Evaporating black holes in the presence of a minimal length

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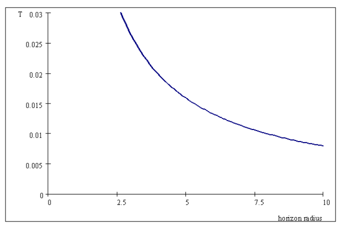
Black hole evaporation

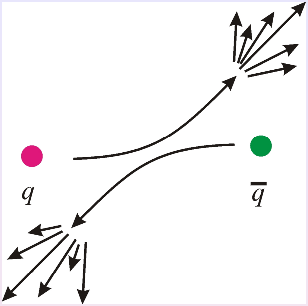
(Mini) Black hole life

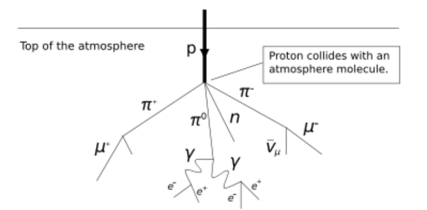


T_H vs r_H

- Balding phase
- Spin down phase
- Schwarzschild phase $T_H \sim 1/r_H$
- Planck phase (?)

(Mini) Black holes @ LHC & Cosmic ray showers





Black Hole Spacetimes

Problem

- Curvature Singularity
- Divergent temperature at the evaporation endpoint
- Ill defined thermodynamics
- Breakdown of General Relativity at short scales

Solution

- We must invoke Quantum Gravity
- Viable approaches
  - String Theory induced Noncommutative Geometry
  - Generalized Uncertainty Principle
  - Loop Quantum Gravity
  - Asymptotically Safe Gravity

Noncommutative geometry

New uncertainty principle

- $$[\,\mathbf{x}^{\mu}\ ,\mathbf{x}^{\nu}\,] = i\,\theta^{\mu\nu} \Longrightarrow \Delta x^{\mu}\Delta x^{\nu} \sim \ell^2 \equiv ||\theta^{\mu\nu}|| \qquad (1)$$

Quasi-classical source terms

Delocalization of source terms within an effective minimal length ℓ

$$\delta(\vec{x}) \to \rho_{\ell}(\vec{x}^2) = \frac{1}{(4\pi\ell^2)^{3/2}}\,\exp\left(-\frac{\vec{x}^2}{4\ell^2}\right)$$

Modified gravity field equations

The energy-momentum tensor delocalization

$$T_0^{\ 0} = -M\ \rho_\ell(\ \vec{x}^2\ )$$
$$T^\mu{}_\nu = \mathrm{Diag}\left(-M\rho_\ell\ , p_r\ , p_\perp\ , p_\perp\right)$$
$$T^{\mu\nu}\,;\,\nu = 0$$

GR

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NCGR

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$$\ell \neq 0 \downarrow \uparrow \ell \to 0$$

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The Schwarzshild Geometry in the presence of ℓ

Einstein/fluid equations

- $$ds^2 = -e^{2\Phi(r)}\,(1 - 2m(r)/r)\;dt^2 + \frac{dr^2}{1 - 2m(r)/r} + r^2\,d\Omega^2 \tag{2}$$

- $$\frac{dm}{dr} = 4\pi\, r^2 \rho\ , \tag{3}$$
$$\frac{1}{2g_{00}}\frac{dg_{00}}{dr} = \frac{m(r) + 4\pi\ r^3 p_r}{r(r - 2m(r))}\ , \tag{4}$$
$$\frac{dp_r}{dr} = -\frac{1}{2g_{00}}\frac{dg_{00}}{dr}\,(\rho + p_r) + \frac{2}{r}\,(p_\perp - p_r) \tag{5}$$

- $$p_r = -\rho_\ell \tag{6}$$

The Schwarzshild Geometry in the presence of ℓ

The solution

- $(G_N = 1,\ c = 1)$

$$ds^2 = \left(1 - \frac{4M}{r\sqrt{\pi}}\,\gamma \right) dt^2 - \left(1 - \frac{4M}{r\sqrt{\pi}}\,\gamma \right)^{-1} dr^2 - r^2\, d\Omega^2 \tag{7}$$

- $\gamma \equiv \gamma\left(3/2\ , r^2/4\ell^2\right)$ is the lower incomplete Gamma function:

$$\gamma\left(3/2\ , r^2/4\ell^2\right) \equiv \int_0^{r^2/4\ell^2} dt\, t^{1/2} e^{-t} \tag{8}$$

The Schwarzshild Geometry in the presence of ℓ

The horizon equation $g_{00}(r_H) = -g_{rr}^{-1}(r_H) = 0$

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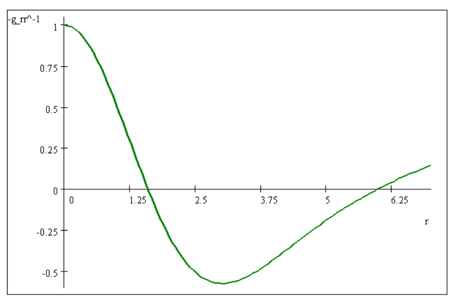

$-g_{rr}^{-1}$ vs r, for various values of M/ℓ.

$M = 3\,\ell \Rightarrow$ two horizons;

$M = \ell \Rightarrow$ ...;

$M = 1.9\,\ell \Rightarrow$ ...;

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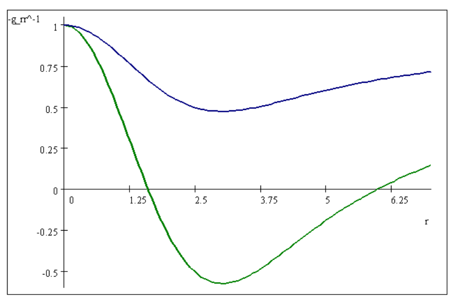

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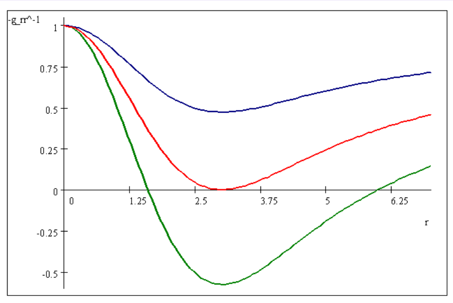


$-g_{rr}^{-1}$ vs r, for various values of M/ℓ.

$M = 3\,\ell \Rightarrow$ two horizons;

$M = \ell \Rightarrow$ no horizon;

$M = 1.9\,\ell \Rightarrow$ one degenerate horizon $r_0 \approx 3.0\,\ell$, extremal BH.

The Schwarzshild Geometry in the presence of ℓ

At the black hole centre

- *The Ricci scalar near the origin is*

$$R\,(\,0\,) = \frac{4M}{\sqrt{\pi}\,\ell^3} \tag{9}$$

- *The curvature is constant and positive (deSitter geometry)*
- *If $M < M_0$ $\Rightarrow$ no BH and no naked singularity (mini-gravastar?)*

Large mass regime, $M \gg M_0$

- inner horizon $\longrightarrow$ origin
- outer horizon $\longrightarrow$ $2M$

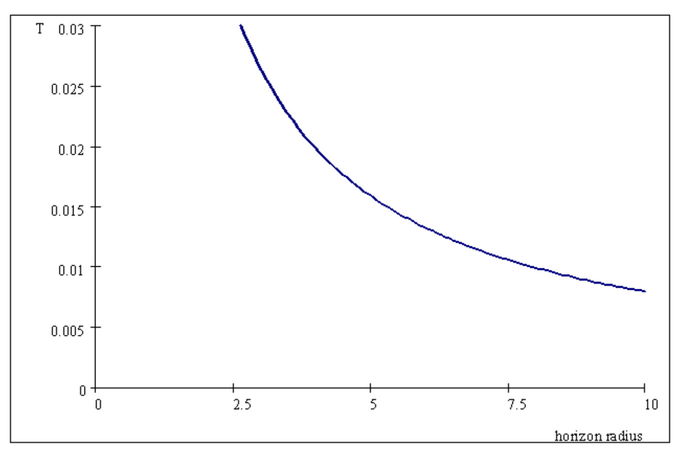
The Schwarzshild Geometry in the presence of ℓ

The Hawking temperature

$$T_H = \frac{1}{4\pi\, r_H}\left[1 - \frac{r_H^3}{4\,\ell^3}\,\frac{e^{-r_H^2/4\ell^2}}{\gamma\left(\,3/2\,;\, r_H^2/4\ell^2\right)}\right] \qquad (10)$$

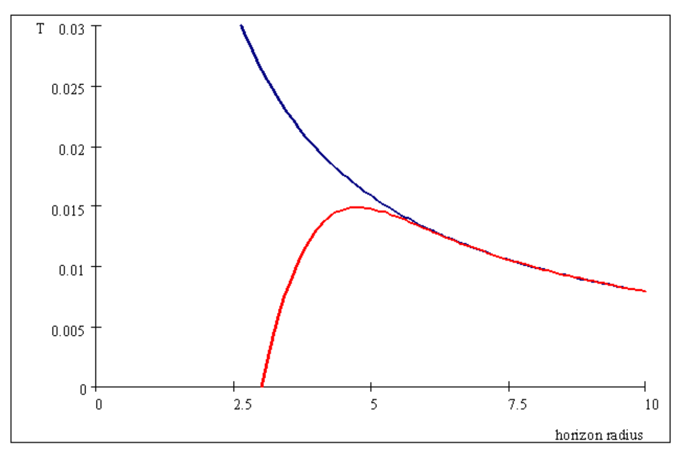
- *If* $r_H^2/4\ell^2 >> 1 \Rightarrow T_H = \frac{1}{4\pi\, r_H}$ *coincides with the Hawking result*
- *If* $r_H \simeq \ell \Rightarrow T_H$ *reaches a maximum* $\simeq 0.015 \times 1/\ell$ *corresponds to a mass* $M \simeq 2.4 \times \ell$ *and* $r_H \simeq 4.7\ell$
- **SCRAM phase**: *cooling down to absolute zero at* $r_H = r_0 = 3.0\ell$ *and* $M = M_0 = 1.9\,\ell$, *the extremal BH*
- *If* $r < r_0$ *there is no black hole.*

The Schwarzshild Geometry in the presence of ℓ



T_H vs r_H for the commutative case

The Schwarzshild Geometry in the presence of ℓ



T_H vs r_H for the commutative and **NC** case.

The Schwarzshild Geometry in the presence of ℓ

Back reaction

- *relevant back-reaction in Planck phase.*
- *SCRAM phase* $\Rightarrow$ *a suppression of quantum back-reaction*
- *At maximum temperature, the thermal energy is* $E = T_H^{Max} \simeq 0.015\,/\ell$, *while the mass is* $M \simeq 2.4\,\ell\, M_P^2$
- $E \sim M \Rightarrow \ell \approx 0.2\, L_P \sim 10^{-34}\, cm.$
- *For this reason we can safely use unmodified form of the metric during all the evaporation process.*

The Dirty solution

- TOV equation

$$\frac{dp_r}{dr} = -\frac{m(r) + 4\pi\ r^3 p_r}{r(r - 2m(r))}\ (\,\rho + p_r\,) + \frac{2}{r}\,(\,p_\perp - p_r\,) \qquad (11)$$

1. p_r and $p_\perp$ must be asymptotically vanishing;
2. p_r and $p_\perp$ must be finite at the horizon(s);
3. p_r and $p_\perp$ must be finite at the origin.

-

$$\rho(r) + p_r(r) \equiv -\ell\ (\,1 - 2m/r)\,\frac{d\rho}{dr} = \frac{1}{2\ell}\,r\,\rho\,(\,1 - 2m/r) \quad (12)$$

-

$$ds^2 \;=\; -e^{2\Phi(\,r\,)}\,(\,1 - 2m\,(\,r\,)\,/\,r\,)\;dt^2 + \frac{dr^2}{1 - 2m\,(\,r\,)\,/\,r} + r^2\,d\Omega^2$$

$$2\Phi\,(\,r\,) = -\frac{MG}{\ell}\left(1 - \frac{2}{\sqrt{\pi}}\gamma\,(\,3/2\,,\,r^2/4\ell^2\,)\right) \qquad (13)$$

The Wormhole solution

- TOV equation

$$\frac{dp_r}{dr} = -\frac{m(r) + 4\pi\ r^3 p_r}{r(r-2m(r))}\ (\rho + p_r) + \frac{2}{r}(p_\perp - p_r) \qquad (14)$$

- $$p_r = -\frac{1}{4\pi r^3} m(r)\,. \qquad (15)$$

- $$ds^2 = -dt^2 + \frac{dr^2}{1 - 4M\gamma(3/2\,; r^2/4\ell^2)/\sqrt{\pi} r} + r^2 d\Omega^2 \qquad (16)$$

The Reissner-Nordström geometry in the presence of ℓ

Properties of the charged source term

- the charge is diffused throughout a region of linear size ℓ

$$\rho_{el.}(r) = \frac{e}{(4\pi\ell^2)^{3/2}} \exp\left(-r^2/4\ell^2\right) \tag{17}$$

  a "point-like object" when a minimal length is considered.

- We find the electric field to be:

$$E(r) = \frac{2Q}{\sqrt{\pi}\, r^2}\, \gamma\left(\frac{3}{2}\, ; \frac{r^2}{4\ell^2}\right) \tag{18}$$

- $F^{\mu\nu} = \delta^{0[\mu|}\delta^{r|\nu]}\, E(r) \Rightarrow T_{el.}{}^{\mu}_{\nu}$

The Reissner-Nordström geometry in the presence of ℓ

The solution

- $ds^2 = g_{00}\, dt^2 - g_{00}^{-1}\, dr^2 - r^2 d\Omega^2$ with

$$g_{00} = 1 - \frac{4M}{r\sqrt{\pi}}\gamma + \frac{Q^2}{\pi\, r^2}\left[\, F\,(\, r\,) + \sqrt{2}\,\frac{r}{\ell}\,\gamma\,\right] \qquad (19)$$

- $M = \oint_\Sigma d\sigma^\mu \left(\, T^0_\mu|_{matt.} + T^0_\mu|_{el.}\,\right)$ where, Σ, is a $t = \mathrm{const.}$, closed three-surface.
- $F(r) \equiv \gamma^2\left(\,1/2\;, r^2/4\ell^2\,\right) - \frac{r}{\sqrt{2}\ell}\gamma\left(\,1/2\;, r^2/2\ell^2\,\right)$

The Reissner-Nordström geometry in the presence of ℓ

The asymptotic behaviors

- small $r \Rightarrow F(r) \sim O\left(r^6\right)$
- again the "singularity" is cured by the vacuum fluctuation of the spacetime fabric

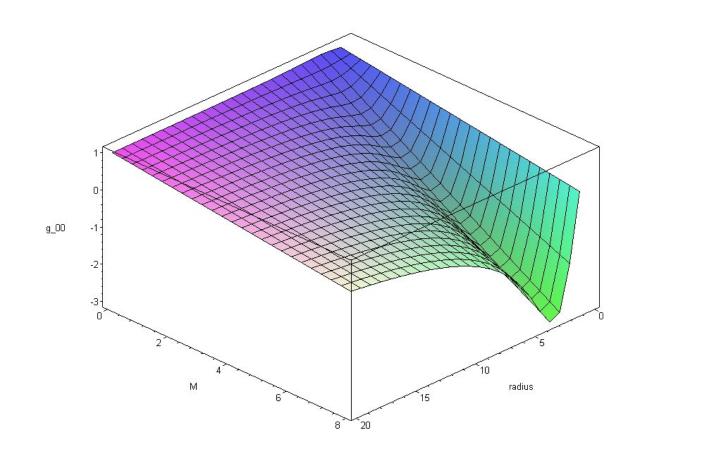
$$g_{00} = 1 - \frac{m_0}{3\sqrt{\pi}\,\ell^3}\, r^2 + O\left(r^4\right) \tag{20}$$

  where m_0 is "*bare mass*" only.

- at large distance, the asymptotic observer measures the *total mass-energy* M and the electric field in the usual way

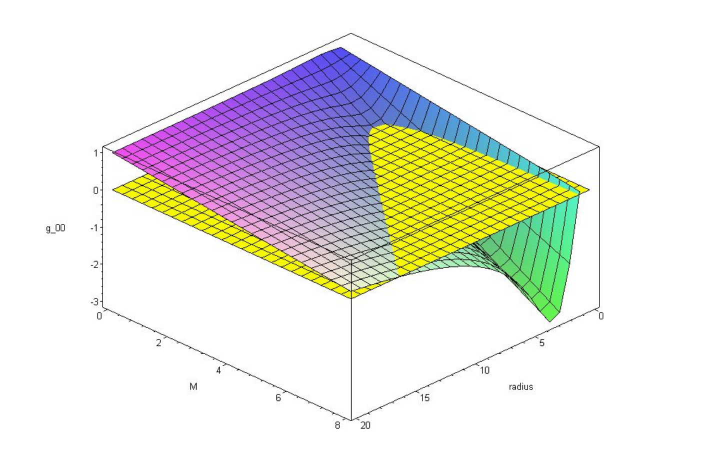
$$g_{00} = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \tag{21}$$

The Reissner-Nordström geometry in the presence of ℓ



g_{00} vs r and M for a charge, $Q = 1$ in ℓ units.

The Reissner-Nordström geometry in the presence of ℓ



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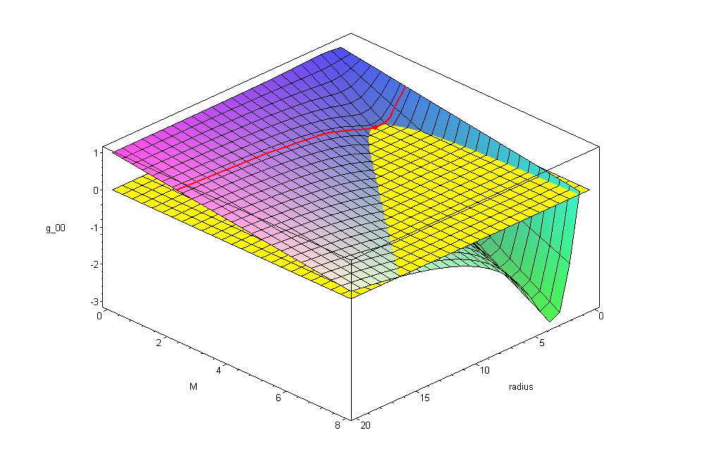

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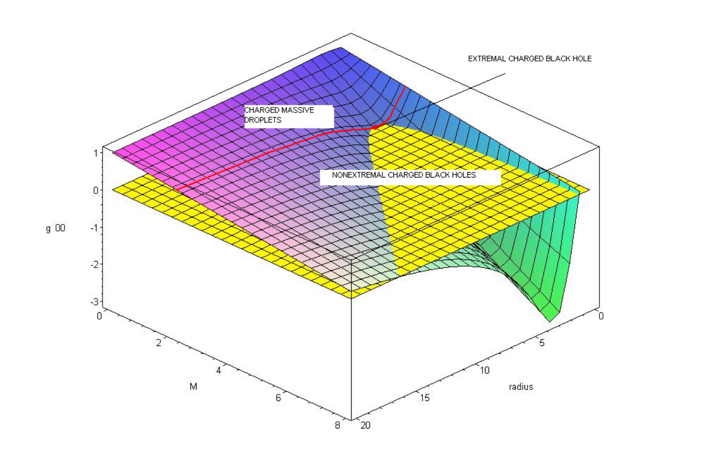


g_{00} vs r and M for a charge, $Q = 1$ in ℓ units.

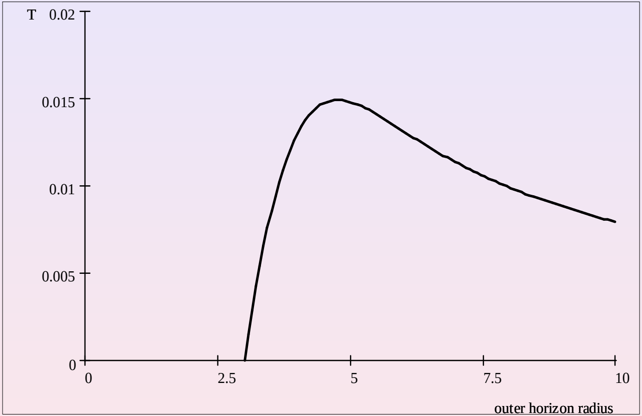
The Reissner-Nordström geometry in the presence of ℓ

The Hawking temperature

$$4\pi\, T_H = \frac{1}{r_+}\left[1 - \frac{r_+^3 \exp(-r_+^2/4\ell^2)}{4\ell^3\gamma\left(3/2\,,\, r_+^2/4\ell^2\right)}\right] +$$
$$-\frac{4Q^2}{\pi r_+^3}\left[\gamma^2\left(3/2\,,\, r_+^2/4\ell^2\right) + \frac{r_+^3 \exp(-r_+^2/4\ell^2)}{16\,\ell^3\gamma\left(3/2\,,\, r_+^2/4\ell^2\right)} F(r_+)\right]$$

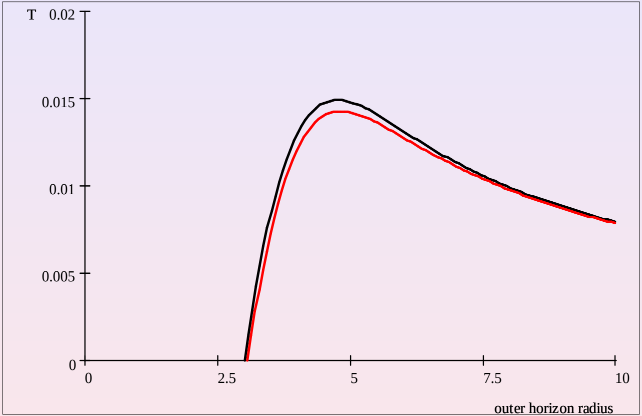
- again instead of growing indefinitely temperature reaches a maximum value and then drops to zero at the extremal BH
- the effect of charge is just to lower the maximum temperature.

The Reissner-Nordström geometry in the presence of ℓ



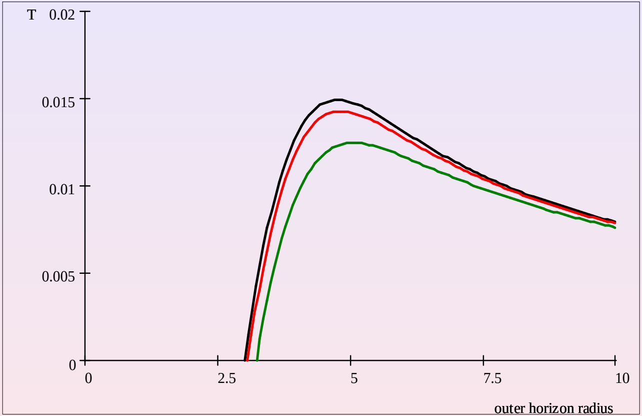
T_H vs r_H for $Q = 0$

The Reissner-Nordström geometry in the presence of ℓ



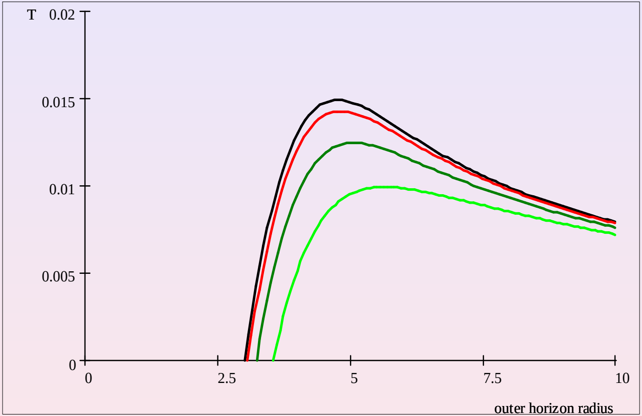
T_H vs r_H for $Q = 0, 1$

The Reissner-Nordström geometry in the presence of ℓ



T_H vs r_H for $Q = 0,1,2$

The Reissner-Nordström geometry in the presence of ℓ



T_H vs r_H for $Q = 0,1,2,3$

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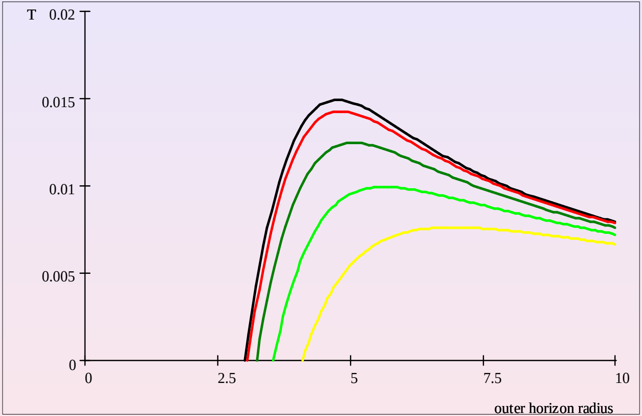

T_H vs r_H for $Q = 0, 1, 2, 3, 4$

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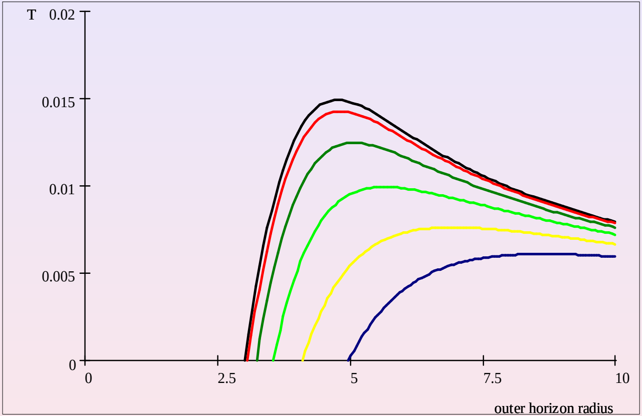

T_H vs r_H for $Q = 0, 1, 2, 3, 4, 5$

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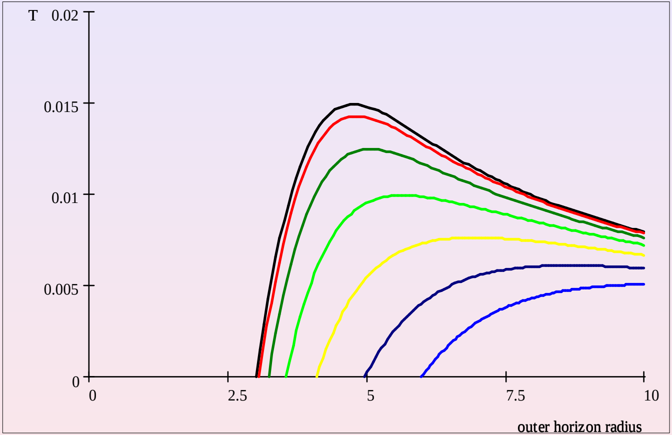
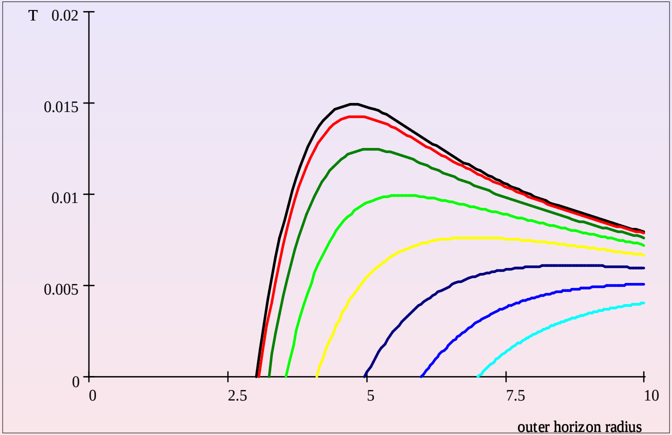


T_H vs r_H for $Q = 0,1,2,3,4,5,6$

The Reissner-Nordström geometry in the presence of ℓ



T_H vs r_H for $Q = 0, 1, 2, 3, 4, 5, 6, 7$

The Reissner-Nordström geometry in the presence of ℓ

Schwinger effect

- $w = \frac{e^2 \ E^2}{\pi^2 \hbar^2 c} \exp\left(-\pi \ m^2 c^3 \ /e \ E \ \hbar\right)$
- being e the electric charge and E the electric field.

BH decay

- $E_{horizon} > E_{critical} = \frac{m^2 \ c^3}{e \ \hbar} \Leftrightarrow Z \geq 1$, where $Q = Ze$
- $r_{dyadosphere} \gg \ell$.
- The Schwinger effect dominates the Hawking effect till a neutral phase.

Extradimensional Solutions

- $ds^2_{(m+1)} = g_{00}\, dt^2 - g_{00}^{-1}\, dr^2 - r^2 d\Omega^2_{m-1}$
-
$$g_{00} = 1 - \frac{1}{M_*^{m-1}} \frac{2M}{r^{m-2}\Gamma\left(\frac{m}{2}\right)}\ \gamma\left(\frac{m}{2}\,,\frac{r^2}{4\ell^2}\right) \qquad (22)$$
- Charged case

$$\begin{aligned} g_{00} &= 1 - \frac{1}{M_*^{m-1}} \frac{2M}{r^{m-2}\Gamma\left(\frac{m}{2}\right)}\ \gamma\left(\frac{m}{2}\,,\frac{r^2}{4\ell^2}\right) \\ &+ \quad \frac{4\,\mathcal{Q}^2\,(m-2)}{M_*^{m-1}\pi^{m-3}\,r^{2m-4}}\left[F_m(r) + c_m\left(\frac{r}{\ell}\right)^{m-2}\gamma\left(\frac{m}{2}\,,\frac{r^2}{4\ell^2}\right)\right] \end{aligned}$$

- $F(r) \equiv \gamma^2\left(\frac{m}{2} - 1\,,\frac{r^2}{4\ell^2}\right) - \frac{2^{(8-3m)/2}\,r^{m-2}}{(m-2)\ell^{(m-2)}}\,\gamma\left(\frac{m}{2} - 1\,,\frac{r^2}{2\ell^2}\right)$

Extradimensional Solutions

Properties of the solutions

- Geometric and thermodynamic behavior equivalent to the 4d one.
  - $\Rightarrow$ there exists a mass threshold M_0 below which BH do not form.
  - $\Rightarrow$ there exists a zero temperature black hole remnant

BH remnants

- $1/\ell \sim M_* \sim 1$ TeV
- remnant cross section $\sigma_{BH} \simeq \pi r_0^2 \sim 10$ nb $\longrightarrow$ 10 BHs per second at LHC.

Extradimensional Solutions

Maximum Temperatures for different m in the neutral case

	3	4	5	6	7	8	9	10
T_H^{max} (GeV)	18×10^{16}	30	43	56	67	78	89	98
T_H^{max} $(10^{15} K)$	$.21 \times 10^{16}$	.35	.50	.65	.78	.91	1.0	1.1

Remnant Masses and radii for different m

	3	4	5	6	7	8	9	10
M_0 (TeV)	2.3×10^{16}	6.7	24	94	3.8×10^2	1.6×10^3	7.3×10^3	3.4×10^4
r_0 (10^{-4} fm)	4.88×10^{-16}	5.29	4.95	4.75	4.62	4.52	4.46	4.40

Potential catastrophic risk @ LHC

Black hole life times

- $$\frac{dM}{dt} = -A_H \Phi, \qquad \Phi = 2\int \frac{d^d p}{(2\pi)^d} \frac{e^{-\frac{1}{8}\ell^2 p^2}\, p}{e^{p\beta_d} - 1} \tag{23}$$

Numerical results

- Assuming $M_{in} = 10$ TeV, for both brane and bulk emission

$$t_{\mathrm{decay}} \lesssim 10^{-16}\,\mathrm{sec}\ , \tag{24}$$

for any $d = 3 - 10$.

Summary and Outlook

Black hole solutions in the presence of ℓ

- one, two or no horizon.
- a deSitter core
- The singular behavior of the Hawking temperature is cured.
- SCRAM phase and zero-temperature final state.
- The quantum back-reaction is unimportant
- Neutral, dirty, wormhole, charged, extradimensional cases

Projects

- Spinning (charged) case
- inflationary cosmology w/o inflaton, Primordial BHs, dark matter.
- Unruh/Hawking (matter fields)
- Analog models (BEC, superfluids)

Summary and Outlook

Asymptotic Safety in QG $\leftrightarrow$ NC Geometry

- Running gravitational constant

$$G_{AS}(p) = \frac{G_0}{1 + \alpha G_0 \; p^2} \tag{25}$$

- The black hole

$$g_{00} = 1 - \frac{2G_{AS}(r)M}{r} \tag{26}$$

$$G_{AS}(r) = \frac{G_0 r^3}{r^3 + \tilde{\alpha} G_0 [r + \beta G_0 M]} \tag{27}$$

-

$$G_\ell(r) = G_0 \frac{2\gamma\left(3/2; r^2/4\ell^2\right)}{\sqrt{\pi}} \tag{28}$$

$$G_\ell(p) = G_0 \; e^{-\ell^2 p^2} \tag{29}$$

Summary and Outlook

LQG↔ NC Geometry

- LQBHs
- regular geometry
- zero temperature final state

	NCBHs	LQBHs	ASBHs
curv. sing.	cured	cured	cured*
gravity eqns	Einstein equations	no	no
max temp.	yes	yes	yes
evap. end	BH remnant	two scenarios	BH remnant
charge	yes	no	no
extradim.	yes	no	no
charge + extra	yes	no	no
angular mom.	yes	no	no

*

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