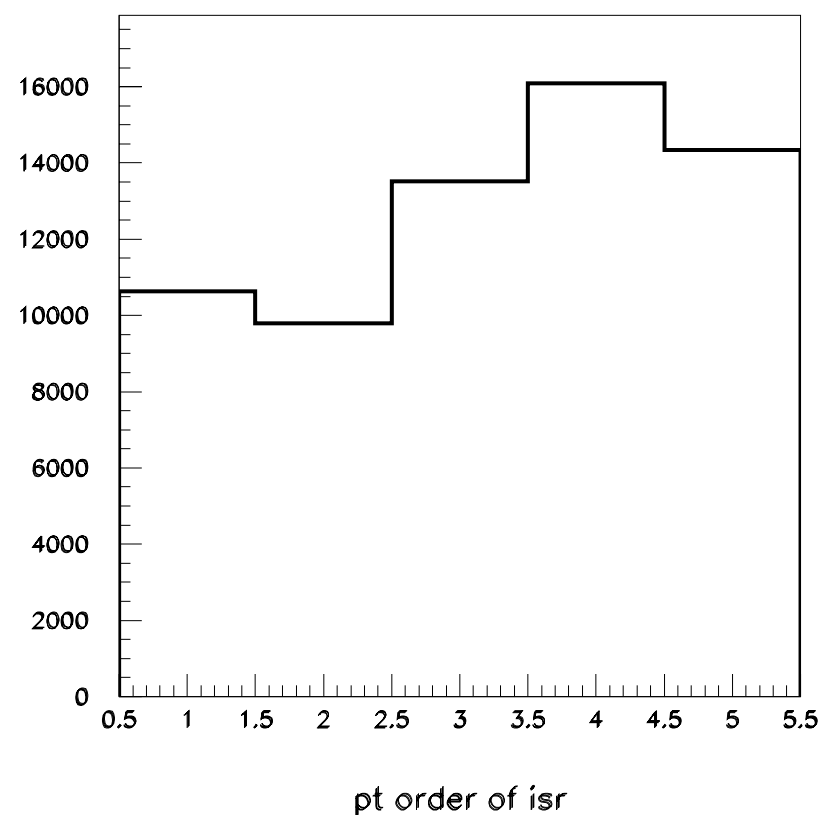
Alwall, Hiramatsu, Nojiri & Shimizu, PRL103(2009)151802

- Consider $gg \rightarrow \tilde{g}\tilde{g}$, $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}_1^0$ at LHC (PYTHIA, 40 fb⁻¹)
 $m_{\tilde{g}} = 685$ GeV, $m_{\tilde{q}} = 1426$ GeV, $m_{\tilde{\chi}_1^0} = 102$ GeV

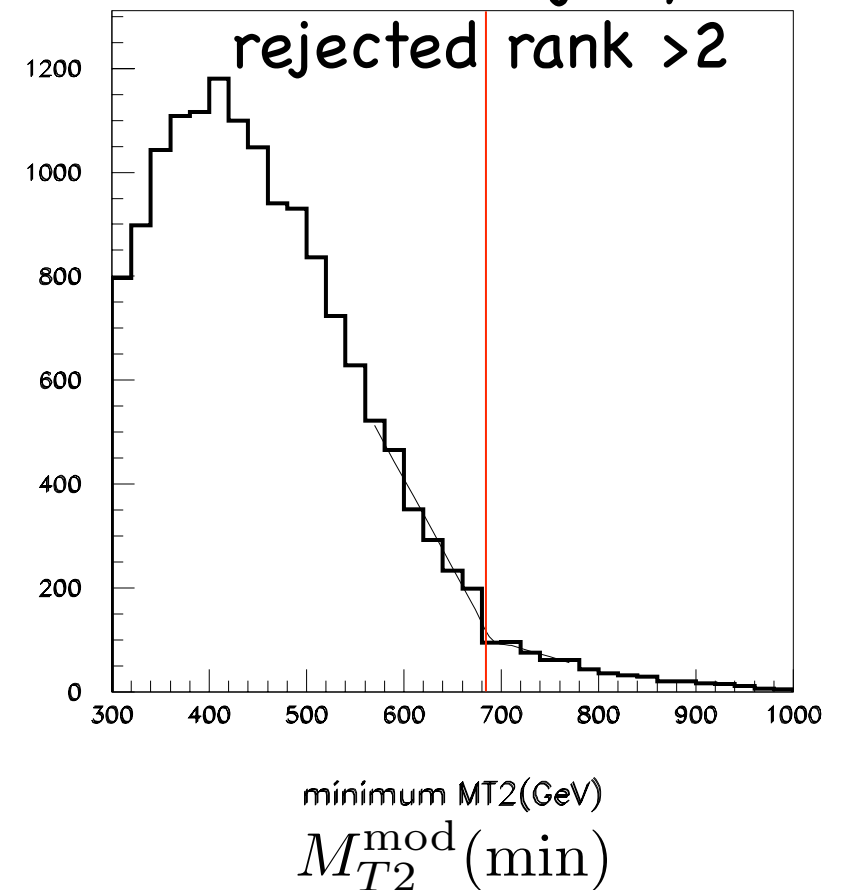
Hardest 4 jets



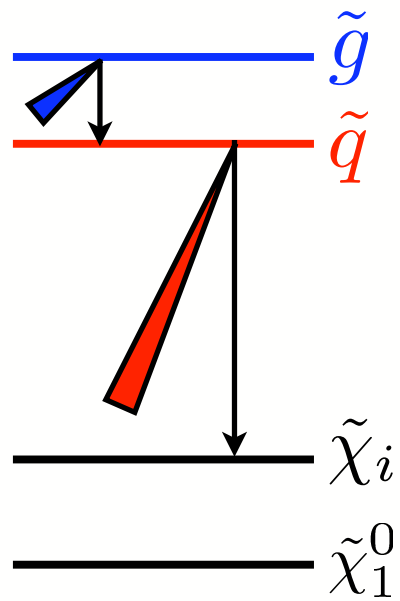
ISR rank order



Hardest 5 jets,
rejected rank >2



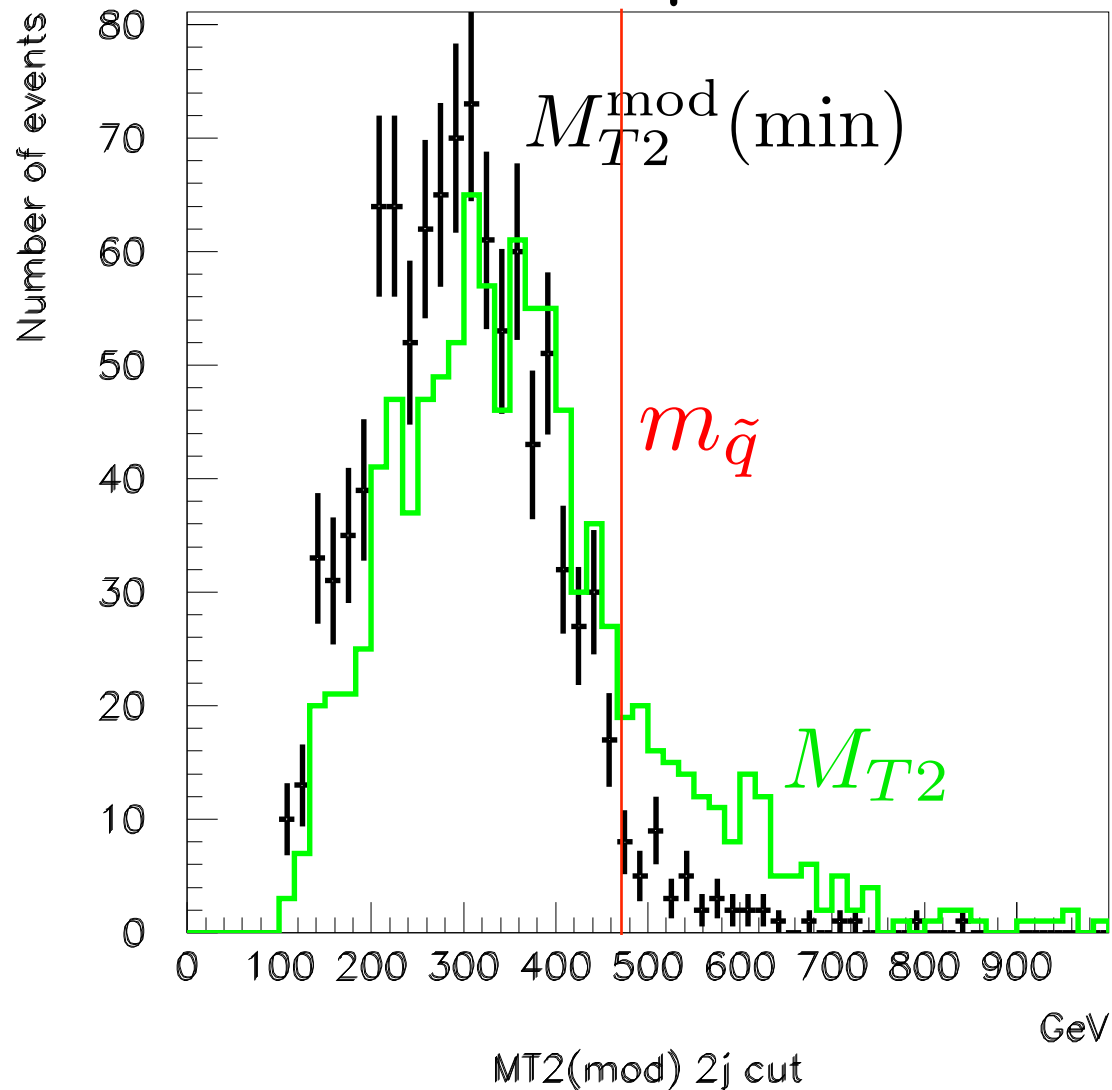
- Again, endpoint is clearer for lowest M_{T2} with extra jet



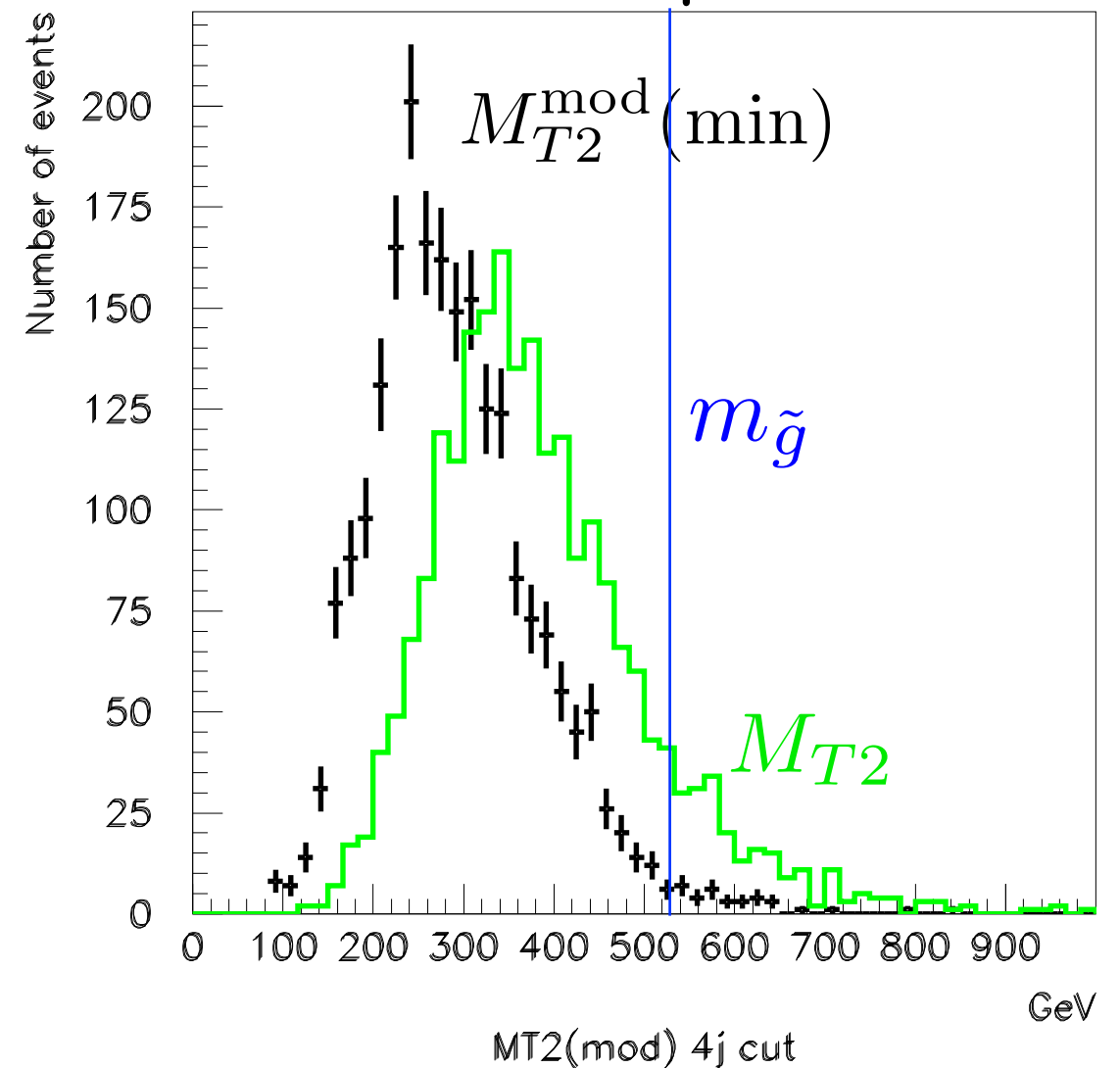
Nojiri & Sakurai, arXiv:1008:1813

- Squark & gluino production at 7 TeV, 1 fb⁻¹
 $m_{\tilde{g}} = 522 \text{ GeV}$, $m_{\tilde{q}} = 472 \text{ GeV}$, $m_{\tilde{\chi}_1^0} = 79 \text{ GeV}$
- 2/4 jet cut favours squark/gluino production

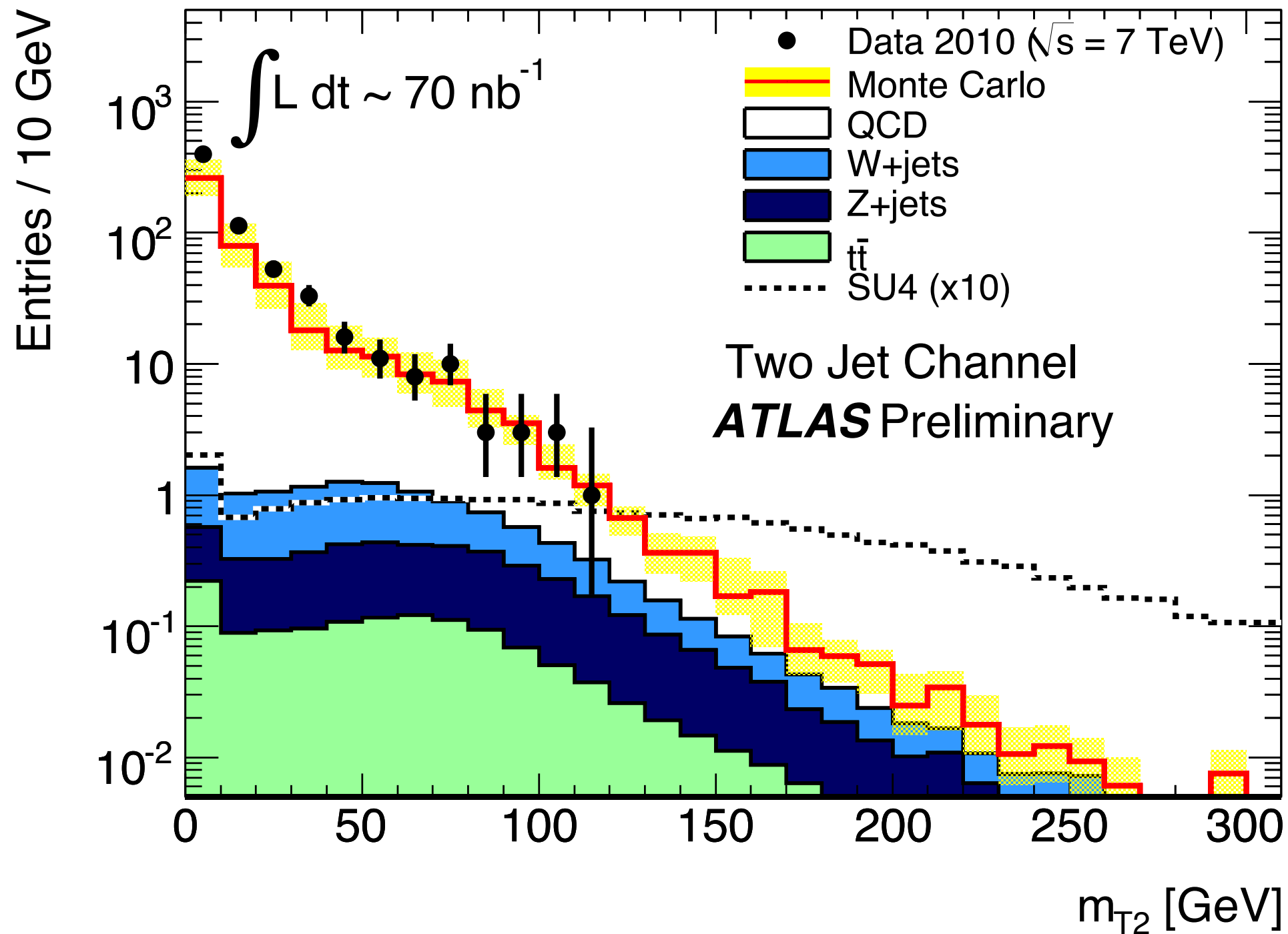
2 jets with $p_{\text{T}} > 200 \text{ GeV}$



4 jets with $p_{\text{T}} > 50 \text{ GeV}$

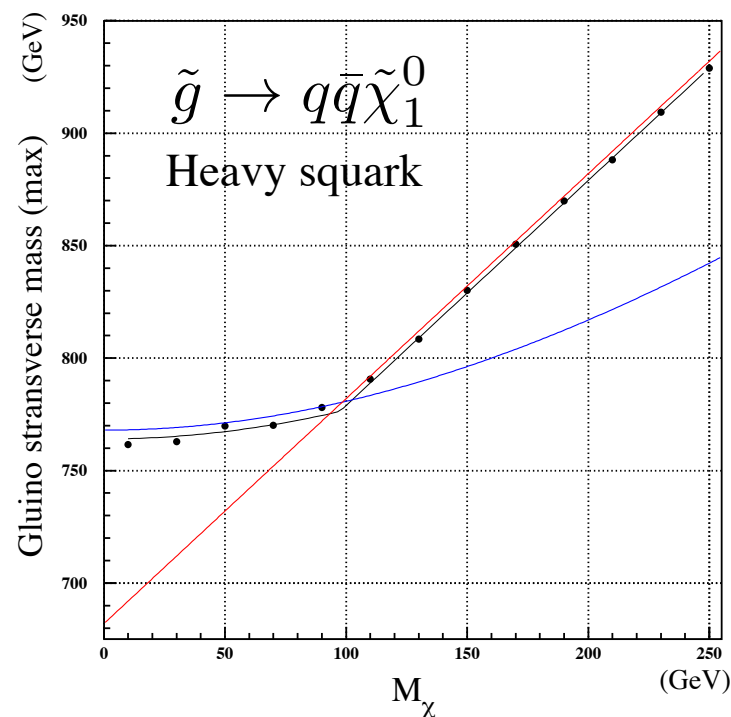


First LHC data



Finding M_{LSP} from M_{T2} ?

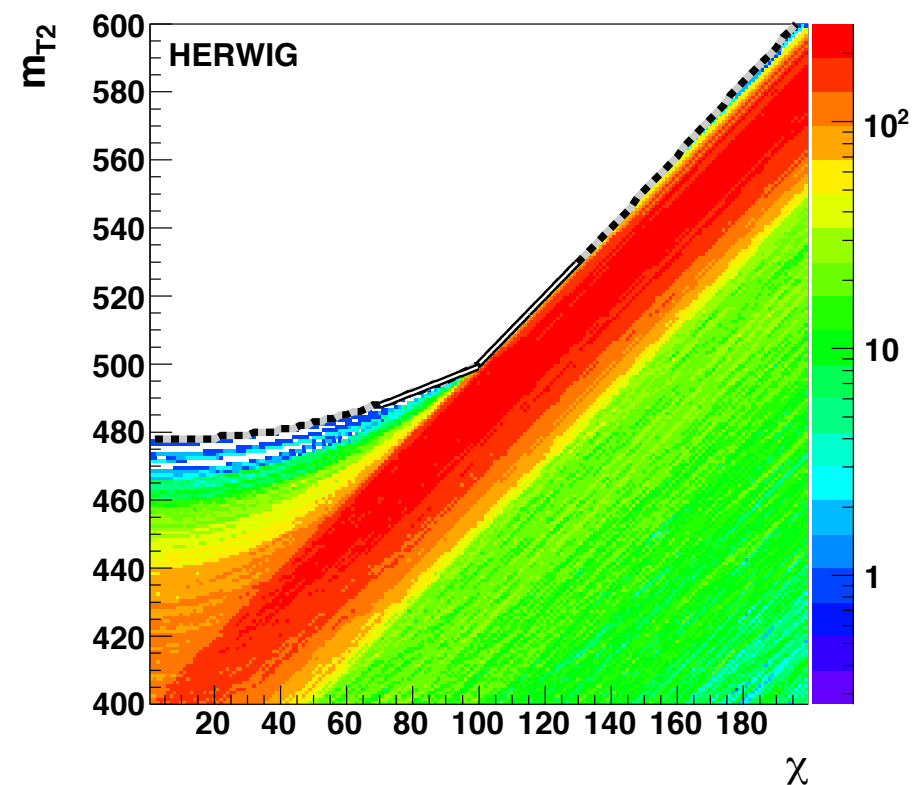
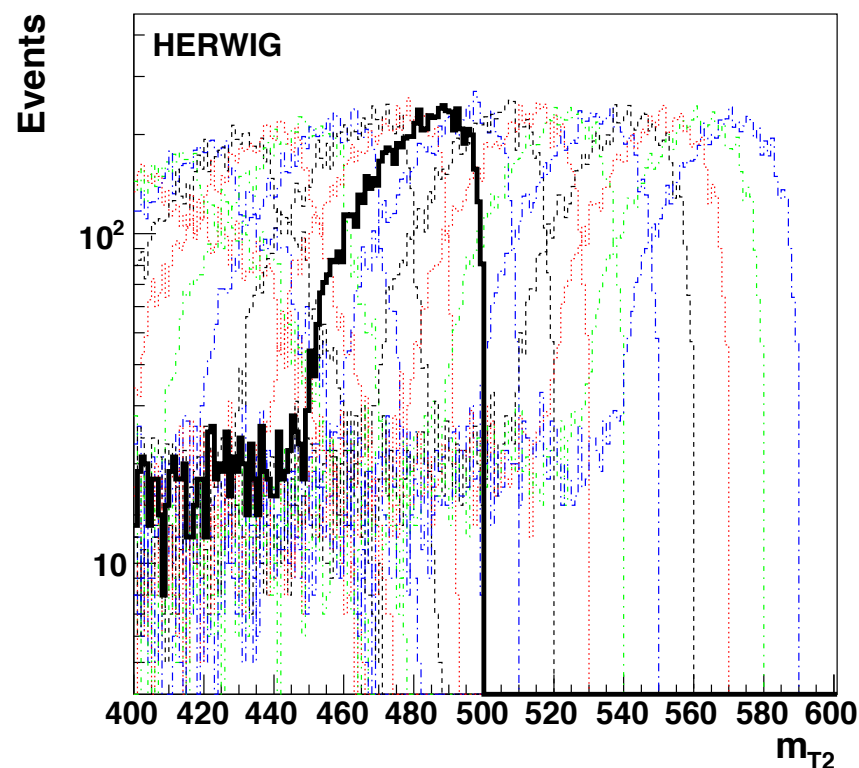
Cho, Choi, Kim, Park, arXiv:1008:1813



● $M_{\text{T2}}^{\text{max}}(\mu_N)$ has a kink at $\mu_N = m_{\tilde{\chi}_1^0}$

● Unfortunately endpoint is weak when $\mu_N < m_{\tilde{\chi}_1^0}$

➔ Difficult to see kink



Barr, Gripaios, Lester, JHEP02(2008)014

Solving decay chains

Solving decay chains

- Measure visible momenta $l \dots n, l' \dots n'$ and missing p_T

- 6 unknown momentum components per event

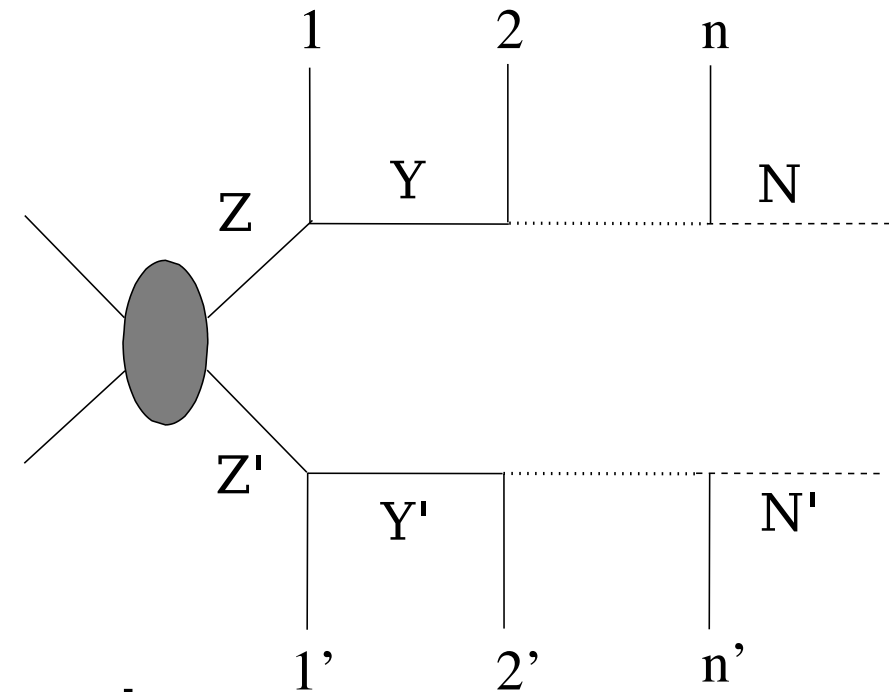
- $n+n'+2$ on-mass-shell constraints per event

- N_m unknown masses \rightarrow we need

$$N_{ev}(n+n'-4) \geq N_m \quad \text{to solve for masses}$$

- Identical chains: $n=n', N_m = n+1 \rightarrow$ need $N_{ev} = 2$ for $n=3,4$

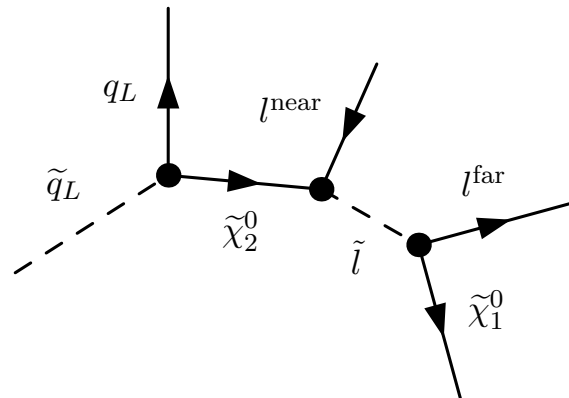
Non-identical ($N=N'$): $N_m = n+n'+1 \rightarrow$ need $N_{ev} = 6$ for $n+n'=5$



Solving pairs of events

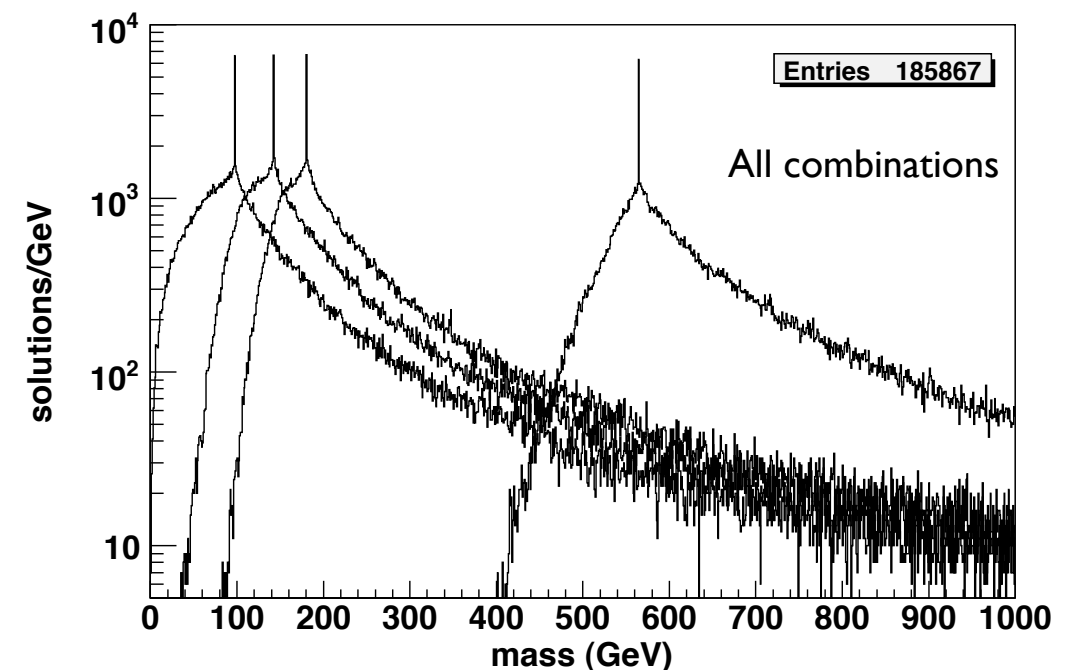
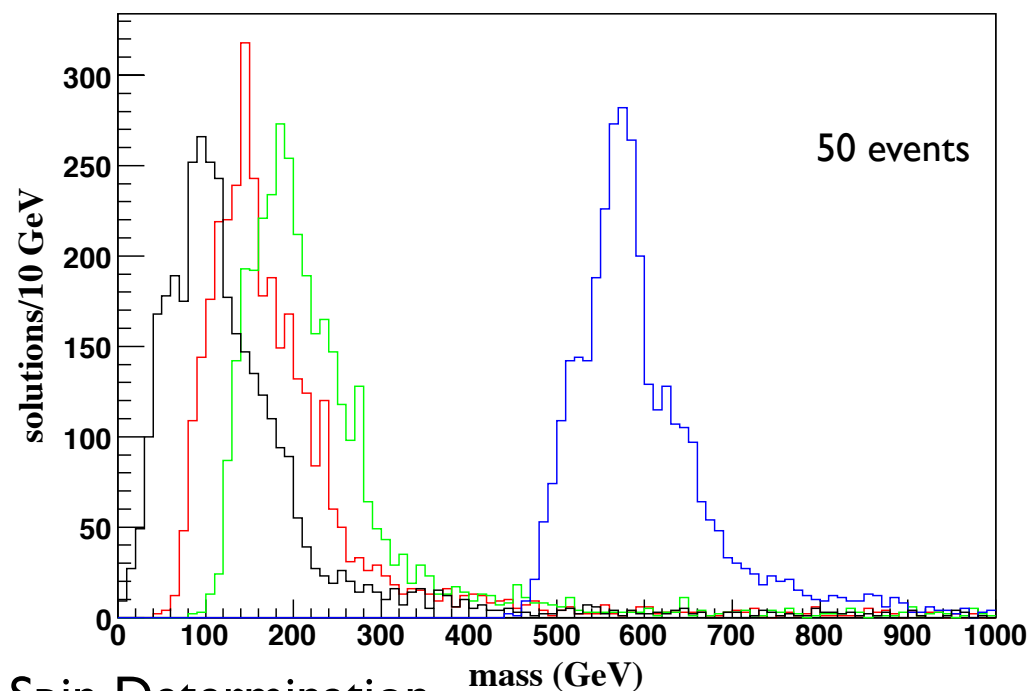
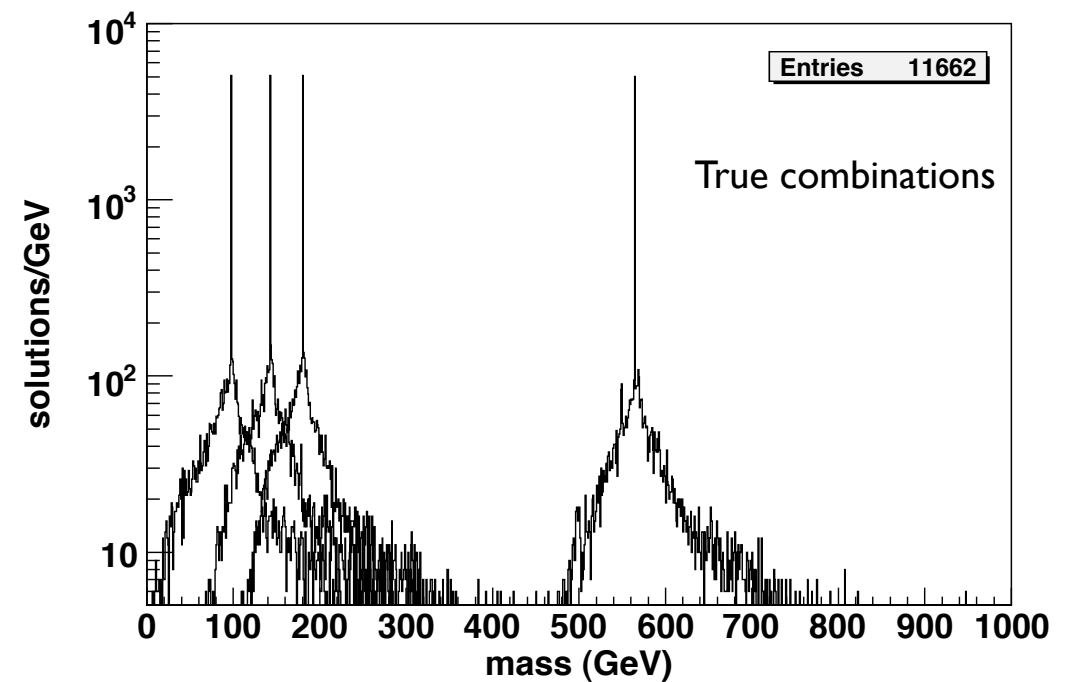
Chen, Gunion, Han, McElrath, PRD 80(2009)035020

- Two identical chains



- SPS Ia masses

$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	\tilde{u}_L	\tilde{e}_R
96	177	537	143



Fitting decay chains

- Assume a mass hypothesis: if $n+n' > 4$ then each event is over-constrained

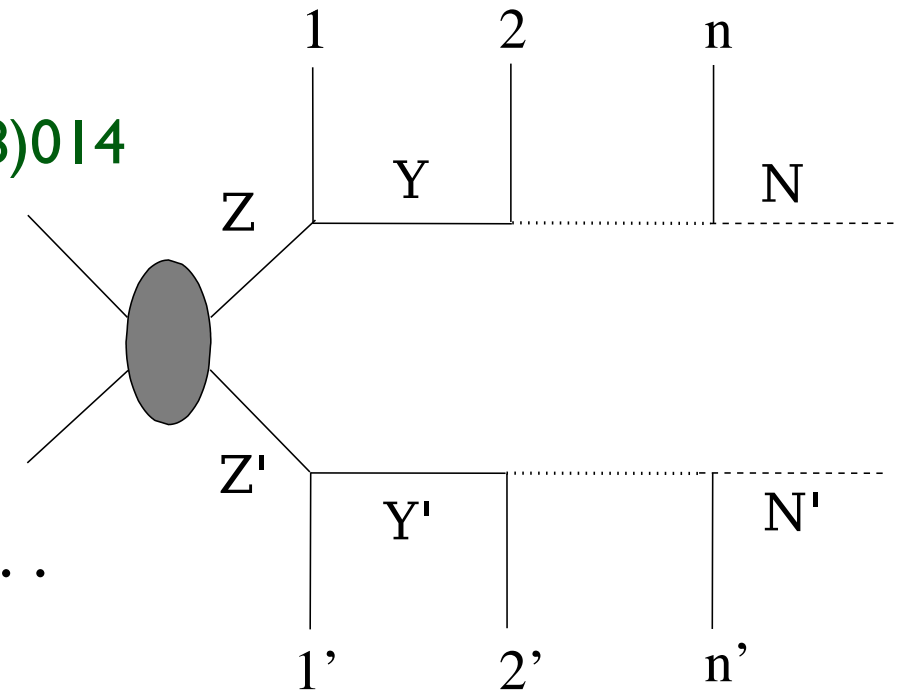
Kawagoe et al., PRD 71(2005)035008
BRW, JHEP 09(2009)124

- E.g. if $n, n'=3$, can solve for $p_N, p_{N'}$

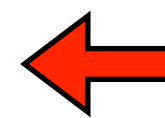
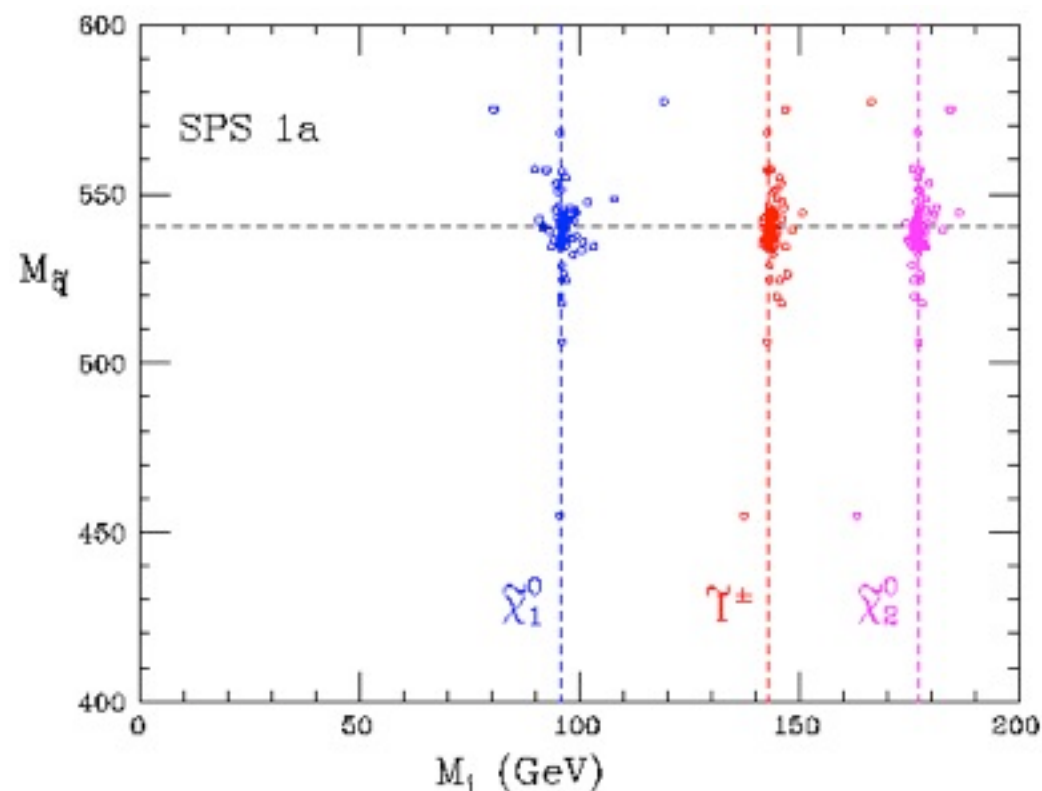
Nojiri, Polesello, Tovey, JHEP05(2008)014

- Measure goodness of fit by

$$\chi^2 = (p_N^2 - M_N^2)/\sigma_N^2 + (p_{N'}^2 - M_{N'}^2)/\sigma_{N'}^2$$



- N.B. $p_N^2 - M_N^2 = p_Z^2 - M_Z^2 = p_Y^2 - M_Y^2 = \dots$

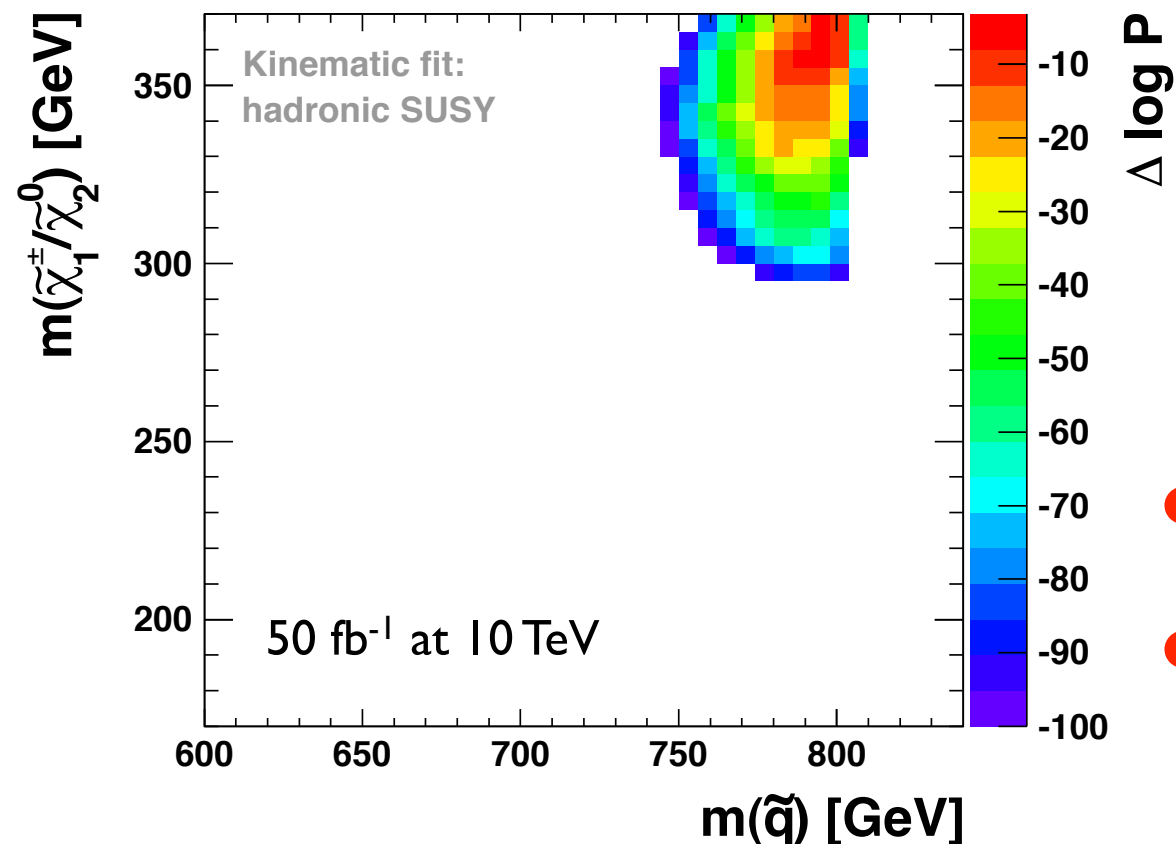
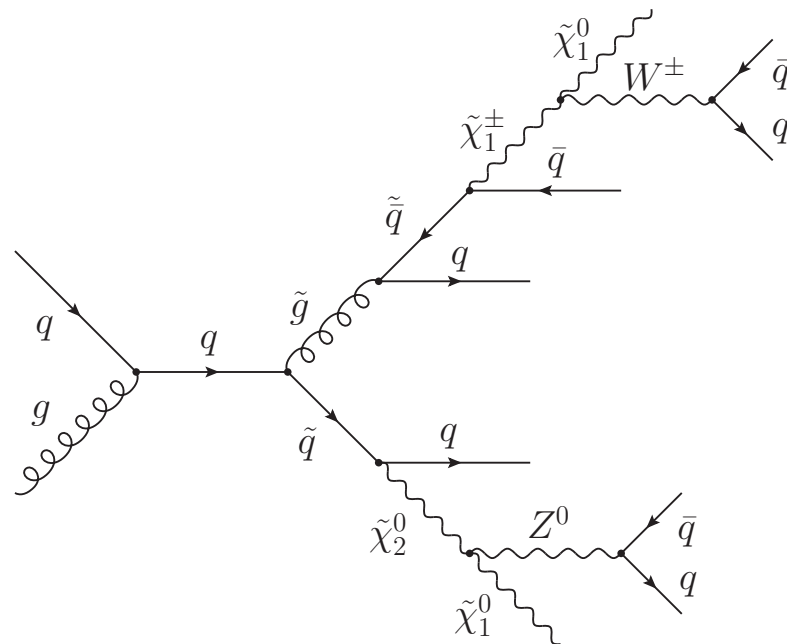


Best-fit points for 100 samples of 25 events (all combinations)

- Effects of jet contamination and background not included

Fully hadronic SUSY decays

Autermann et al., 0911.2607



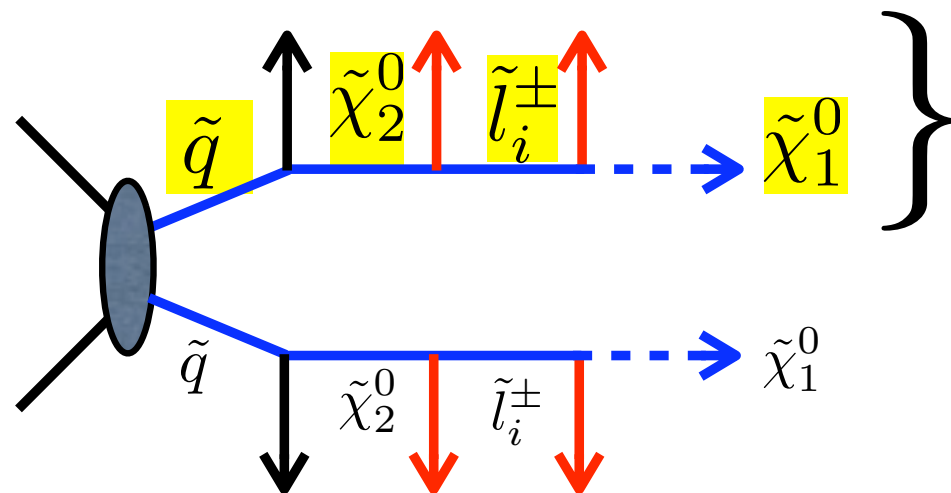
Parameter	Value	Particle	Mass [GeV]
m_0	230 GeV	$\tilde{q}_{ds,uc}^L$	807, 800
$m_{1/2}$	360 GeV	$\tilde{q}_{ds,uc}^R$	775, 782
$\tan \beta$	10	$\tilde{q}_b^{1/2}$	734, 771
A_0	0 GeV	$\tilde{q}_t^{1/2}$	599, 787
$\text{sign} \mu$	+	\tilde{g}	851
		$\tilde{\chi}_{1,2,3,4}^0$	144, 271, 475, 490
		$\tilde{\chi}_{1,2}^\pm$	273, 487

	without ISR & FSR		with ISR & FSR	
	Bg	Sig	Bg	Sig
selection efficiency	4.2%	29%	6.9%	30%
S/B		1/10.9		1/16.4
S/B (complete)		1/11.3		1/33.4

- $m(\tilde{g}), m(\tilde{\chi}_1^0)$ held fixed
- wrong combinations shift $m(\tilde{\chi}_1^\pm / \chi_2^0)$

Including dilepton edge

Nojiri, Sakurai, BRW, JHEP 06(2010)069

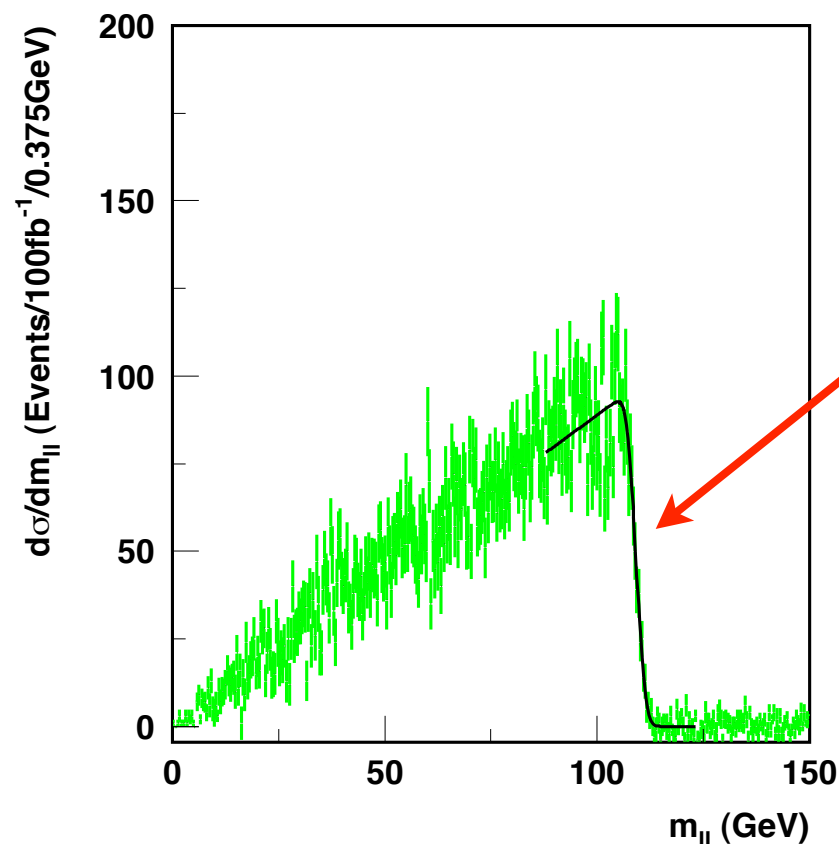


$$M_1 = m_{\tilde{l}}^2 - m_{\tilde{\chi}_1^0}^2$$

$$M_2 = m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}}^2$$

$$M_3 = m_{\tilde{q}}^2 - m_{\tilde{\chi}_2^0}^2$$

$$M_4 = M_{ll}^{\max} = (m_{\tilde{\chi}_2^0}^2 - m_{\tilde{l}}^2)(m_{\tilde{l}}^2 - m_{\tilde{\chi}_1^0}^2)/m_{\tilde{l}}^2$$

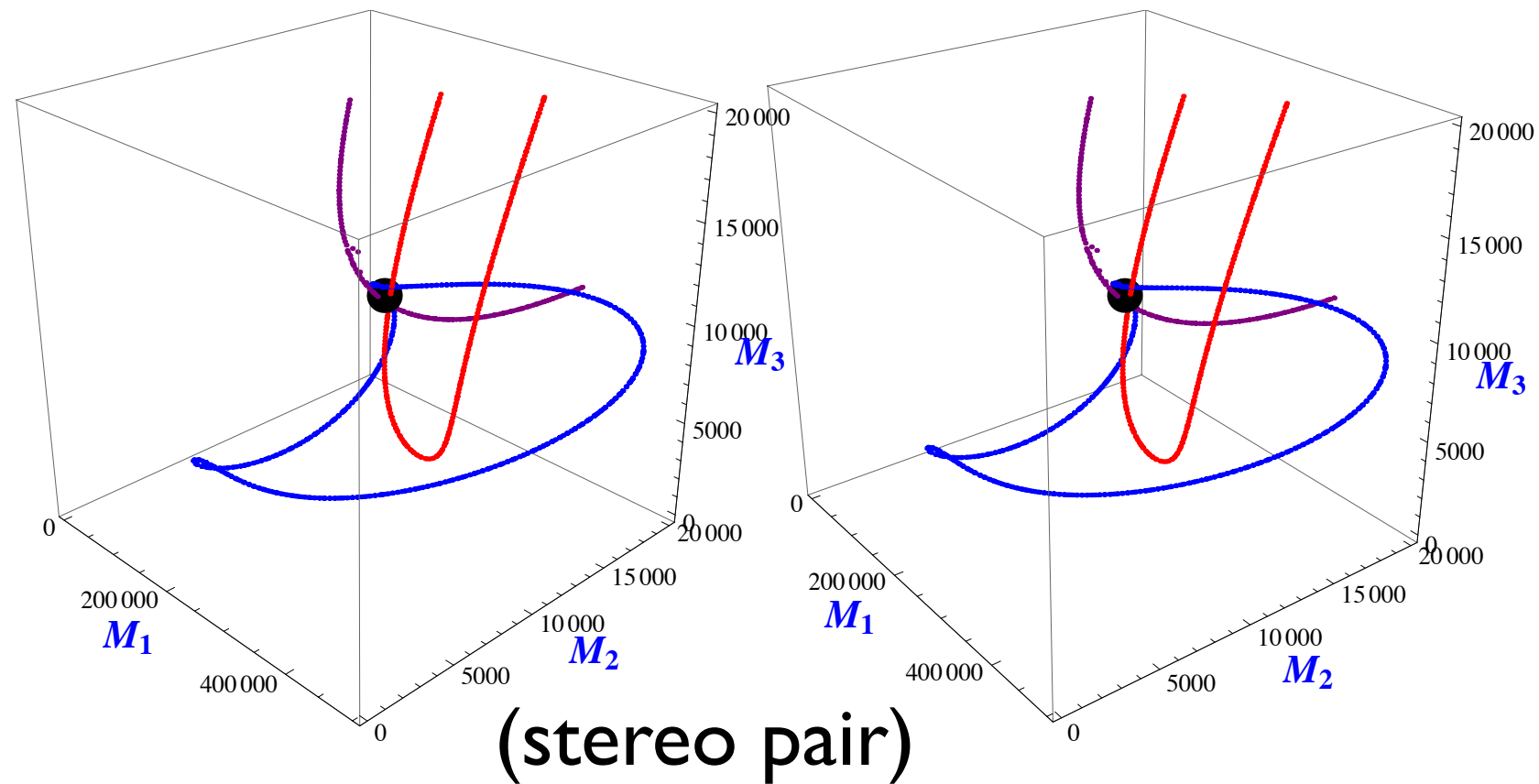


fix M_4 by dilepton mass measurement

- Number of unknowns reduced to 3

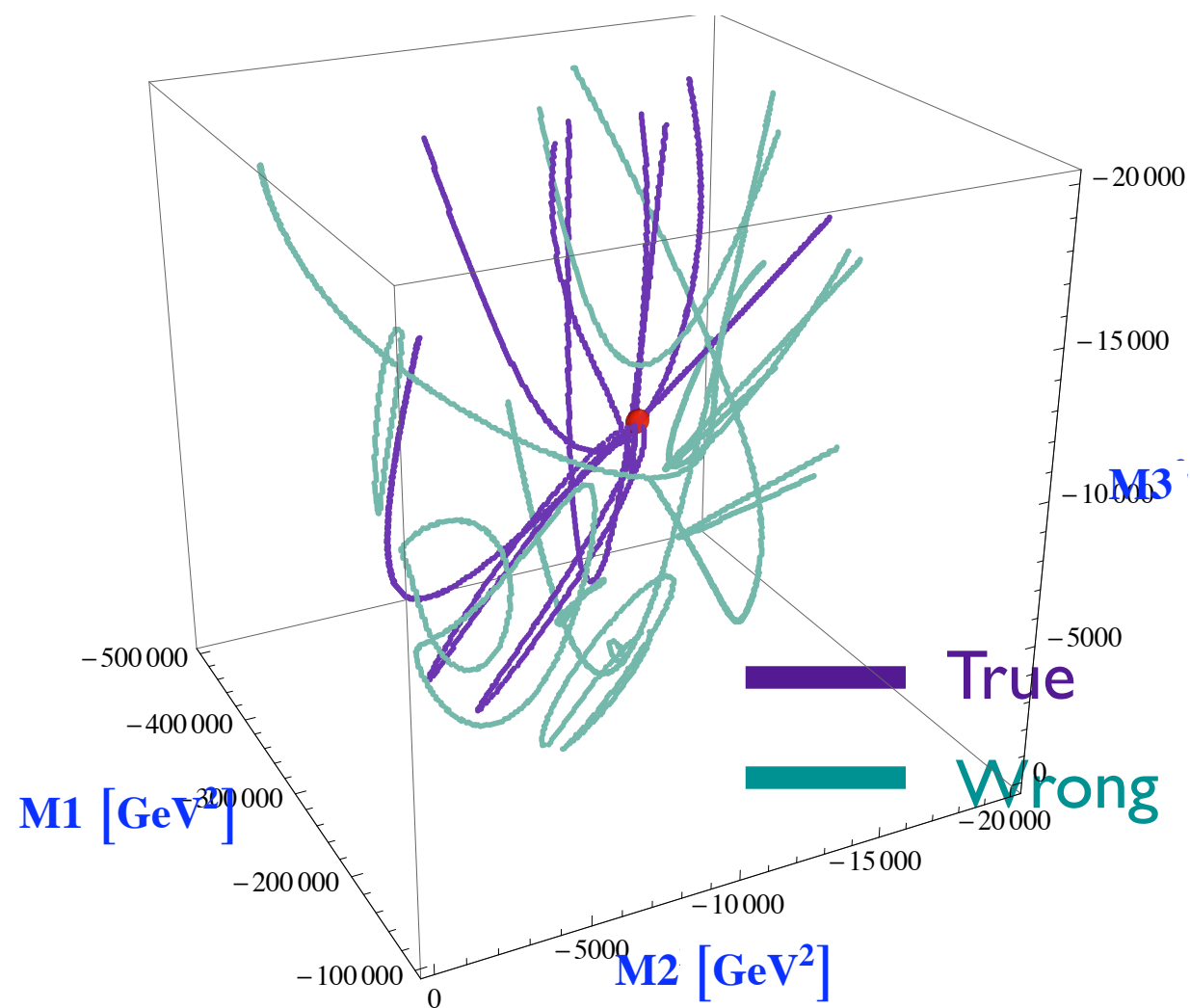
➔ Visualize solution curves in 3D

Mass solution curves

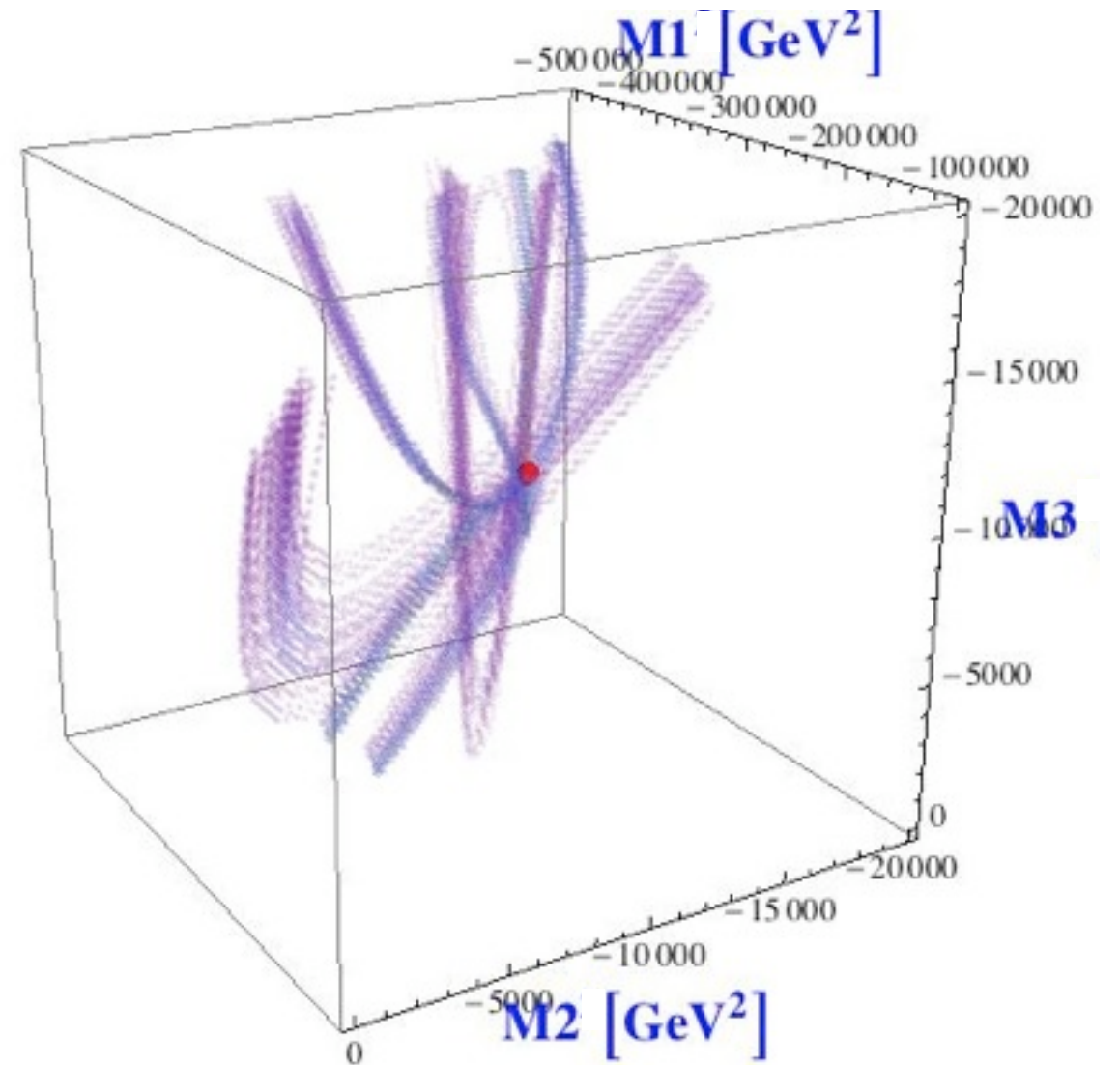


- 3 events, parton level, correct combinations
- ➔ Curves intersect at correct values

Mass solution curves (2)



- Wrong combinations don't intersect



- Resolution smearing (right combinations)

Herwig MC study

	m_0	$m_{1/2}$	A_0	$\tilde{\chi}_1^0$	\tilde{e}_R	$\tilde{\chi}_2^0$	\tilde{u}_L
Point A	110	220	0	86	142	161	504
Point B	100	250	-100	99	141	186	563
Point C	140	260	0	103	174	193	592

Table 2. Parameters and mass spectra in GeV for non-CMSSM model points A, B and C. Parameters common to all points are $m_0^{\text{3rd gen.}} = 300 \text{ GeV}$, $\tan \beta = 10$, $\text{sign}(\mu) = +$.

The following cuts are applied in order to select signal events:

- (i) $M_{\text{eff}} \equiv \sum_{i=1}^4 p_T^{\text{jet},i} + \sum_{i=1}^4 p_T^{\text{lep},i} + E_T^{\text{miss}} > 400 \text{ GeV}$;
- (ii) $E_T^{\text{miss}} > \max(200 \text{ GeV}, 0.2 M_{\text{eff}})$;
- (iii) At least two jets with $p_T^{\text{jet},1} > 100 \text{ GeV}$ and $p_T^{\text{jet},2} > 50 \text{ GeV}$ within $|\eta| < 2.5$;
- (iv) Two pairs of opposite sign same flavour leptons with $p_T > 20 \text{ GeV}$ and $|\eta| < 3$;
- (v) No b jet with $p_T > 30 \text{ GeV}$ and $|\eta| < 3$.

 $\sim 250 \text{ events for } 20 \text{ fb}^{-1} \text{ at } 14 \text{ TeV}$

MC results

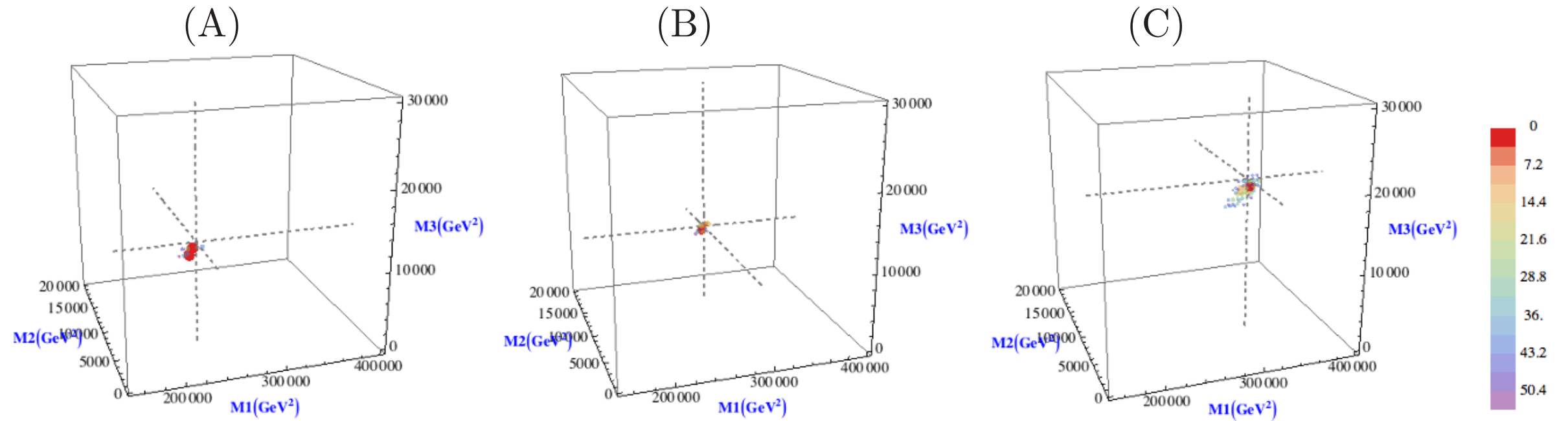


Figure 4. Distribution of $\Delta\chi^2(\mathbf{M})$ for each model point at detector level. The true mass point is at the intersection of the three dashed lines.

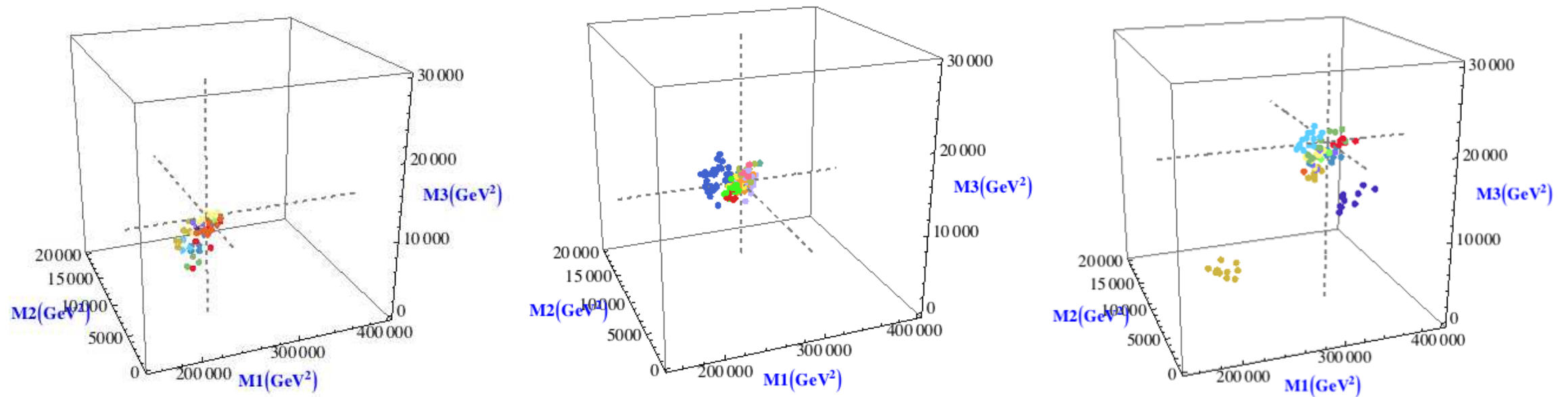


Figure 5. One-sigma regions of 10 sub-samples, distinguished by their colours. Each sub-sample contains 25 events.

Global Inclusive Observables

Inclusive observables

- How can jets from hard subprocess be distinguished from ISR jets?
- In principle, there is no way! So let's look at "global inclusive" observables
- Consider e.g. the total invariant mass M visible in the detector:

$$M = \sqrt{E^2 - P_z^2 - \cancel{E}_T^2}$$

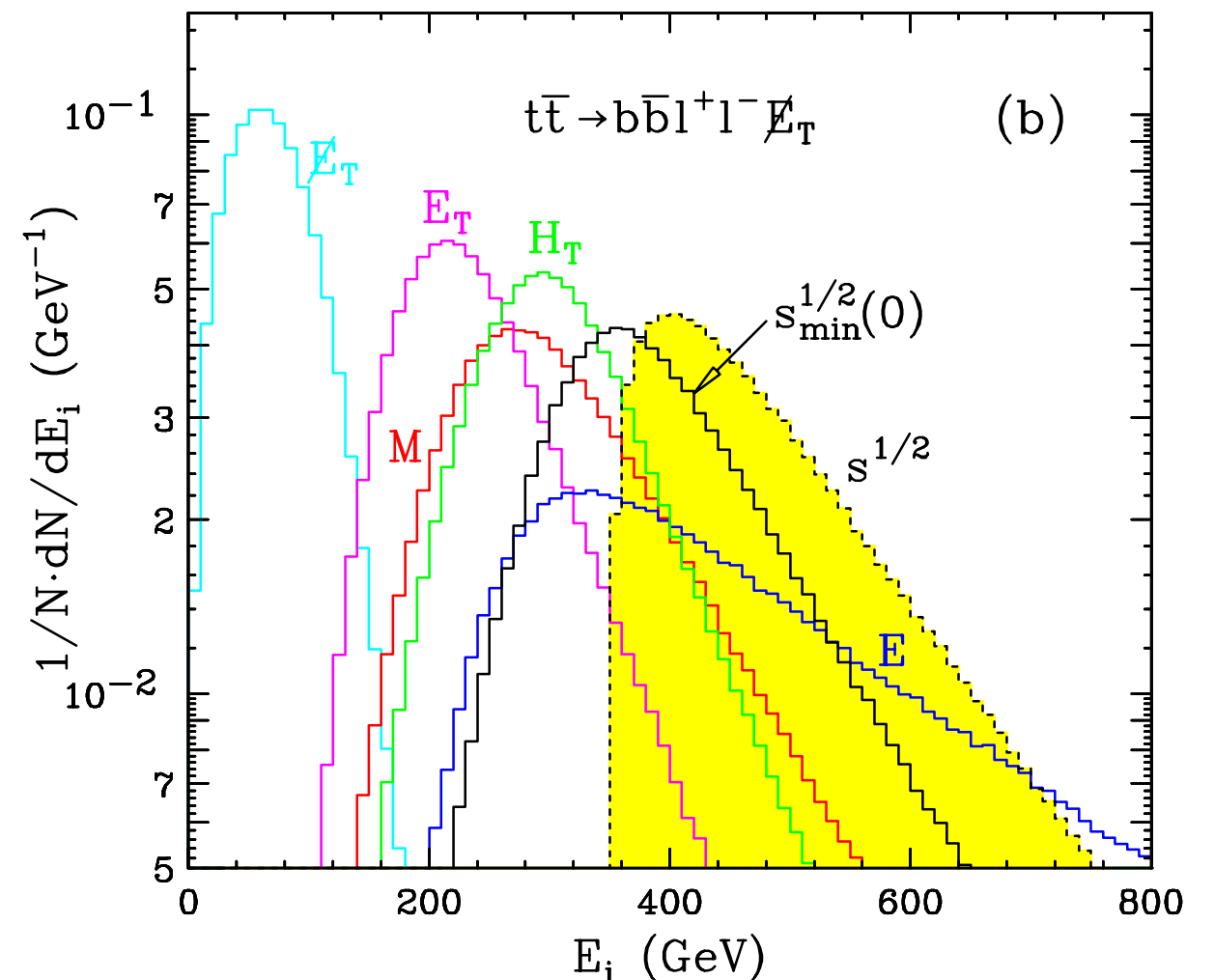
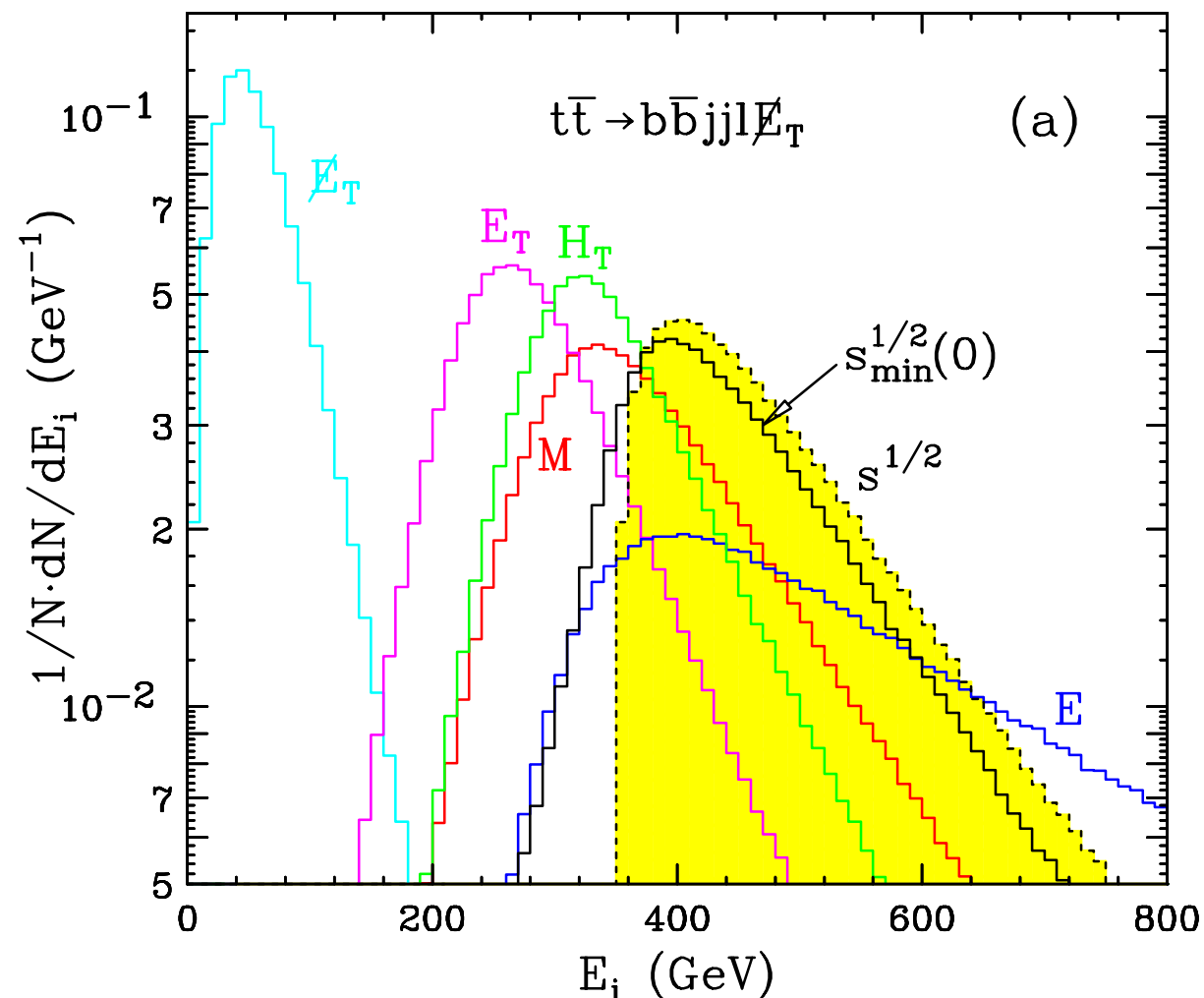
or (Konar, Kong & Matchev, JHEP 03(2009)085)

$$\hat{s}_{\min}^{1/2}(M_{\text{inv}}) = \sqrt{M^2 + \cancel{E}_T^2} + \sqrt{M_{\text{inv}}^2 + \cancel{E}_T^2}$$

Inclusive observables: MC results

$$\hat{s}_{\min}^{1/2}(M_{\text{inv}}) = \sqrt{M^2 + \cancel{E}_T^2} + \sqrt{M_{\text{inv}}^2 + \cancel{E}_T^2}$$

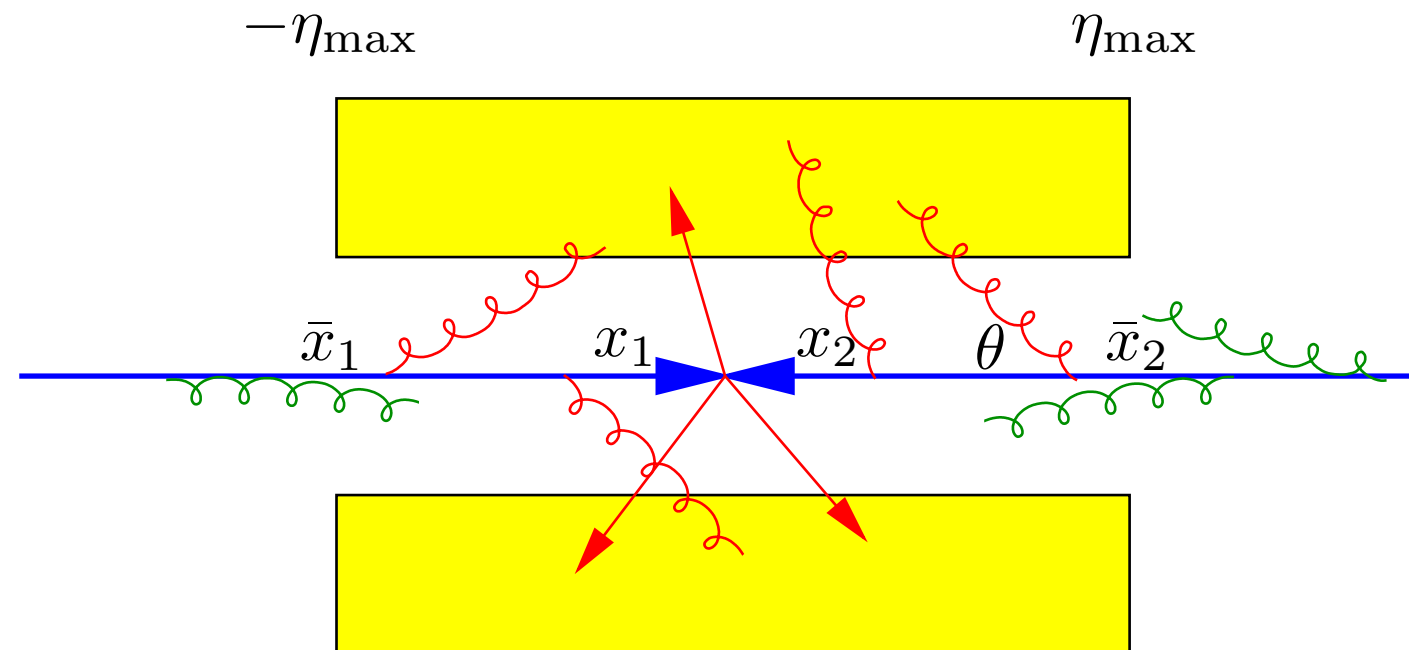
$$M = \sqrt{E^2 - P_z^2 - \cancel{E}_T^2} \quad H_T = E_T + \cancel{E}_T$$



● Pythia MC: ISR turned off!

Konar, Kong, Matchev, JHEP03(2009)085

ISR effects on inclusive observables



$$\frac{d\sigma}{dM^2} = \int \frac{d\bar{x}_1}{\bar{x}_1} \frac{d\bar{x}_2}{\bar{x}_2} dx_1 dx_2 f(\bar{x}_1, Q_c) f(\bar{x}_2, Q_c) K\left(\frac{x_1}{\bar{x}_1}; Q_c, Q\right) K\left(\frac{x_2}{\bar{x}_2}; Q_c, Q\right) \hat{\sigma}(x_1 x_2 S) \delta(M^2 - \bar{x}_1 \bar{x}_2 S)$$

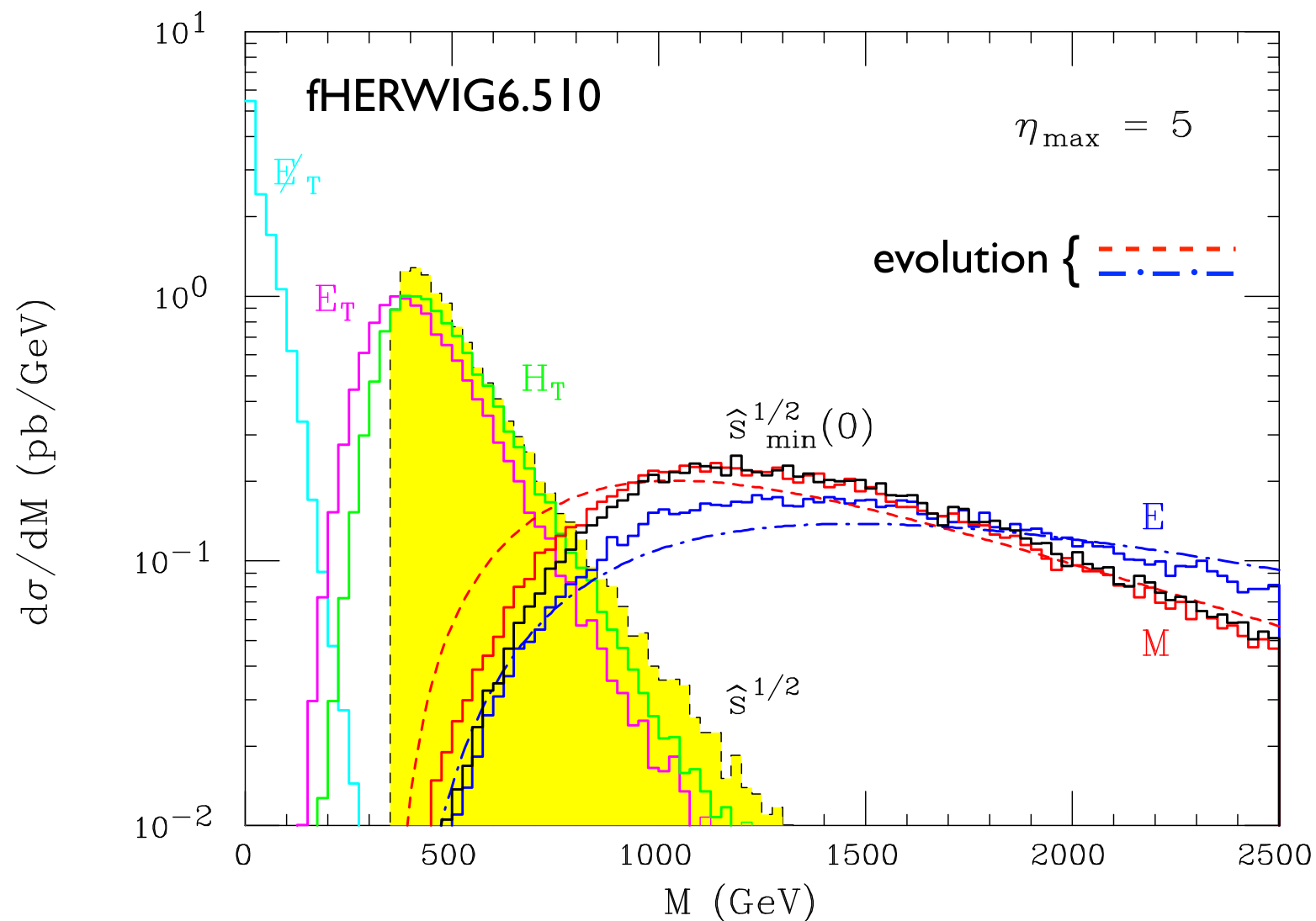
- ISR at $\theta > \theta_c \sim \exp(-\eta_{\max})$ enters detector
- Hard scale $Q^2 \sim \hat{s} = x_1 x_2 S$ but $M^2 = \bar{x}_1 \bar{x}_2 S$
- PDFs sampled at $Q_c \sim \theta_c Q$

A Papaefstathiou & BW, JHEP 04(2010)084

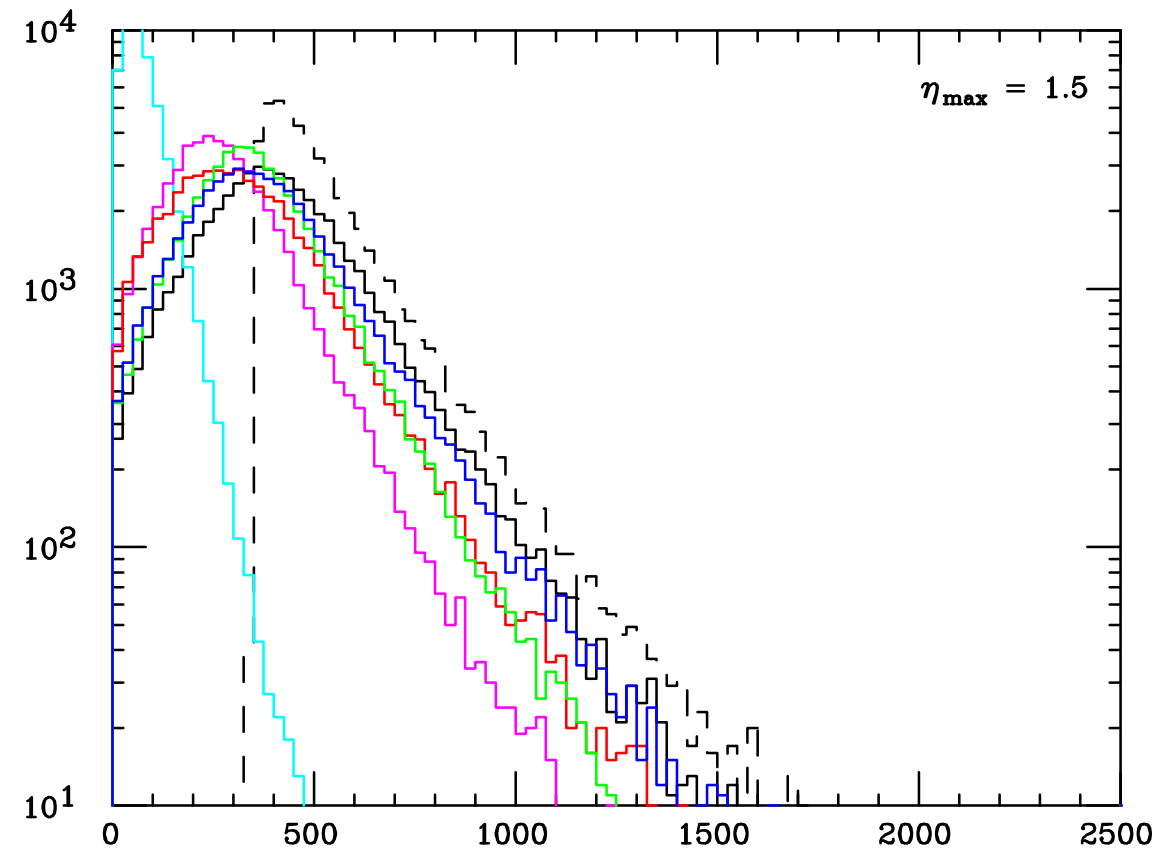
ISR Effects: MC Results

$$\hat{s}_{\min}^{1/2}(M_{\text{inv}}) = \sqrt{M^2 + \cancel{E}_T^2} + \sqrt{M_{\text{inv}}^2 + \cancel{E}_T^2}$$

$$M = \sqrt{E^2 - P_z^2 - \cancel{E}_T^2} \quad H_T = E_T + \cancel{E}_T$$

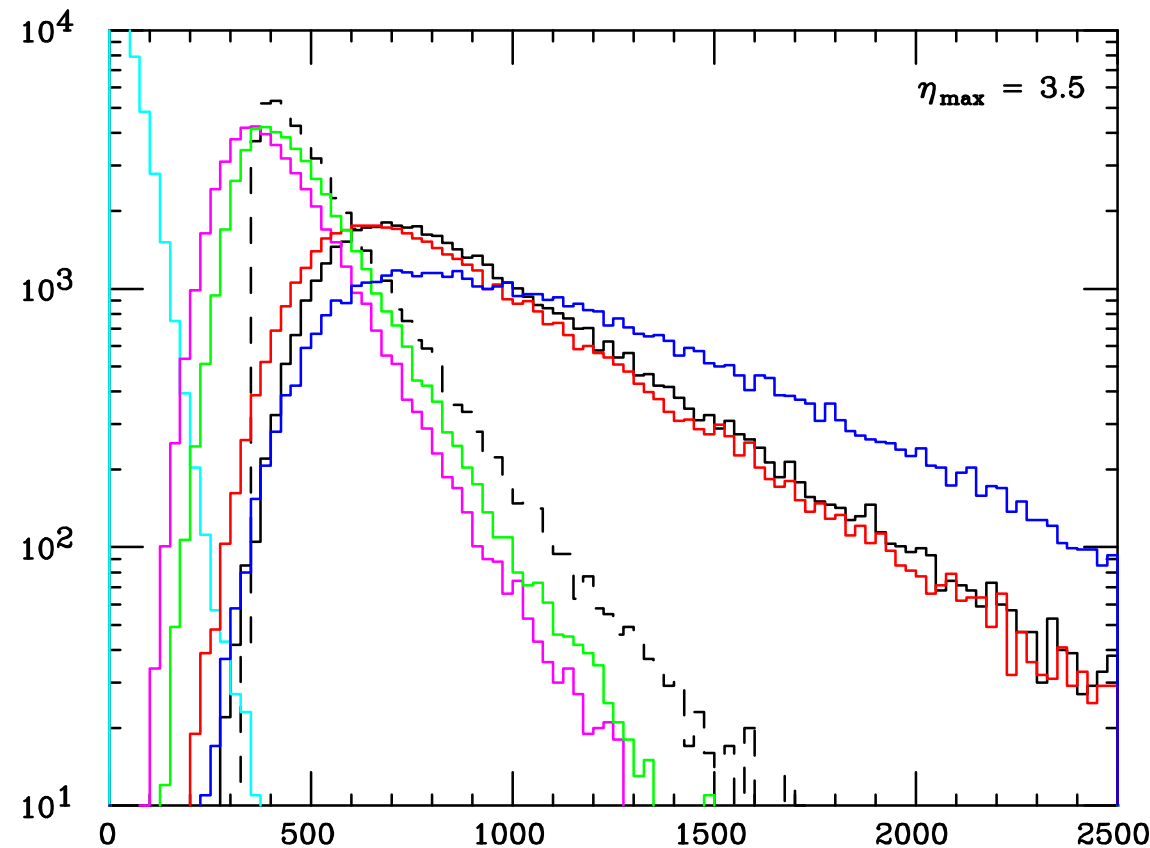


Dependence on η_{\max}



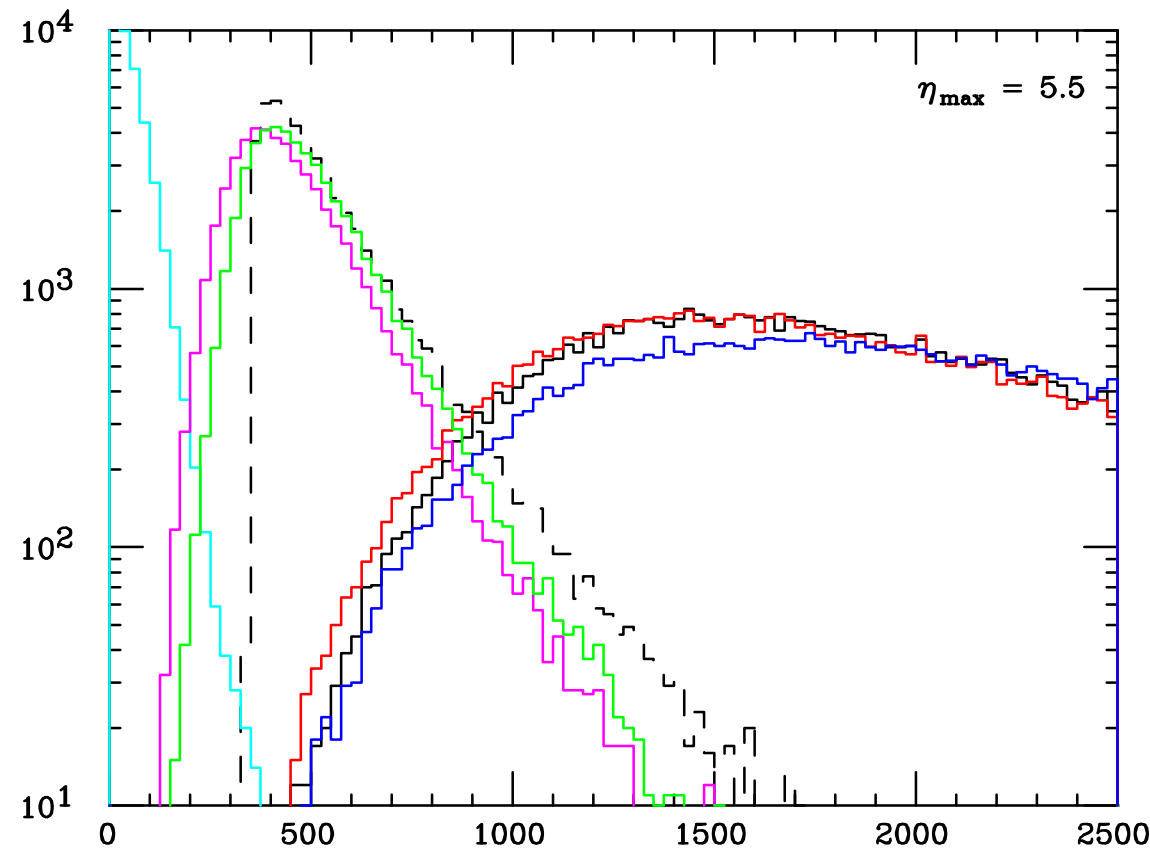
- E , M , \hat{s}_{\min} strongly dependent; \cancel{E}_T , E_T , H_T not

Dependence on η_{\max}



- E , M , \hat{s}_{\min} strongly dependent; \cancel{E}_T , E_T , H_T not

Dependence on η_{\max}



- E , M , \hat{s}_{\min} strongly dependent; E_T , E_T , H_T not

Conclusions on Masses

- M_{T2} will be an important observable
 - New ideas on reducing ISR jet contamination
- Decay chains: solving and fitting
 - Include edge information when solving
- Global inclusive observables
 - Only transverse observables are robust
 - Scale information from others?

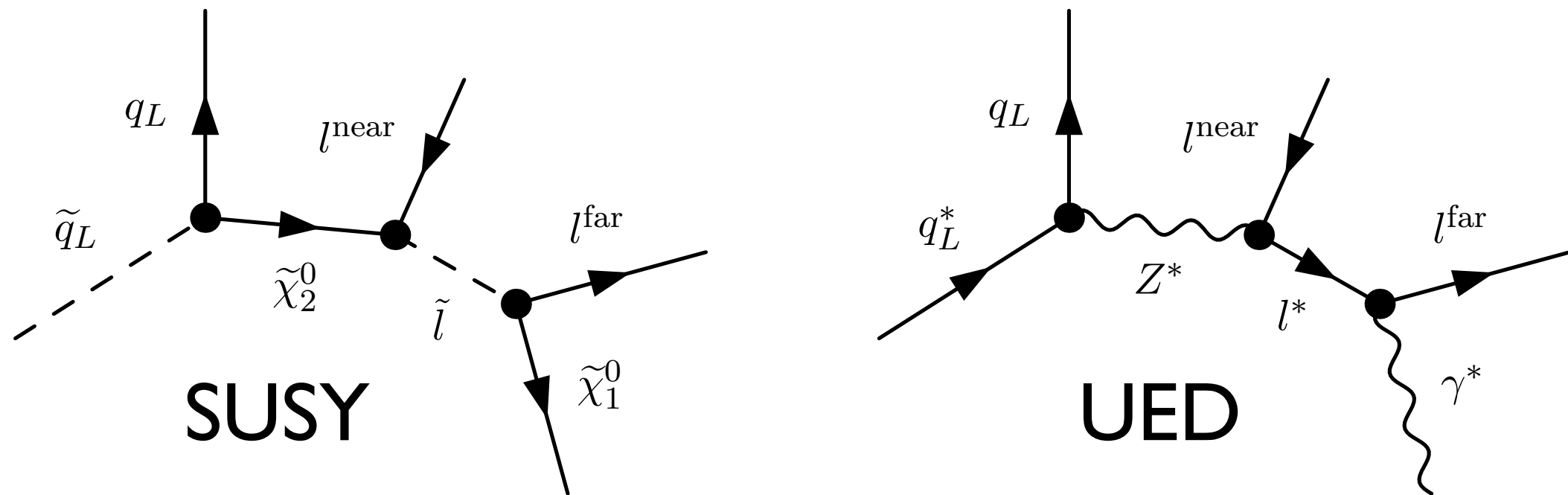
Spin Determination with ...

- Sequential decay chains
- Dileptons
- Three-body decays
- Cross sections

See also: review by Wang & Yavin,
Int.J.Mod.Phys. A23(2008)4647

Decay chains

“Classic” decay chain again

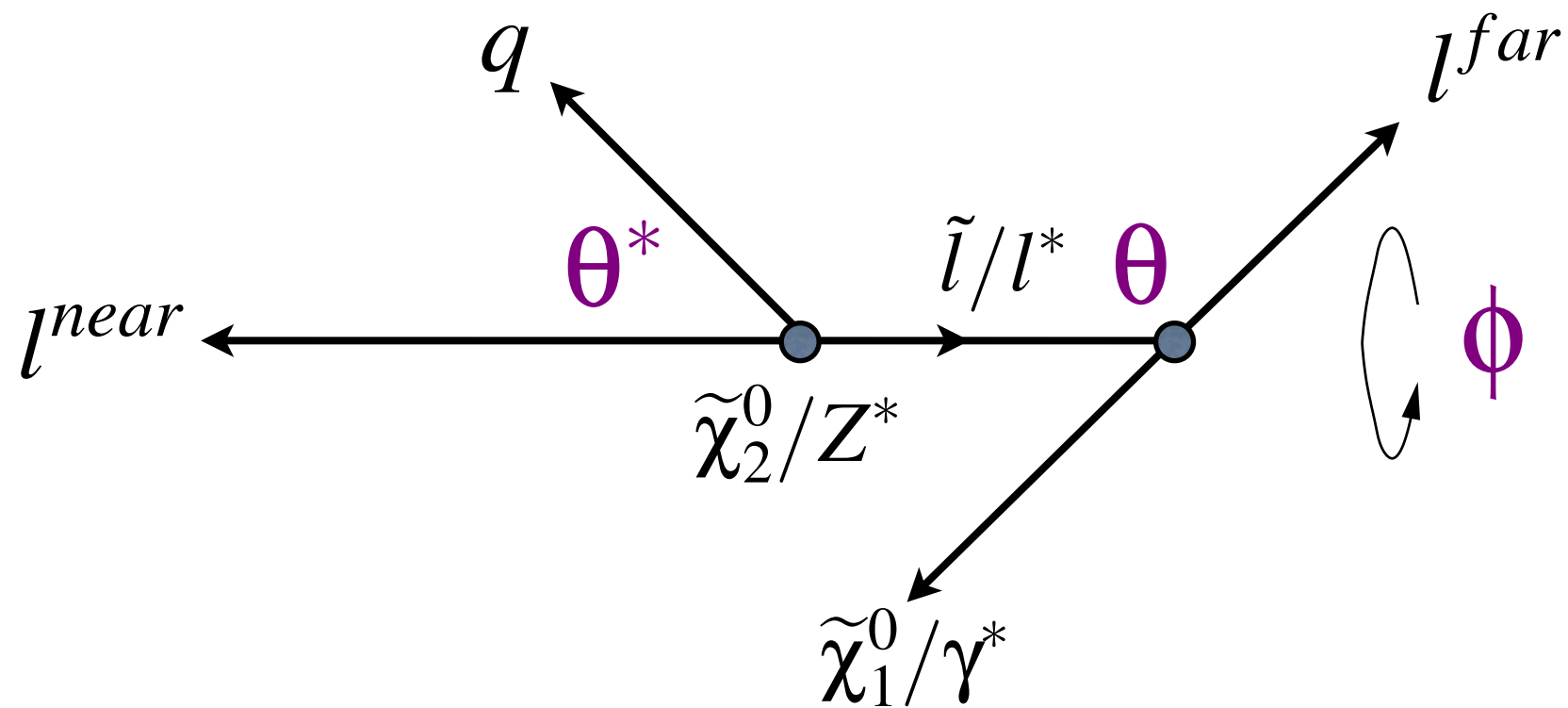


- Two distinct helicity structures, with different spin correlations:
 - Process 1: $\{q, l^{\text{near}}, l^{\text{far}}\} = \{q_L, l_L^-, l_L^+\}$ or $\{\bar{q}_L, l_L^+, l_L^-\}$ or $\{q_L, l_R^+, l_R^-\}$ or $\{\bar{q}_L, l_R^-, l_R^+\}$;
 - Process 2: $\{q, l^{\text{near}}, l^{\text{far}}\} = \{q_L, l_L^+, l_L^-\}$ or $\{\bar{q}_L, l_L^-, l_L^+\}$ or $\{q_L, l_R^-, l_R^+\}$ or $\{\bar{q}_L, l_R^+, l_R^-\}$.

Smillie & BW, JHEP 10(2005)069

Datta, Kong, Matchev, PR D72(2005)096006

Angular variables



➔ θ^* defined in $\tilde{\chi}_2^0/Z^*$ rest frame

➔ θ, ϕ defined in \tilde{l}/l^* rest frame

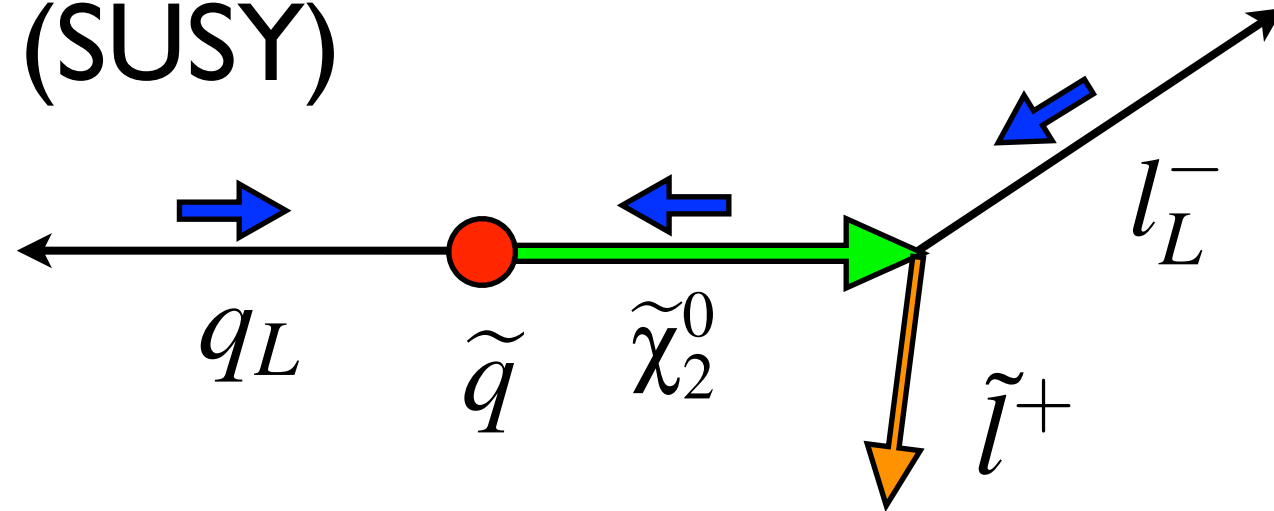
Invariant masses

- ql^{near} : $m_{ql}/(m_{ql})_{max} = \sin(\theta^*/2)$
- $l^{near}l^{far}$: $m_{ll}/(m_{ll})_{max} = \sin(\theta/2)$
- ql^{far} : $m_{ql}/(m_{ql})_{max} = \frac{1}{2} \left[(1-y)(1 - \cos \theta^* \cos \theta) + (1-y)(\cos \theta^* - \cos \theta) - 2\sqrt{y} \sin \theta^* \sin \theta \cos \phi \right]^{\frac{1}{2}}$

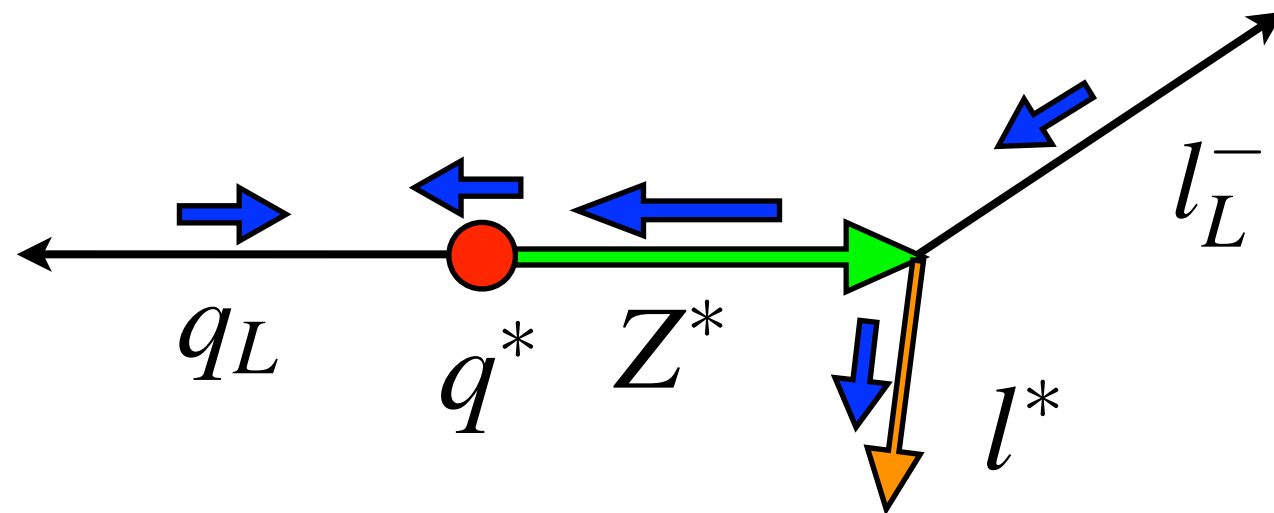
where $x = m_{Z^*}^2/m_{q^*}^2$, $y = m_{l^*}^2/m_{Z^*}^2$, $z = m_{\gamma^*}^2/m_{l^*}^2$

Helicity dependence

- Process I (SUSY)



- Process I (UED, transverse Z^* : $P_T/P_L = 2x$)



➔ Both prefer high $(ql^-)^{near}$ invariant mass

UED and SUSY mass spectra

- UED models tend to have quasi-degenerate spectra

γ^*	Z^*	q_L^*	l_R^*	l_L^*
501	536	598	505	515

Table 1: UED masses in GeV, for $R^{-1} = 500\text{GeV}$, $\Lambda R = 20$, $m_h = 120\text{GeV}$, $\overline{m}_h^2 = 0$ and vanishing boundary terms at cut-off scale Λ .

($M_n \sim n/R$
broken by boundary
terms and loops, with
low cutoff)

- SUSY spectra typically more hierarchical

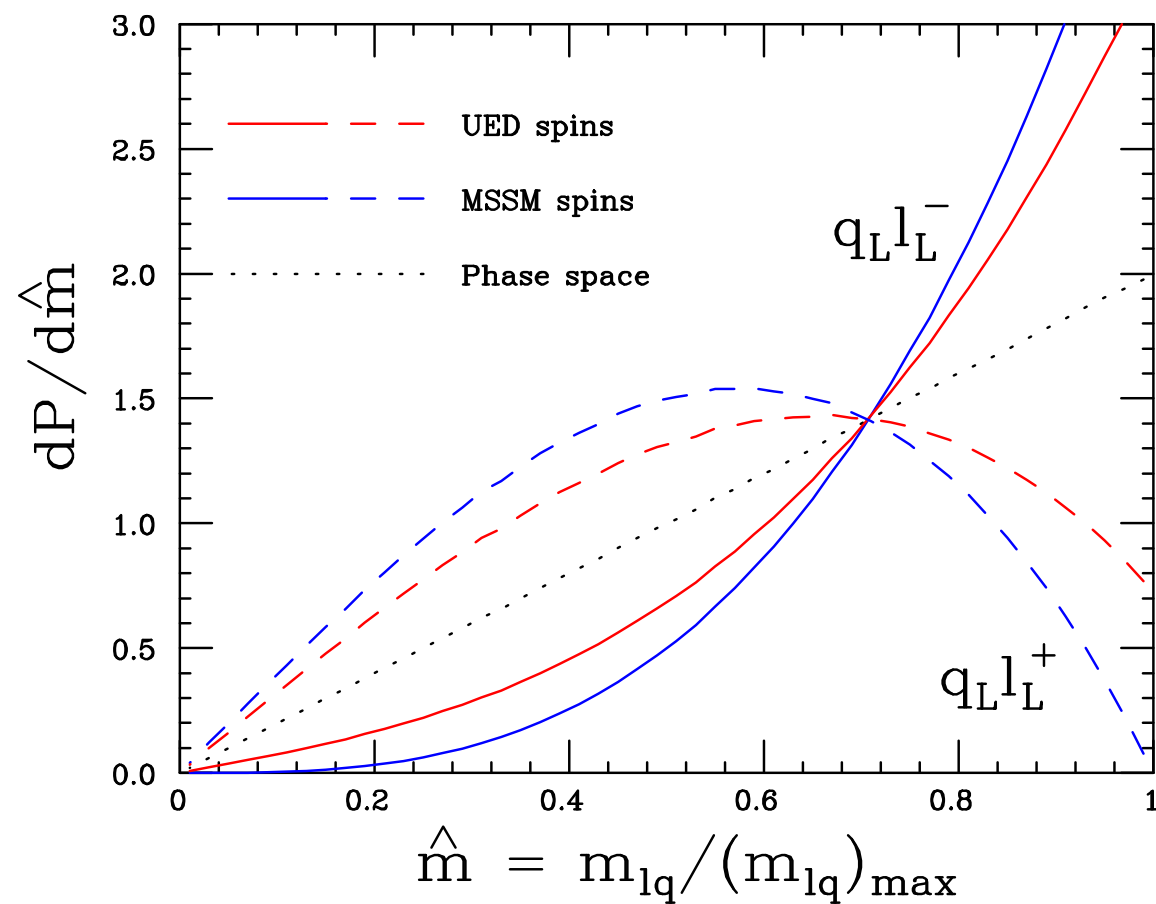
$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	\tilde{u}_L	\tilde{e}_R	\tilde{e}_L
96	177	537	143	202

Table 2: SUSY masses in GeV, for SPS point 1a.

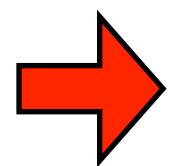
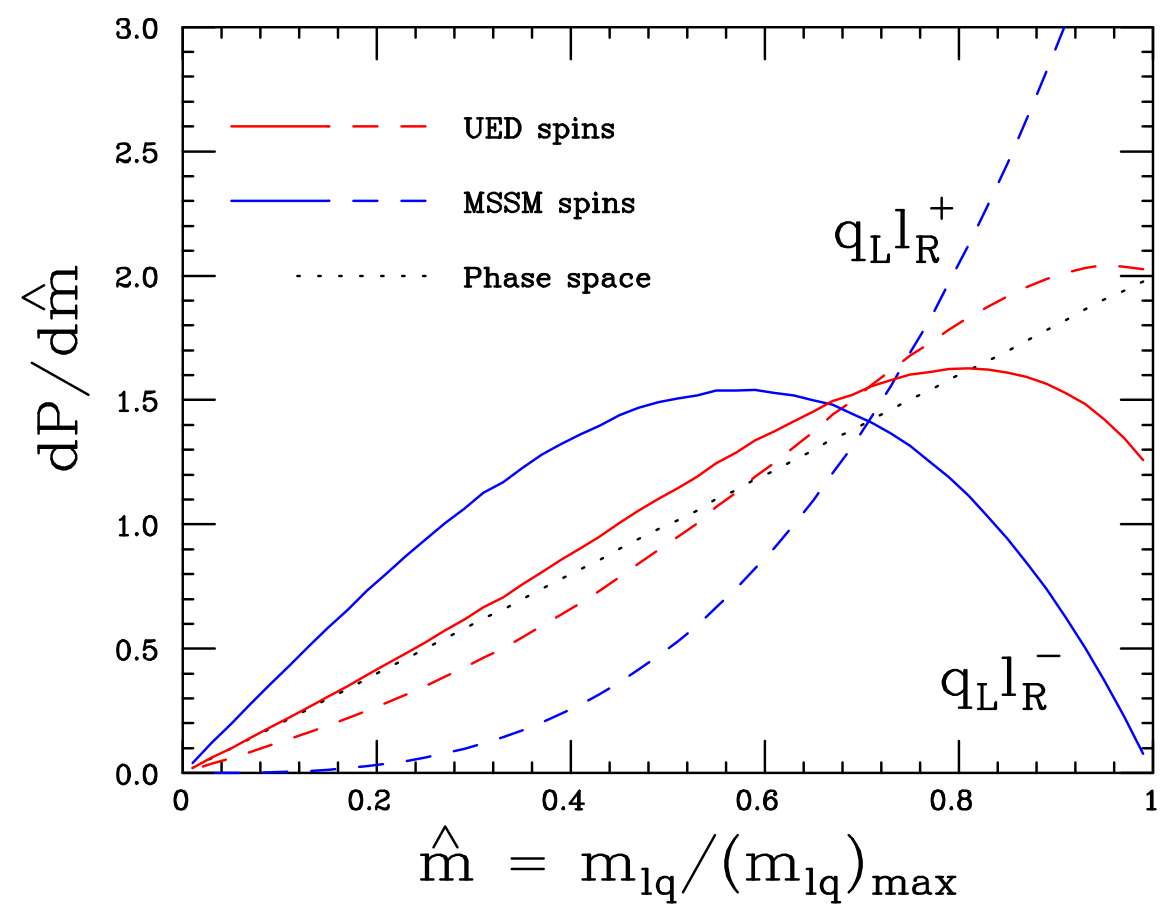
(high-scale universality)

ql^{near} mass distribution

UED masses



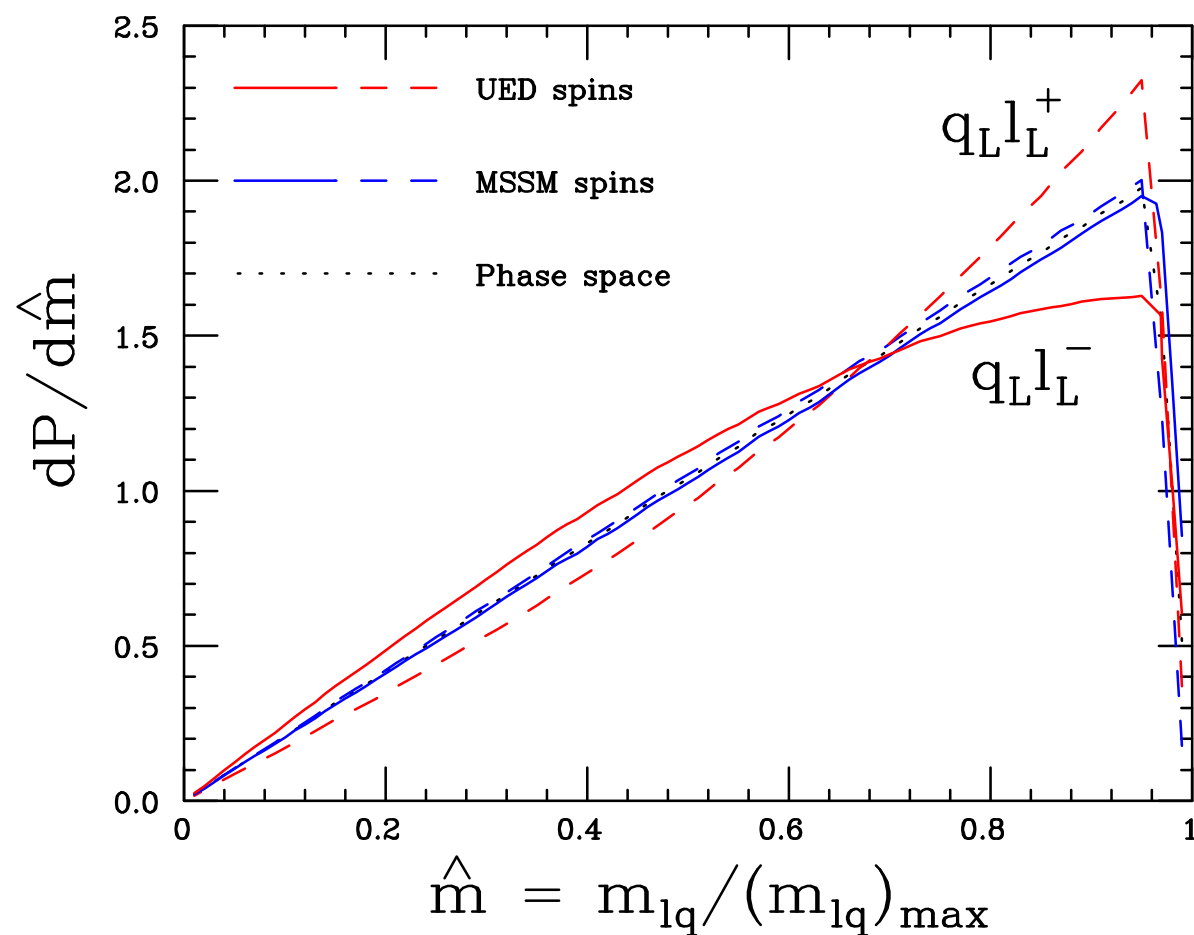
SPS Ia masses



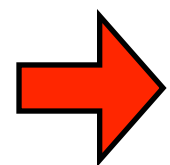
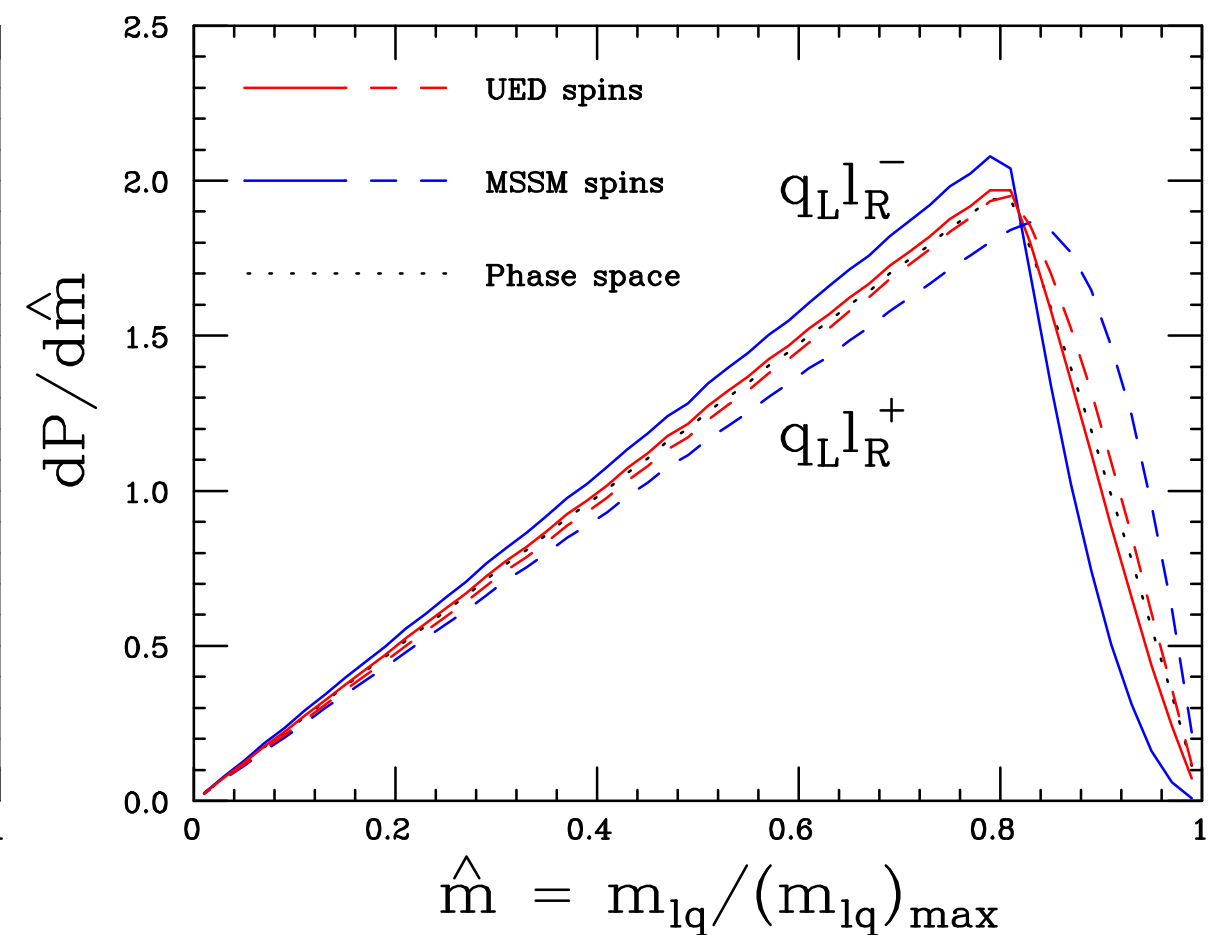
UED and SUSY not distinguishable for UED masses

ql^{far} mass distribution

UED masses



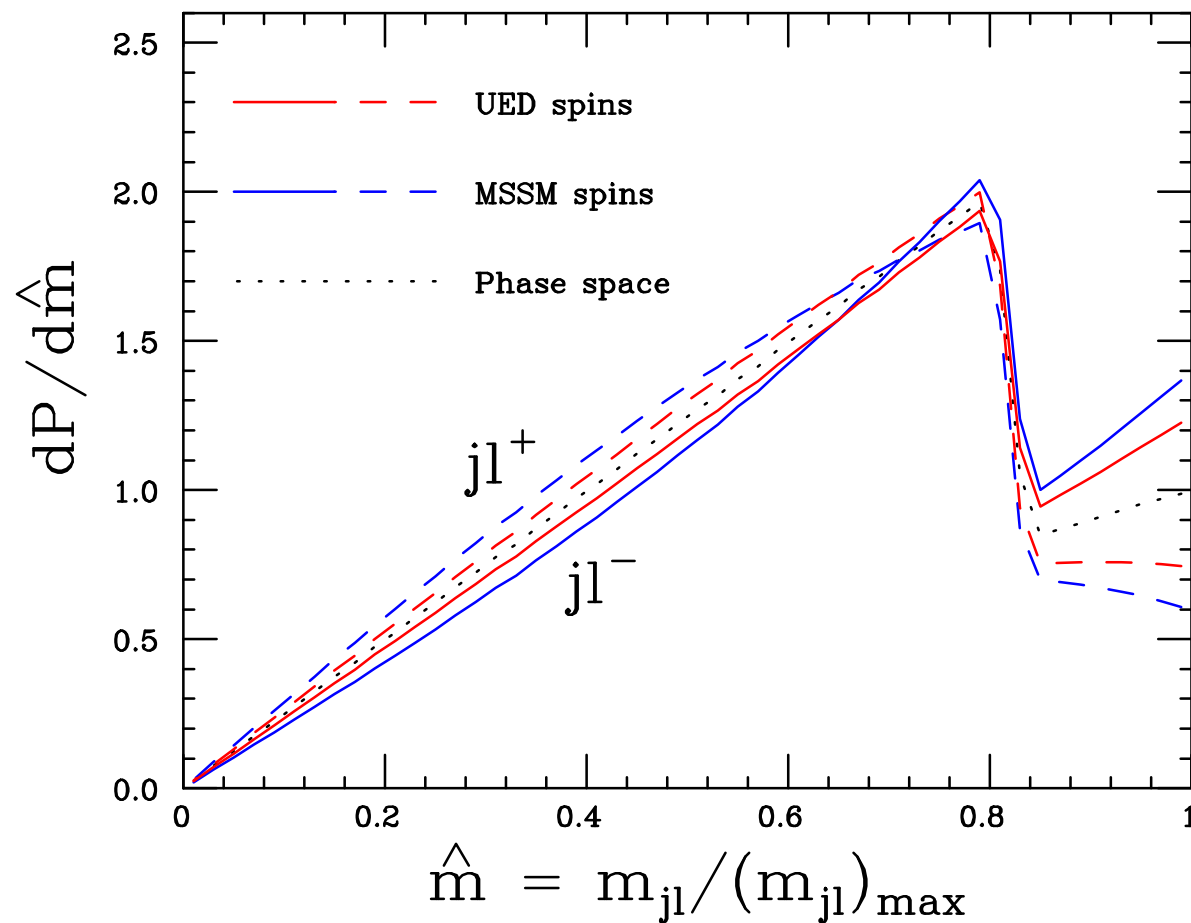
SPS Ia masses



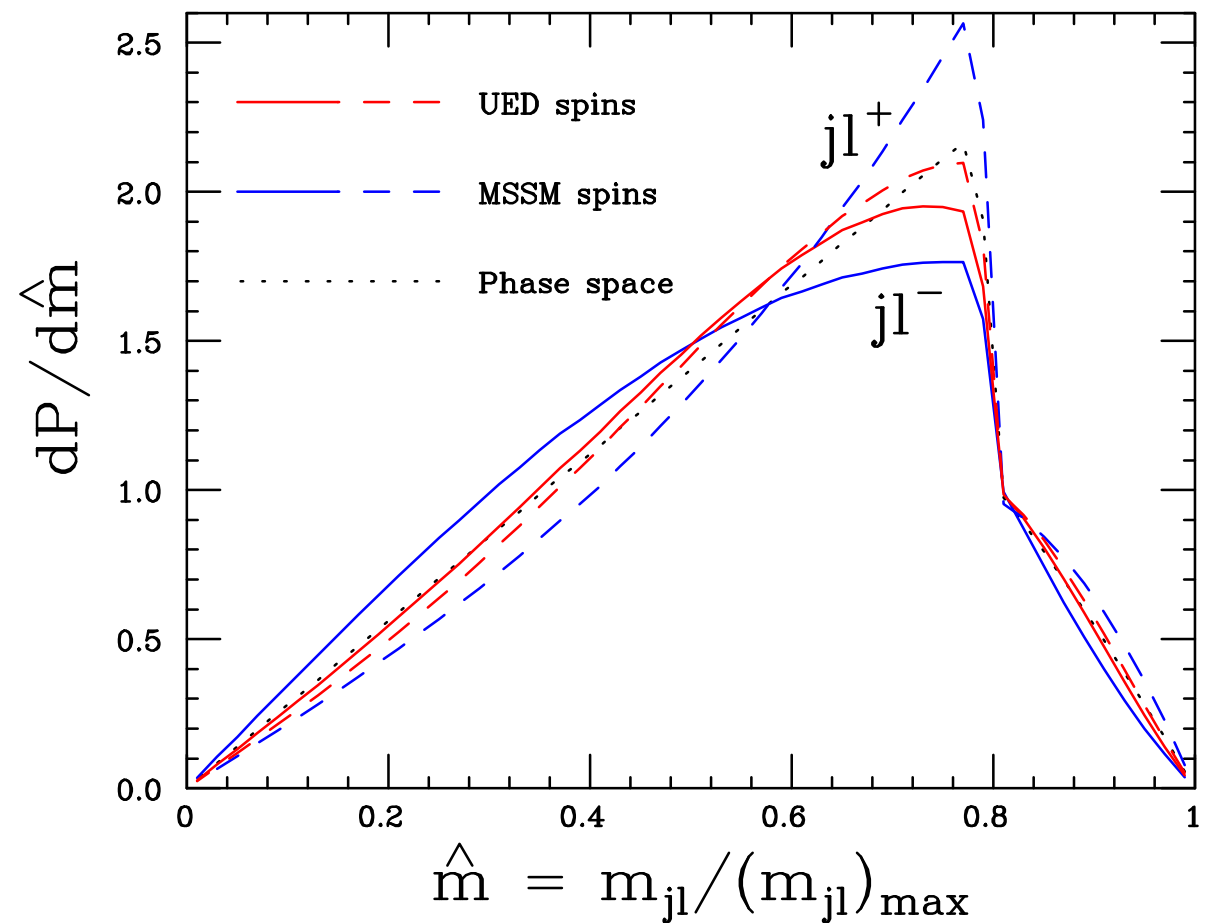
Correlation weak but slightly enhances UED-SUSY difference

Jet + lepton mass distribution

UED masses



SPS Ia masses



- ➡ Not resolvable for UED masses, maybe for SUSY masses
- ➡ Charge asymmetry due to **quark vs antiquark excess**

Production cross sections (pb)

Masses	Model	σ_{all}	σ_{q^*}	$\sigma_{\bar{q}^*}$	f_q
UED	UED	253	163	84	0.66
UED	SUSY	28	18	9	0.65
SPS 1a	UED	433	224	80	0.74
SPS 1a	SUSY	55	26	11	0.70

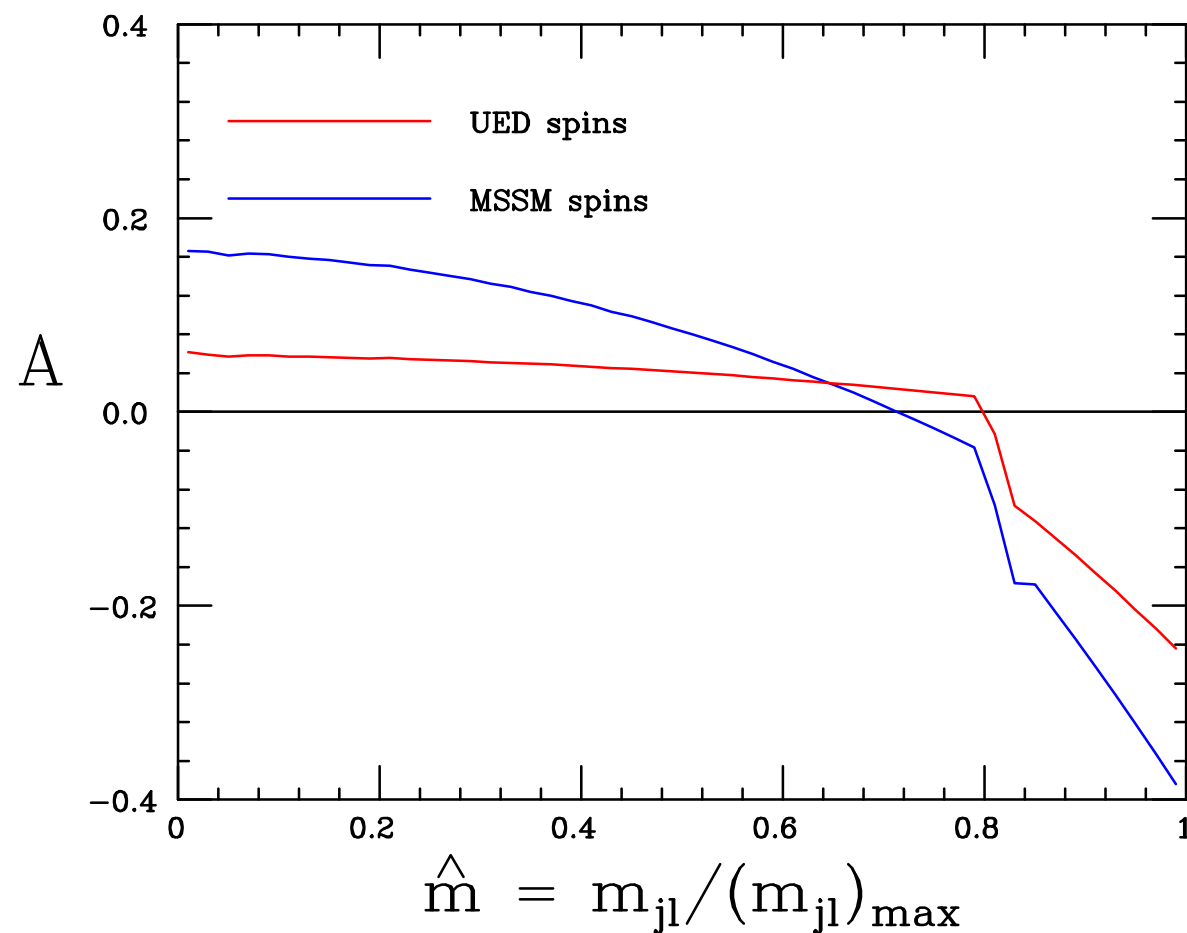
➔ $\sigma_{\text{UED}} \gg \sigma_{\text{SUSY}}$ for same masses (100 pb = 1/sec)

➔ $q^*/\bar{q}^* \sim 2 \Rightarrow$ charge asymmetry

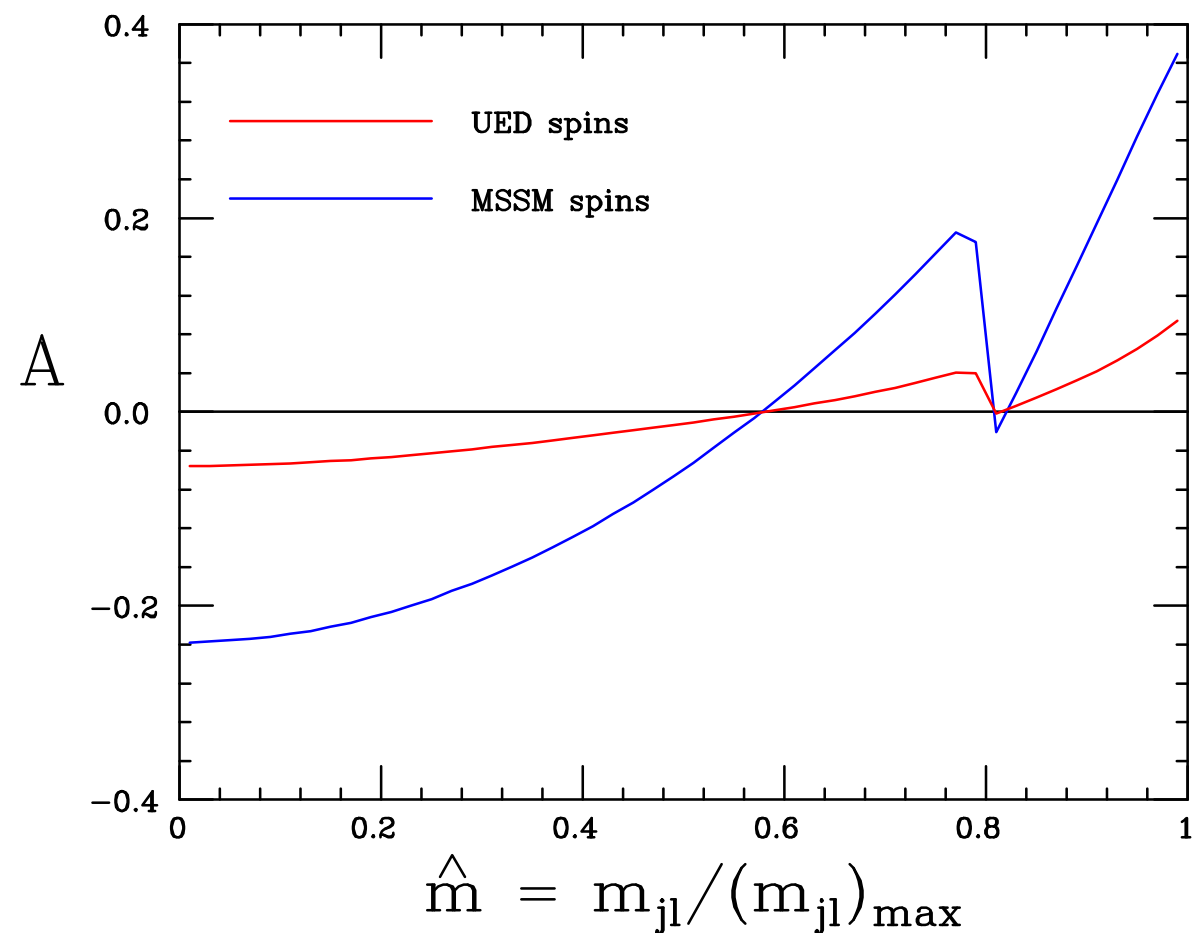
Charge asymmetry

$$A = \frac{(jl^+) - (jl^-)}{(jl^+) + (jl^-)}$$

UED masses



SPS Ia masses

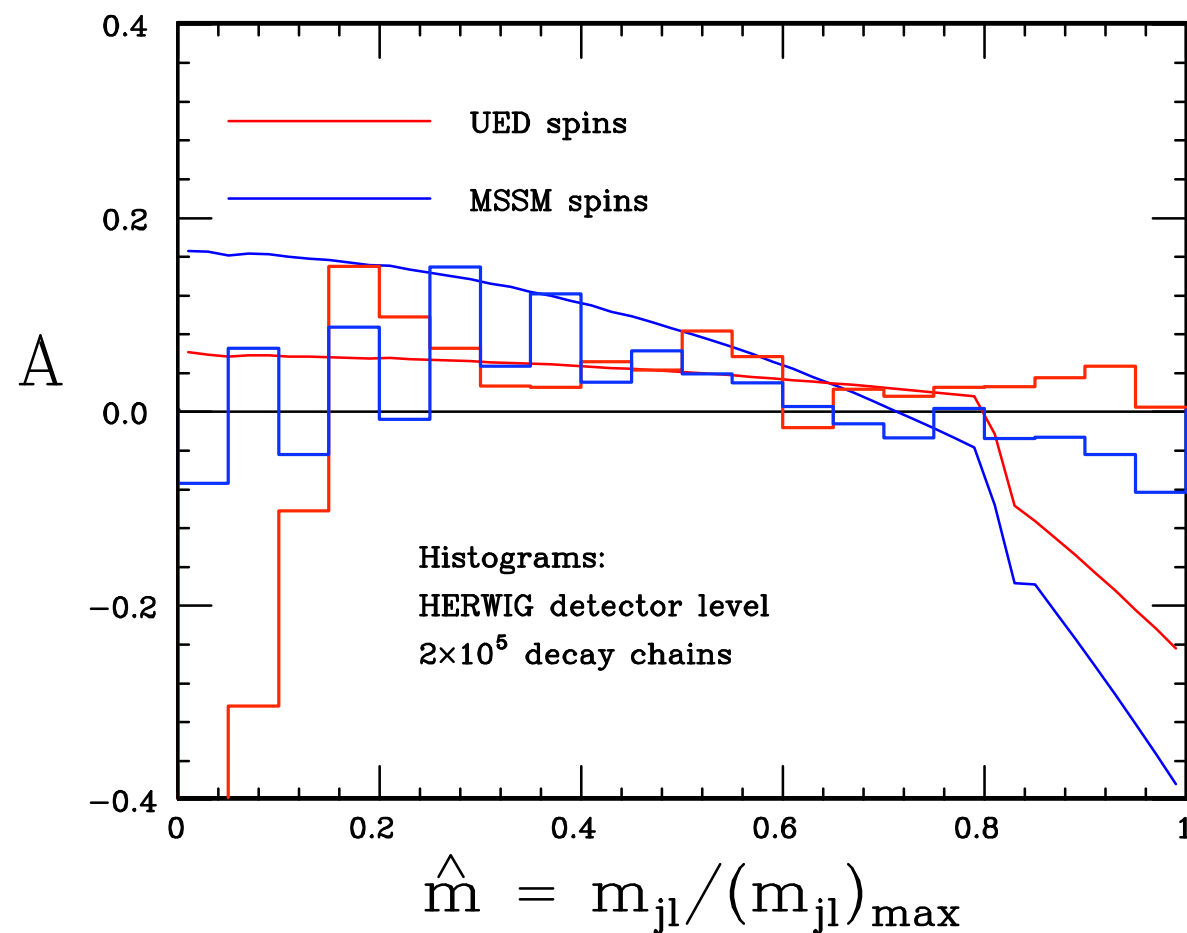


➔ Similar form, different magnitude

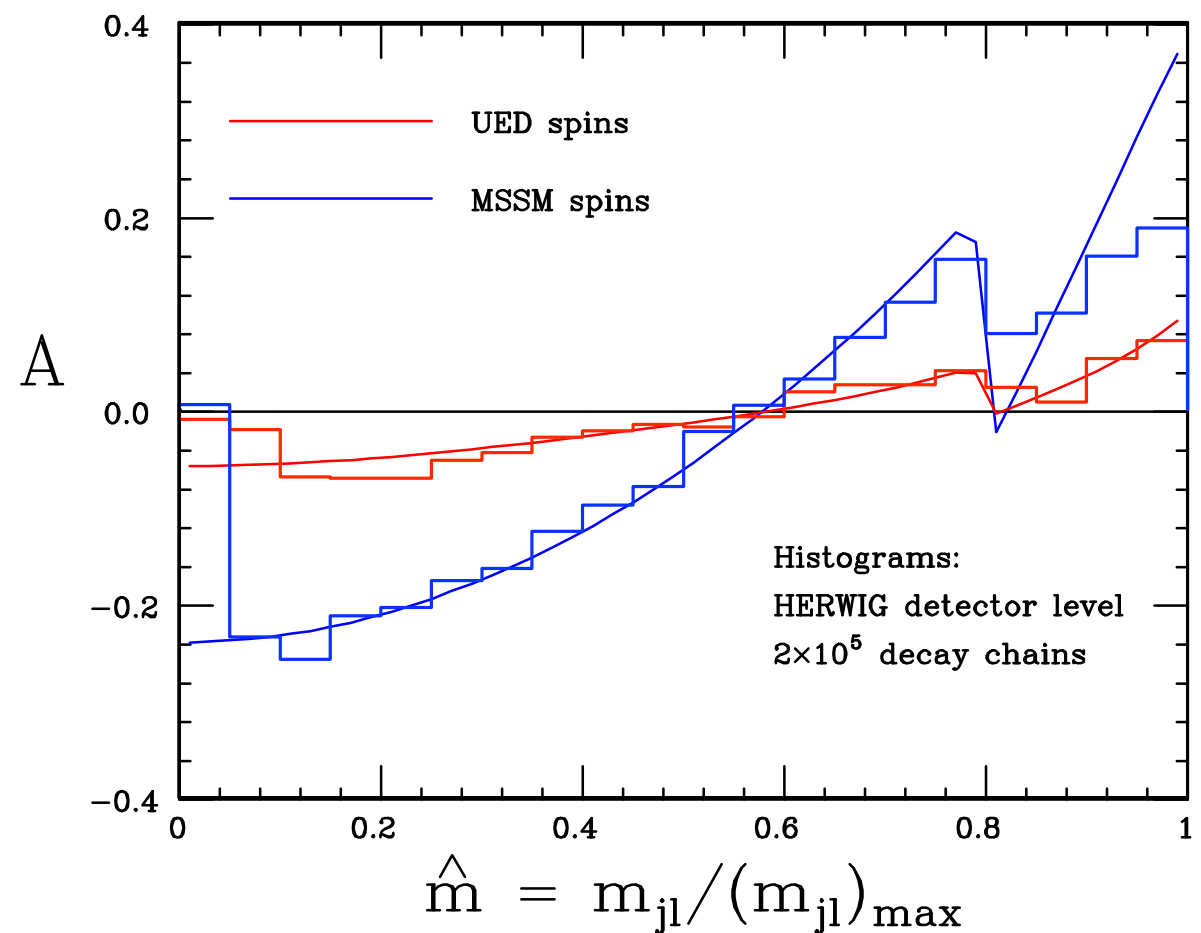
Charge asymmetry

$$A = \frac{(jl^+) - (jl^-)}{(jl^+) + (jl^-)}$$

UED masses



SPS Ia masses

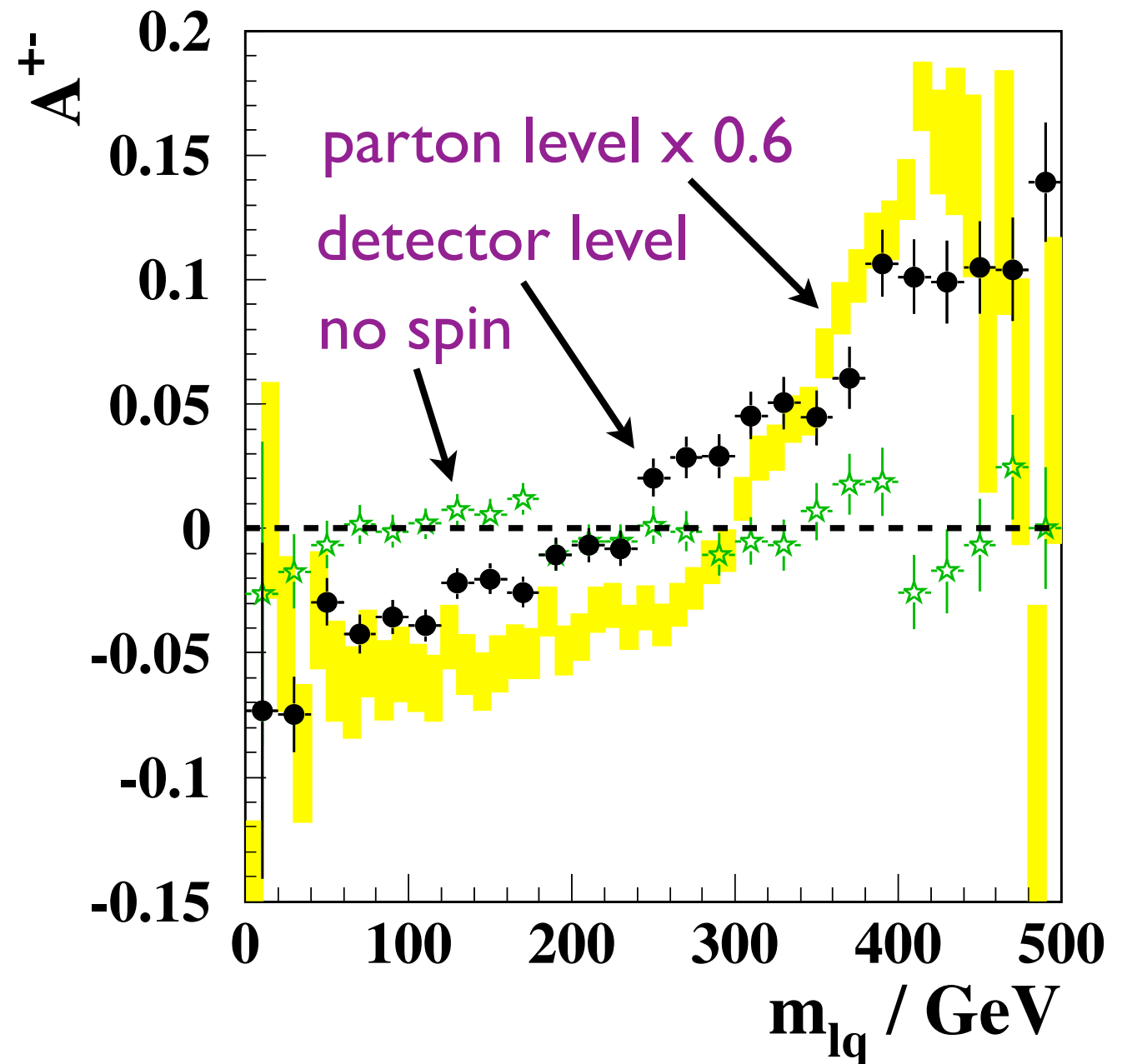


- ➡ Similar form, different magnitude
- ➡ Not detectable for UED masses

Charge asymmetry at detector level

A Barr, hep-ph/0405052

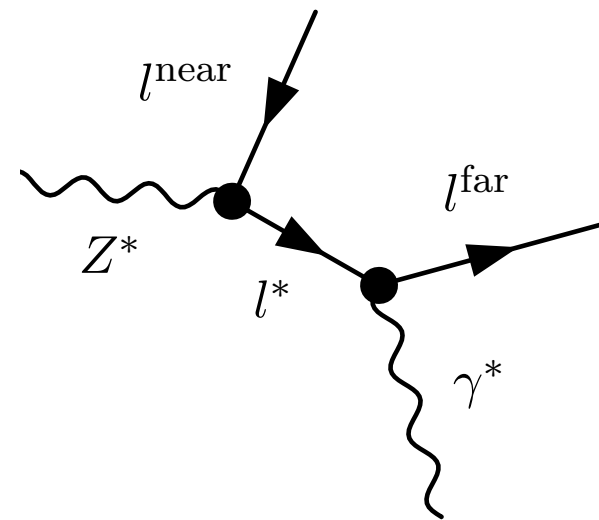
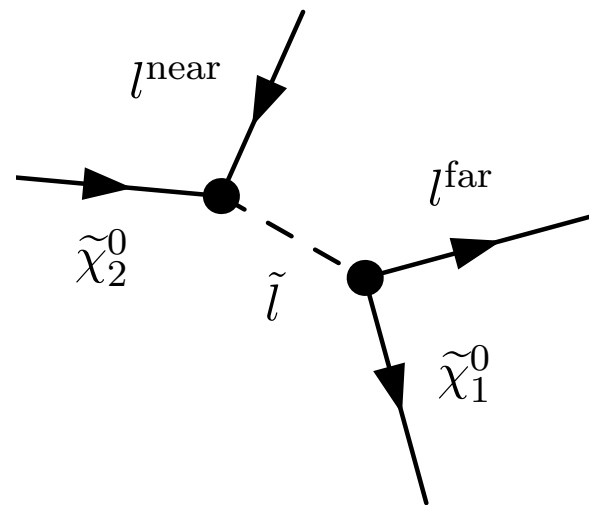
- Same decay chain:
 $\tilde{q}_L \rightarrow \tilde{\chi}_2^0 q_L \rightarrow \tilde{l}_R^\pm l^\mp q_L$
- Different MSSM point
(now excluded)
- Compared with no spin
(i.e. phase space) only
- More careful study of background and detector effects
- Points are for 500 fb^{-1}
- Used HERWIG



See also: Goto, Kawagoe, Nojiri, PR D70(2004)075016

Dileptons

Dileptons in “classic” chain



$$\frac{dP^{UED}}{d\hat{m}_{ll}} = \frac{4\hat{m}_{ll}}{(2+y)(1+2z)} [y + 4z + (2-y)(1-2z)\hat{m}_{ll}^2]$$

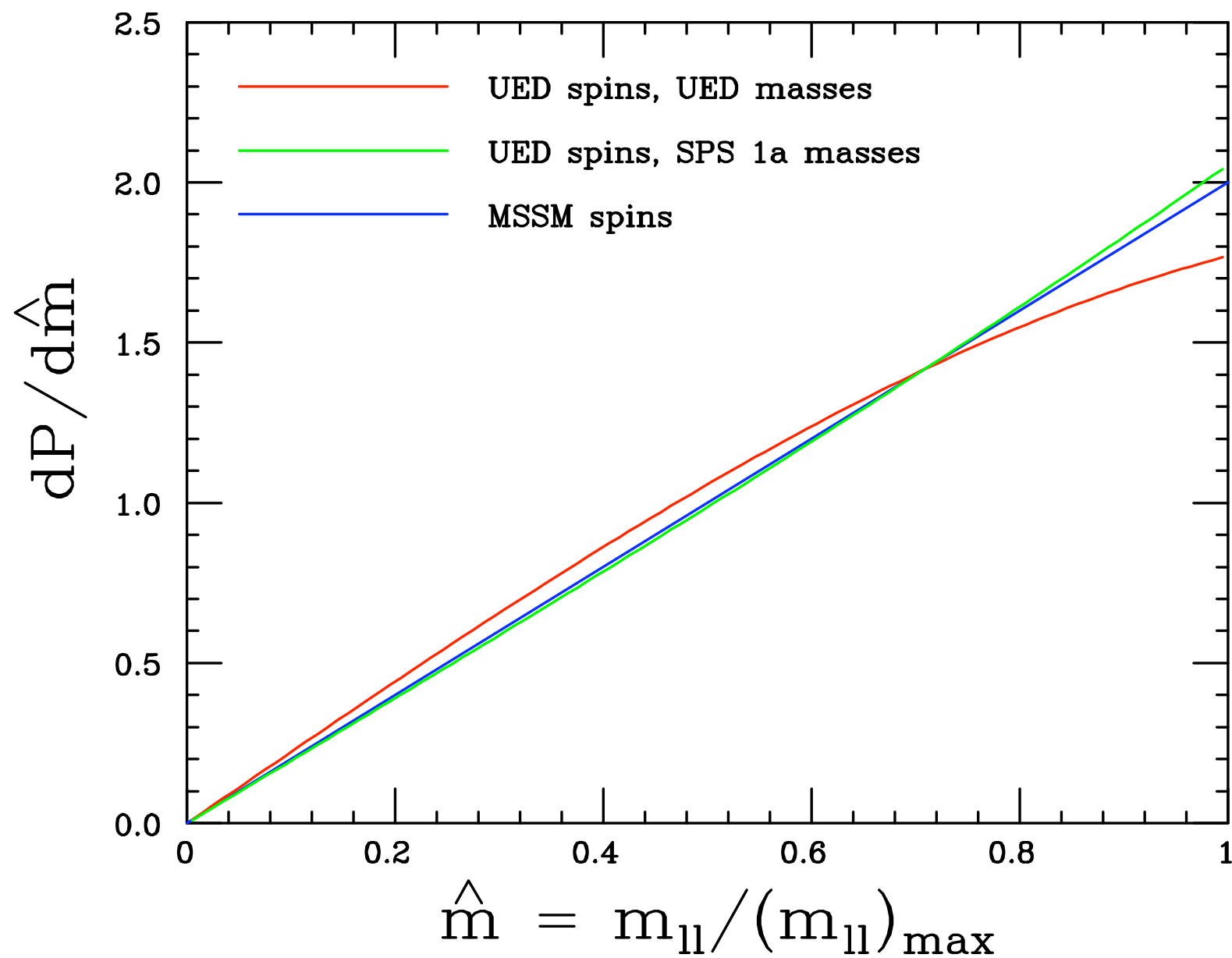
- $y = m_{l^*}^2/m_{Z^*}^2$ and $z = m_{\gamma^*}^2/m_{l^*}^2$

- UED: $y = 0.92$ $z = 0.95$

- SPS Ia: $y = 0.65$ $z = 0.45$

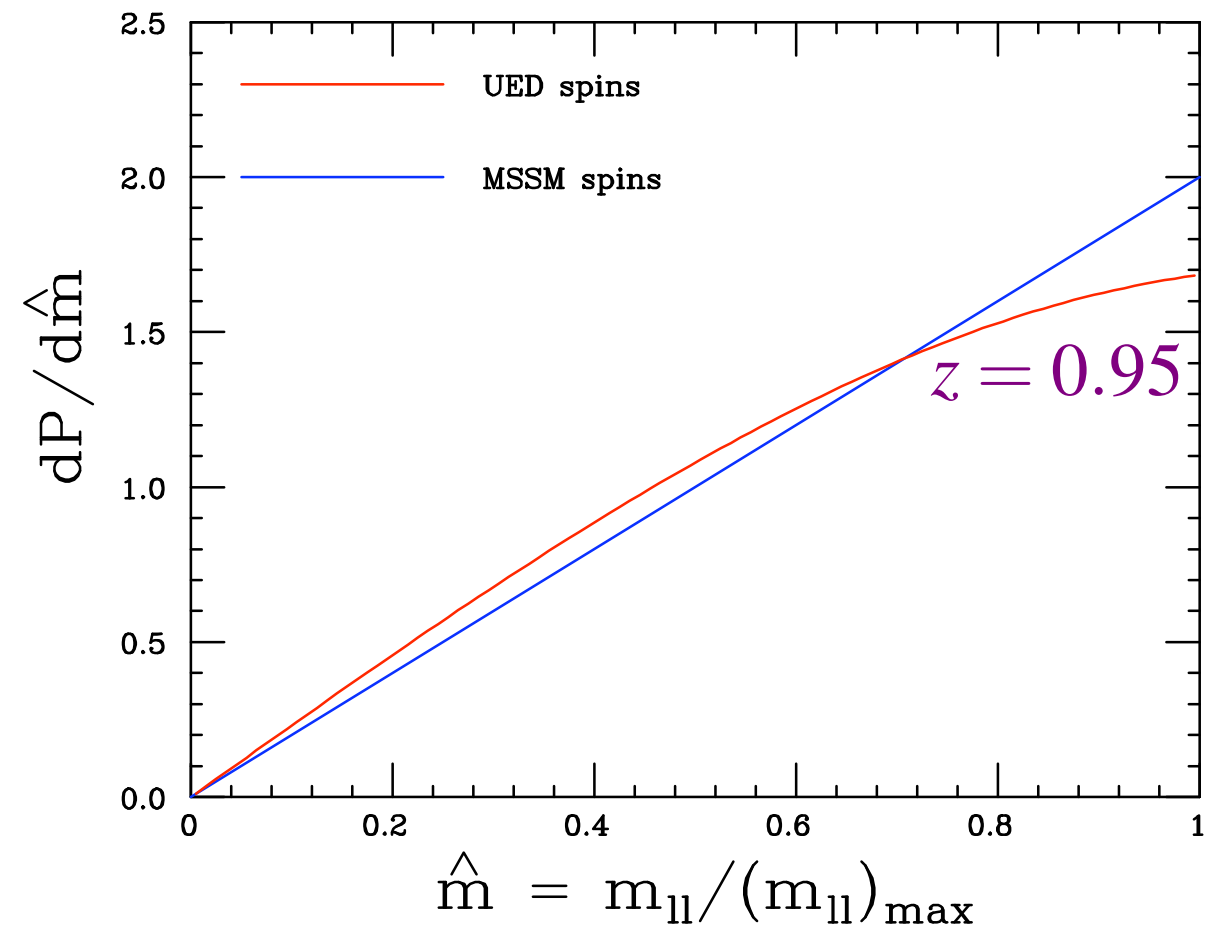
➔ Sensitivity greatest at small y and z

Dilepton mass distribution



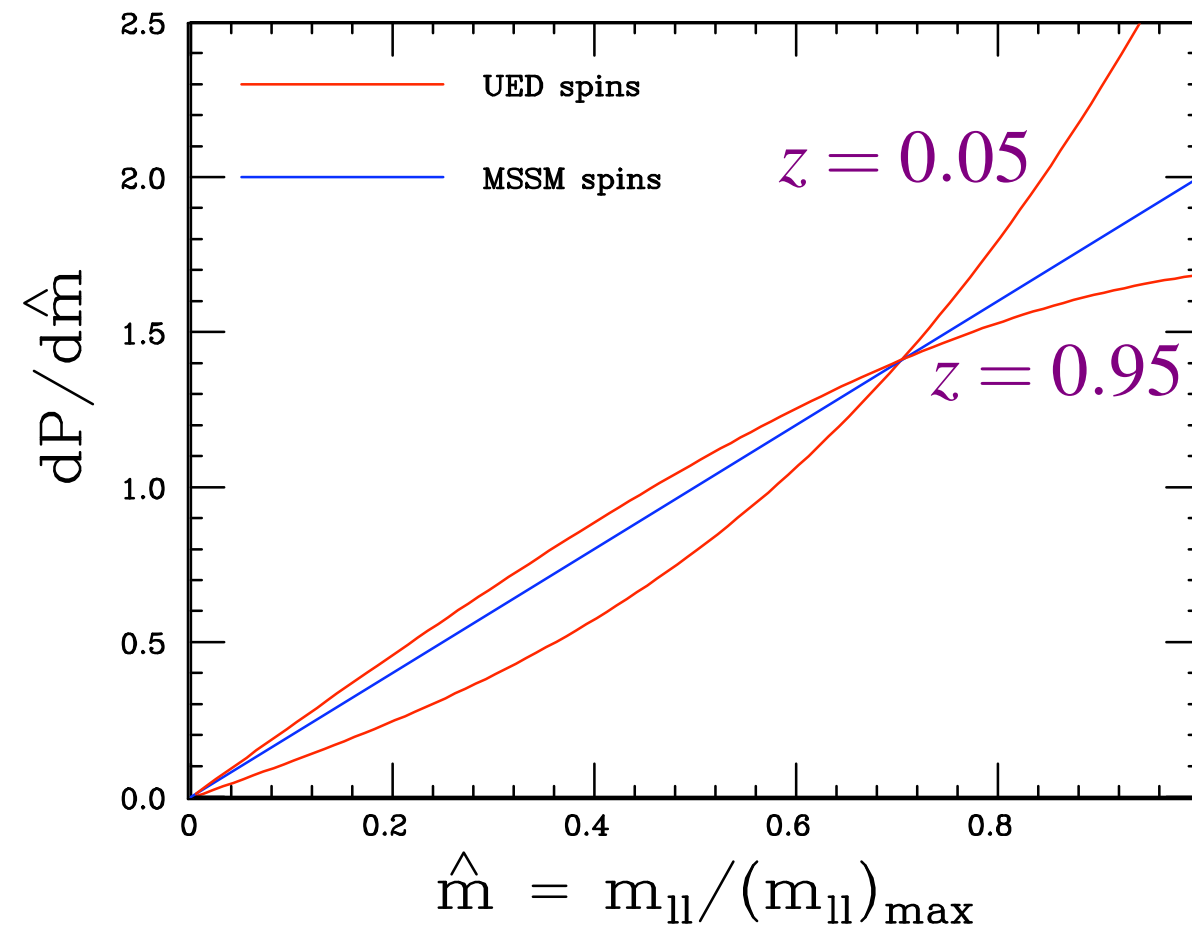
➔ No sensitivity for these masses!

Dilepton mass distribution (2)



$$y = m_{l^*}^2/m_{Z^*}^2 = 0.65, \quad z = m_{\gamma^*}^2/m_{l^*}^2 = 0.95 - 0.05$$

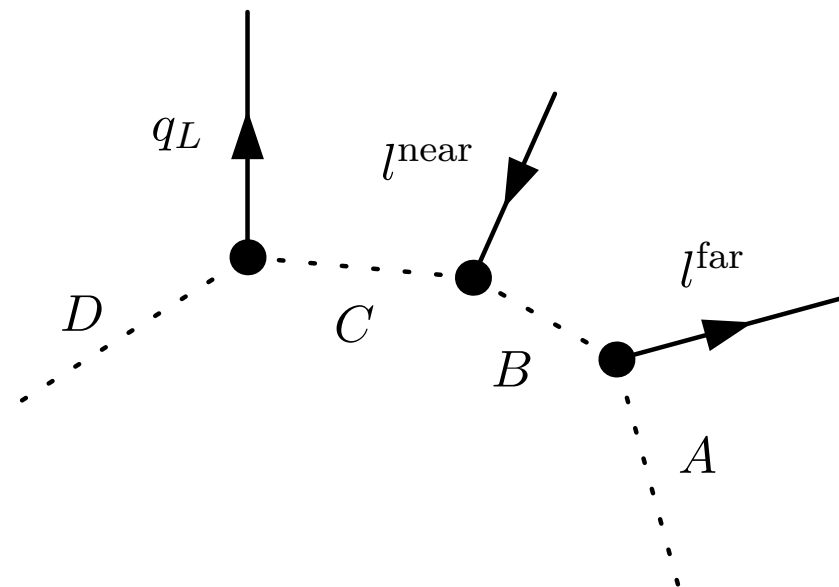
Dilepton mass distribution (2)



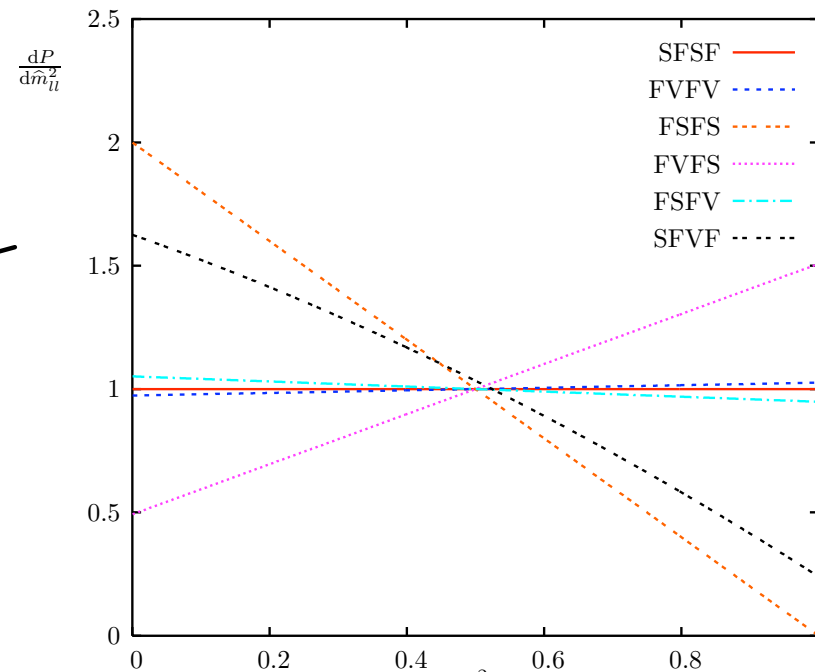
$$y = m_{l^*}^2/m_{Z^*}^2 = 0.65, \quad z = m_{\gamma^*}^2/m_{l^*}^2 = 0.95 - 0.05$$

➔ Independent of masses and spins at $\hat{m} = 1/\sqrt{2}$ ($\theta = \pi/2$)

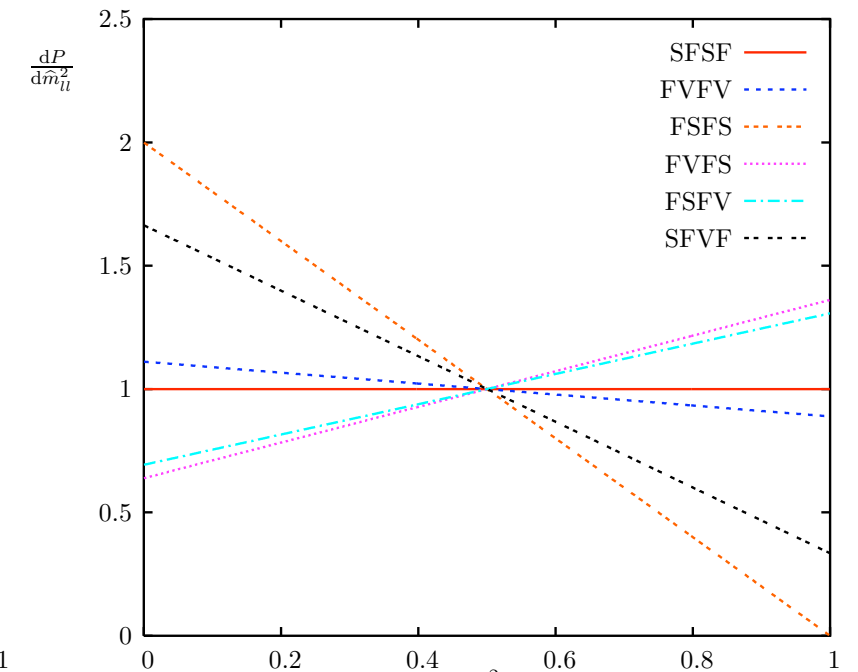
All possible spin assignments



A	B	C	D
$\tilde{\chi}_1^0$	\tilde{e}_R	$\tilde{\chi}_2^0$	\tilde{u}_L
96	143	177	537



A	B	C	D
γ^*	l_L^*	Z^*	q_L^*
800	824	851	956



Dilepton invariant mass-squared

D	C	B	A
Scalar	Fermion	Scalar	Fermion
Fermion	Vector	Fermion	Vector
Fermion	Scalar	Fermion	Scalar
Fermion	Vector	Fermion	Scalar
Fermion	Scalar	Fermion	Vector
Scalar	Fermion	Vector	Fermion

← SUSY } not distinguishable
← UED }
 ... but some others are.

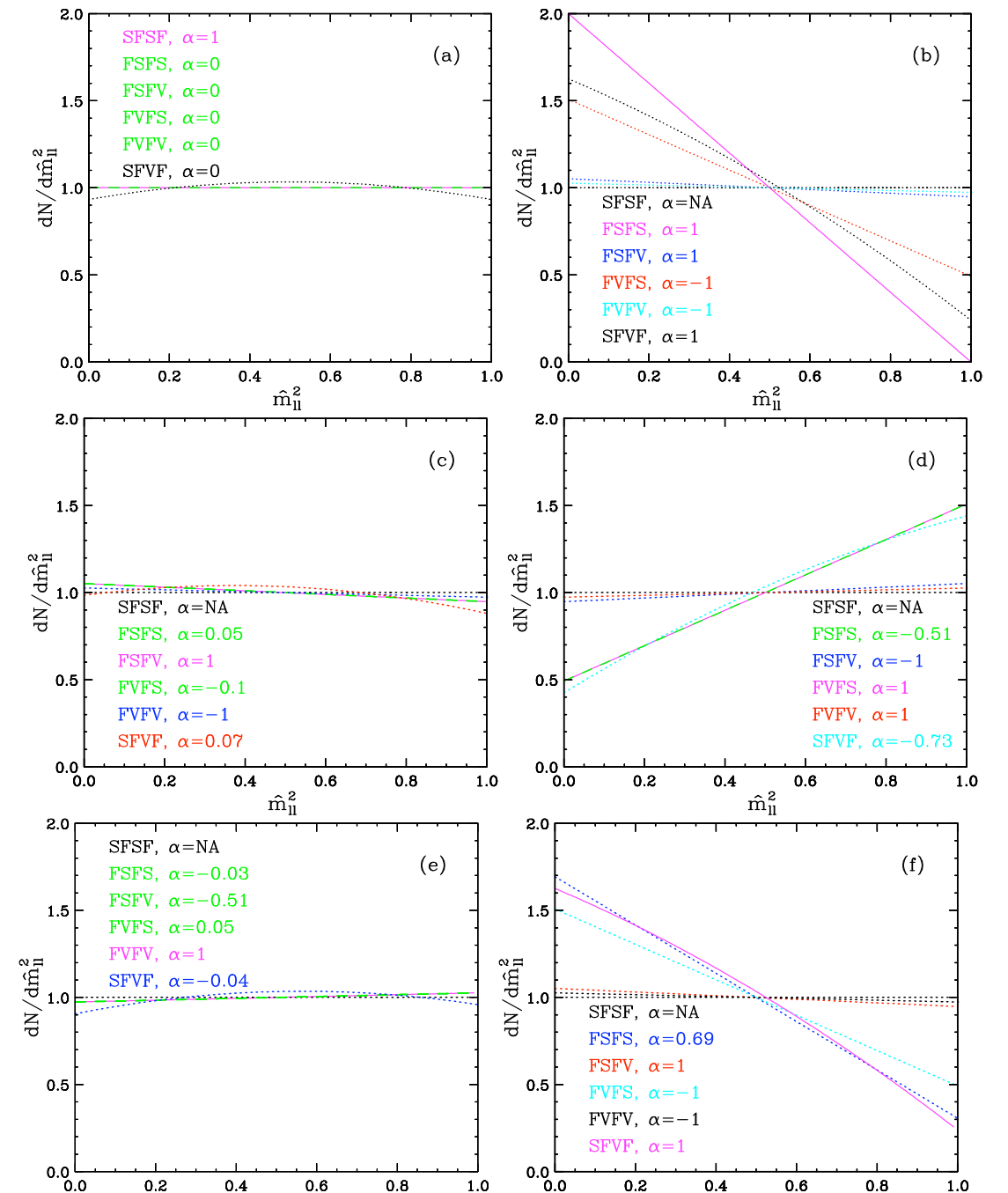
Athanasίου, Lester, Smillie, BW, JHEP 08(2006)055

All possible assignments (2)

Allowing arbitrary mixtures of L and R couplings:

Processes P_{11}		Processes P_{12}	
$\{q_L, \ell_L^-, \ell_L^+\}$ $f c_L ^2 b_L ^2 a_L ^2$	$\{\bar{q}_L, \ell_L^+, \ell_L^-\}$ $\bar{f} c_L ^2 b_L ^2 a_L ^2$	$\{q_L, \ell_L^-, \ell_R^+\}$ $f c_L ^2 b_L ^2 a_R ^2$	$\{\bar{q}_L, \ell_L^+, \ell_R^-\}$ $\bar{f} c_L ^2 b_L ^2 a_R ^2$
$\{\bar{q}_L, \ell_R^-, \ell_R^+\}$ $\bar{f} c_L ^2 b_R ^2 a_R ^2$	$\{q_L, \ell_R^+, \ell_R^-\}$ $f c_L ^2 b_R ^2 a_R ^2$	$\{\bar{q}_L, \ell_R^-, \ell_L^+\}$ $\bar{f} c_L ^2 b_R ^2 a_L ^2$	$\{q_L, \ell_R^+, \ell_L^-\}$ $f c_L ^2 b_R ^2 a_L ^2$
$\{q_R, \ell_R^-, \ell_R^+\}$ $f c_R ^2 b_R ^2 a_R ^2$	$\{\bar{q}_R, \ell_R^+, \ell_R^-\}$ $\bar{f} c_R ^2 b_R ^2 a_R ^2$	$\{q_R, \ell_R^-, \ell_L^+\}$ $f c_R ^2 b_R ^2 a_L ^2$	$\{\bar{q}_R, \ell_R^+, \ell_L^-\}$ $\bar{f} c_R ^2 b_R ^2 a_L ^2$
$\{\bar{q}_R, \ell_L^-, \ell_L^+\}$ $\bar{f} c_R ^2 b_L ^2 a_L ^2$	$\{q_R, \ell_L^+, \ell_L^-\}$ $f c_R ^2 b_L ^2 a_L ^2$	$\{\bar{q}_R, \ell_L^-, \ell_R^+\}$ $\bar{f} c_R ^2 b_L ^2 a_R ^2$	$\{q_R, \ell_L^+, \ell_R^-\}$ $f c_R ^2 b_L ^2 a_R ^2$
$\{\bar{q}_L, \ell_L^-, \ell_L^+\}$ $\bar{f} c_L ^2 b_L ^2 a_L ^2$	$\{q_L, \ell_L^+, \ell_L^-\}$ $f c_L ^2 b_L ^2 a_L ^2$	$\{\bar{q}_L, \ell_L^-, \ell_R^+\}$ $\bar{f} c_L ^2 b_L ^2 a_R ^2$	$\{q_L, \ell_L^+, \ell_R^-\}$ $f c_L ^2 b_L ^2 a_R ^2$
$\{q_L, \ell_R^-, \ell_R^+\}$ $f c_L ^2 b_R ^2 a_R ^2$	$\{\bar{q}_L, \ell_R^+, \ell_R^-\}$ $\bar{f} c_L ^2 b_R ^2 a_R ^2$	$\{q_L, \ell_R^-, \ell_L^+\}$ $f c_L ^2 b_R ^2 a_L ^2$	$\{\bar{q}_L, \ell_R^+, \ell_L^-\}$ $\bar{f} c_L ^2 b_R ^2 a_L ^2$
$\{\bar{q}_R, \ell_R^-, \ell_R^+\}$ $\bar{f} c_R ^2 b_R ^2 a_R ^2$	$\{q_R, \ell_R^+, \ell_R^-\}$ $f c_R ^2 b_R ^2 a_R ^2$	$\{\bar{q}_R, \ell_R^-, \ell_L^+\}$ $\bar{f} c_R ^2 b_R ^2 a_L ^2$	$\{q_R, \ell_R^+, \ell_L^-\}$ $f c_R ^2 b_R ^2 a_L ^2$
$\{q_R, \ell_L^-, \ell_L^+\}$ $f c_R ^2 b_L ^2 a_L ^2$	$\{\bar{q}_R, \ell_L^+, \ell_L^-\}$ $\bar{f} c_R ^2 b_L ^2 a_L ^2$	$\{q_R, \ell_L^-, \ell_R^+\}$ $f c_R ^2 b_L ^2 a_R ^2$	$\{\bar{q}_R, \ell_L^+, \ell_R^-\}$ $\bar{f} c_R ^2 b_L ^2 a_R ^2$
Processes P_{21}		Processes P_{22}	

Data from	Can this data be fitted by model					
	SFSF	FSFS	FSFV	FVFS	FVFV	SFVF
SFSF	yes	no	no	no	no	no
FSFS	no	yes	maybe	no	no	no
FSFV	no	yes	yes	no	no	no
FVFS	no	no	no	yes	maybe	no
FVFV	no	no	no	yes	yes	no
SFVF	no	no	no	no	no	yes



Burns, Kong, Matchev, Park, 0808.2472

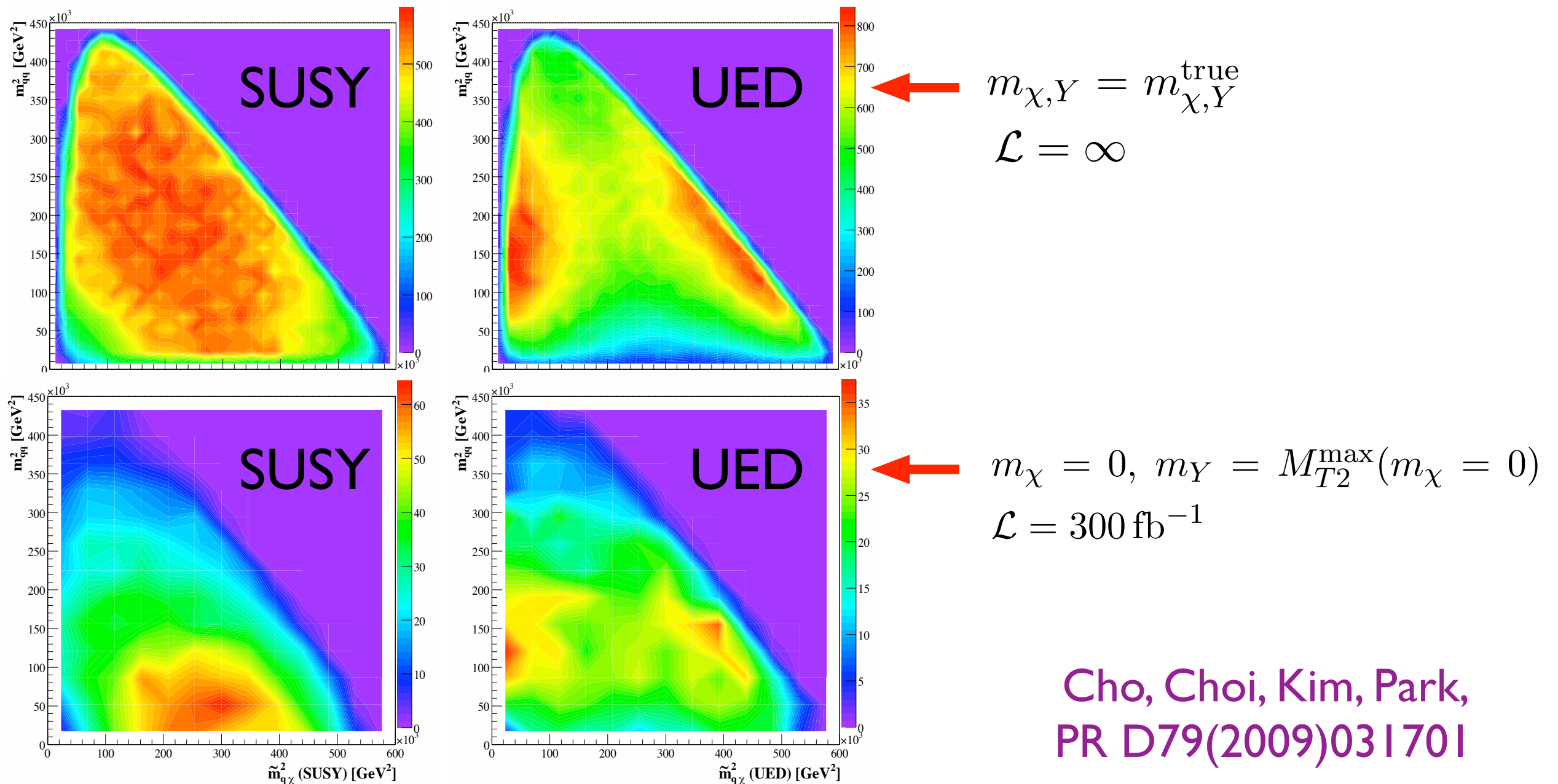
Dilepton invariant mass-squared

Three-body decays

M_{T2} -assisted spin determination

$$pp \rightarrow Y(1) + \bar{Y}(2) \rightarrow V(p_1)\chi(k_1) + V(p_2)\chi(k_2), \quad Y \rightarrow q(p_q)\bar{q}(p_{\bar{q}})\chi(k)$$

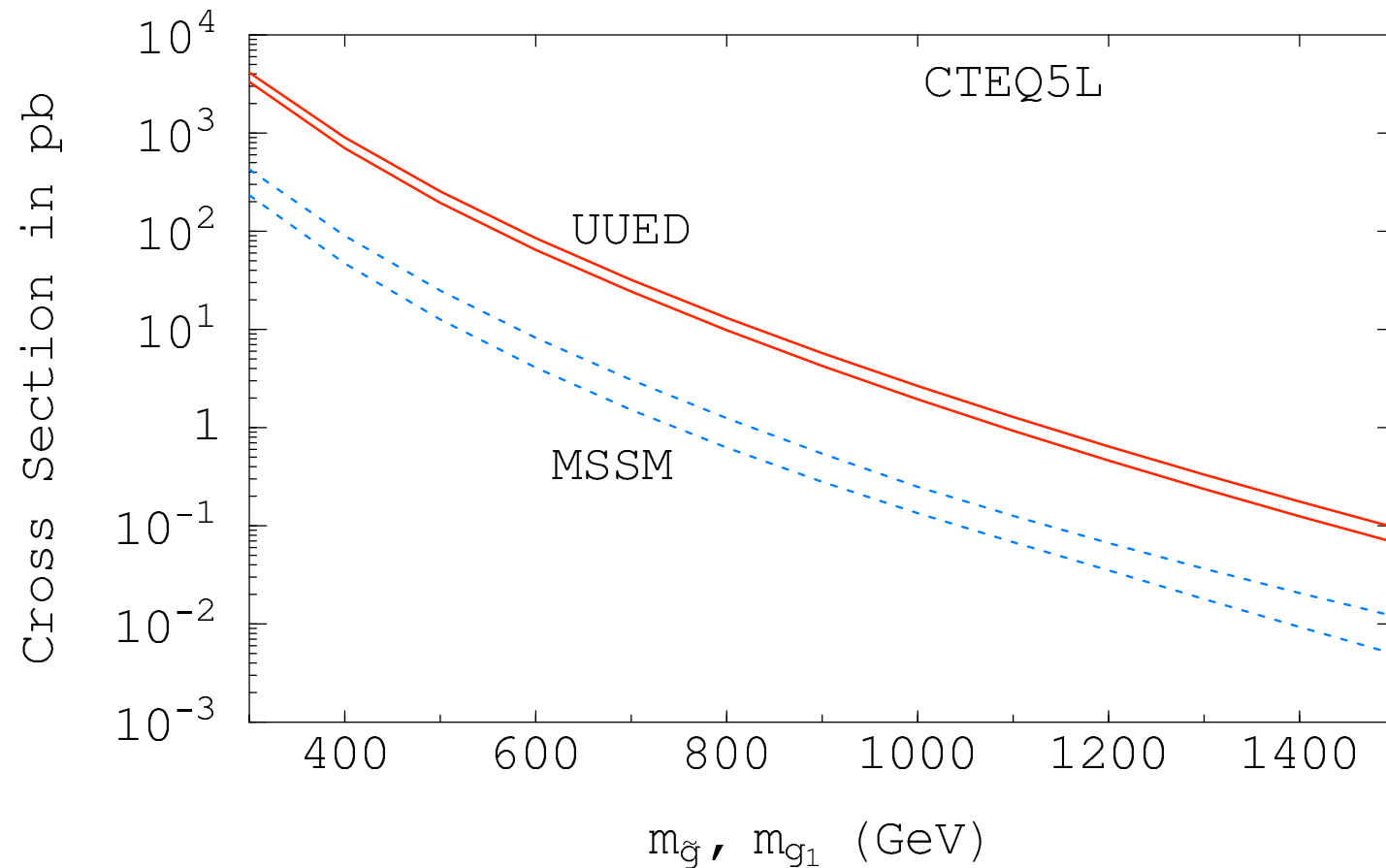
$$M_{T2}(p_i, m_\chi) \equiv \min_{\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{p}_T^{\text{miss}}} \left[\max\{M_T^{(1)}, M_T^{(2)}\} \right] \rightarrow \text{assign 4-momenta}$$



Cho, Choi, Kim, Park,
PR D79(2009)031701

Cross sections

Cross sections imply spins

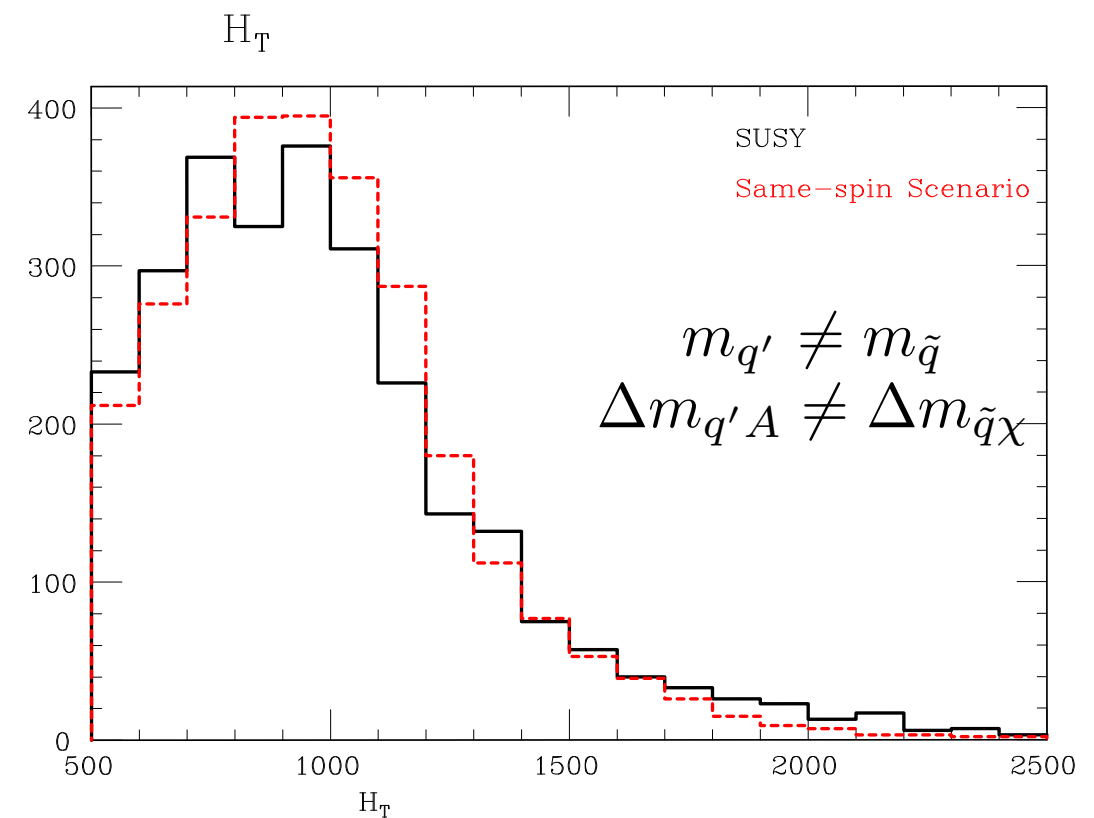
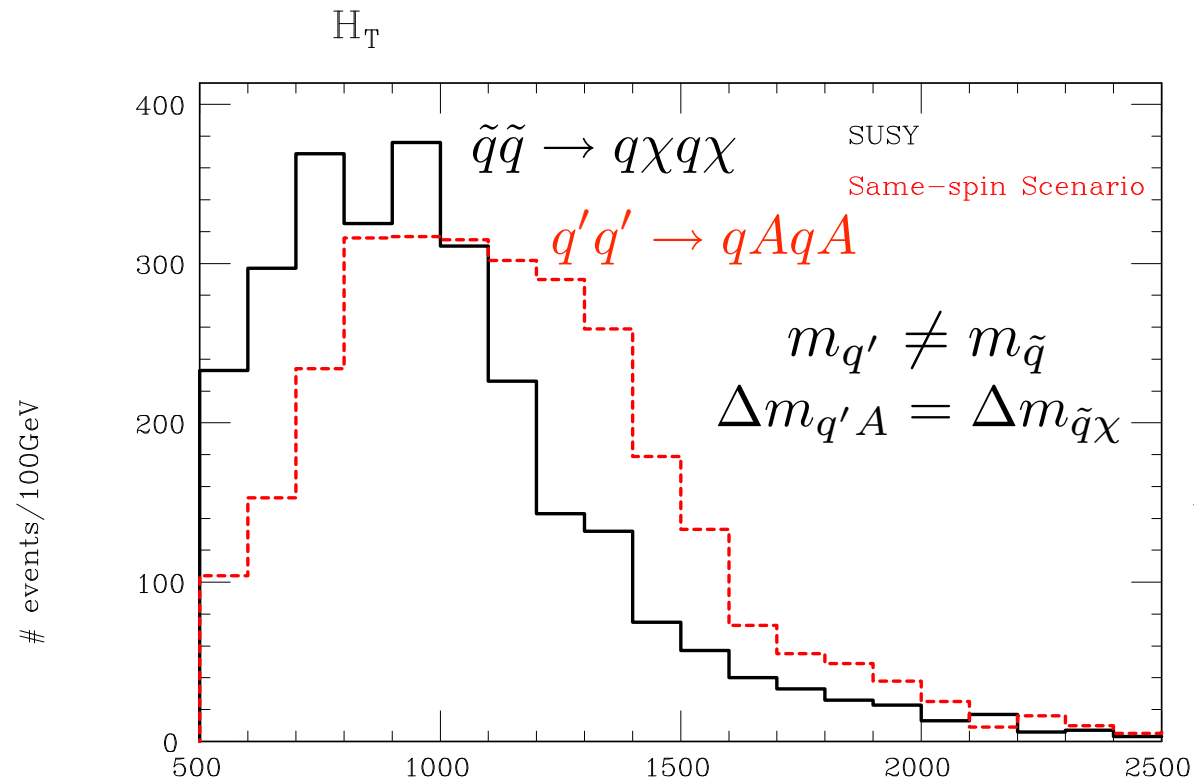


➔ Higher spins mean higher cross sections (for given masses)

Datta, Kane, Toharia hep-ph/0510204

	MSSM	U-UED
Production Cross sections	$\sigma_{\tilde{g}\tilde{g}} = 4.51$ pb	$\sigma_{g_1 g_1} = 65.95$ pb
Branching Fractions	$\tilde{g} \rightarrow q\bar{q}'\chi_1^\pm = 0.45$ $\tilde{g} \rightarrow q\bar{q}\chi_2^0 = 0.28$ $\tilde{g} \rightarrow q\bar{q}\chi_1^0 = 0.27$	$g_1 \rightarrow q\bar{q}'W_1^\pm = 0.45$ $g_1 \rightarrow q\bar{q}'Z_1 = 0.28$ $g_1 \rightarrow q\bar{q}'B_1 = 0.27$
	$\chi_1^\pm \rightarrow q\bar{q}'\chi_1^0 = 0.67$ $\chi_1^\pm \rightarrow \ell\nu\chi_1^0 = 0.33$ $\chi_2^0 \rightarrow q\bar{q}\chi_1^0 = 0.94$ $\chi_2^0 \rightarrow \ell\bar{\ell}\chi_1^0 = 0.04$ $\chi_2^0 \rightarrow \nu\bar{\nu}\chi_1^0 = 0.01$	$W_1^\pm \rightarrow q\bar{q}'B_1 = 0.18$ $W_1^\pm \rightarrow \ell\nu B_1 = 0.82$ $Z_1^\pm \rightarrow q\bar{q}B_1 = 0.22$ $Z_1^\pm \rightarrow \ell\bar{\ell}B_1 = 0.39$ $Z_1^\pm \rightarrow \nu\bar{\nu}B_1 = 0.39$
Cascade Fractions		
1-lepton	0.248	0.385
OS 2-lepton	0.030	0.183
SS 2-lepton	0.011	0.068
3-lepton	0.003	0.081
Cascade Rates		
1-lepton	1.12 pb	25.39 pb
OS 2-lepton	0.13 pb	12.06 pb
SS 2-lepton	0.05 pb	4.48 pb
3-lepton	0.014 pb	5.34 pb

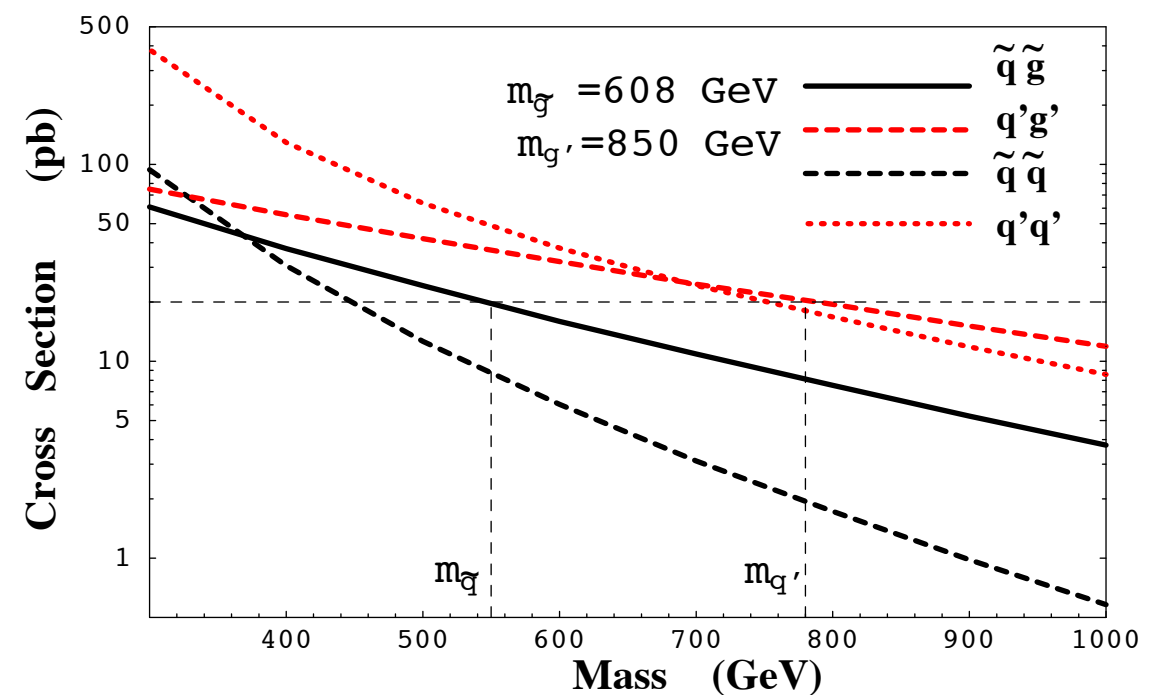
Cross sections imply spins (2)



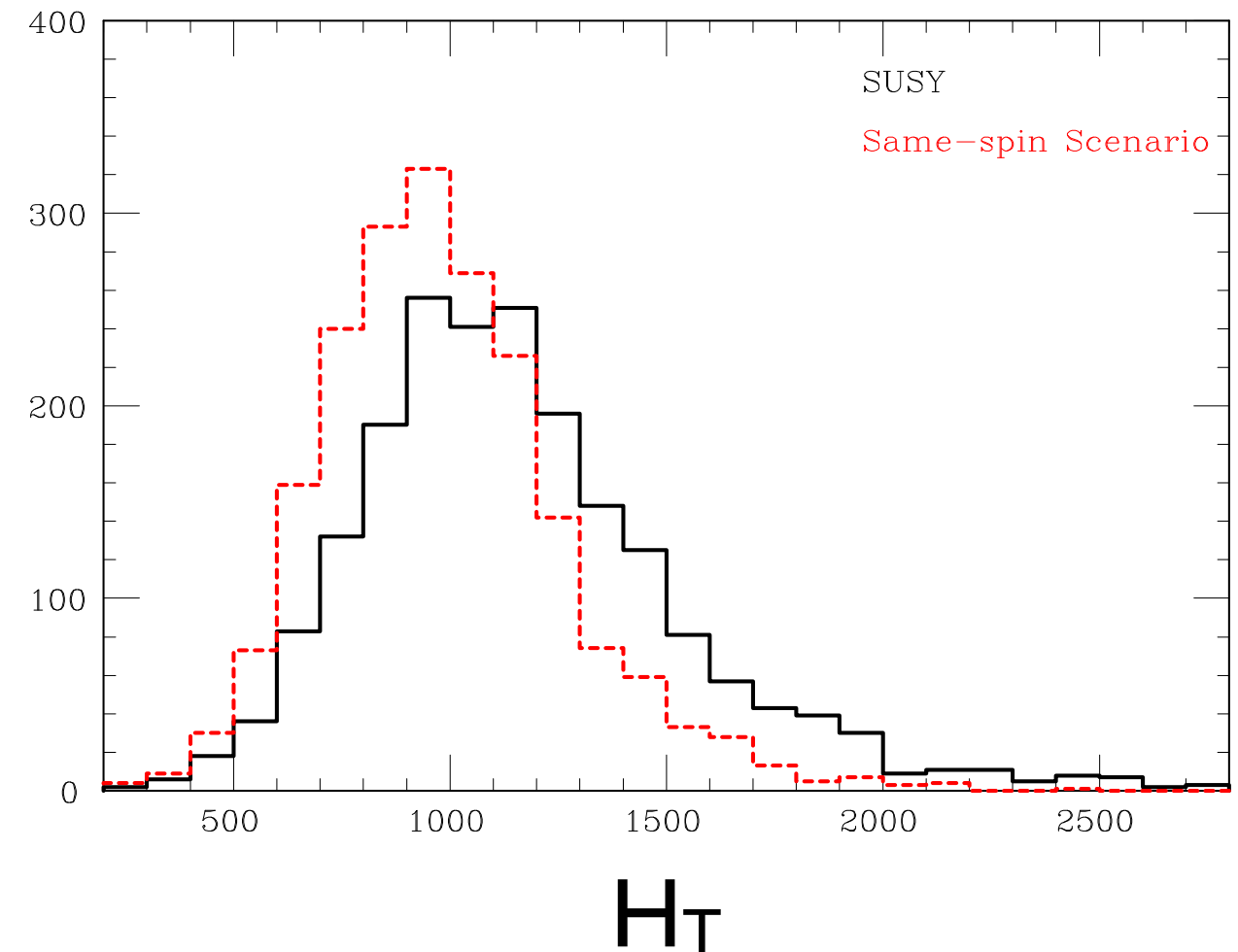
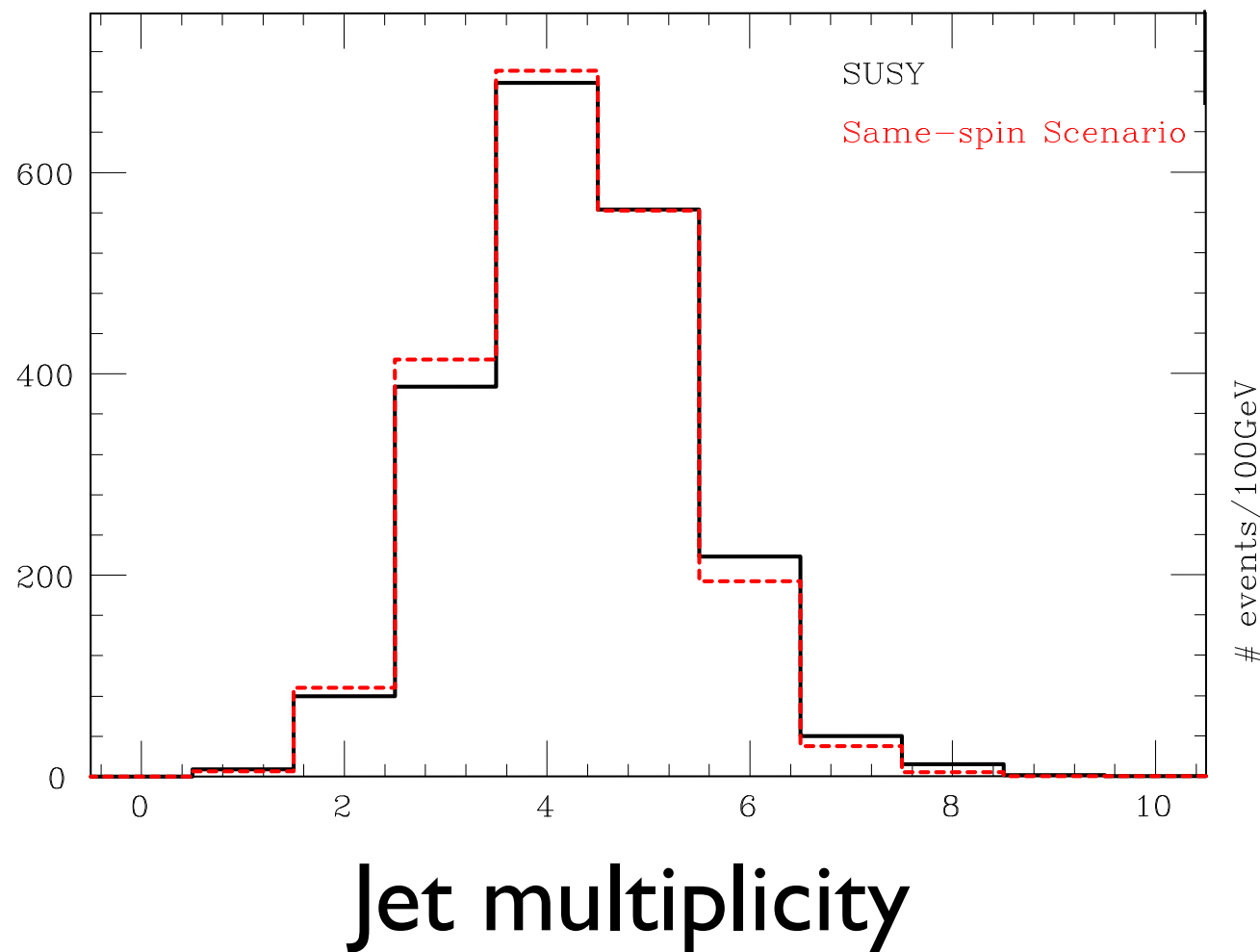
$$H_T = \sum E_T + \cancel{E}_T$$

- Can match cross section and one distribution by adjusting masses
- Cannot match several cross sections or distributions ...

Kane, Petrov, Shao, Wang,
J.Phys.G37(2010)045004



Cross sections imply spins (3)



- Can vary masses to fit cross section and one distribution
- E.g. match jet counts \rightarrow H_T doesn't match \rightarrow ambiguity resolved

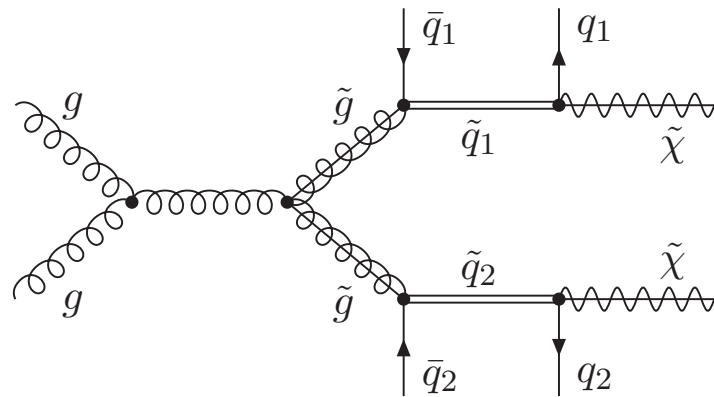
Conclusions on Spins

- Sequential decay chains
 - Possibilities -- but difficult for degenerate masses
- Dileptons
 - SUSY vs UED difficult at LHC -- other cases possible
- Three-body decays
 - M_{T2} assistance looks useful here (and elsewhere?)
- Cross sections
 - Should be included

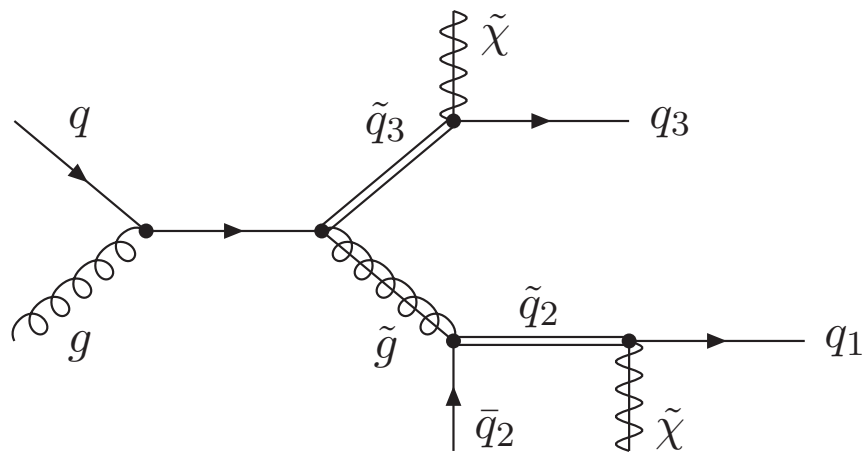
 Full simulations (and data) needed!

Backup slides

Gluino spin correlations



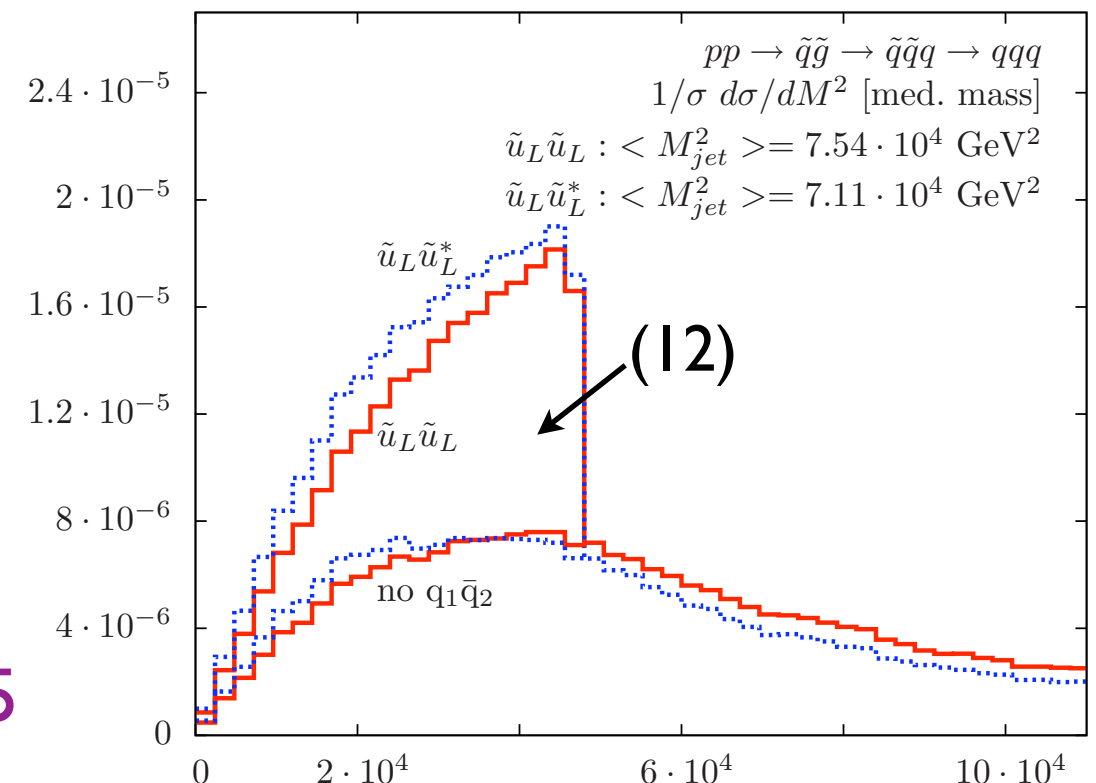
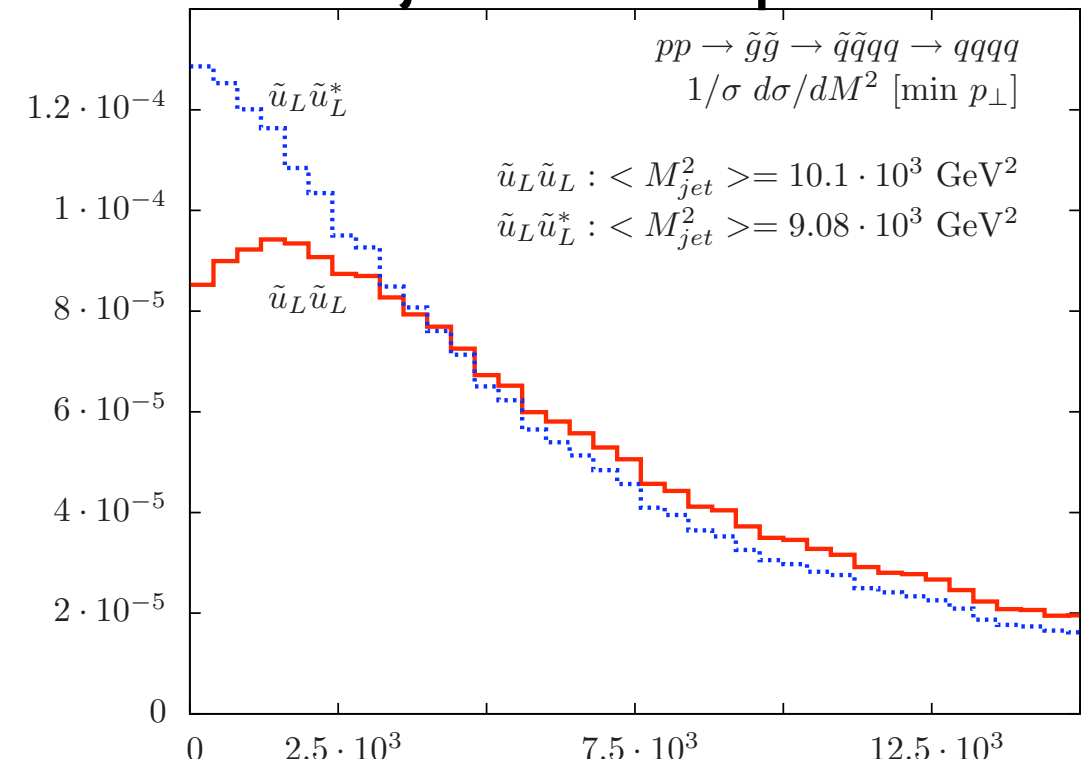
- Lowest p_T jets from gluinos



- Lowest mass dijet \sim (12)
- Medium mass dijet \sim (23)

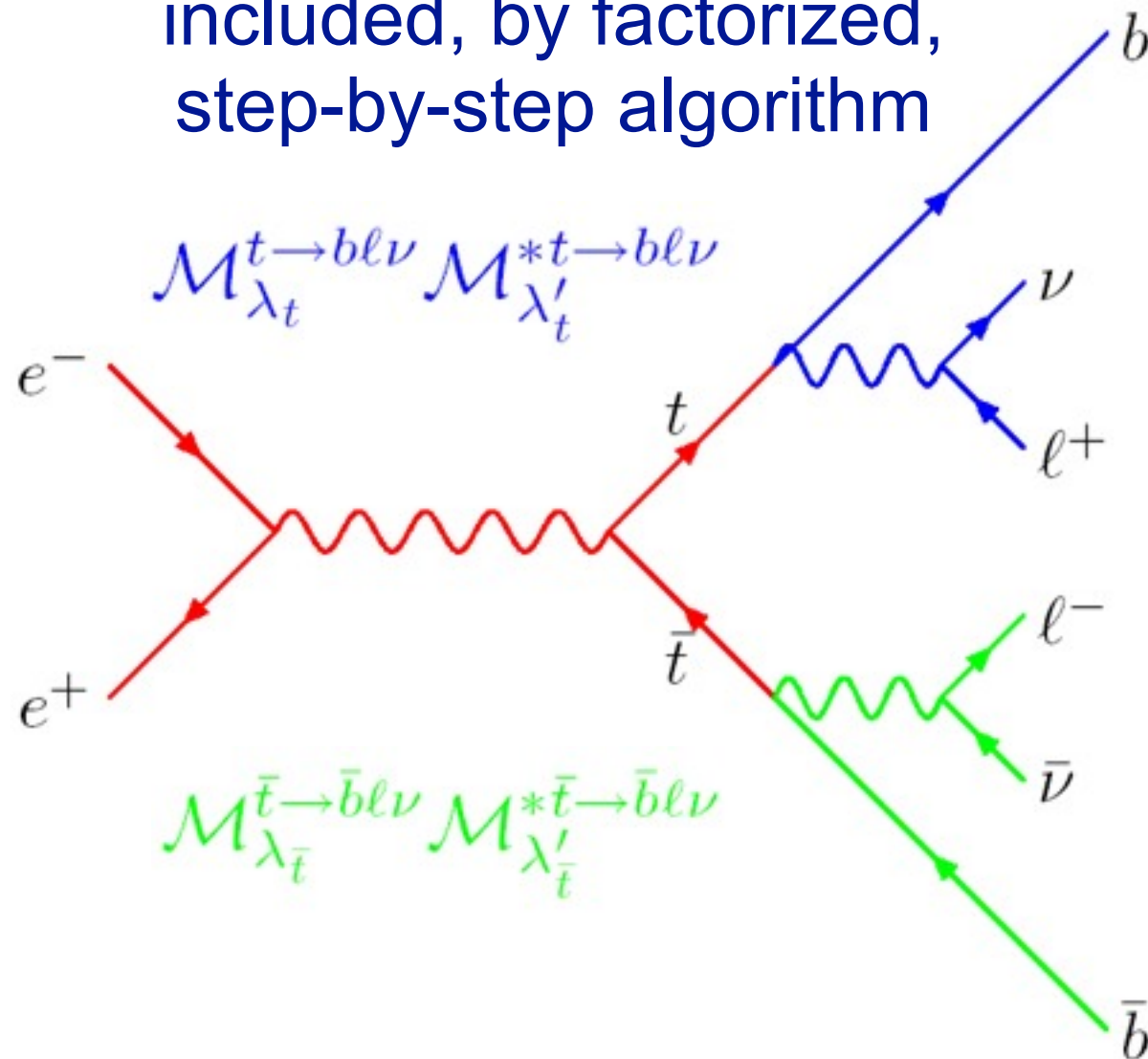
Krämer, Popenda, Spira, Zerwas, 0902.3795

Dijet mass-squared



Production/Decay Spin Correlations in Herwig

Full spin correlations included, by factorized, step-by-step algorithm



$$\rho_{\text{prod}}^{\lambda_c \lambda'_c \lambda_d \lambda'_d} = \mathcal{M}_{ab \rightarrow cd}^{\lambda_c \lambda_d} \mathcal{M}_{ab \rightarrow cd}^{* \lambda'_c \lambda'_d},$$

$$D_c^{\lambda_c \lambda'_c} = \mathcal{M}_{c \text{ decay}}^{\lambda_c} \mathcal{M}_{c \text{ decay}}^{* \lambda'_c},$$

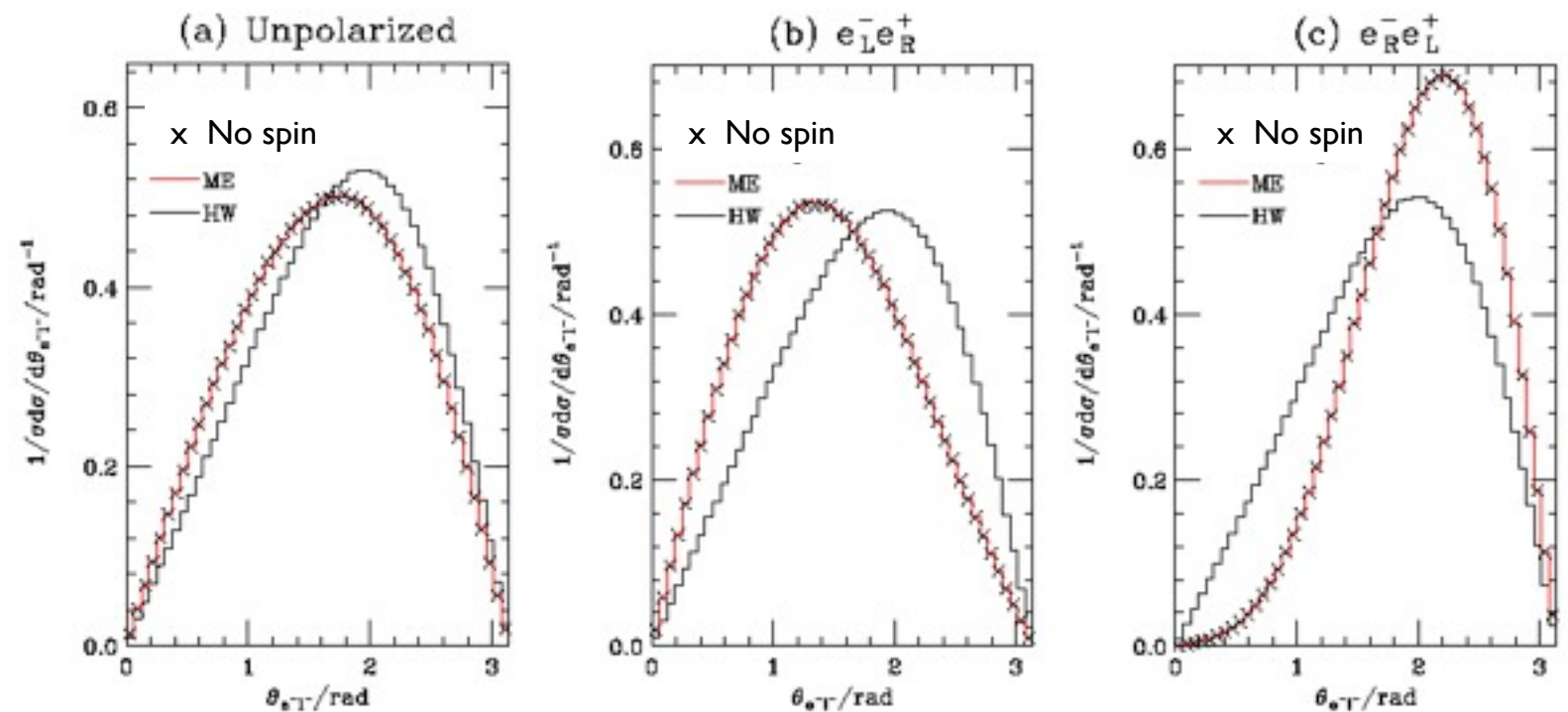
$$|\mathcal{M}|^2 = \rho_{\text{prod}}^{\lambda_c \lambda'_c \lambda_d \lambda'_d} D_c^{\lambda_c \lambda'_c} D_d^{\lambda_d \lambda'_d}$$

$$= \rho_{\text{prod}}^{\lambda_c \lambda_c \lambda_d \lambda_d} \left(\frac{\rho_{\text{prod}}^{\lambda_c \lambda'_c \lambda_d \lambda_d} D_c^{\lambda_c \lambda'_c}}{\rho_{\text{prod}}^{\lambda_c \lambda_c \lambda_d \lambda_d}} \right) \times \left(\frac{\rho_{\text{prod}}^{\lambda_c \lambda'_c \lambda_d \lambda'_d} D_c^{\lambda_c \lambda'_c} D_d^{\lambda_d \lambda'_d}}{\rho_{\text{prod}}^{\lambda_c \lambda'_c \lambda_d \lambda_d} D_c^{\lambda_c \lambda'_c}} \right)$$

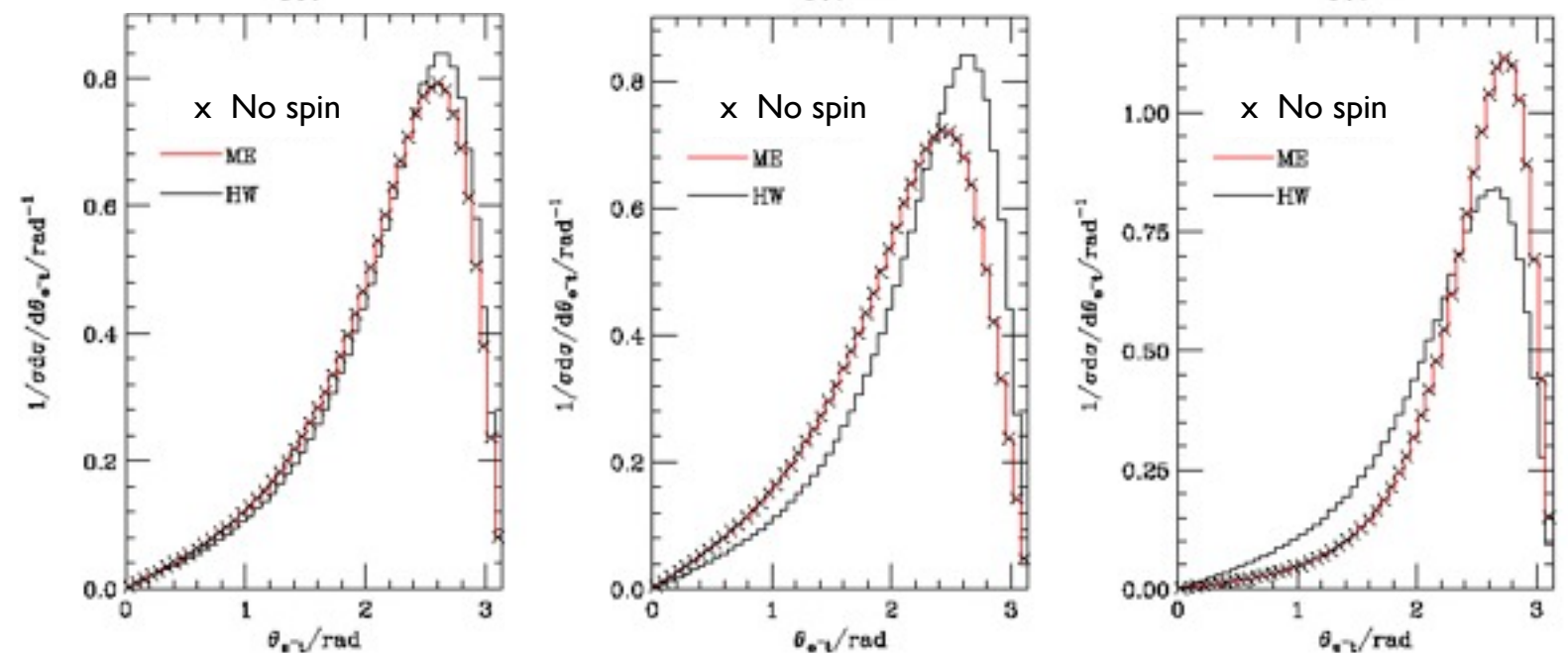
Richardson, hep-ph/0110108

Top spin correlations in Herwig

Lepton-beam
correlation



Top-beam
correlation

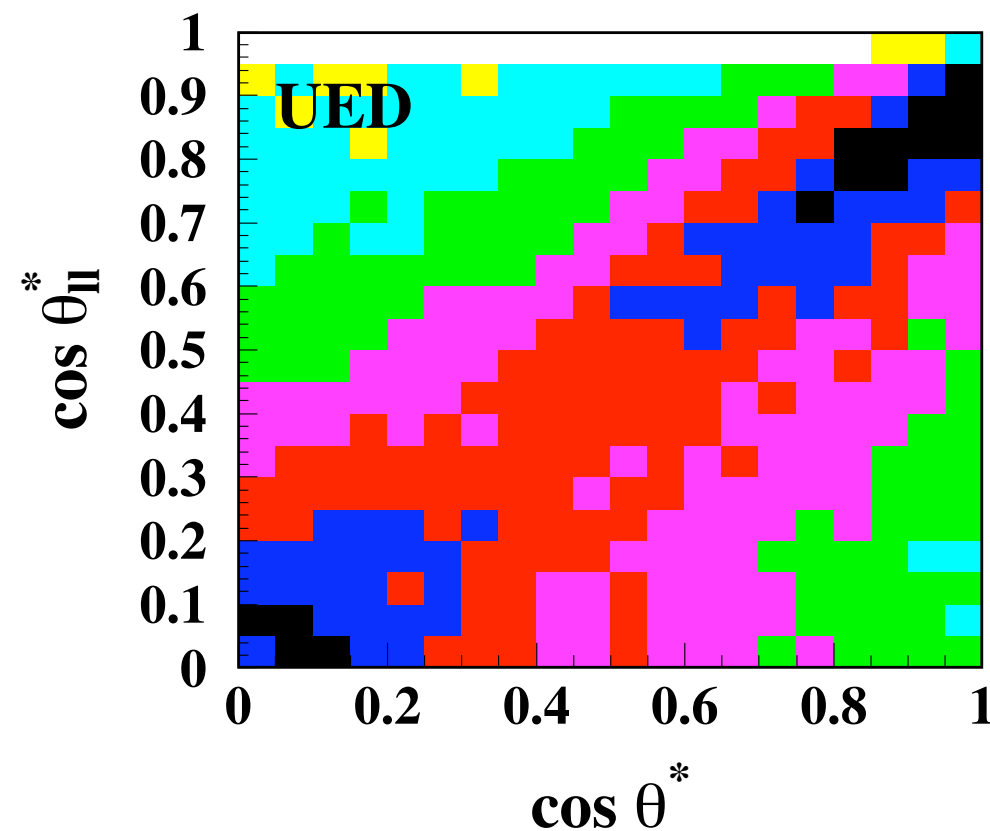
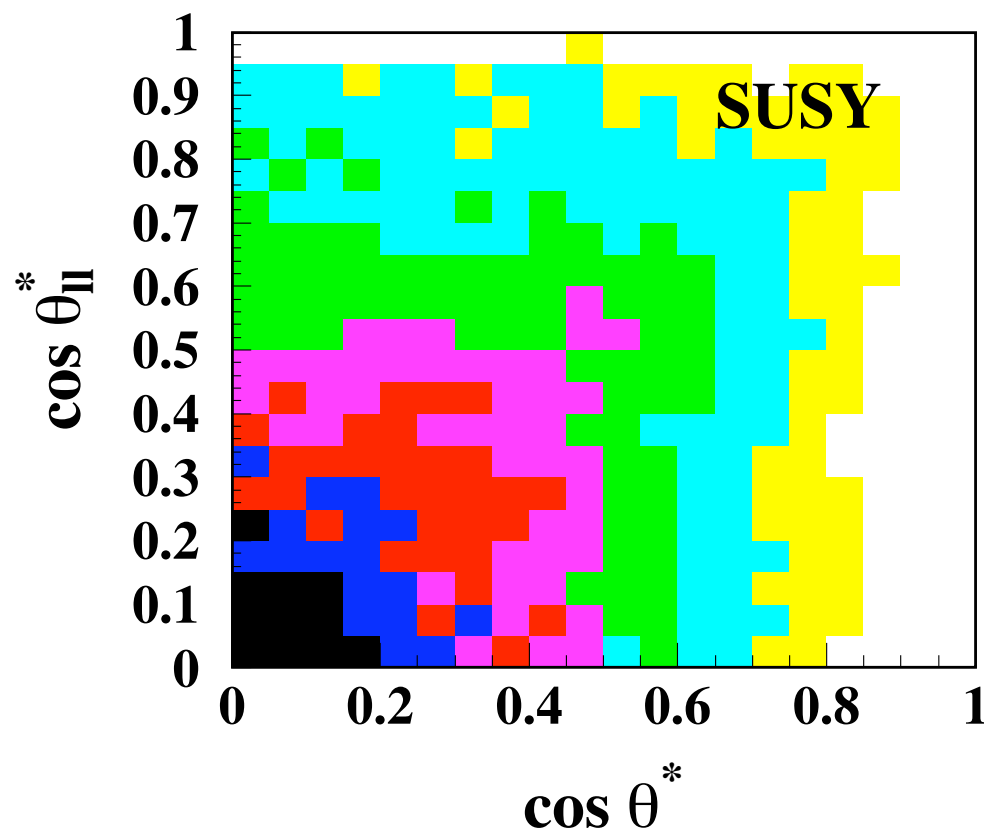


● SM, SUSY & UED in Herwig++

Hw++ manual: Bähr et al., 0803.0883

Dislepton production

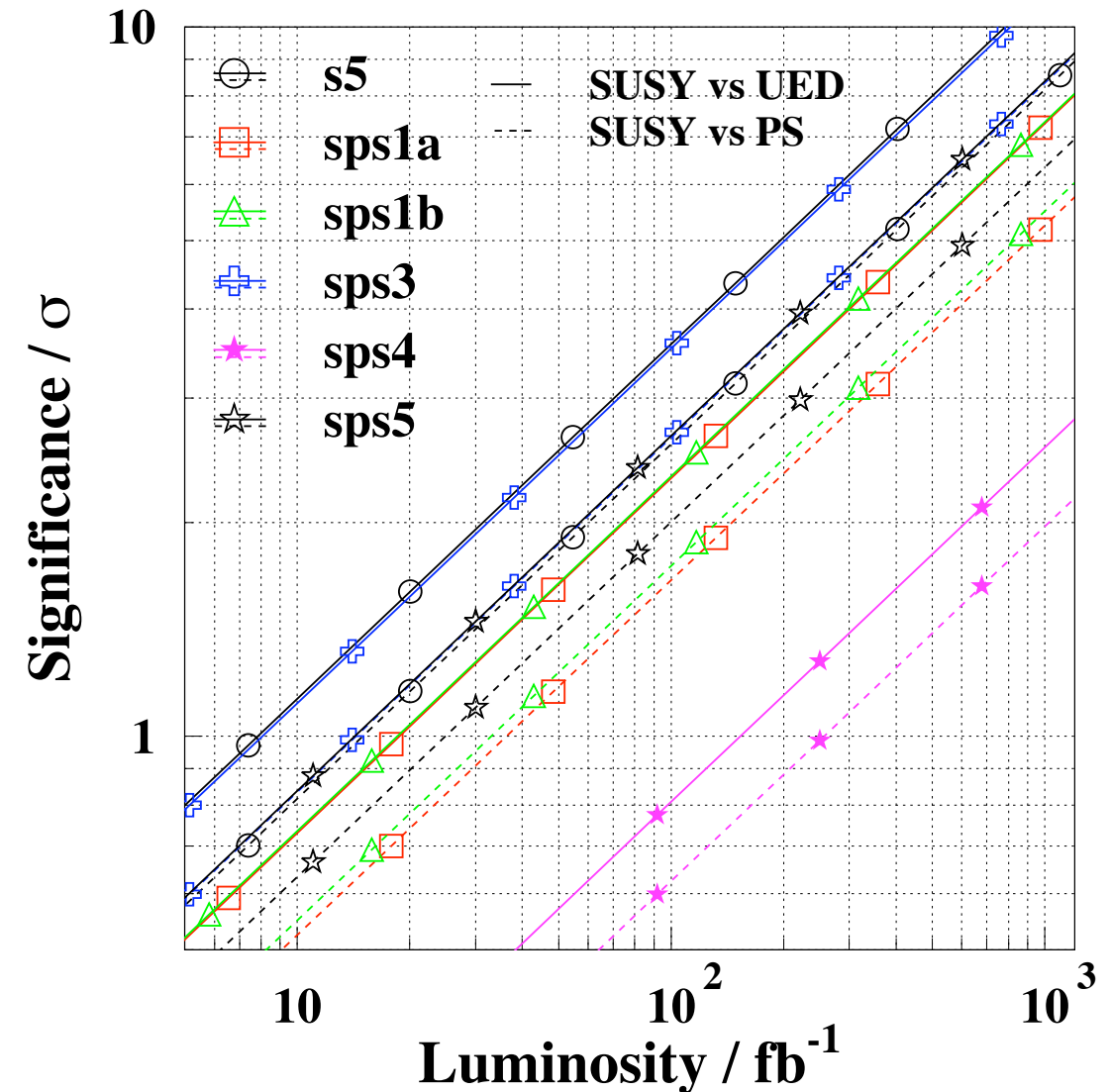
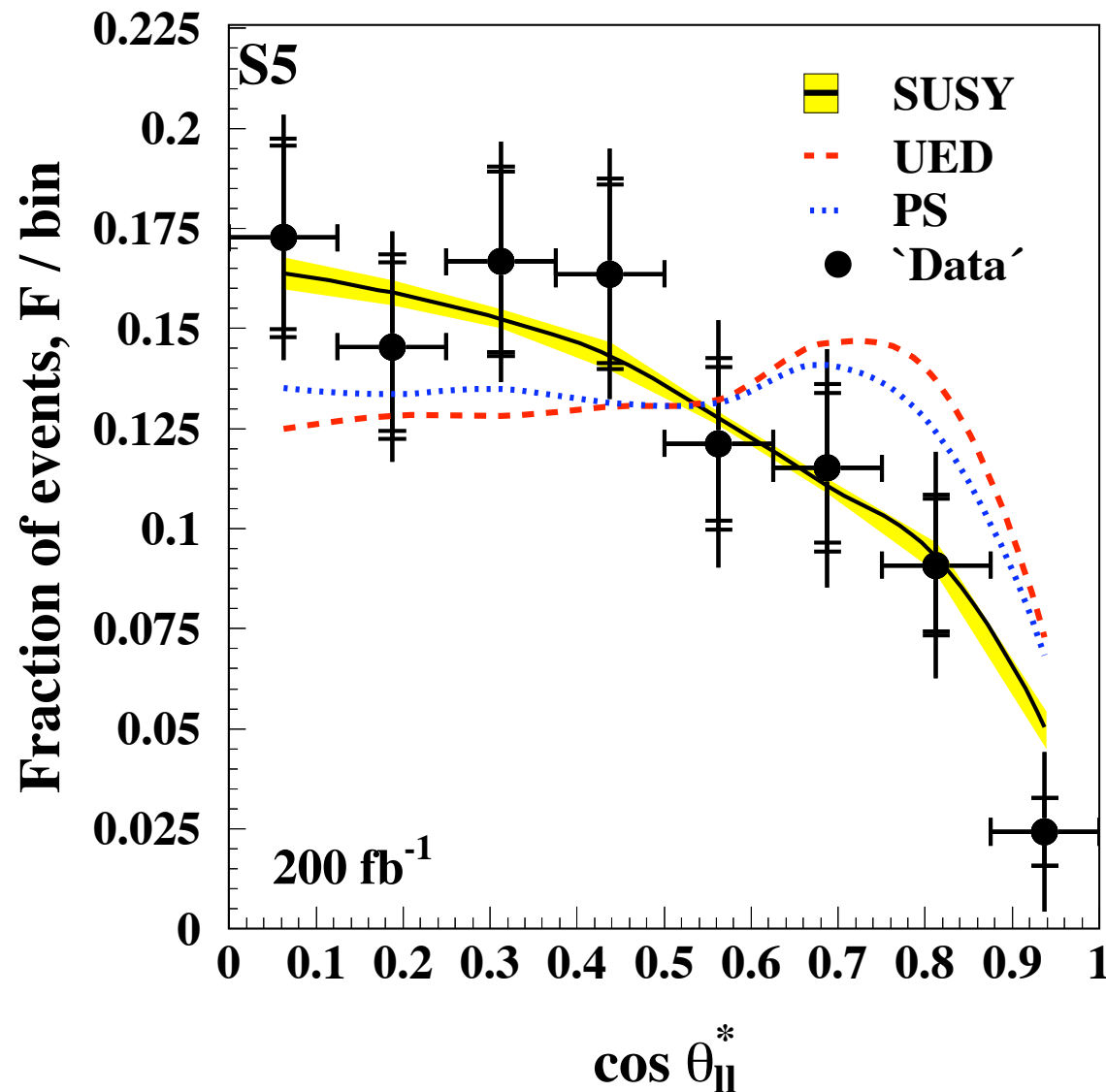
- $q\bar{q} \rightarrow Z^0/\gamma \rightarrow \tilde{\ell}^+ \tilde{\ell}^- \rightarrow \tilde{\chi}_1^0 \ell^+ \tilde{\chi}_1^0 \ell^-$
- Distribution of $\cos \theta_{ll}^* \equiv \tanh(\Delta\eta_{\ell^+ \ell^-}/2)$
is correlated with Z^0/γ decay angle θ^*



(neglects KKlepton polarisation)

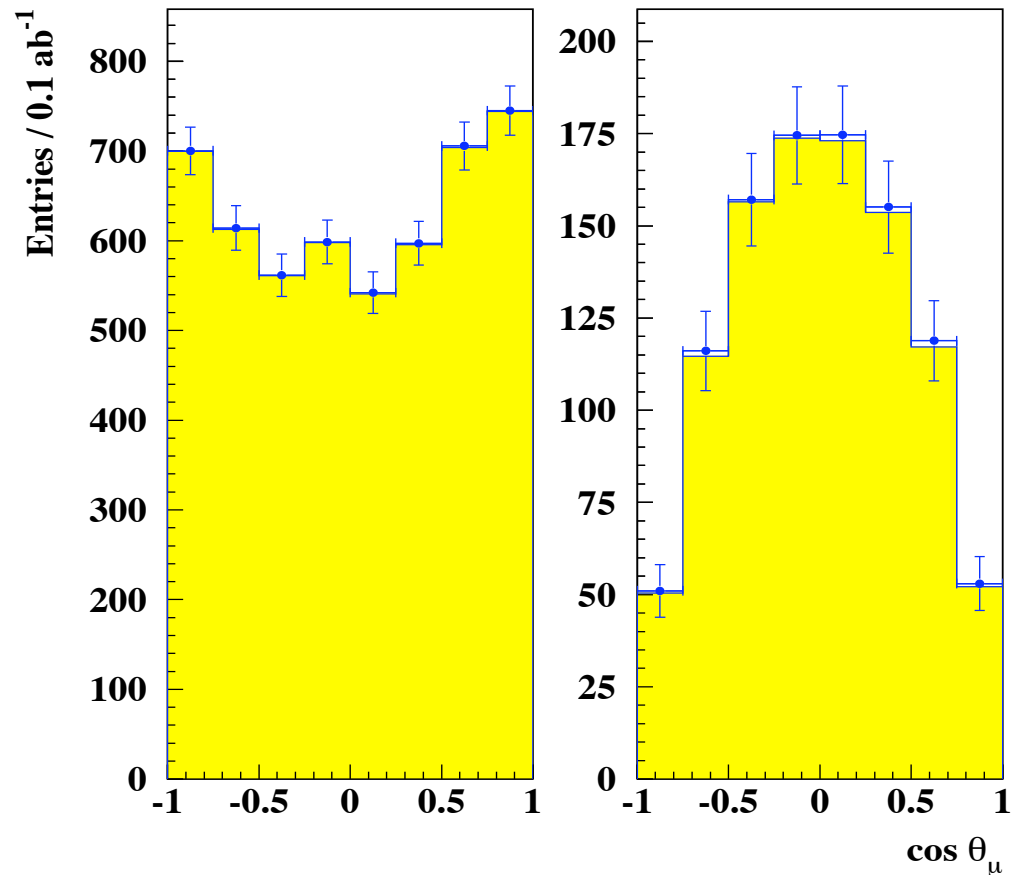
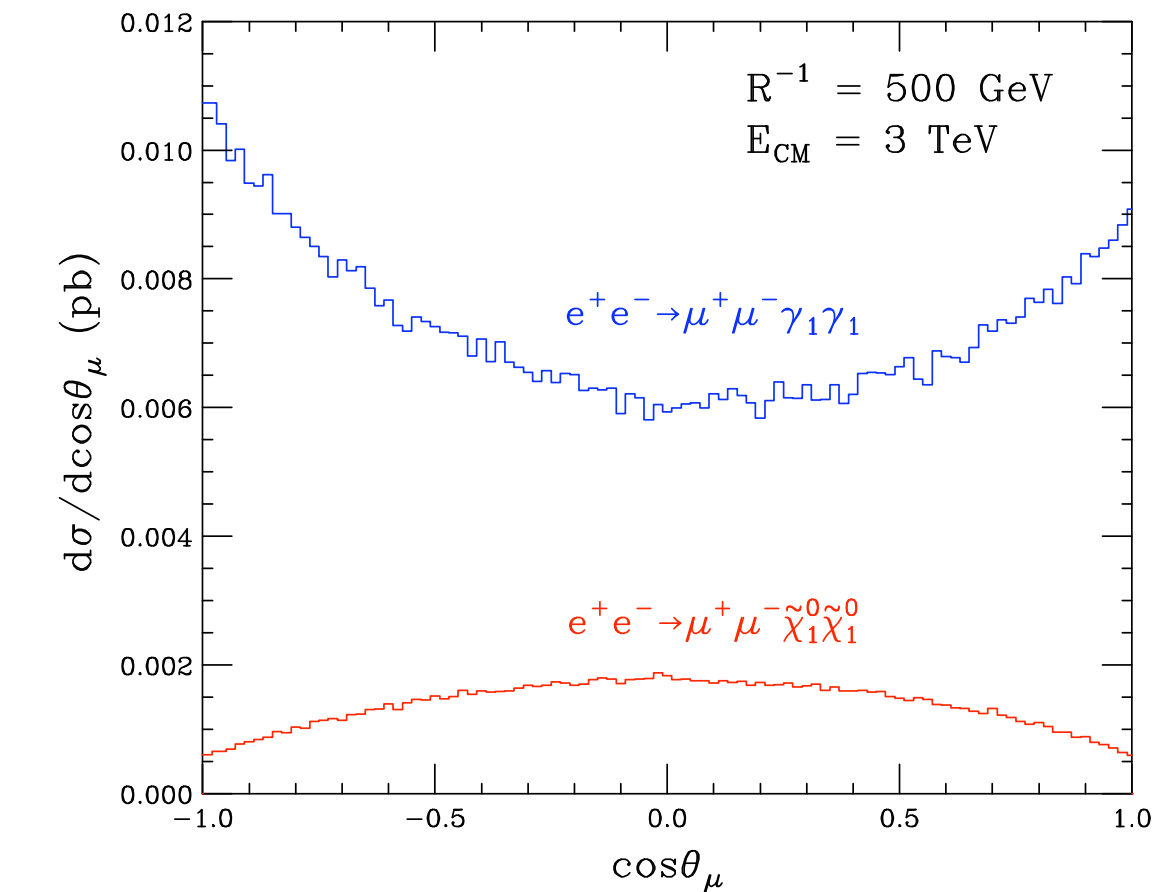
A Barr, hep-ph/0511115

Dislepton production (2)

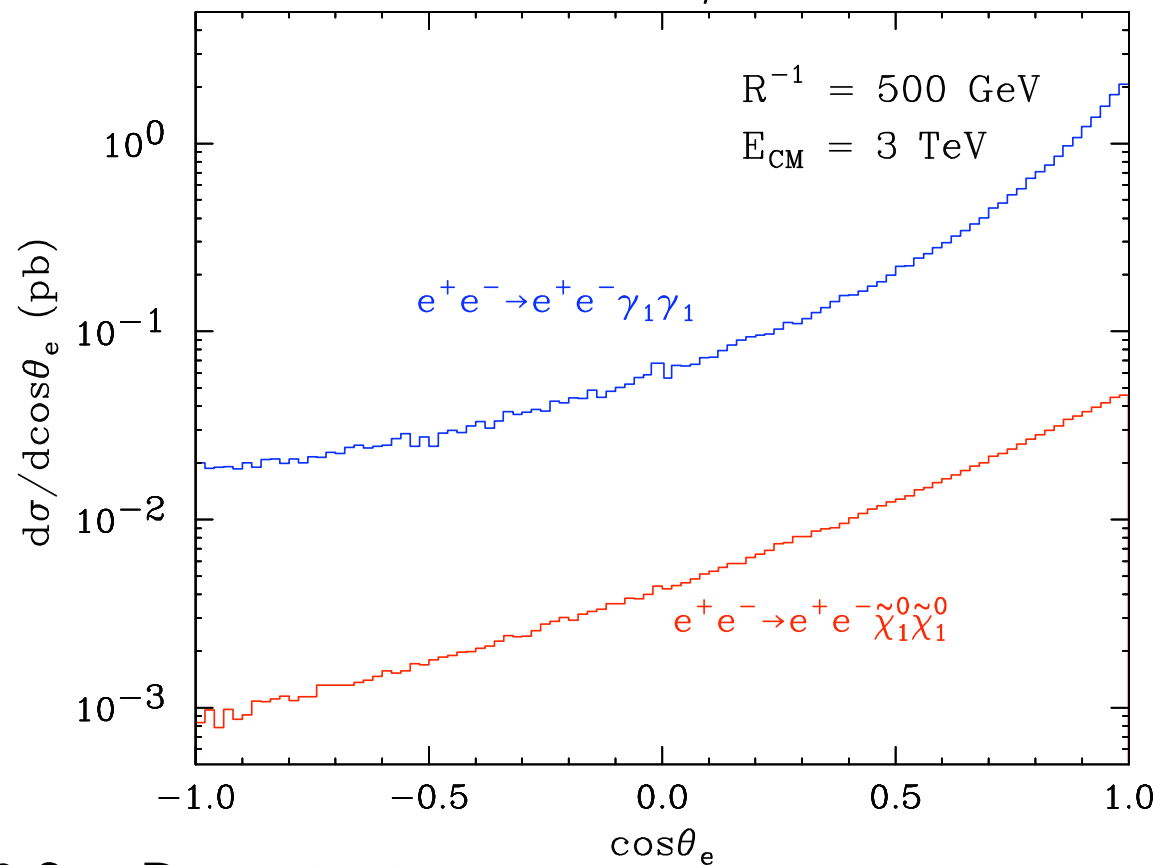


- Outer error bars: after SUSY & SM background subtraction
- Significance strongly dependent on mass spectrum

Disleptons at CLIC



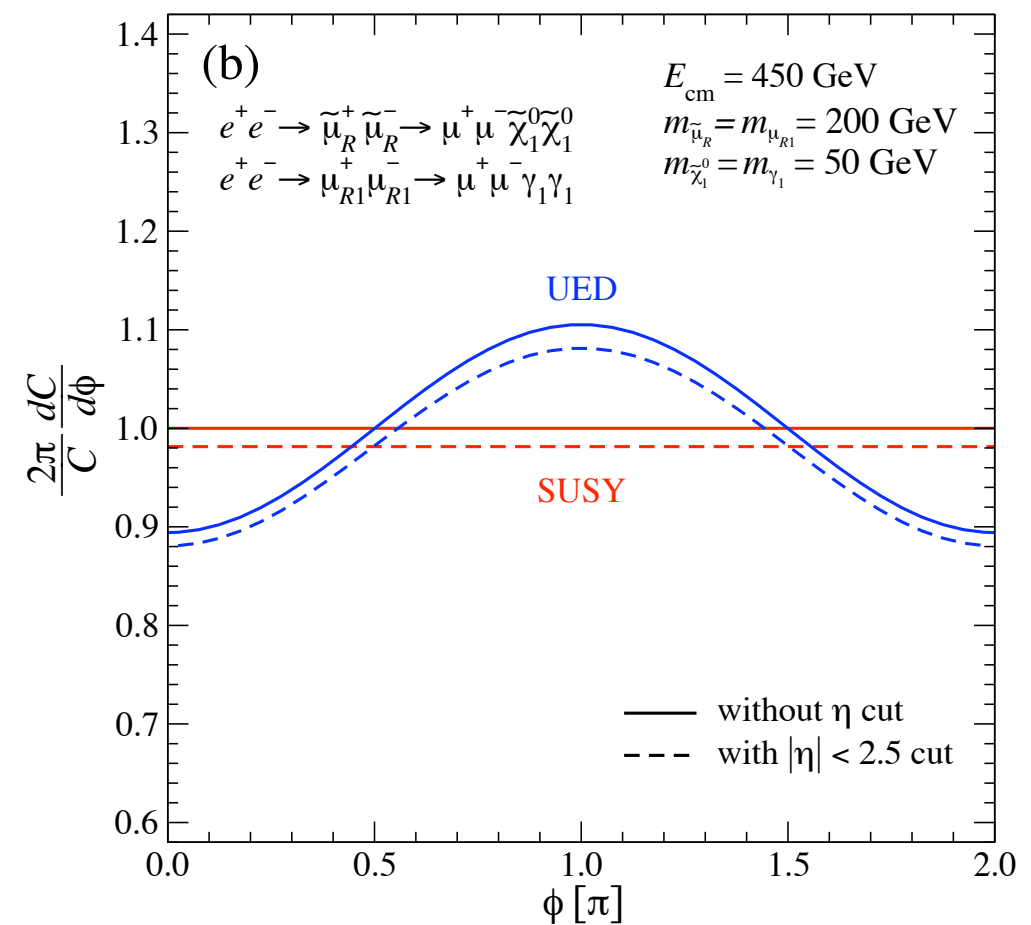
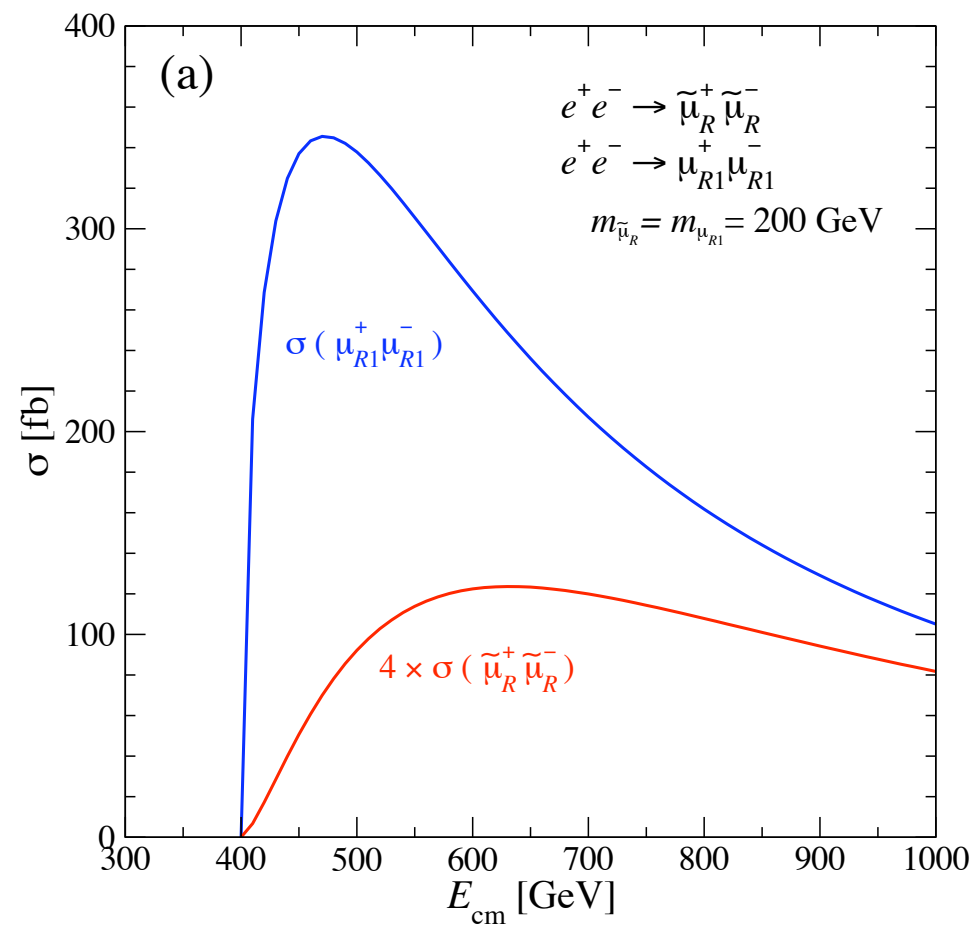
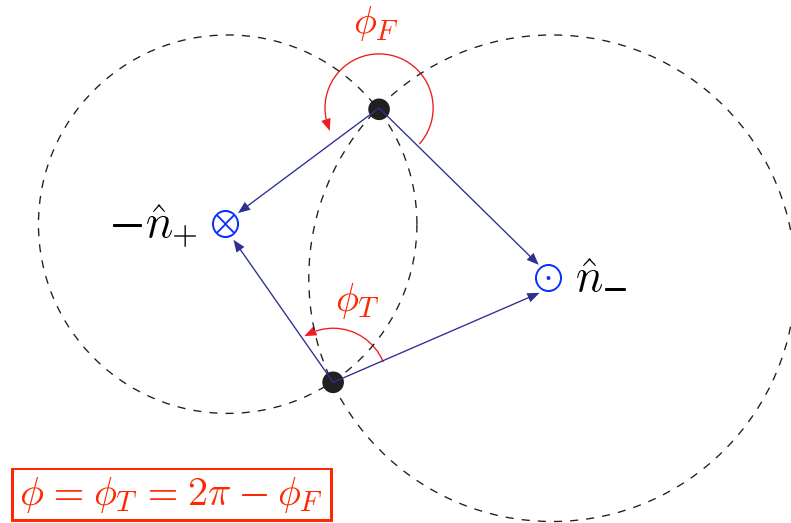
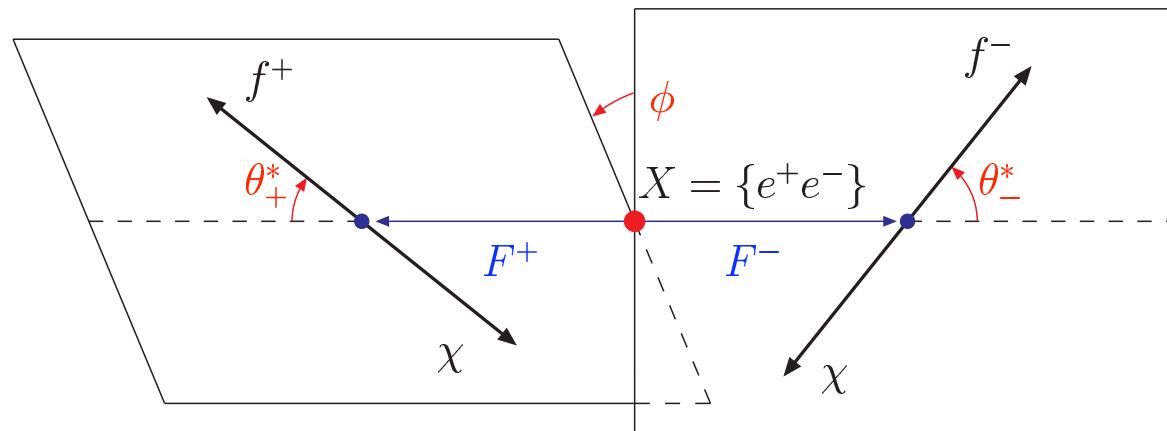
Detector level



Battaglia, Datta, DeRoeck, Kong,
 Matchev, hep-ph/0502041, 0507084

UED: Bhattacharya, Dey, Kundu,
 Raychaudhuri, hep-ph/0502031

Azimuthal correlations in e^+e^-



Buckley, Choi, Mawatari, Murayama, 0811.3030

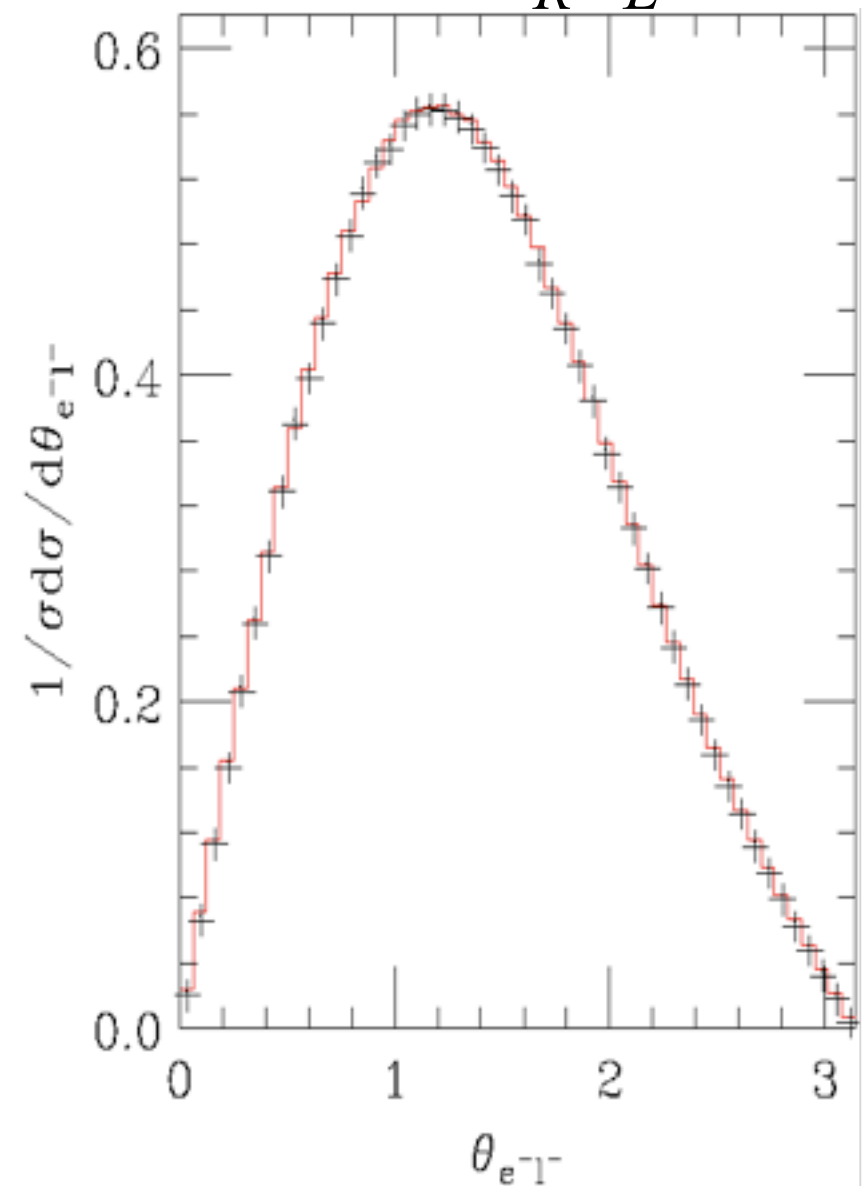
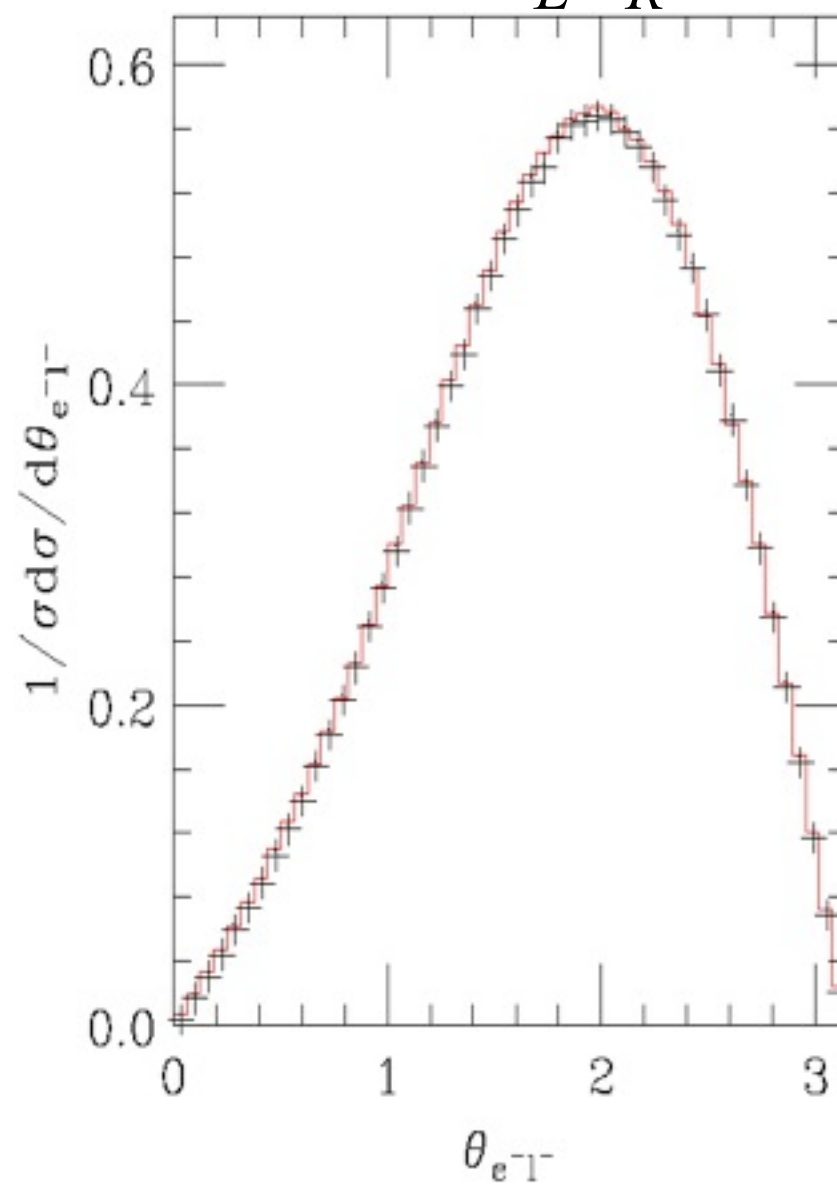
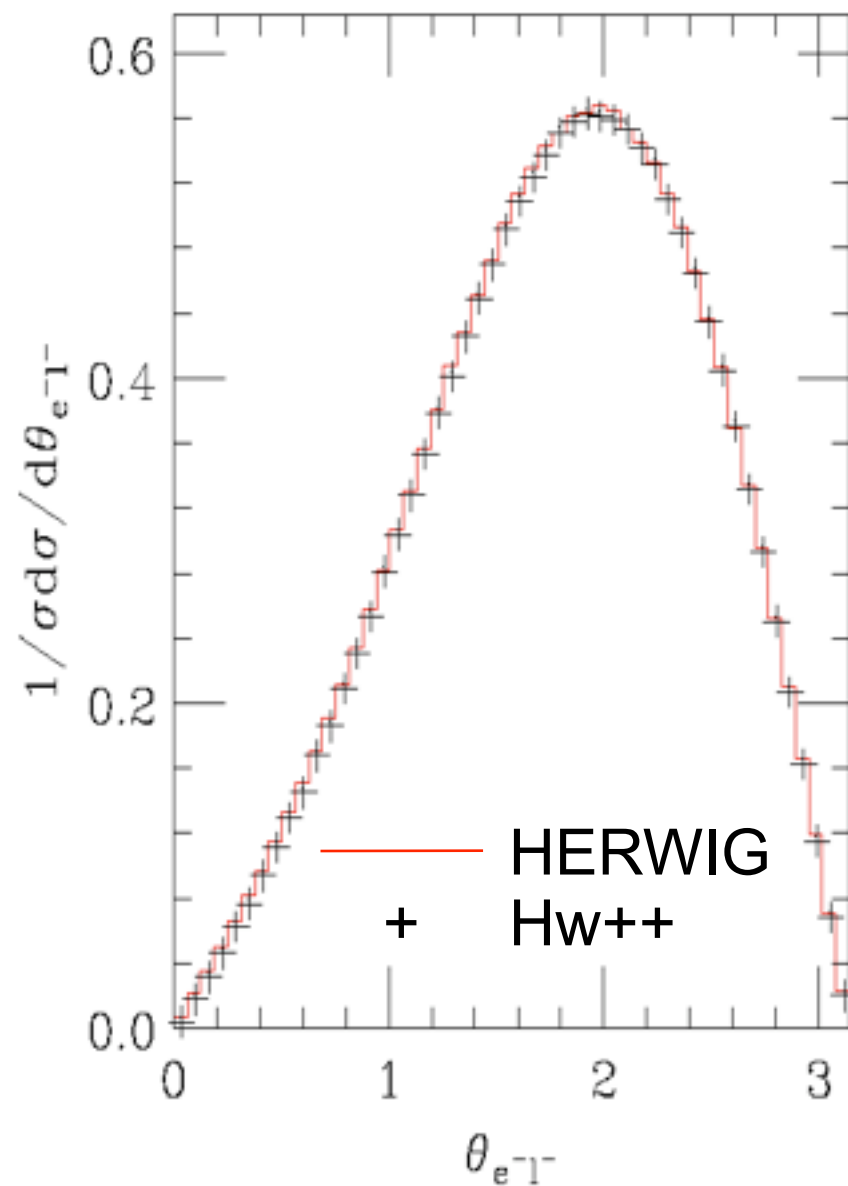
Spin Correlations in HERWIG

$$e^+e^- \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_1^0 \rightarrow \tilde{l}_R^+ l^- \tilde{\chi}_1^0 \rightarrow l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

Unpolarised

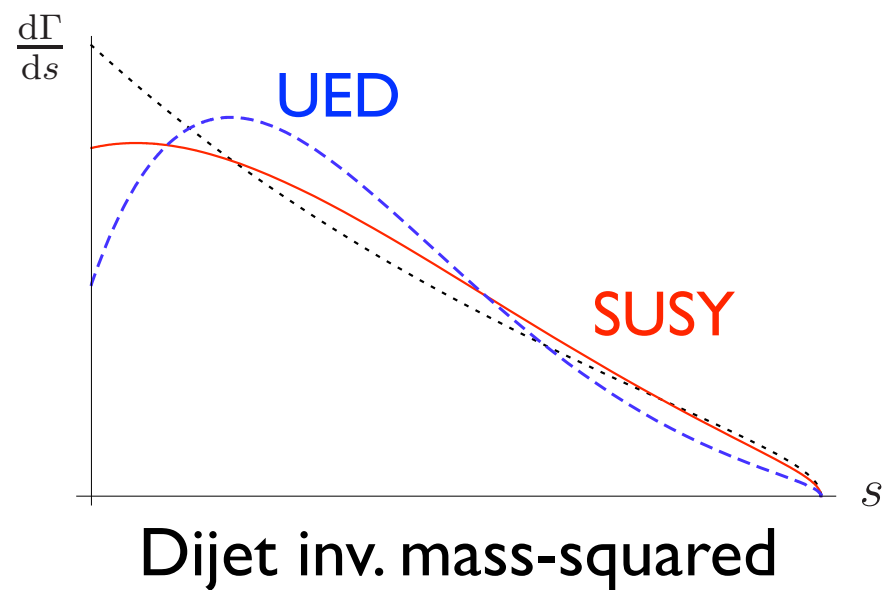
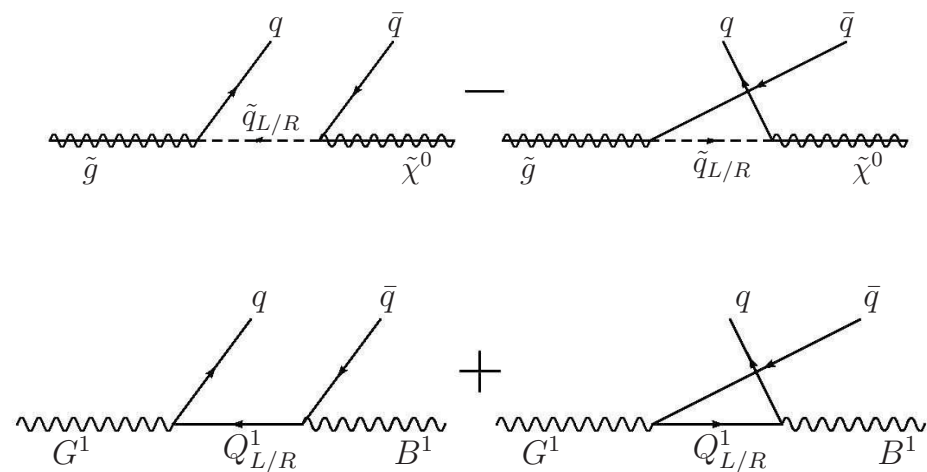
$e_L^- e_R^+$

$e_R^- e_L^+$



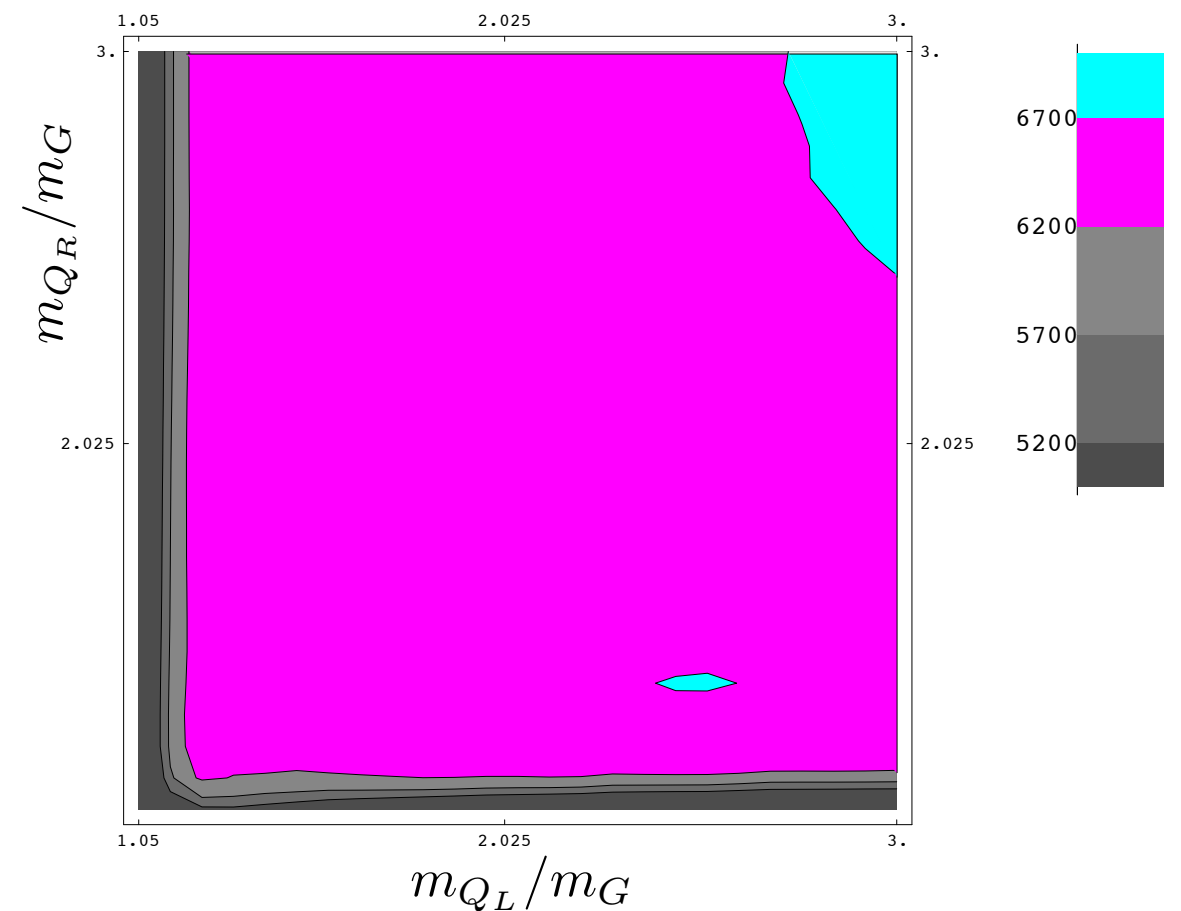
Gigg, Richardson, hep-ph/0703199

Three-body gluino decays



Csaki, Heinonen, Perelstein, 0707.0014

Number of events needed to discriminate



Kullback-Leibler measure:

$$N \sim \log R/\text{KL}(T, S)$$

$$\text{KL}(T, S) = \int_m \log \left(\frac{p(m|T)}{p(m|S)} \right) p(m|T) dm$$