Sussex University November $15^{th}$ 2010

# Non-linear perturbations from cosmological inflation

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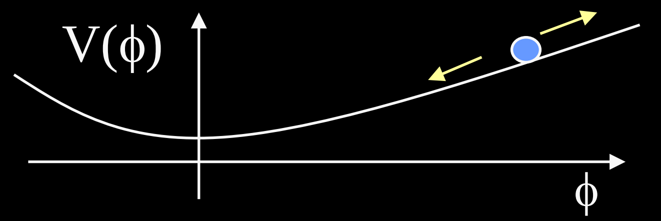
***and SEPnet!***

# summary:

- linear perturbations (quantum fluctuations) of free field(s) during inflation provide excellent fit to current data

- non-linear perturbations offer distinctive observational signatures of physics of the very early universe

# Vacuum fluctuations




- *small-scale/underdamped zero-point fluctuations (k>aH)*

$$\left\langle \delta\phi_k \delta\dot{\phi}^*_{k'} \right\rangle \quad = \quad i\hbar\delta\left(k-k'\right)$$

- *large-scale/overdamped perturbations in growing mode (k<aH)*

*linear evolution* $\Rightarrow$ *Gaussian random field*

$$\Rightarrow \quad \mathcal{P}\left(\delta\phi\right)_{k=aH} \quad \approx \quad \frac{4\pi k^3 \left|\delta\phi_k{}^2\right|}{(2\pi)^3} \quad = \quad \left(\frac{H}{2\pi}\right)^2$$

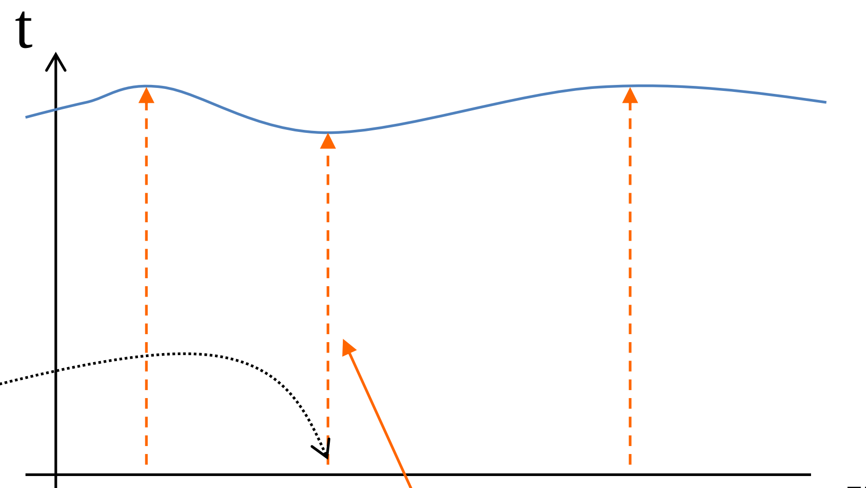
*fluctuations of any light scalar fields (m<3H/2) ʽfrozen-in' on large scales*

*interactions = non-linearity = non-Gaussianity*

*suppressed during slow-roll inflation* $\left\langle \delta\phi_k \delta\phi_{k'} \delta\phi_{k''} \right\rangle \quad \approx \quad \varepsilon \left|\delta\phi_k{}^4\right| \delta\left(k+k'+k''\right)$

# the δN formalism for primordial perturbations

**in radiation-dominated era**
curvature perturbation $\zeta$ on
uniform-density hypersurface




$$N = \int_{initial}^{final} H\, dt$$

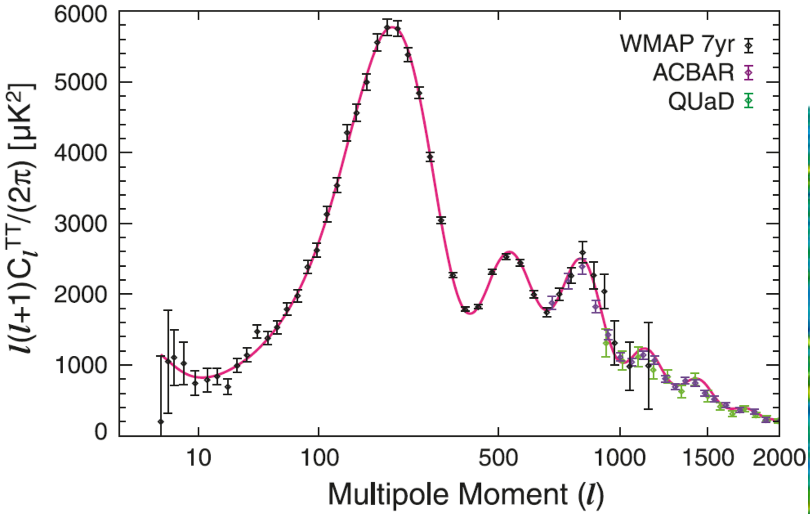
**during inflation**
field perturbations $\delta\phi_I(x,t_i)$ on
initial spatially-flat hypersurface

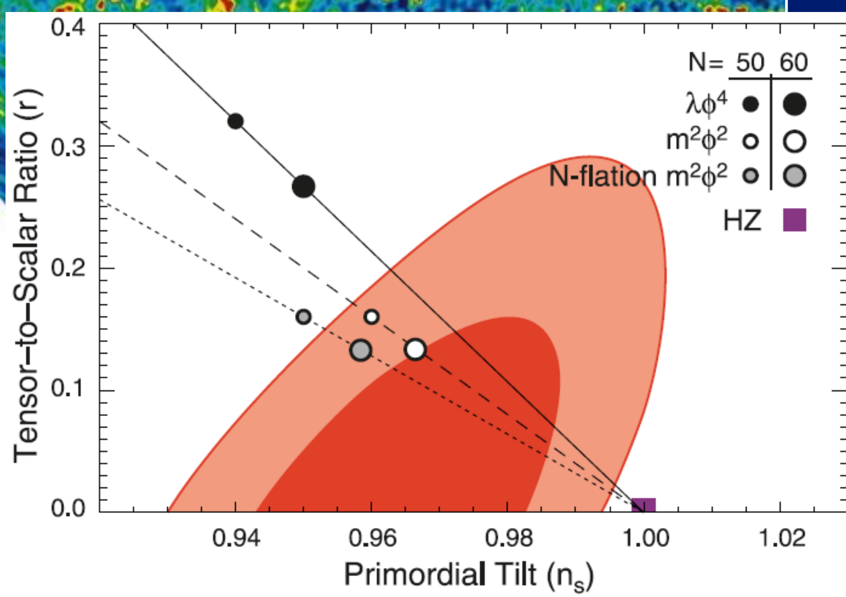
on large scales, neglect spatial gradients, treat as "separate universes"

$$\zeta = N(\phi_{initial}) - \bar{N} \approx \sum_I \frac{\partial N}{\partial \phi_I}\delta\phi_I$$

Starobinsky \`85; Sasaki & Stewart \`96

# WMAP 7 year data February 2010




WMAP 7-year data (Komatsu et al 2010)

- scale dependence

  $0.944 < n < 0.992$

- gravitational waves

  tensor-scalar ratio

  $r < 0.36$

  non-Gaussianity

  $-10 < f_{NL} < 74$

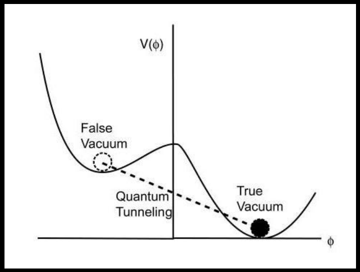
- how could we identify the correct model of inflation?
  - local non-Gaussianity (fNL-local) from curvaton?
  - equilateral non-Gaussianity (fNL-equilateral) from DBI inflation?
  - topological defects from hybrid inflation?
  - gravitational waves from preheating?
  - bubbles from first-order inflation?
  - …

*non-linear perturbations from inflation*

# extended inflation

La & Steinhardt 1989
Barrow & Maeda 1990
Steinhardt & Accetta 1990

- simple, compelling model for gravity-driven inflation...
- false vacuum + first-order transition
- + Brans-Dicke gravity




$$L = \Phi R - \frac{\omega}{\Phi}\left(\nabla\Phi\right)^2 - V_{falsevacuum}$$

- solves graceful exit problem of Guth's old inflation
  - $\Phi$ grows during inflation, Hubble rate decreases, until first-order transition completes
- scale-free gravity (one dimensionless parameter)

# dynamical solution to hierarchy problem

E. Weinberg 1989
Liddle & Wands 1992

- start near Planck scale

$$H^2 \approx \frac{M_{GUT}^4}{M_{Pl}^2(\Phi)} \approx M_{Pl}^2(\Phi) \approx M_{GUT}^2$$

- bubble nucleation rate

$$\Gamma = M_{GUT}^4 \exp\left(-S_E\right)$$

  - $S_E >> 1$ is dimensionless Euclidean action ("shape parameter")

- percolation parameter

$$P = \frac{\Gamma}{H^4} \approx \frac{M_{Pl}^4(\Phi)}{M_{GUT}^4} \exp\left(-S_E\right)$$

  - $P$ grows as $\Phi$ grows (gravity gets weaker) and $H$ decreases

- phase transition completes / inflation ends when $p=1$

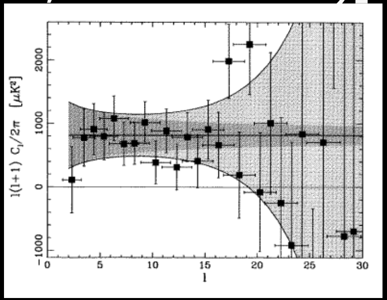
$$\Rightarrow \quad M_{Pl} \approx M_{GUT} \exp\left(S_E / 4\right) >> M_{GUT}$$

# power law inflation

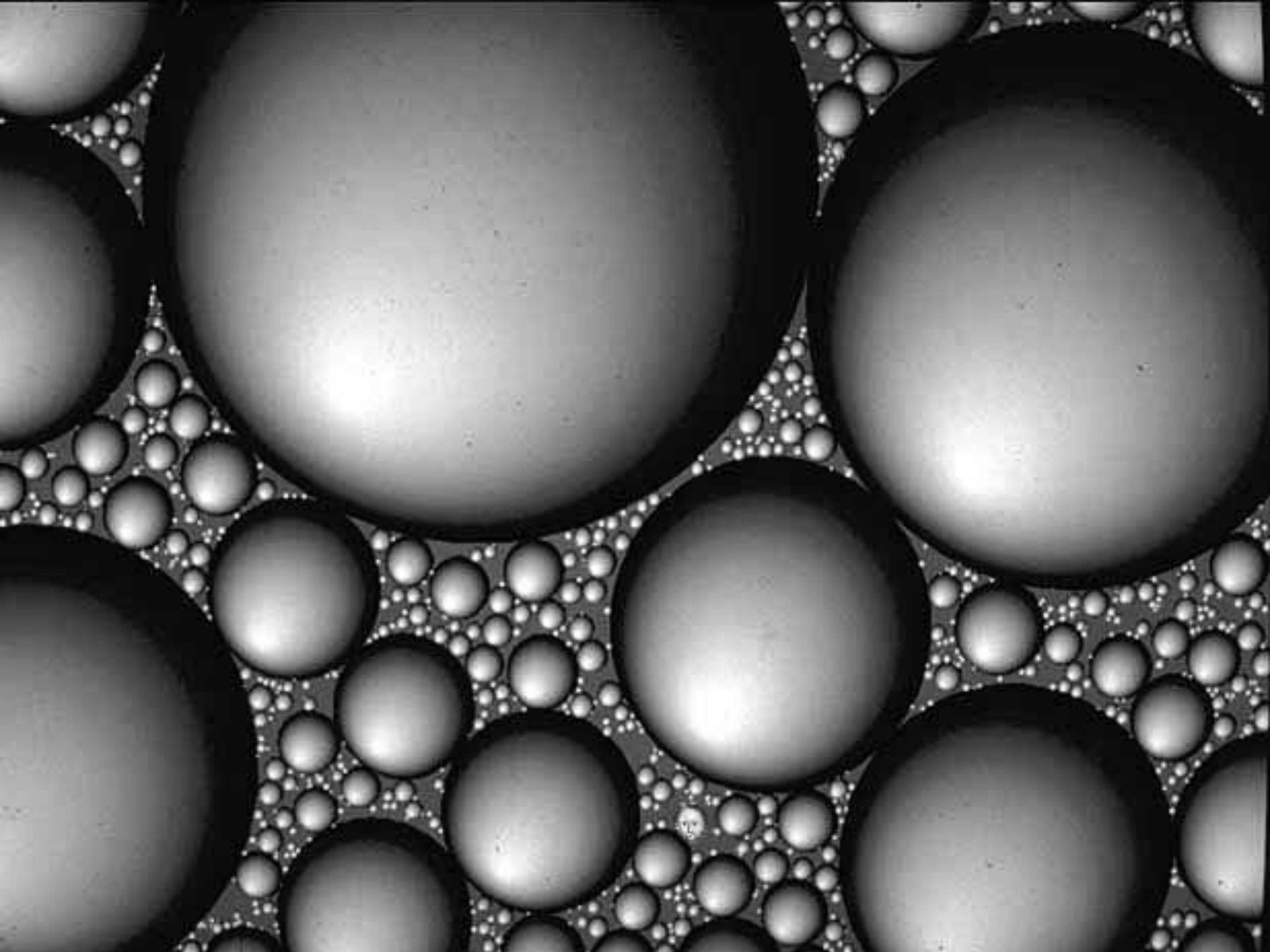
- power-law inflation
$$a \propto t^{p} \quad ; \quad p = \frac{2\omega + 3}{4}$$

- linear perturbations
  - [conformal transform to Einstein frame (Brans 1962, Maeda 1989)]
  - reproduces scale-invariant spectrum as $\omega \to \infty$

CoBE (1994)



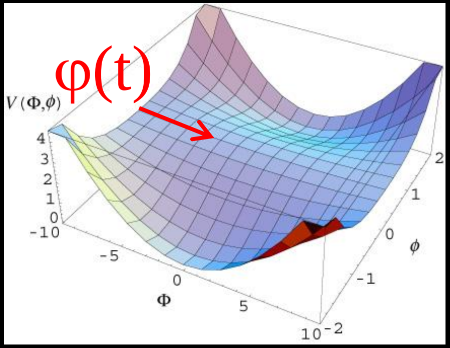
- **non-linear perturbations**
  - first-order transition leads to distribution of bubbles
  - spectrum of bubbles also becomes scale-invariant as $\omega \to \infty$
  - ***"big bubble problem"*** Weinberg (1989); Liddle & Wands; Maeda & Sakai (1992)



# hybrid inflation

Linde 1993
Copeland, Liddle, Lyth, Stewart & Wands 1994

- inflaton field changes *shape* of false vacuum potential
  - $S_E(t) => \Gamma(t) \sim M^4 exp[- S_E(t)]$
  - ends by *sudden* phase transition
  - first- or second-order




- non-linear perturbations only on small scales

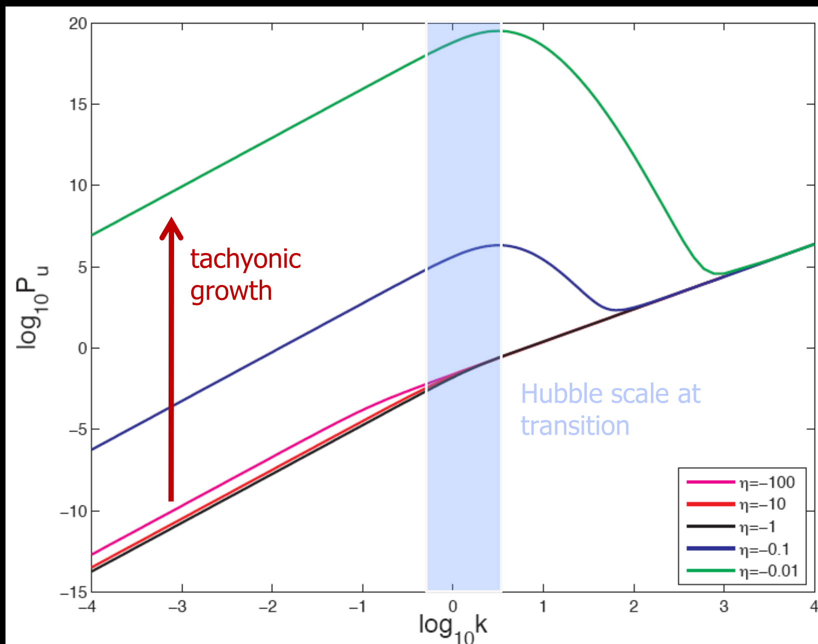
Mulryne, Seery & Wesley 2009
Abolhasani & Firouzjahi; Lyth; Fonseca, Sasaki & Wands; Gong & Sasaki 2010

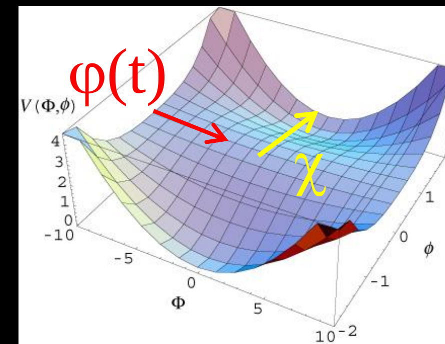
  - inhomogeneous bubbles or tachyonic preheating

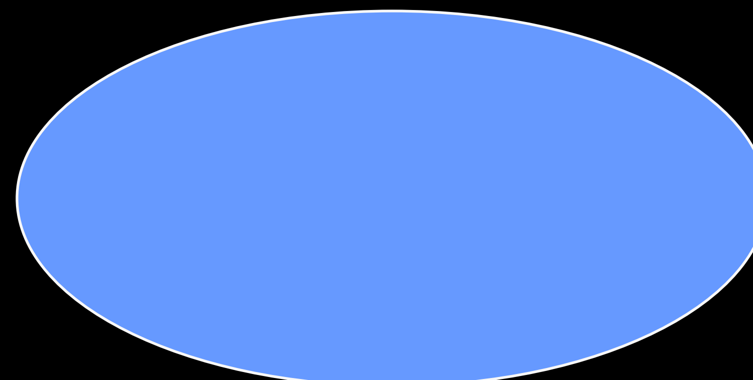
# tachyonic instability of long-wavelengths

Fonseca, Sasaki & Wands 2010

spectrum of perturbations in "waterfall" field



long-wavelength delta-N calculation normalised by small-wavelength fluctuations

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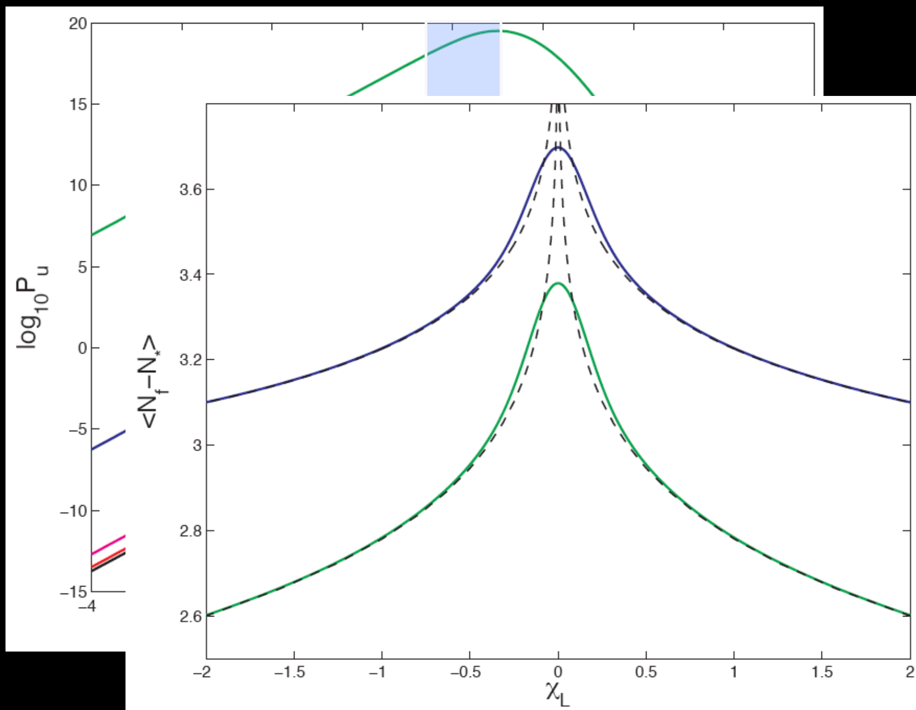


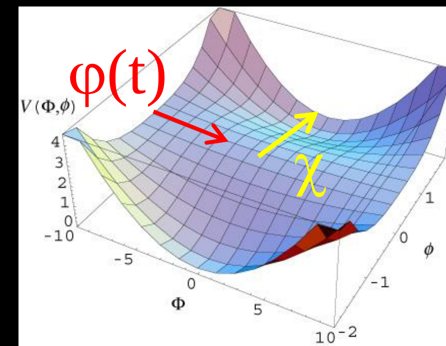


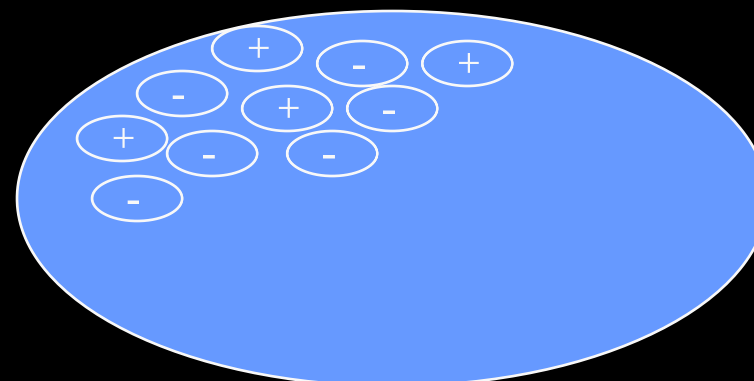

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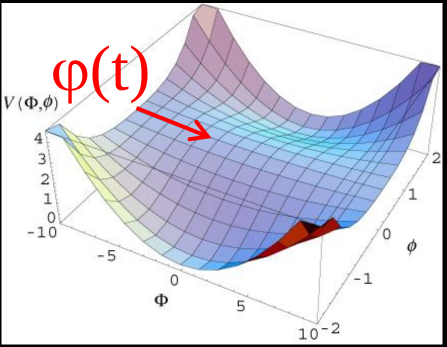


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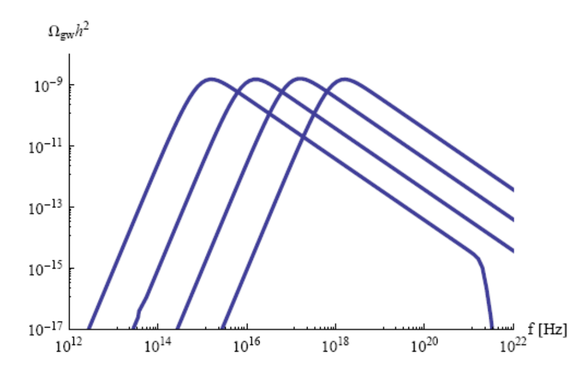
  - inhomogeneous bubbles or tachyonic preheating
  - spectrum of relic gravitational waves on characteristic scale




Easther '09

# Sources of primordial gravitational waves:

Quantum fluctuations of gravitational field

First-order phase transitions?

Preheating after inflation?

Cosmic string cusps?

*Primordial density perturbations*

# Second-order GW from first-order density perturbations

Tomita (1967); Matarrese et al (1994); Hwang; K. Nakamura; Ananda, Clarkson & Wands (2006)

- scalar, vector and tensor modes couple at second and higher order
- tensor perturbations become gauge-dependent
- in longitudinal gauge for general FRW cosmology $w=P/\rho$, $c_s^2=dP/d\rho$

$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S_{ij}^{TT}}$$

*where second-order source is transverse-tracefree part of*

$$\begin{aligned} S_{ij} \; = \; & 2\Phi\partial_i\partial_j\Phi - 2\Psi\partial_i\partial_j\Phi + 4\Psi\partial_i\partial_j\Psi + \partial_i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & -\frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) \\ & -\frac{2c_s^2}{3w\mathcal{H}}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi) \end{aligned}$$

*Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290*

# GW from density perturbations in radiation era

*Ananda, Clarkson & Wands, gr-qc/0612013*

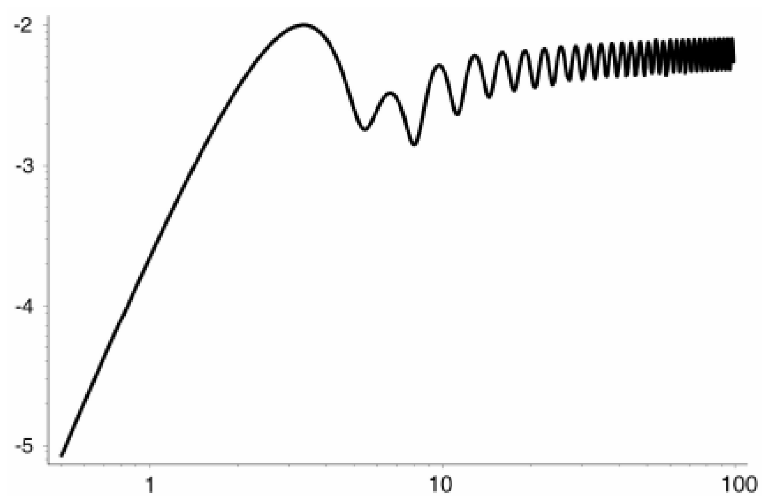
$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}$$

- almost scale-invariant primordial density power spectrum

$$P_\Phi(k) = \frac{4}{9}\Delta^2_R(k) \quad \text{for } k << aH$$

- generates almost scale-invariant gravitational wave background

$$\boxed{\begin{aligned}\Omega_{\mathrm{GW},0}(k) &\equiv \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k} \\ &\approx 30\Omega_{\gamma,0}\Delta^4_R(k) \\ &\text{for } k >> aH\end{aligned}}$$




$k/aH$

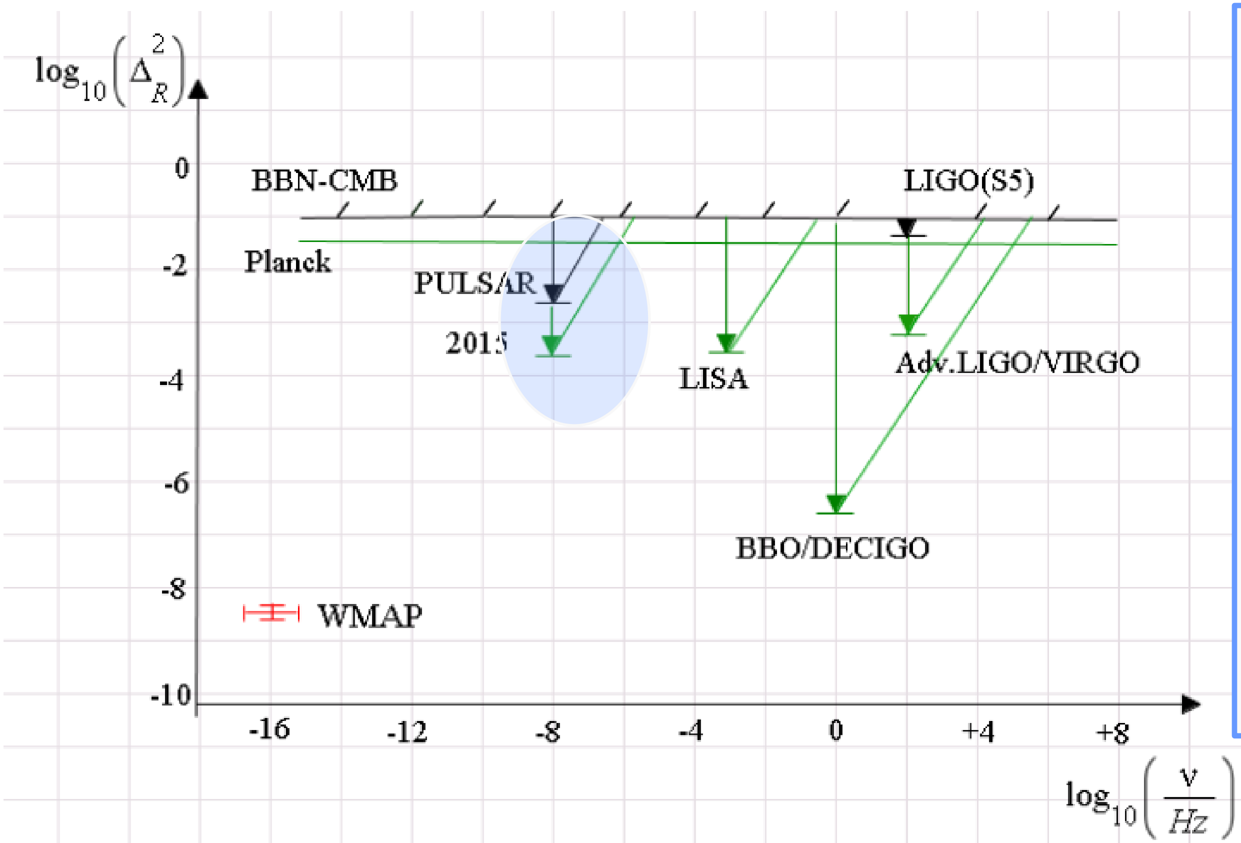
- e.g.,

$$\Omega_{\mathrm{GW},0} \approx 10^{-20} \quad \text{for } \Delta^2_R \approx 10^{-9}$$

# Constraints on primordial density perturbations

*Assadullahi & Wands, arXiv:0907.4073*

$$\Omega_{\mathrm{GW},0}(k) \approx 30\,\Omega_{\gamma,0}\,\Delta_R^4(k)$$




- LIGO/VIRGO
  $\Delta_R^2 < 0.07\,, \quad \nu \approx 100\mathrm{Hz}$
- Advanced LIGO/VIRGO
  $\Delta_R^2 < 8\times10^{-4}$
- LISA
  $\Delta_R^2 < 3\times10^{-4}\,, \ \nu \approx \mathrm{mHz}$
- BBO/DECIGO
  $\Delta_R^2 < 3\times10^{-7}\,, \quad \nu \approx 1\mathrm{Hz}$

•Pulsar timing data rules out intermediate mass primordial black holes

*Saito & Yokoyama, arXiv:0812.4339 (Phys Rev Lett)*
*Bugaev & Klimai, arXiv:09080664*

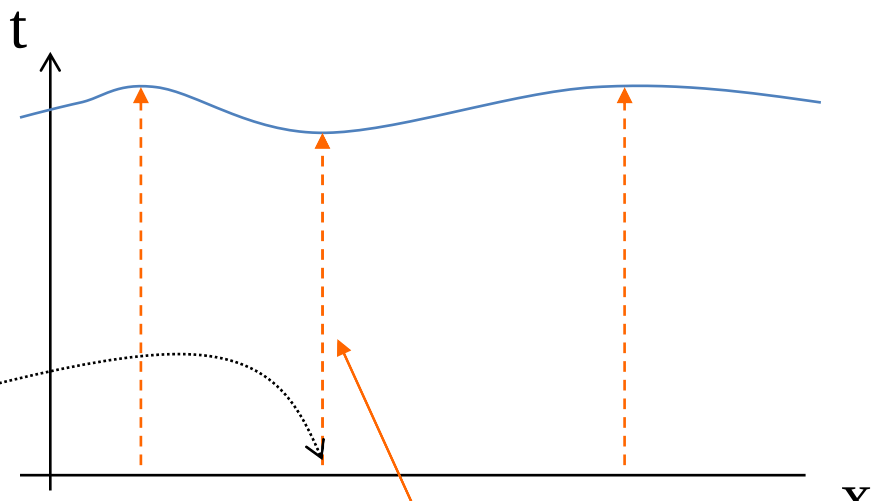
# second-order density perturbations

- non-linear evolution lead to **non-Gaussian distribution**
  - **non-zero bispectrum and higher-order correlators**

  - Local-type non-Gaussianity
    - super-Hubble evolution of Gaussian random field from multi-field inflation
  - Equilateral-type non-Gaussianity
    - sub-Hubble interactions in k-inflation/DBI inflation

# the δN formalism for primordial perturbations

**in radiation-dominated era**
curvature perturbation $\zeta$ on
uniform-density hypersurface




**during inflation**
field perturbations $\delta\phi_I(x,t_i)$ on
initial spatially-flat hypersurface

$$\boxed{N = \int_{initial}^{final} H\, dt}$$

on large scales, neglect spatial gradients, treat as "separate universes"

$$\zeta = N\left(\phi_{initial}\right) - \bar{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Starobinsky `85; Sasaki & Stewart `96

**Lyth & Rodriguez '05 – *works to any order***

# the δN formalism order by order at Hubble exit

t

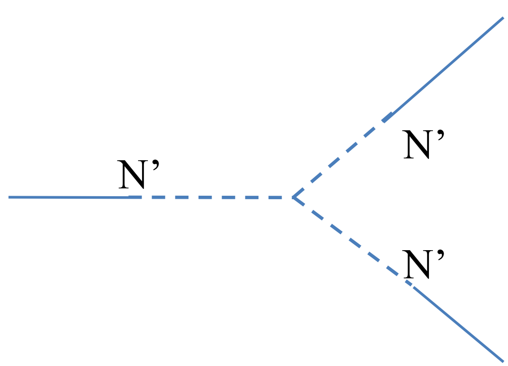
$$\delta\phi_I = \delta_1\phi_I + \frac{1}{2}\delta_2\phi_I + \ldots$$

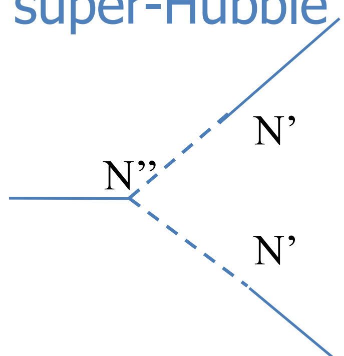
$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \ldots$$

$$= \left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_1\phi_I\right] + \frac{1}{2}\left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_2\phi_I + \sum_{I,J}\frac{\partial^2 N}{\partial\phi_I\partial\phi_I}\delta_1\phi_I\delta_1\phi_J\right] + \ldots$$

sub-Hubble quantum interactions

super-Hubble classical evolution



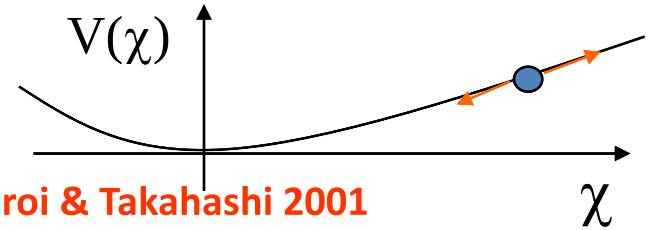




Byrnes, Koyama, Sasaki & DW

# large non-Gaussianity from inflation?

- **single inflaton field**
  - adiabatic perturbations => $\zeta$ constant on large scales
  - during conventional slow-roll inflation $f_{NL}{}^{local} \approx N''/N'^2 = \eta - 2\varepsilon \ll 1$
  - for any adiabatic model (Creminelli&Zaldarriaga 2004)
    $$f_{NL}{}^{local} = -\frac{5}{12}(n-1)$$
  - k/DBI - inflation $f_{NL}{}^{equil} \approx 1/c_s$
- **multi-field models**
  - typically $f_{NL}$~1 for slow-roll inflation
  - could be much larger from sudden transition at end of inflation ?
  - modulated reheating
  - curvaton $f_{NL}$~$1/\Omega_{decay}$ >>1 ?
  - new ekpyrotic models $|f_{NL}|$>>1

# curvaton scenario:




**Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001**

curvaton $\chi$ = a weakly-coupled, late-decaying scalar field

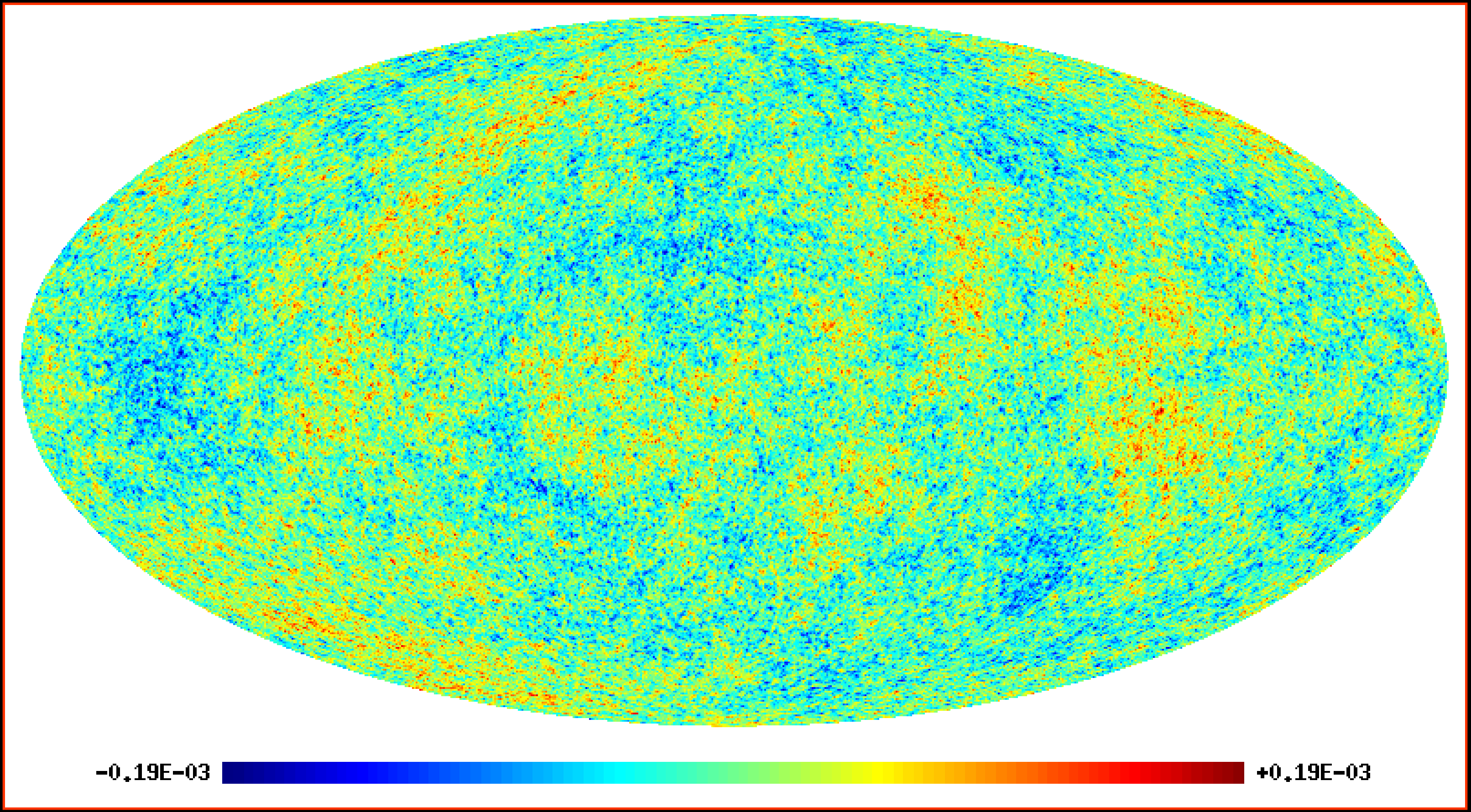
- light during inflation (m<<H) hence acquires an almost scale-invariant, *Gaussian distribution of field fluctuations* on large scales
- energy density for massive field, $\rho_\chi = m^2\chi^2/2$
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi} \approx \frac{1}{3}\left(\frac{2\chi\delta\chi + \delta\chi^2}{\chi^2}\right)$$

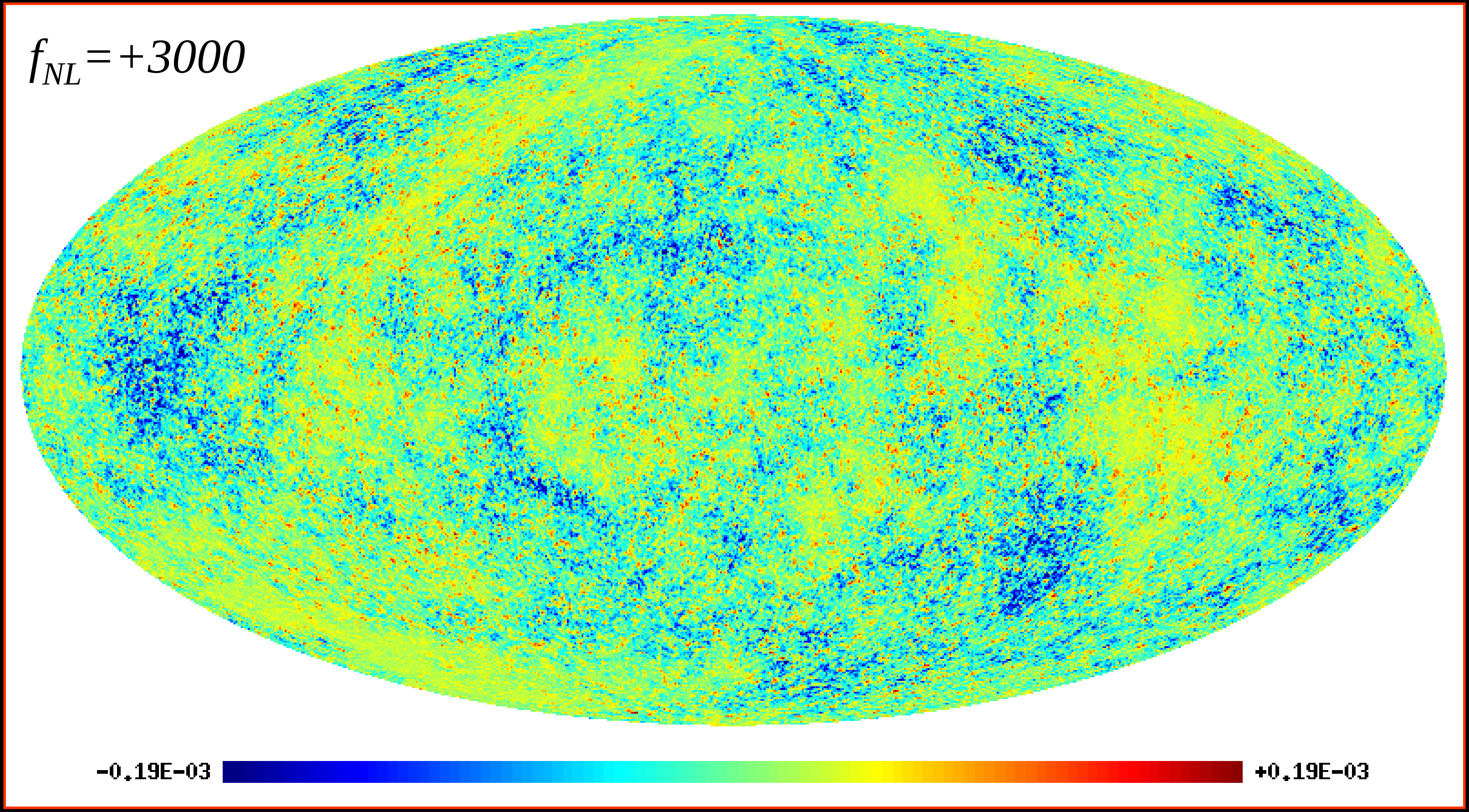
- transferred to radiation when curvaton decays with some efficiency $\approx \Omega_{\chi,decay}$

$$\zeta = \Omega_{\chi,decay}\zeta_\chi$$

$$= \zeta_G + \frac{3}{4\Omega_{\chi,decay}}\zeta_G^2 \quad \Rightarrow \quad f_{NL} = \frac{5}{4\Omega_{\chi,decay}}$$




Liguori, Matarrese and  Moscardini (2003

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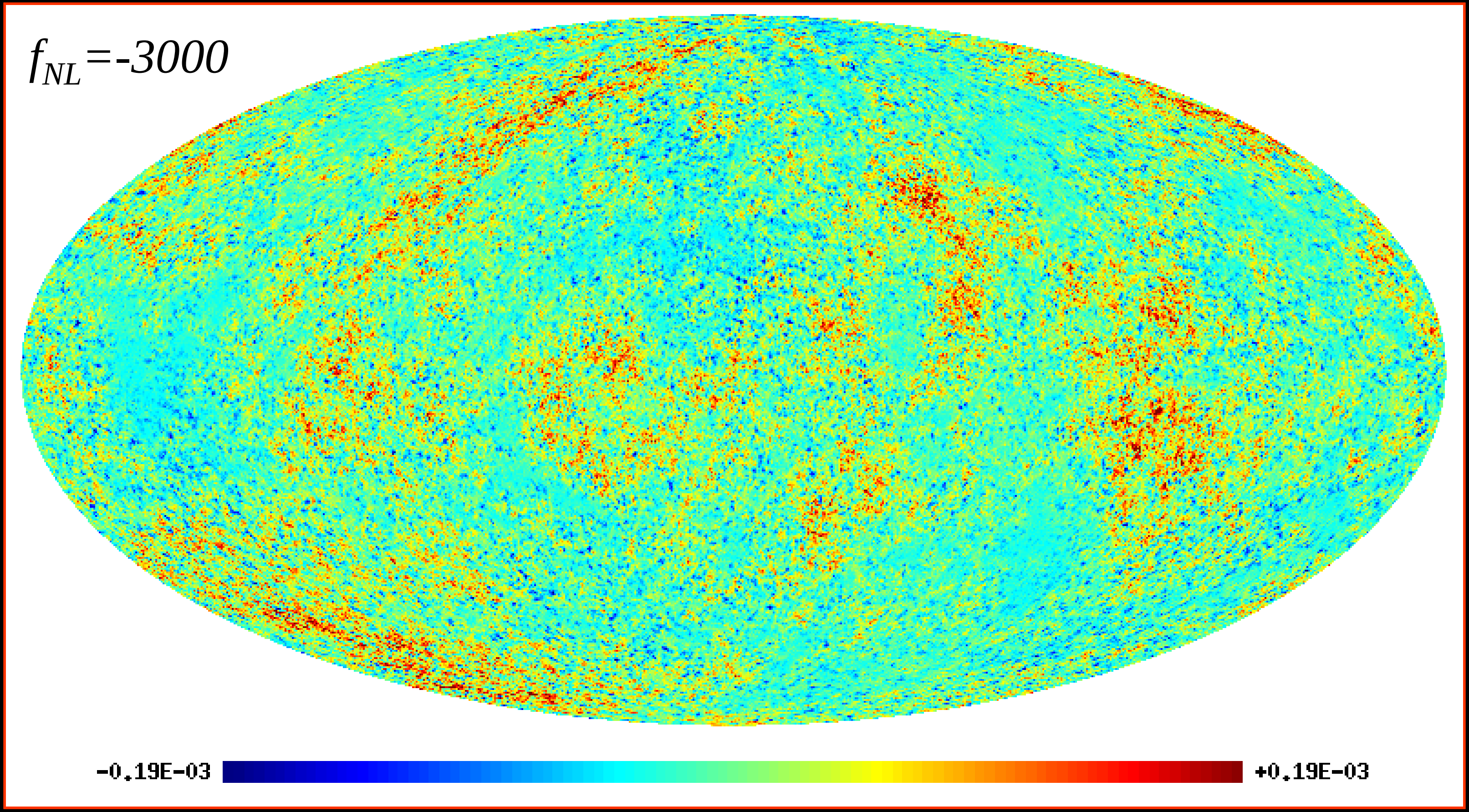


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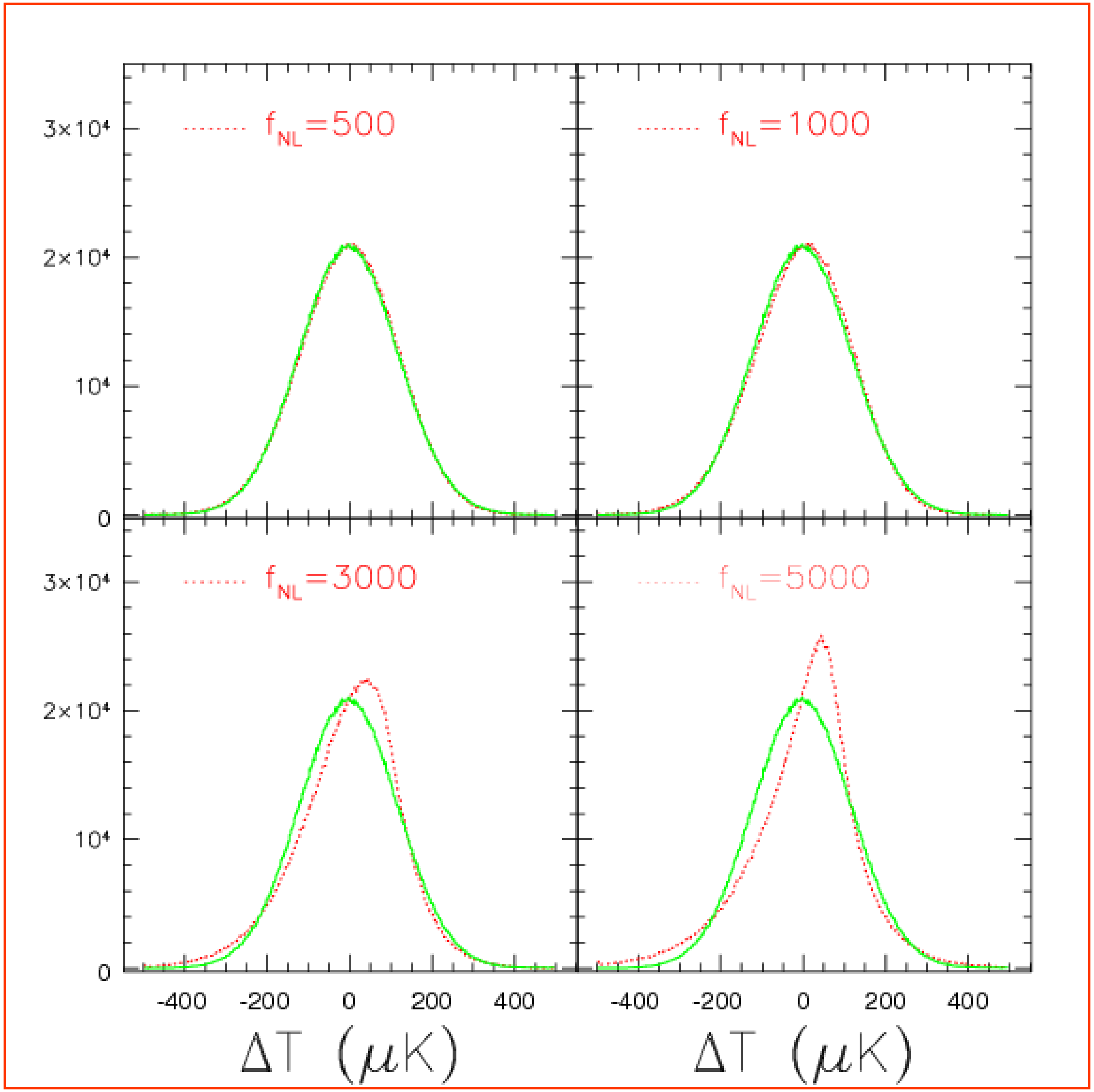



note: $f_{NL}$ < 100 implies Gaussian to better than 0.1%

# evidence for local non-Gaussianity?

- $\Delta T/T \approx -\Phi/3$, so positive $f_{NL} \Rightarrow$ more cold spots in CMB

- various groups have attempted to measure this with the WMAP CMB data using estimators based on matched filtering (all 95% CL) :

| | | |
|---|---|---|
| $27 < f_{NL} < 147$ | Yadav & Wandelt | WMAP3 data |
| $-9 < f_{NL} < 111$ | Komatsu et al | WMAP5 |
| $-4 < f_{NL} < 80$ | Smith et al. | Optimal WMAP5 |
| $-10 < f_{NL} < 74$ | Komatsu et al | WMAP7 |

- Large scale structure observations have recently given independent indications due to non-local bias on large scales (Dalal et al 2007):
  - $-29 < f_{NL} < 70$ (95% CL) Slosar et al 2008
  - $27 < f_{NL} < 117$ (95% CL) Xia et al 2010 [NVSS survey of AGNs]

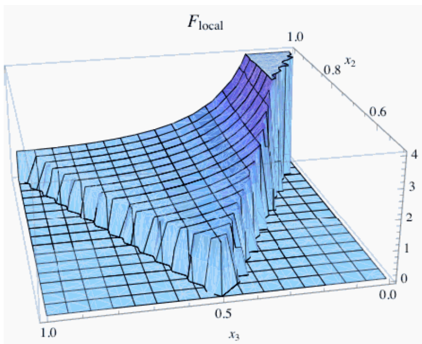
# non-Gaussianity from inflation

inflation

| | Source of non-Gaussianity | Bispectrum type |
|---|---|---|
| Initial vacuum | Excited state | Folded? |
| Sub-Hubble evolution | Higher-derivative interactions<br>e.g. k-inflation, DBI, ghost | Equilateral<br>(+orthogonal?) |
| Hubble-exit | Features in potential | |
| Super-Hubble evolution | Self-interactions+gravity | Local |
| End of inflation | Tachyonic instability | Local |
| (p)Reheating | Modulated (p)reheating | Local |
| After inflation | Curvaton decay | Local |
| **primordial non-Gaussianity** | | |
| Radiation + matter<br>+ last-scattering | Primary anisotropies | Local+equilateral |
| ISW/lensing | Secondary anisotropies | Local+equilateral |

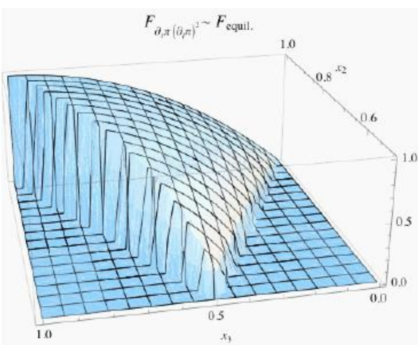
# templates for primordial bispectra

$$P_\zeta(k) = \mathcal{P}(k)/k^3, \quad B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}(k_1,k_2,k_3)\big(P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\big)$$



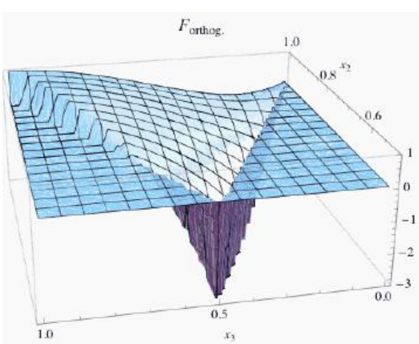

- local type (Komatsu&Spergel 2001)
  - local in real space (fNL=constant)
  - max for squeezed triangles: k<<k',k''

$$B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}^{local}\left(\mathcal{P}(k_1)\right)^2 \left(\frac{1}{k_1^3k_2^3} + \frac{1}{k_2^3k_3^3} + \frac{1}{k_3^3k_1^3}\right)$$




- equilateral type (Creminelli et al 2005)
  - peaks for k1~k2~k3

$$B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}^{equil}\left(\mathcal{P}(k_1)\right)^2 \left(\frac{3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{k_1^3k_2^3k_3^3}\right)$$

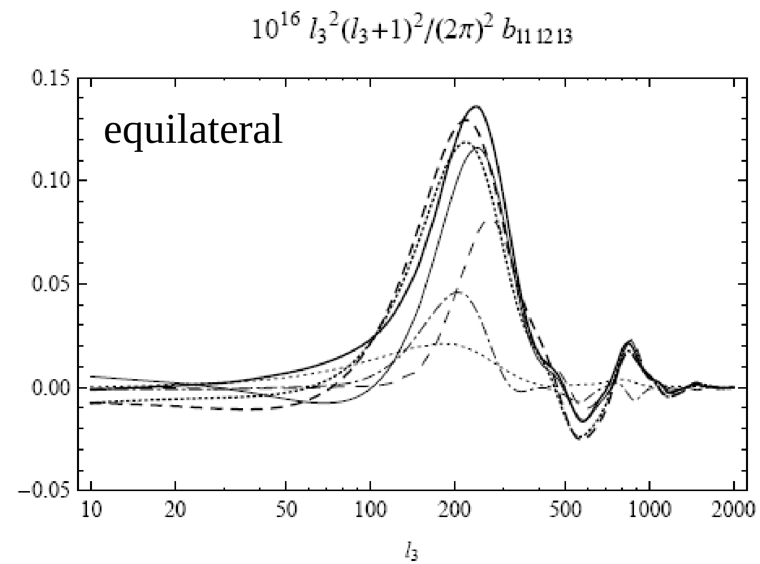


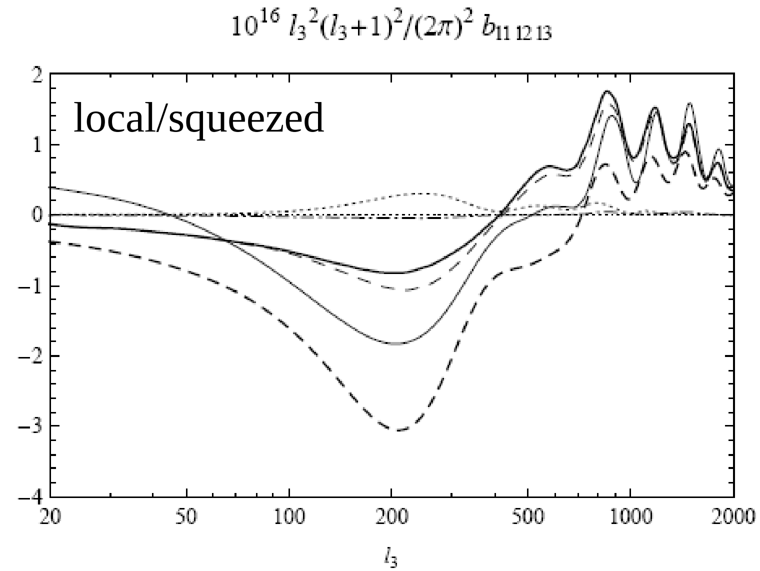

- orthogonal type (Senatore et al 2009)

$$B_\zeta(k_1,k_2,k_3) = (6/5)\, f_{NL}^{orthog}\left(\mathcal{P}(k_1)\right)^2 \left(\frac{81}{k_1k_2k_3(k_1+k_2+k_3)^3}\right)$$

# non-linear Boltzmann-equation

- Pitrou, Uzan & Bernardeau (2010)
  - equilateral + local contribution to bispectrum
  - contributes effective fNL≈5



# ESA Planck satellite launched!


next all-sky survey

data end 2012

$r \approx 0.1$?

$\mathbf{f_{NL} < 10}$

# summary:

- linear perturbations (quantum fluctuations) of free field(s) during inflation provide excellent fit to current data
- non-linear perturbations offer distinctive observational signatures of physics of the very early universe