On the infrared behaviour of QCD Green functions in the Landau and the Maximally Abelian gauge

Functional methods: scaling and decoupling solutions for *n*-point functions

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U. of Sussex, Nov. 17, 2011

Outline

Motivation

CONFINEMENT

implies

- a non–perturbative RG invariant confinement scale

$$\Lambda = \mu \exp\left(-\int^{g} \frac{dg'}{\beta(g')}\right) \stackrel{g\to 0}{\to} \mu \exp\left(-\frac{1}{2\beta_0 g^2}\right)$$

- infrared singularities $\Longleftrightarrow$ continuum approach

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Confinement mechanism(s) for **Gluons** & **Quarks?**

Minimal requirement: No colored asymptotic physical states!

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Covariant Gauge Theory and BRST quartet mechanism

Gauge theory: Unphysical degrees of freedom!

QED: Physical states obey Lorentz condition.

$$\partial_\mu A^\mu |\Psi\rangle = 0 \quad (\mathrm{Gupta-Bleuler}).$$

$\Rightarrow$ Two physical massless photons.

Time-like photon (i.e. negative norm state!) cancels

longitudinal photon in S–matrix elements!

Time–like photon “confines” longitudinal photon!

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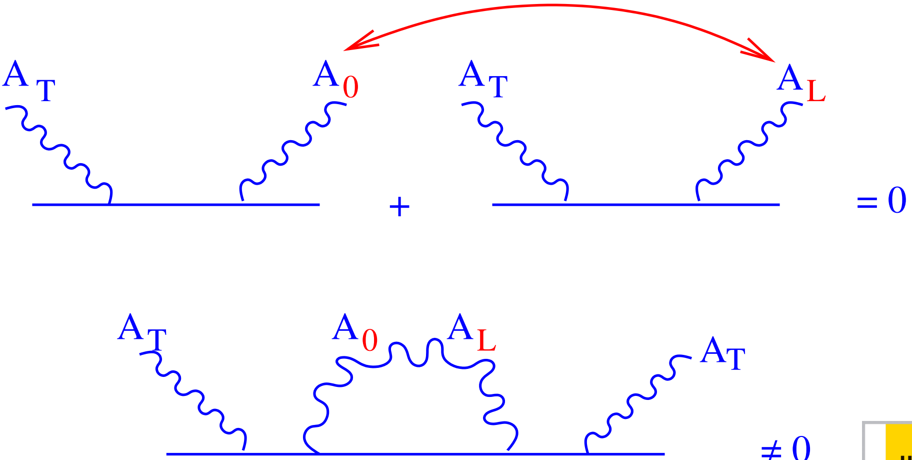
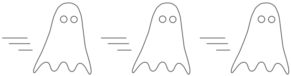
In S-Matrix



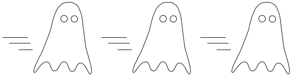
Covariant Gauge Theory and BRST quartet mechanism

Quantization of QCD:

Selfinteraction of gluons,
cancelation process much more complicated!



⇒ Faddeev–Popov ghosts = anticomm. scalar fields.



Ghosts are **unphysical**
(anti-commuting scalar)
Yang–Mills degrees of freedom!
Important in quantum fluct., but no associated particles!

Global ghost field as ‘gauge parameter‘:
BRST symmetry of the gauge-fixed action!

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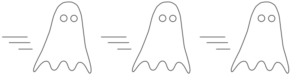

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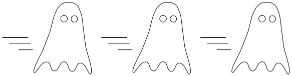

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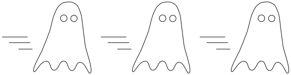

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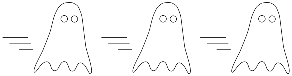

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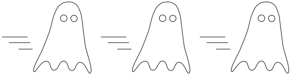

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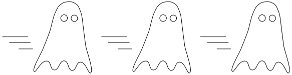

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Symmetry of the gauge-fixed generating functional:

$$\delta_B A_\mu^a = D_\mu^{ab} c^b \lambda \,, \qquad \delta_B q = -igt^a c^a q \lambda \,,$$
$$\delta_B c^a = -\frac{g}{2} f^{abc} c^b c^c \lambda \,, \qquad \delta_B \bar{c}^a = \frac{1}{\xi} \partial_\mu A_\mu^a \lambda \,,$$

Becchi–Rouet–Stora & Tyutin (BRST), 1975

- Parameter $\lambda \in$ Grassmann algebra of the ghost fields
- λ carries ghost number $N_{\text{FP}} = -1$
- Via Noether theorem: BRST charge operator Q_B
- generates ghost # graded algebra $\delta_B \Phi = \{iQ_B, \Phi\}$

Covariant Gauge Theory and BRST quartet mechanism

BRST algebra: $Q_B^2 = 0$, $[iQ_c, Q_B] = Q_B$,

- complete in indefinite metric state space $\mathcal{V}$.
- generates ghost # graded $\delta_B \Phi = \{iQ_B, \Phi\}$.
- $\mathcal{L}_{GF} = \delta_B \left(\bar{c}\left(\partial_\mu A^\mu + \frac{\alpha}{2} B\right)\right)$ BRST exact.

Positive definite subspace $\mathcal{V}_{\rm pos} = {\rm Ker}(Q_B)$
(*i.e.* all states $|\psi\rangle \in \mathcal{V}$ with $Q_B|\psi\rangle = 0$)
contains ${\rm Im} Q_B$ (*i.e.* all states $Q_B|\phi\rangle$),
c.f. exterior derivative in differential geometry.

Hilbert space: cohomology $\mathcal{H} = \frac{{\rm Ker} Q_B}{{\rm Im} Q_B} \simeq \mathcal{V}_s$ BRST singlet

longitudinal & timelike gluons, ghosts : elementary BRST quartet

(c.f. Gupta–Bleuler mechanism in QED)

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Kugo–Ojima confinement criterion

$\Rightarrow$ Physical states are BRST singlets!
(BRST cohomology: Hilbert space $\mathcal{H} = \frac{\mathrm{Ker}\, Q_{BRST}}{\mathrm{Im}\, Q_{BRST}}$.)

Time–like and longitudinal gluons (in elementary BRST quartet) removed from asymptotic states as in QED, but:

Transverse gluons and quarks also members of BRST quartets, *i.e.* confined,

if **ghost propagator is highly infrared singular!**
($\Rightarrow$ **Kugo–Ojima confinement criterion**)

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Kugo–Ojima confinement criterion

Realization of Confinement depends on global gauge structure:
Globally conserved current ($\partial^\mu J^a_\mu = 0$)

$$J^a_\mu = \partial^\nu F^a_{\mu\nu} + \{Q_B, D^{ab}_\mu \bar{c}^b\}$$

with charge $$Q^a = G^a \quad + \quad N^a.$$

QED: MASSLESS PHOTON states in both terms.
Two different combinations yield:
unbroken global charge $\tilde{Q}^a = G^a + \xi N^a$.
spont. broken displacements (photons as Goldstone bosons).

No massless gauge bosons in $\partial^\nu F^a_{\mu\nu}$: $G^a \equiv 0$.
(QCD, e.w. Higgs phase, ...)

Kugo–Ojima confinement criterion

QCD: Well-defined (in $\mathcal{V}$) unbroken global charge

$$Q^a = N^a = \{Q_B, \int d^3x\, D_0^{ab}\bar{c}^b\}$$

With $D_\mu^{ab}\bar{c}^b(x) \overset{x^0\to\pm\infty}{\longrightarrow} (\delta^{ab} + u^{ab})\partial_\mu\bar{\gamma}^b + \ldots$

$\Rightarrow$ Kugo-Ojima Confinement Criterion: $u^{ab}(0) = -\delta^{ab}$

where

$$\int dx e^{ip(x-y)}\langle 0|T\, D_\mu c^a(x) g(A_\nu \times \bar{c})^b(y)|0\rangle =: (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2),$$

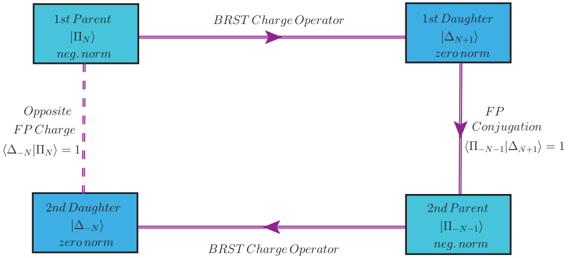
Sufficient condition in Landau gauge:

Ghost propagator more sing. than simple pole!

If fulfilled: **Physical States $\equiv$ BRST singlets $\equiv$ color singlets!**

Kugo–Ojima confinement criterion

Non-perturbative BRST quartets of transverse gluons, resp., quarks:

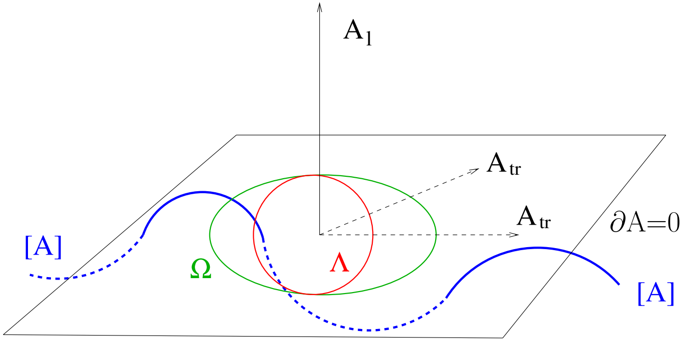


$\|\Pi_0\rangle$	transverse gluons	quarks
$\|\Delta_1\rangle$	gluon-ghost bound states	quark-ghost bound states
$\|\Pi_{-1}\rangle$	gluon-antighost bound states	quark-antighost bound states
$\|\Delta_0\rangle$	gluon-ghost-antigh./gluonic b.s.	quark-gh.-antigh./quark-gluon

N. Alkofer and R.A., Phys. Lett. B **702** (2011) 158 [arXiv:1102.2753 [hep-th]];
PoS **FACESQCD** (2011) 043 [arXiv:1102.3119 [hep-th]].

Gribov horizon & Zwanziger condition

Gauge fixing in YM theories never completely unique:



Λ topologically non-trivial as complete config. space also is!

Landau gauge: $\Gamma = \{A : \partial \cdot A = 0\}$

Minimal Landau gauge: $\Omega = \{A : ||A||^2 \; minimal\}$

First Gribov region: $\Omega = \{A : \partial \cdot A = 0, \partial \cdot D(A) \geq 0\}$

Fundam. Modular Region: $\Lambda = \{A : global\ extrema\}$

NO GRIBOV COPIES

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Relevant configuration space: $\Lambda / SU(N_c)$
Gribov: Cut off integral at boundary $\partial\Omega$
Zwanziger: Ambiguities resolved due to additional
IR boundary condition on ghost prop.

$$\lim_{k^2 \to 0} \left(k^2 D_{\text{Ghost}}(k^2)\right)^{-1} = 0.$$

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Kugo-Ojima vs. Gribov-Zwanziger

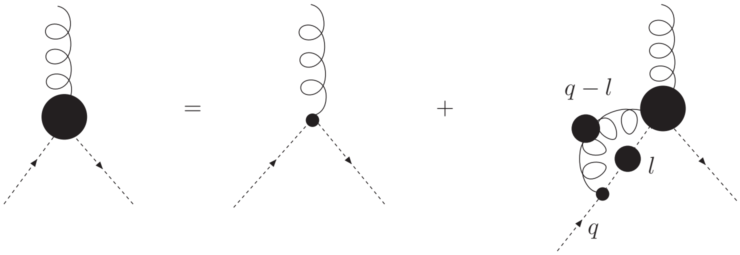
- In Landau gauge: Kugo-Ojima and Gribov-Zwanziger lead to practically same infrared constraints.
- Results in positivity violation for transverse gluons: non-pert. realization of Oehme–Zimmermann superconvergence relation (antiscreening contradicts positivity of gluon spectral density).
  R. Oehme and W. Zimmermann, Phys. Rev. **D21** (1980) 471.

- Kugo-Ojima requires Lorentz-covariant gauge,
  but fails *e.g.* also in the Lorentz-cov. Maximally Abelian gauge.
- Gribov-Zwanziger applies to Landau and Coulomb gauge (where Λ is compact and convex),
  but *e.g.* not to Maximally Abelian gauge (where Gribov region is unbounded).

- Recently: generalized ("refined") Gribov-Zwanziger scheme which results in infrared finite Landau gauge Green functions,
  D. Dudal, S. Sorella, N. Vandersickel, *et al.* .

Infrared Structure of Landau gauge Yang-Mills theory

- Starting point in gauges with transverse gluon propagator: Ghost-Gluon-Vertex fulfills Dyson-Schwinger equation

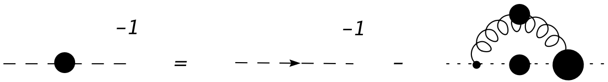


- Transversality of gluon ⇒ Bare Vertex for $q_\mu \to 0$
- No anomalous dimensions in the IR

J. C. Taylor, Nucl. Phys. B **33** (1971) 436.
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.
A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004).
W. Schleifenbaum, A. Maas, J. Wambach and R. A., Phys.Rev.D72 (2005) 014017.

Infrared Exponents for Gluons and Ghosts

- Dyson-Schwinger equation (DSE) for the ghost-propagator:



<u>Ansatz</u> for Gluon, $Z(p^2) \sim (p^2)^\alpha$,
and Ghost Ren. Fct., $G(p^2) \sim (p^2)^\beta$.

- Selfconsistency $\Rightarrow$ $-\beta = \alpha + \beta =: \kappa$ i.e.

$$Z(p^2) \sim (p^2)^{2\kappa}\,, \quad G(p^2) \sim (p^2)^{-\kappa}$$

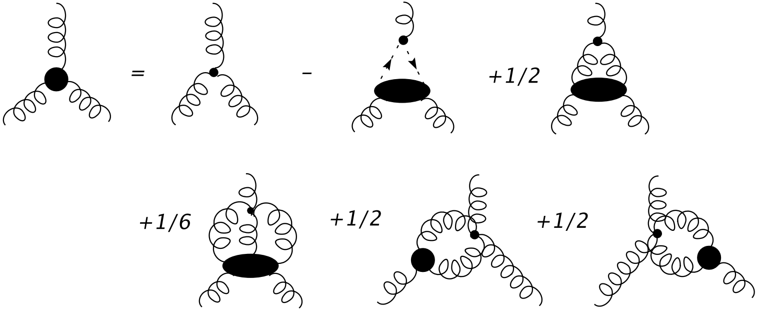
L. v. Smekal, A. Hauck, R. A., Phys. Rev. Lett. **79** (1997) 3591

- IR enhanced ghost propagator: $0.5 \leq \kappa < 1$
Kugo–Ojima confinement criterion,
Oehme–Zimmermann superconvergence relation,
and Gribov–Zwanziger horizon condition fulfilled!
P. Watson and R.A., Phys. Rev. Lett. **86** (2001) 5239

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Example: DSE for 3-gluon-vertex



Use DSEs and ERGEs:

→ Two different towers of equations for Green functions

E.g. ghost propagator

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MATHEMATICA:

R. A., M. Q. Huber, K. Schwenzer, Comp. Phys. Comm. **180** (2009) 965 [arXiv:0808.2939 [hep-th]]

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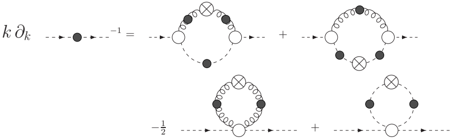

IR-Analysis of whole tower of equations $\Rightarrow$

Solution unique [C.S. Fischer and J.M. Pawlowski, PRD **80** (2009) 025023]

except a solution with IR trivial Green functions

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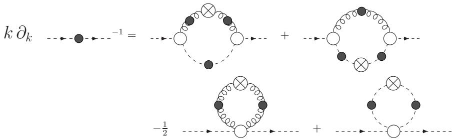

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Infrared Exponents for Gluons and Ghosts:

Scaling vs. decoupling solution:

- Lattice calculations for gluon propagator favor decoupling solution.
- Scaling solution respects BRST,
  decoupling solution breaks BRST.
- Strong coupling lattice calculations: two different IR exponents, contradicts decoupling, resp., massive gluon.
  (L.v. Smekal, A. Sternbeck, arXiv:0811.4300 [hep-lat];
  A. Cucchieri, T. Mendes, Phys. Rev. **D80** (2010) 016005.)
- IR behaviour depends on non-perturbative completion of gauge.
  (A. Maas, Phys. Lett. **B689** (2010) 107.)

General Infrared Exponents for Gluons and Ghosts

Scaling solution:
n external ghost & antighost legs and m external gluon legs
(one external scale p^2; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Ghost propagator IR divergent
- Gluon propagator IR suppressed
- Ghost-Gluon vertex IR finite
- 3- & 4- Gluon vertex IR divergent

★ IR fixed point for the coupling from each vertex
★ Conformal nature of Infrared Yang-Mills theory!
★ Ghost sector of YM-theory dominates IR!
D. Zwanziger, Phys. Rev. D **69** (2004) 016002

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Yang-Mills Running Coupling: Infrared fixed point

Ghost-Gluon-Vertex UV finite:

$$\alpha_S(\mu^2) = \frac{g^2(\mu^2)}{4\pi} = \frac{1}{4\pi\beta_0} g_0^2 Z(\mu^2) G^2(\mu^2)$$

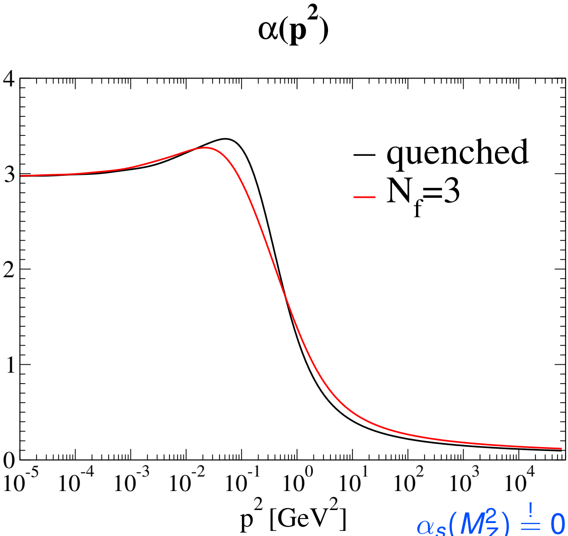
With known IR behavior of gluon (Z) and ghost (G) function:

IR fixed point

$$\alpha_c = \alpha_S(k^2 \to 0) \simeq 2.972^*$$

$^*\alpha_S(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)}$

Running Coupling



$\alpha_s(M_Z^2) \stackrel{!}{=} 0.118$

YM Running Coupling: IR fixed point

$$G(p^2) \sim (p^2)^{-\kappa} , \quad Z(p^2) \sim (p^2)^{2\kappa}$$
$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa} , \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$

$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2) \sim \frac{const_{gh-gl}}{N_c}$$

$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2) \sim \frac{const_{3g}}{N_c}$$

$$\alpha^{4g}(p^2) = \alpha_\mu \, [\Gamma^{4g}(p^2)]^2 \, Z^4(p^2) \sim \frac{const_{4g}}{N_c}$$

Running Coupling

- These constants are DIFFERENT!
- The coupling from the Quark-Gluon vertex will show a qualitatively different behaviour!

There is NO universal QCD coupling in the infrared domain of QCD!

Positivity violation for the gluon propagator

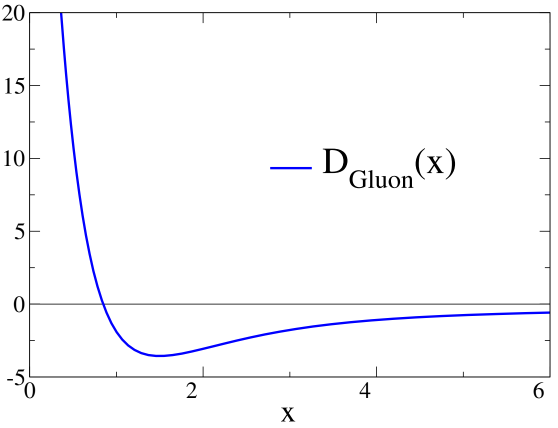
Simple argument [Zwanziger]:
IR vanishing gluon propagator implies

$$0 = D_{gluon}(k^2 = 0) = \int d^4x D_{gluon}(x)$$

$\Longrightarrow$ $D_{gluon}(x)$ has to be negative for some values of x.

Positivity violation for the gluon propagator

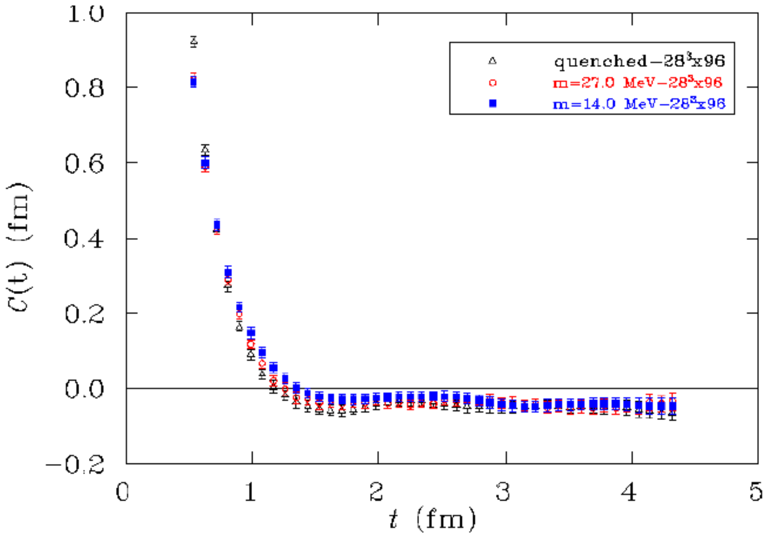
Fourier transform of DSE result:



Gluons unobservable $\Longrightarrow$ **Gluon Confinement!**

Positivity violation for the gluon propagator

Lattice (P. Bowman et al., Phys.Rev.**D76** (2007) 094505):



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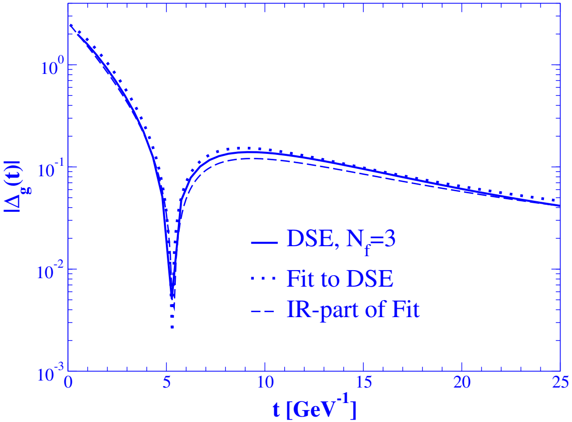

R.A., W. Detmold, C.S. Fischer and P. Maris, PR**D70** (2004) 014014

Positivity violation for the gluon propagator

$$D^{\rm fit}_{gluon}(p^2) = w \frac{1}{p^2} \left(\frac{p^2}{\Lambda^2_{\rm QCD} + p^2} \right)^{2\kappa} \left(\alpha_{\rm fit}(p^2) \right)^{-\gamma}$$

- IR part: cut for $-\Lambda^2_{\rm QCD} < p^2 < 0$
- $D^{\rm fit}_{gluon}$: cut along negative, i.e. timelike, half-axis!

  Wick rotation possible!

- w arbitrary normalization parameter
- $\kappa = \frac{93-\sqrt{1201}}{98}$ fixed from IR analysis
- $\gamma = \frac{-13N_c+4N_f}{22N_c-4N_f}$ from perturbation theory
- **Effectively one parameter**[†]**: $\Lambda_{\rm QCD}$=520 MeV!**

[†] from fits to lattice data: $\Lambda_{\rm QCD} \approx 380$ MeV

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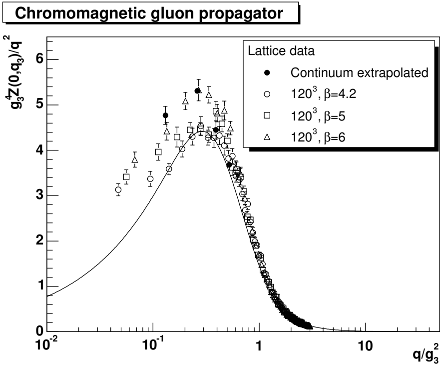
Partial gluon confinement at any T

Gluon propagator at high T:

A. Maas, J. Wambach, RA, EPJ **C37** (2004) 335; **C42** (2005) 93.

A. Cucchieri, A. Maas and T. Mendes, PR **D75** (2007) 076003.



Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation

Partial gluon confinement at any T

Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation at any T:

No infrared singularities, *c.f.* Linde (1980),
because no chromomagnetic mass of type $\omega_m(\vec{k}=0) = m_m(T)$!

K. Lichtenegger, D. Zwanziger, Phys. Rev. D **78** (2008) 034038.

No surprise:

- three-dimensional YM theory confining
- area law for spatial Wilson loop
- Coulomb string tension $\neq 0$ at any T

Static chromomagnetic sector is never deconfined!

Picturing Gluon Confinement

DSE scaling solution of Yang-Mills theory:

- Gluon propagator vanishes on the light cone, and
- *n*-point gluon vertex functions diverge on the light cone!

⇒ Attempts to kick a gluon free (*i.e.* to produce a real gluon) immediately results in production of infinitely many virtual soft gluons!

⇒ perfect color charge screening
+ positivity violation (which implies BRST quartet cancelation):

Gluon confinement!

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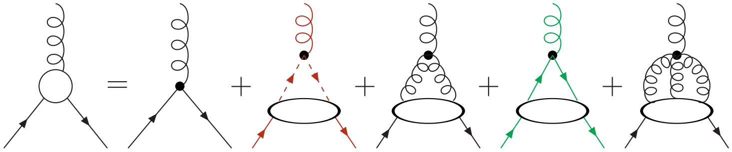
Dynamically induced scalar quark confinement

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals Phys. **324** (2009) 106.

Quark-gluon vertex:



Quark diagram: Hadronic contributions ('unquenching')

Ghost diagram: **Infrared leading!**

Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitely broken:
quark propagator infrared finite

$$S(p) = \frac{\not p + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not p}{M^2} + \frac{Z_f}{M}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{12} \lambda_i G^i_\mu \,, \quad G^1_\mu = \gamma_\mu \,, \quad G^2_\mu = \hat{p}_\mu \,, \quad G^3_\mu = \ldots$$

WITH $\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa}$

INFRARED DIVERGENT

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INFRARED DIVERGENT

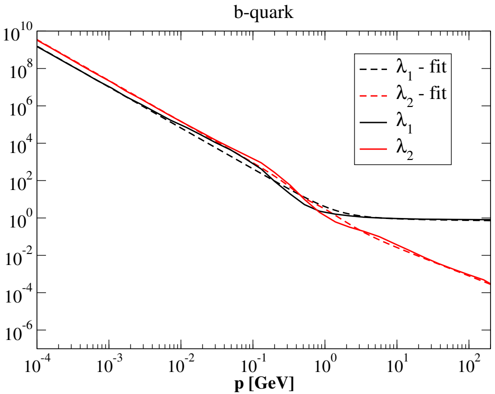
Dynamically induced scalar quark confinement

Chiral symmetry dynamically or explicitely broken:

$$\lambda_{1,2,\ldots} \sim (p^2)^{-1/2-\kappa} \text{ i.e.}$$

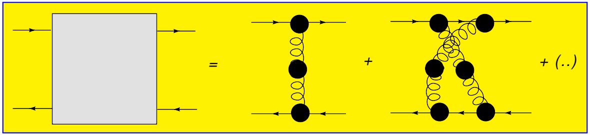
Quark-Gluon vertex IR divergent!

Scalar component λ_2 in IR even **larger** than vector component λ_1!



Dynamically induced scalar quark confinement

“Quenched´´ quark-antiquark potential



infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{pr}} \quad \sim \quad |\mathbf{r}|$$

i.e. linear, dominantly scalar, quark confinement!

Dynamically induced scalar quark confinement

Chiral symmetry artificially enforced:

$$S(p) = \left(\frac{\not p + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \right)_{M \to 0} \to \frac{Z_f \not p}{p^2}$$

AND

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Dynamically induced scalar quark confinement

Quark-antiquark potential: No confinement

$$\Gamma^{0,0,2}(p^2) \sim const.$$

$$V(\mathbf{r}) \sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} e^{i\mathbf{pr}} \quad \sim \quad \frac{\mathbf{1}}{|\mathbf{r}|}$$

Picturing quark confinement

DSE scaling solution for quark sector:

- quark propagator IR trivial (DχSB),
- quark-gluon vertex functions including a self-consistently generated scalar quark-gluon coupling (DχSB!) diverge on the quark "mass´´ shell!

$\Rightarrow$ Attempts to kick a quark free (*i.e.* to produce a real quark) immediately results in production of infinitely many virtual soft gluons!

$\Rightarrow$ linearly rising potential
i.e., infrared slavery:

Quark confinement!

String formation? Properties of confining field configuration? ...? ...?

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η' mass from IR divergent Green functions

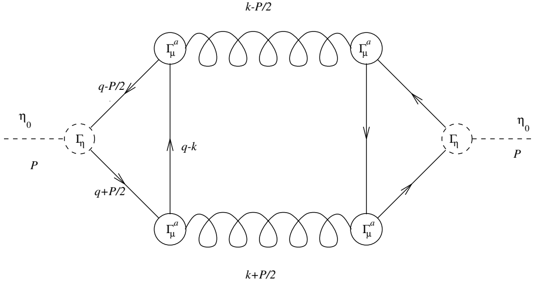
R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

$U_A(1)$ symmetry anomalous $\Rightarrow \eta'$ mass $\gg \pi$ mass

Where is this encoded in the Green functions?

J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

E.g. in:



$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

R.A., C. S. Fischer, R. Williams, Eur. Phys. J. A **38** (2008) 53.

$U_A(1)$ symmetry anomalous $\Rightarrow \eta'$ mass $\gg \pi$ mass

QCD vacuum: winding number spots as, *e.g.*, instantons, couple

to chiral quark zero modes $\Rightarrow U_A(1)$ symmetry broken!

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J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

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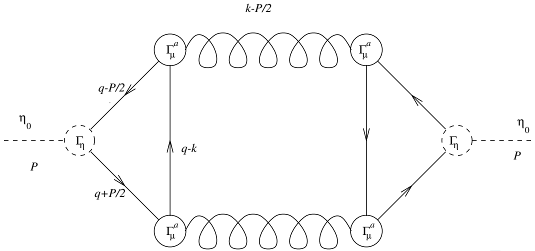

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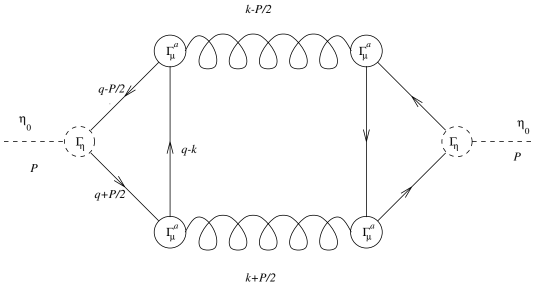

$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

η' mass from IR divergent Green functions

However: Infinitely many diagrams (n-gluon exchange) contribute!

Nevertheless:
Calculate contribution from **diamond diagram only** employing DSE results for the gluon and quark propagators and quark-gluon vertex (provides correct pseudoscalar and vector meson masses):

$\chi^2 \approx (160\mathrm{MeV})^4$ vs. phenomenological value $(180\mathrm{MeV})^4$

results in: $m_\eta = 479$MeV, $m_{\eta'} = 906$MeV, $\theta = -23^0$.

Conclusion:
(Fluct.) topologically non-trivial fields $\Leftrightarrow$ IR singularities of GF!
...another view to generate the Witten-Venezanio mechanism ...

Motivation: Different view on Confinement

Chromomagnetic Monopoles in Max. Abelian Gauge

- Dual superconductor picture of Confinement
- String formation
  - Casimir scaling at intermediate distances
  - N-ality at large distances
- Chiral symmetry dynamically broken by quark zero modes
- Topological susceptibility: $U_A(1)$ anomaly
- Area law for spatial Wilson loop at any temperature

Picturing Gluon and Quark Confinement
Chromomagnetic Monopoles in Max. Abelian Gauge

- screening currents of chromomagnetic monopoles
- $\longrightarrow$ chromoelectric fields cannot penetrate monopole vacuum
  $\longrightarrow$ chromoelectric fields squeezed to flux tubes
  (as magnetic field in type-II-superconductors by superconducting electric currents)
- string formation & linearly rising potential

Gluon & Quark confinement!

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Maximally IR Divergent Solution and Scaling Relations

M.Q. Huber, K. Schwenzer and R.A., Eur. Phys. J. C **68** (2010) 581 [arXiv:0904.1873];
PoS **FACESQCD** (2011) 001 [arXiv:1103.0236]

M.Q. Huber, V. Mader, A. Windisch and R.A., in preparation

Maximally IR Divergent Solution and Scaling Relations

Note: The IR exponent of at least one diagram must equal the IR exponent of the vertex function on the l.h.s.

BUT: Analysis diagram by diagram???

Arbitrary Diagram v

Numbers of vertices and propagators related
$\Rightarrow$ formula for the IR exponent by pure combinatorics.

Function of:

- propagator IR exponents δ_{x_i}
- number of external legs m^{x_i}
- number of vertices.

$$\delta_v = -\frac{1}{2}\sum_i m^{X_i}\delta_{X_i} + \sum_i (\text{\# of dressed vertices})_i C_1^i + \sum_i (\text{\# of bare vertices})_i C_2^i$$

depends only on external legs $\Rightarrow$ equal for all diagrams in a DSE/RGE.

Maximally IR Divergent Solution and Scaling Relations

Use:

dressed vertices	$C_1^i = \delta_{vertex} + \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{vertex}}} \delta_j \geq 0$	from RGEs
prim. divergent vertices	$C_2^i = \frac{1}{2} \sum_{\substack{\text{legs } j \text{ of} \\ \text{prim. div.} \\ \text{vertex}}} \delta_j \geq 0$	from DSEs/RGEs

- restrictive inequalities from RGEs/DSEs
- lower bound on IR exponents
- propagator DSEs: at least one inequality has to be saturated
- scaling relations as e.g. $\delta_{gl} = -2\delta_{gh}$ in Landau gauge

Agrees with (same formula with different arguments):

C.S. Fischer, J.M. Pawlowski, PRD 80 (2009) 025023 [arXiv:0903.2193 [hep-th]]

The Maximally Abelian Gauge

The Maximally Abelian Gauge minimizes the off-diagonal gluon field!

1. Identify Abelian subalgebra $[T^i, T^j] = 0$ (diagonal matrices)
   e.g., $T^1 = \frac{1}{2}\lambda^3$ and $T^2 = \frac{1}{2}\lambda^8$ in SU(3)
2. Split the gauge field$A_\mu = A_\mu^r T^r, \quad r = 1, \ldots, N^2 - 1$:
   Abelian/Diagonal and non-Abelian/off-diagonal fields

$$A_\mu = A_\mu^i T^i + B_\mu^a T^a, \quad i = 1, \ldots, N-1, \quad a = N, \ldots, N^2 - 1$$

3. Gauge fixing:

$$D_\mu^{rb} B_\mu^b = 0 \qquad D_\mu^{rb} \text{ cov. deriv. w.r.t. diag. gluon}$$

$$\partial_\mu A_\mu^i = 0 \qquad \text{[for lattice optional]}$$

Hypothesis of Abelian dominance: Abelian part dominates IR?

Ezawa, Iwazaki, PRD 25 (1982) 2681

The Maximally Abelian Gauge

- Non-linear gauge fixing condition
  depends on diag. gluon A, add. t.l. vertices $A\bar{c}c$, $A\bar{c}c$, $BB\bar{c}c$
- Yang-Mills vertices split into ABB, $AABB$, $BBBB$, etc.
- Renormalizability: quartic ghost interaction
- two gauge fixing parameters: $\alpha_A = 0$ (Landau gauge), α_B.

Note difference in interaction terms between $SU(2)$ and $SU(N > 2)$!

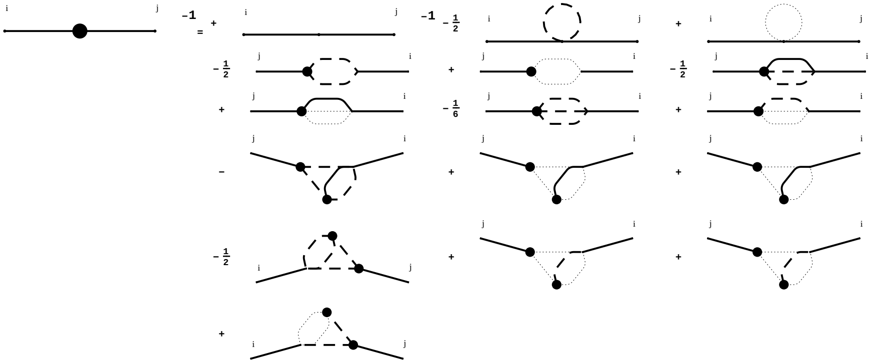
▶ Number of diagrams in DSEs/RGEs is **large**!

Employ DoDSE / DoFun package in MATHEMATICA!

(R.A., M.Q. Huber, K. Schwenzer, CPC **180** (2009) 965 [arXiv:0808.2939];
M.Q. Huber, J. Braun, arXiv:1102:5307)

The Maximally Abelian Gauge

E.g. DoDSE output for diagonal gluon propagator:



R.A., M.Q. Huber, K. Schwenzer, CPC **180** (2009) 965 [arXiv:0808.2939];
M.Q. Huber, J. Braun, arXiv:1102:5307

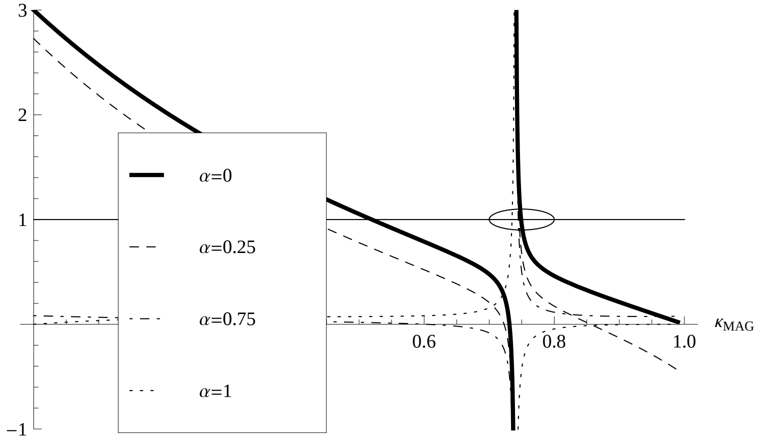
IR Scaling Solution for the Maximally Abelian Gauge

Employ the machinery of the IR power counting technique ...:

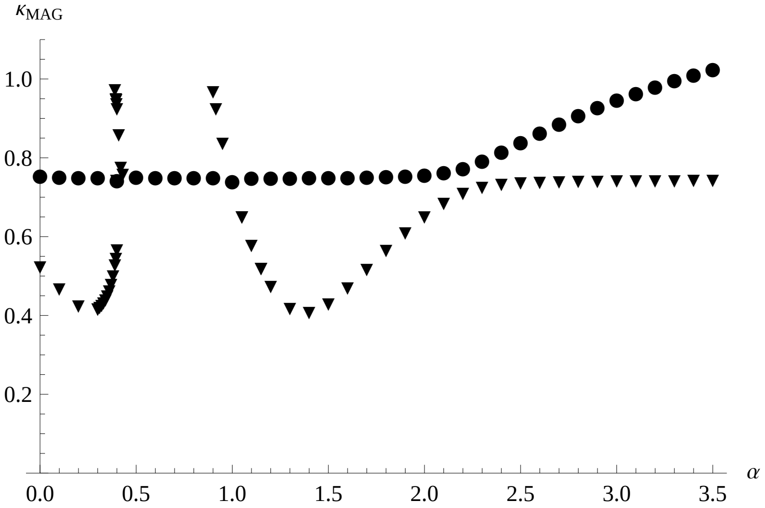
- Dichotomy of decoupling vs. scaling exists also in the MAG!
- If scaling 2-loop diagrams IR leading
- IR leading terms for $SU(2)$ and $SU(N > 2)$ identical
- $\kappa_{\mathrm{MAG}} := -\delta_A = \delta_B = \delta_c \geq 0$
- IRE$= \frac{1}{2}(n_A - n_B - n_c)\kappa_{\mathrm{MAG}}$ for n_A even
- IRE$= \frac{1}{2}(n_A - n_B - n_c + \frac{1}{2} \pm \frac{1}{2})\kappa_{\mathrm{MAG}}$ for n_A odd
- Note $\delta_A \leq 0$!!! **Diagonal gluon propagator IR enhanced!**

IR Scaling Solution for the Maximally Abelian Gauge

$\kappa_{MAG} \approx 0.75$:



IR Scaling Solution for the Maximally Abelian Gauge



Connection between decoupling solutions and the one scaling solution?

- Born term (non-)cancelation via renormalization of diagonal gluon propagator decisive.
- If cancelation then IR dvgt. renormalization function, i.e. scaling solution.
- If Born term then IR finite propagator, i.e. one of the decoupling solutions:
  If any of the three propagators is IR finite ("massive") then the other two have to be massive too due to the tadpole diagrams.

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Towards a numerical solution . . .

2-loop diagrams dominate!

In scaling solution IR leading, for decoupling numerically large.

- DSE truncation with sunset diagram included.
- Overlapping divgcs.: MiniMOM (as in Landau gauge) impossible! Dimensional regularization / renormalization to expensive!
- BPHZ renormalization pert. verified and semi-pert. tested.
- IR exponents reproducable.
- Ready for numerical solution of truncated DSE system . . .

Conclusions and Outlook

Landau gauge QCD Green functions:

- Gluons confined by ghosts: Positivity violated!
  Gluons removed from S–matrix!
- Infrared-finite strong running coupling in Yang-Mills theory!
  Conformal Nature of Infrared Yang-Mills theory!
- Analytic structure of gluon propagator:
  effectively one parameter!
- Positivity violation at any temperature!
- Chiral symmetry dynamically broken! In 2- and **3**-point function!
- Quark confinement: In IR dominantly scalar!
- η' mass generated ($U_A(1)$ anomaly)

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  (Kugo–Ojima Confinement,
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  Gribov–Zwanziger horizon condition, . . .)
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Maximally Abelian Gauge:

- Potential scaling solution: IR exponents of and scaling relations for all *n*-point functions in maximally Abelian gauge
- Same IR behaviour of SU(N>2) and SU(2)
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Color algebra undistorted & direct coupling to IR enhanced diagonal gluon . . .

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