Interacting Fixed-Points in Chiral Yukawa Systems - Critical behaviour and Asymptotic Safety

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Outline

1. Chiral Yukawa systems in d=4
   - Motivation: Triviality and Hierarchy in the standard model
   - Asymptotic safety and the Wetterich equation
   - Truncation for chiral Yukawa systems
   - Fixed-points, critical exponents & predictivity
2. Chiral fermion models in d=3
   - Motivation: quantitative control and statistical systems
   - Four-fermion interactions and truncation
   - Fixed-point mechanisms and results

CHIRAL YUKAWA SYSTEMS IN d=4
with Holger Gies and Stefan Rechenberger

Standard model Higgs sector

- Higgs sector parametrizes masses of matter fields and weak gauge bosons
- will be directly tested at LHC
- Higgs sector plagued by two problems:

triviality & hierarchy problem

Triviality

- Parametrize Higgs field as

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4 + \text{ i.a. with fermions}$$

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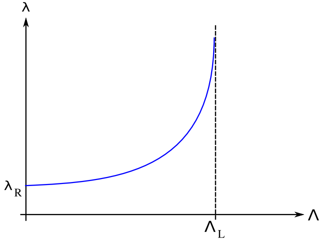

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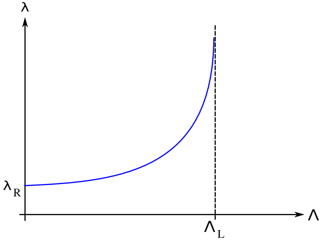

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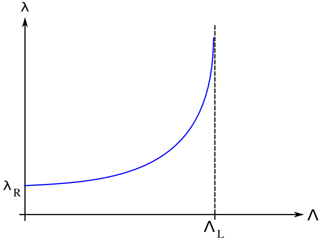

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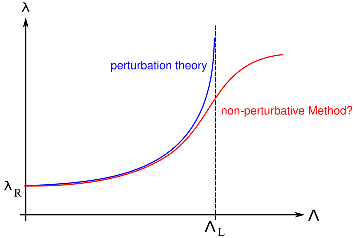

- Landau-pole $\rightarrow$ breakdown of perturbative QFT $\rightarrow$ new d.o.f.?
- Near Λ_L PT looses validity since λ grows large
- Need non-perturbative tool to study triviality & take into account fermions!

Hierarchy problem

Huge hierarchy in SM

$$\Lambda_{\mathrm{EW}} \sim 10^{2}\,\mathrm{GeV} \ll \Lambda_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$$

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$$\underbrace{m_{\mathrm{R}}^2}_{\sim 10^4\,\mathrm{GeV}^2} \sim \underbrace{m_{\Lambda,\mathrm{UV}}^2}_{\sim 10^{32}(X + \ldots 10^{-28})\mathrm{GeV}^2} - \delta m^2$$

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Hierarchy problem corresponds to existence of a large critical exponent $\Theta_I > 0$ at a fixed point (e.g. in ϕ^4-theory at the GFP $\Theta = 2$)

Asymptotic safety

- Triviality and the Hierarchy problem arise in perturbative QFT
- Non-perturbative scenario, which can circumvent these problems:

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- AS scenario mainly discussed in the context of a quantum theory for gravity
- Recently, AS scenario for QG has received strong support thanks to new techniques in non-perturbative quantum field theory
- Very active reasearch on realisation of AS in QG as well as observational consequences (BH, Cosmology, LHC physics,...)

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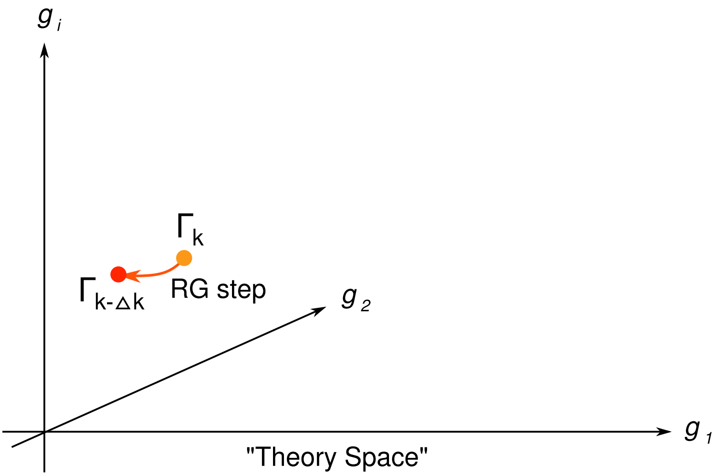
- Recently, AS scenario for QG has received strong support thanks to new techniques in non-perturbative quantum field theory
- Very active reasearch on realisation of AS in QG as well as observational consequences (BH, Cosmology, LHC physics,...)
- Setting of the AS scenario is more general and might also be applied to other QFTs that have problems with non-renormalizability
- As a toy model for the SM we will investigate a class of chiral Yukawa systems

General notion of asymptotic safety

Effective average action: $\Gamma_k[\chi] = \sum_i g_{i,k}\mathcal{O}_i$, Scale dependence: $k\partial_k\Gamma_k[\chi] = \sum_i \beta_{i,k}\mathcal{O}_i$.



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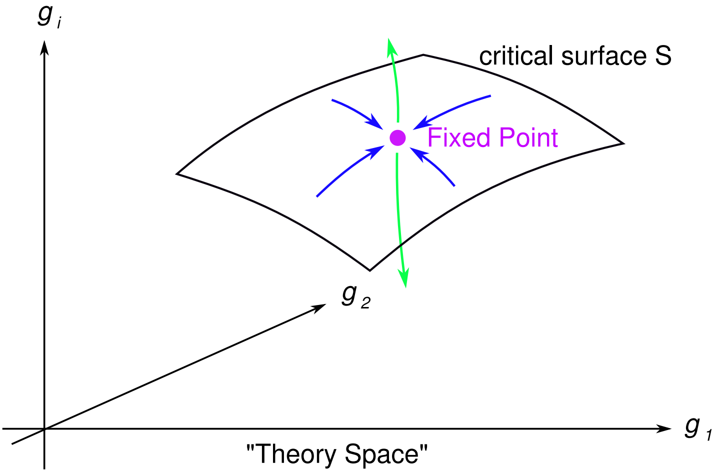

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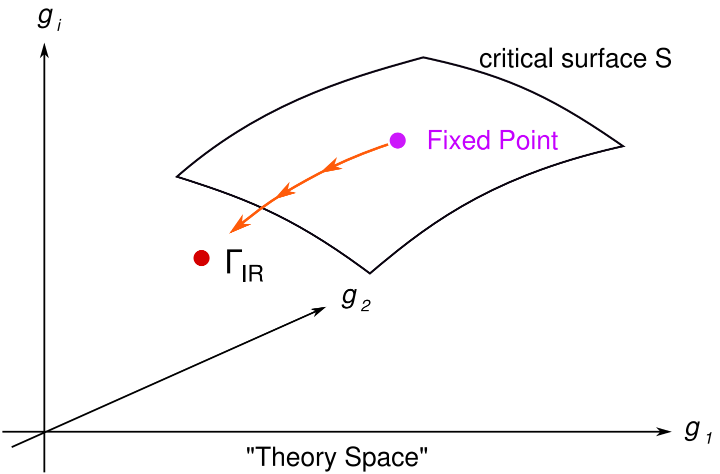
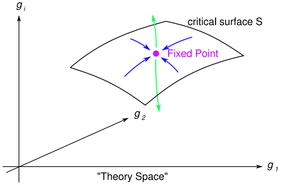


Triviality & hierarchy problem in the asymptotic safety scenario



- If NGFP exists we can draw the limit $\Lambda \to \infty \to$ system independent from UV cutoff $\to$ no triviality problem
- Dimension of the critical surface: $\Delta = \dim\mathcal{S} =$ number of relevant directions
- If $\Delta < \infty \to$ system predictive
- Hierarchy problem $\sim$ large critical exponents $\Theta_I > 0$.
- RG computation will show how large the Θ_I are at a NGFP (GFP $\Theta = 2$)

Flow equation & asymptotic safety

Exact RG equations (ERGE) derived from path-integral representation (Wetterich '93)

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\{[\Gamma_k^{(2)}[\Phi] + R_k]^{-1}(\partial_t R_k)\}, \quad \partial_t = k\frac{d}{dk}$$

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$$\partial_t g_{i,k} = B_i^j(g_{j,k} - g_j^*), \;\; B_i^j = \frac{\partial \beta_i}{\partial g_{j,k}}\Big|_{g^*}$$

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General solution of the linearized fixed-point equation

$$g_{i,k} = g_i^* + \sum_I C_I V_i^I \left(\frac{k_0}{k}\right)^{\Theta_I}$$

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- Re $\Theta_I > 0$: relevant coupling (to be fixed by experiment)
- Re $\Theta_I < 0$: irrelevant coupling (prediction for physical observable in the IR)

Toy model - chiral Yukawa system

Derivative expansion, leading-order truncation

$$\Gamma_k = \int d^4x \Big\{ i(\bar{\psi}_L^a \partial\!\!\!/ \psi_L^a + \bar{\psi}_R \partial\!\!\!/ \psi_R) + (\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a) + U_k(\rho) + \bar{h}_k \bar{\psi}_R \phi^{a\dagger} \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^a \psi_R \Big\}$$

- N_{L} left-handed fermions ψ_{L}^a, 1right-handed fermion ψ_{R}
- N_{L} complex bosons ϕ^a

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- N_L complex bosons ϕ^a
- invariant under chiral $U(N_\mathrm{L})_\mathrm{L} \otimes U(1)_\mathrm{R}$ transformations
- define $\rho = \phi^{a\dagger}\phi^a$
- dimensionless quantities: $\tilde{\rho} = k^{2-d}\rho, \quad h^2 = k^{d-4}\bar{h}_k^2, \quad u(\tilde{\rho}) = k^{-d} U_k(\rho)|_{\rho = k^{d-2}\tilde{\rho}}$

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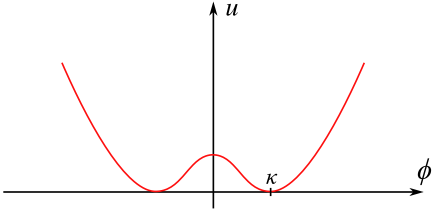
Expand effective potential about minimum $\kappa := \tilde\rho_{\text{min}} > 0$ (SSB)

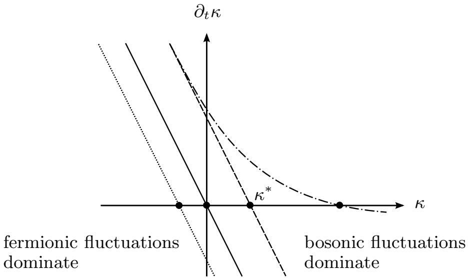
$$u = \frac{\lambda_2}{2!}(\tilde\rho - \kappa)^2 + \frac{\lambda_3}{3!}(\tilde\rho - \kappa)^3 + \ldots$$
$$\kappa, \ \lambda_{n_{\max}}, \ \lambda_2 > 0.$$



Fixed-point mechanism

Loop contributions to the running of κ

$$\partial_t \kappa = -2\kappa + \text{bosonic interactions} - \text{fermionic interactions}$$



- Dominating fluctuations of boson field allow for positive κ^*
- Suitable κ-dependence flattens the β-function near fixed-point (reduces the hierachy problem)
- near FP the vev exhibits a conformal behaviour $v \sim k$

Fixed-point analysis for our toy model

- Whether or not the balancing possible crucially depends on d.o.f. of the model.
- LO truncation can be parametrized by three couplings: h^2, λ, κ.

$$\begin{aligned} \partial_t h^2 &= \beta_h(h^2, \lambda, \kappa) = 0, \\ \partial_t \lambda &= \beta_\lambda(h^2, \lambda, \kappa) = 0. \end{aligned}$$

$\Rightarrow$ obtain a conditional fixed-point

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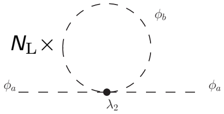
β_κ-function receives the contributions

$\beta_\kappa = -2\kappa + N_{\mathrm{L}} \times$



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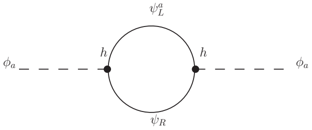
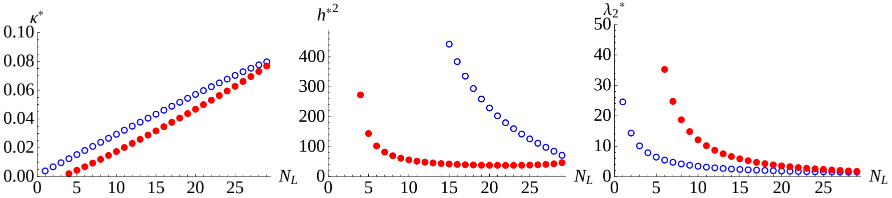


Fixed-points and critical exponents

We find a NGFPs for $1 \leq N_L \leq 29$



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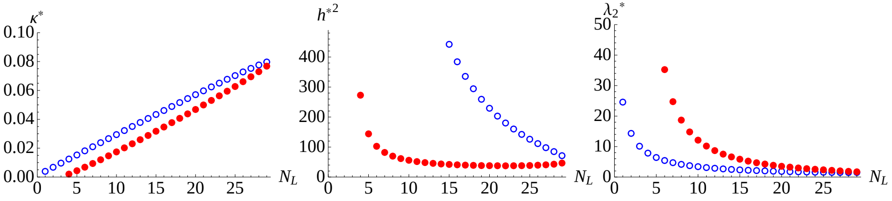
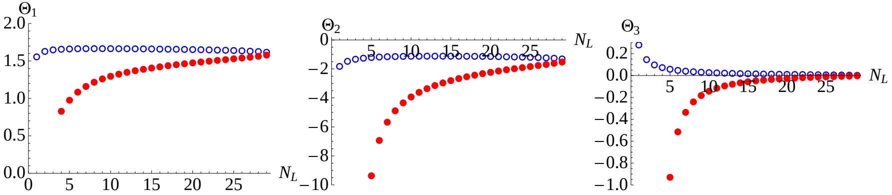
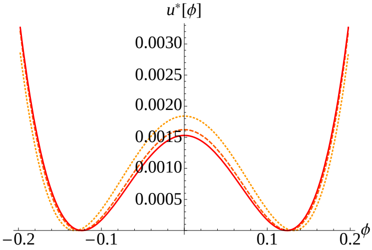


and the CRITICAL EXPONENTS



UV fixed-point regime for $N_{\mathrm{L}} = 10$

- Convergence of the fixed-point potential u^* at LO



$$
\begin{aligned}
\text{FP :} \quad & \kappa^* = 0.0152, \quad \lambda^* = 12.13, \quad h^{*2} = 57.41\,, \\
\text{Critical exponents :} \quad & \Theta_1 = 1.056, \quad \Theta_2 = -0.175, \quad \Theta_3 = -2.350
\end{aligned}
$$

- 1 relevant direction $\longrightarrow$ 1 physical parameter to be fixed
- All other parameters are predictions from the theory
- The real part of the relevant direction is 1.056$\longrightarrow$ Hierarchy problem weaker

(Toy-)Higgs mass and (Toy-)Top mass from asymptotic safety

- Flow is fixed by IR value of κ
- In realistic model this corresponds to the Higgs vev

$$v = \lim_{k\to 0} \sqrt{2\kappa}\, k$$

- IR values of other two parameters are predictions related to the Higgs and the top mass

$$m_{\text{Higgs}} = \sqrt{\lambda_2}\, v, \quad m_{\text{top}} = \sqrt{h^2}\, v.$$

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- Choosing $v = 246\mathrm{GeV}$ and $N_{\mathrm{L}} = 10$ as an example, we find

$$m_{\text{Higgs}} = 0.97 v = 239\mathrm{GeV}, \quad m_{\text{top}} = 5.56 v = 1422\mathrm{GeV}.$$

CHIRAL YUKAWA SYSTEMS IN d=3
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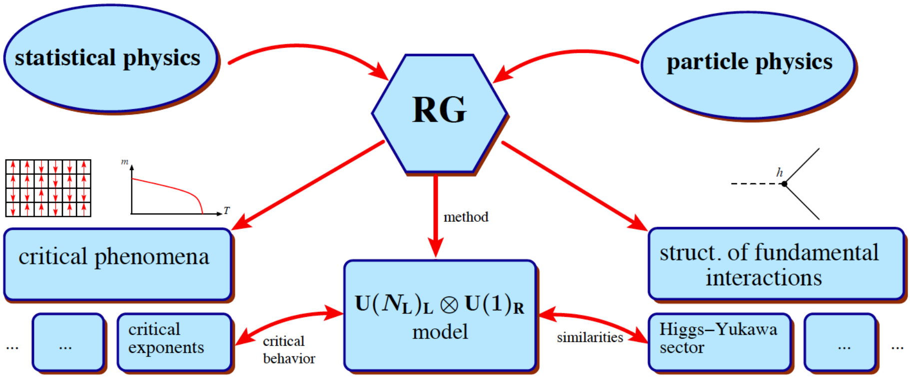
Chiral fermion models in d=3, Motivation

- Understanding EW SSB requires quantitative control of fluctuating chiral fermions and bosons beyond PT
- 3-d chiral fermion models (QED_3, Thirring model): high-T_c cuprates, graphene



Chiral fermion models in d=3, Motivation

Quantitative comparisons between (non-perturbative) field-theoretical models

1. Example: O(N)-models (table from Blaizot *et al.*, Phys. Rev. E80: 030103, 2009)

TABLE I: Coefficient c and critical exponents of the $O(N)$ models for $d = 3$.

N	BMW				Resummed perturbative expansions					Monte-Carlo and high-temperature series				
	η	ν	ω	c	η	ν	ω	c	Ref.[a]	η	ν	ω	c	Ref.[a]
0	0.034	0.589	0.83		0.0284(25)	0.5882(11)	0.812(16)		[17]	0.030(3)	0.5872(5)	0.88		[18][19]
1	0.039	0.632	0.78	1.15	0.0335(25)	0.6304(13)	0.799(11)	1.07(10)	[17][14]	0.0368(2)	0.6302(1)	0.821(5)	1.09(9)	[20][21]
2	0.041	0.674	0.75	1.37	0.0354(25)	0.6703(15)	0.789(11)	1.27(10)	[17][14]	0.0381(2)	0.6717(1)	0.785(20)	1.32(2)	[22][23]
3	0.040	0.715	0.73	1.50	0.0355(25)	0.7073(35)	0.782(13)	1.43(11)	[17][14]	0.0375(5)	0.7112(5)	0.773		[24, 25]
4	0.038	0.754	0.72	1.63	0.035(4)	0.741(6)	0.774(20)	1.54(11)	[17][14]	0.0365(10)	0.749(2)	0.765	1.6(1)	[25][21]
10	0.022	0.889	0.80		0.024	0.859			[26]					

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1	0.039	0.632	0.78	1.15	0.0335(25)	0.6304(13)	0.799(11)	1.07(10)	[17][14]	0.0368(2)	0.6302(1)	0.821(5)	1.09(9)	[20][21]
2	0.041	0.674	0.75	1.37	0.0354(25)	0.6703(15)	0.789(11)	1.27(10)	[17][14]	0.0381(2)	0.6717(1)	0.785(20)	1.32(2)	[22][23]
3	0.040	0.715	0.73	1.50	0.0355(25)	0.7073(35)	0.782(13)	1.43(11)	[17][14]	0.0375(5)	0.7112(5)	0.773		[24, 25]
4	0.038	0.754	0.72	1.63	0.035(4)	0.741(6)	0.774(20)	1.54(11)	[17][14]	0.0365(10)	0.749(2)	0.765	1.6(1)	[25][21]
10	0.022	0.889	0.80		0.024	0.859			[26]					

2. Example: Gross-Neveu-model with N fermions in d=3 (e.g. $N=12$):

- MC simluation (S. Hands, A. Kocic & J. B. Kogut, 1993): $\nu = 1.022$, $\eta_\sigma = 0.913$
- FRG (L. Rosa, P. Vitale & C. Wetterich, 2002): $\nu = 1.023$, $\eta_\sigma = 0.936$

Classification of 3d fermionic models

$$S = \int d^3x \{\bar{\psi}^a \gamma^\mu \partial_\mu \psi^a + (\bar{\psi}^a \mathcal{O}_X \psi^a)(\bar{\psi}^b \mathcal{O}_Y \psi^b)\}$$

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- γ_μ in usual Euclidean chiral rep., i.e. 4×4
- $\psi^a, \bar{\psi}^a$ are 4-component spinors
- reducible rep. gives rise to 2 "fifth-γ" matrices γ_4 & γ_5: $\{\gamma_{4,5}, \gamma_\mu\} = 0, \quad \{\gamma_4, \gamma_5\} = 0$
- 4×4 Dirac algebra $\mathcal{O}_X, \mathcal{O}_Y \in \{\mathbb{1}, \gamma_\mu, \gamma_4, \sigma_{\mu\nu}, \mathrm{i}\gamma_\mu\gamma_4, \mathrm{i}\gamma_\mu\gamma_5, \mathrm{i}\gamma_4\gamma_5, \gamma_5\}$
- define chiral projectors $P_{\mathrm{L/R}} = \frac{1}{2}(\mathbb{1} \pm \gamma_5)$
- Weyl spinors: $\psi_{\mathrm{L/R}} = P_{\mathrm{L/R}}\psi, \quad \bar{\psi}_{\mathrm{L/R}} = \bar{\psi} P_{\mathrm{R/L}}$

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- Now impose chiral $\mathrm{U}(N_\mathrm{L})_\mathrm{L} \otimes \mathrm{U}(N_\mathrm{R})_\mathrm{R}$ symmetry

$$\begin{aligned} \mathrm{U}(N_\mathrm{L})_\mathrm{L}: &\quad \psi^a_\mathrm{L} \mapsto U^{ab}_\mathrm{L}\psi^b_\mathrm{L}, \quad \bar\psi^a_\mathrm{L} \mapsto \bar\psi^b_\mathrm{L}(U^\dagger_\mathrm{L})^{ba}, \\ \mathrm{U}(N_\mathrm{R})_\mathrm{R}: &\quad \psi^a_\mathrm{R} \mapsto U^{ab}_\mathrm{R}\psi^b_\mathrm{R}, \quad \bar\psi^a_\mathrm{R} \mapsto \bar\psi^b_\mathrm{R}(U^\dagger_\mathrm{R})^{ba} \end{aligned}$$

- includes $\mathrm{U}(1)_\mathrm{A}$ axial transformations and $\mathrm{U}(1)_\mathrm{V}$ phase rotations, thus

$$\mathrm{U}(N_\mathrm{L})_\mathrm{L} \otimes \mathrm{U}(N_\mathrm{R})_\mathrm{R} \cong \mathrm{SU}(N_\mathrm{L})_\mathrm{L} \otimes \mathrm{SU}(N_\mathrm{R})_\mathrm{R} \otimes \mathrm{U}(1)_\mathrm{A} \otimes \mathrm{U}(1)_\mathrm{V}$$

Discrete symmetries

Reducible rep $\Rightarrow$ discrete trafos not unique

- Charge conjugation $C = \frac{1}{2}[(1+\epsilon)\gamma_2\gamma_5 + i(1-\epsilon)\gamma_2\gamma_4]$ and ϵ arbitrary complex phase

$$\mathcal{C}: \psi^a \mapsto (\bar{\psi}^a C)^T, \quad \bar{\psi}^a \mapsto -\left(C^\dagger \psi^a\right)^T$$

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- time reversal $(x_1, x_2, x_3) \mapsto (x_1, x_2, -x_3) =: \hat{x}$ with $T = \frac{1}{2}[(1+\eta)\gamma_2\gamma_3 + i(1-\eta)\gamma_1]$

$$\mathcal{T}: \psi^a(x) \mapsto T\psi^a(\hat{x}), \quad \bar{\psi}^a(x) \mapsto \bar{\psi}^a(\hat{x})T^\dagger$$

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Transformation properties of bilinears:

	$\mathcal{C}(\epsilon=1)$	$\mathcal{C}(\epsilon=-1)$	$\mathcal{P}(\zeta=1)$	$\mathcal{P}(\zeta=-1)$	$\mathcal{T}(\eta=1)$	$\mathcal{T}(\eta=-1)$
$\bar{\psi}^a_{\mathrm{L}}\psi^b_{\mathrm{R}}$	$\bar{\psi}^b_{\mathrm{R}}\psi^a_{\mathrm{L}}$	$\bar{\psi}^b_{\mathrm{L}}\psi^a_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{L}}\psi^b_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{R}}\psi^b_{\mathrm{L}}$	$\bar{\psi}^a_{\mathrm{L}}\psi^b_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{R}}\psi^b_{\mathrm{L}}$
$\bar{\psi}^a_{\mathrm{L}}\gamma_\mu\psi^b_{\mathrm{L}}$	$-\bar{\psi}^b_{\mathrm{L}}\gamma_\mu\psi^a_{\mathrm{L}}$	$-\bar{\psi}^b_{\mathrm{R}}\gamma_\mu\psi^a_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{L}}\tilde{\gamma}_\mu\psi^b_{\mathrm{L}}$	$\bar{\psi}^a_{\mathrm{R}}\tilde{\gamma}_\mu\psi^b_{\mathrm{R}}$	$-\bar{\psi}^a_{\mathrm{L}}\hat{\gamma}_\mu\psi^b_{\mathrm{L}}$	$-\bar{\psi}^a_{\mathrm{R}}\hat{\gamma}_\mu\psi^b_{\mathrm{R}}$
$\bar{\psi}^a_{\mathrm{L}}\sigma_{\mu\nu}\psi^b_{\mathrm{R}}$	$-\bar{\psi}^b_{\mathrm{R}}\sigma_{\mu\nu}\psi^a_{\mathrm{L}}$	$-\bar{\psi}^b_{\mathrm{L}}\sigma_{\mu\nu}\psi^a_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{L}}\tilde{\sigma}_{\mu\nu}\psi^b_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{R}}\tilde{\sigma}_{\mu\nu}\psi^b_{\mathrm{L}}$	$-\bar{\psi}^a_{\mathrm{L}}\hat{\sigma}_{\mu\nu}\psi^b_{\mathrm{R}}$	$-\bar{\psi}^a_{\mathrm{R}}\hat{\sigma}_{\mu\nu}\psi^b_{\mathrm{L}}$
$\bar{\psi}^a_{\mathrm{L}}\gamma_4\psi^b_{\mathrm{L}}$	$\bar{\psi}^b_{\mathrm{L}}\gamma_4\psi^a_{\mathrm{L}}$	$-\bar{\psi}^b_{\mathrm{R}}\gamma_4\psi^a_{\mathrm{R}}$	$-\bar{\psi}^a_{\mathrm{L}}\gamma_4\psi^b_{\mathrm{L}}$	$\bar{\psi}^a_{\mathrm{R}}\gamma_4\psi^b_{\mathrm{R}}$	$\bar{\psi}^a_{\mathrm{L}}\gamma_4\psi^b_{\mathrm{L}}$	$-\bar{\psi}^a_{\mathrm{R}}\gamma_4\psi^b_{\mathrm{R}}$
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Classification

Invariant terms under $\mathrm{U}(N_{\mathrm{L}})_{\mathrm{L}} \otimes \mathrm{U}(N_{\mathrm{R}})_{\mathrm{R}} \otimes \mathcal{C}(\epsilon = 1) \otimes \mathcal{P}(\zeta = 1) \otimes \mathcal{T}(\eta = 1)$:

- no invariant mass terms
- invariant standard kinetic terms
- 12 invariant 4-Fermi interaction terms

$$
\begin{gathered}
\left(\bar{\psi}^a_{\mathrm{L}} \psi^b_{\mathrm{R}}\right)\left(\bar{\psi}^b_{\mathrm{R}} \psi^a_{\mathrm{L}}\right), \\
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\left(\bar{\psi}^a_{\mathrm{L}} \gamma_\mu \psi^b_{\mathrm{L}}\right)\left(\bar{\psi}^b_{\mathrm{L}} \gamma_\mu \psi^a_{\mathrm{L}}\right), \quad \left(\bar{\psi}^a_{\mathrm{R}} \gamma_\mu \psi^b_{\mathrm{R}}\right)\left(\bar{\psi}^b_{\mathrm{R}} \gamma_\mu \psi^a_{\mathrm{R}}\right), \\
\left(\bar{\psi}^a_{\mathrm{L}} \mathrm{i} \gamma_\mu \gamma_4 \psi^b_{\mathrm{R}}\right)\left(\bar{\psi}^b_{\mathrm{R}} \mathrm{i} \gamma_\mu \gamma_4 \psi^a_{\mathrm{L}}\right) \\
+6 \text{ terms with inverse flavour structure}
\end{gathered}
$$

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\end{gathered}
$$

- terms with inverse flavour structure are not independent: Fierz transformations $= 6$ equations $\Rightarrow 12 - 6 = 6$ invariant terms

Classification includes NJL-type models, Thirring model, Gross-Neveu model and effetive models for various parts of the cuprate phase diagram

Partial bosonization

Hubbard-Stratonovich trafo introduces 6 boson-fermion i.a.

- 1 scalar boson $\sim \bar{\psi}^a_{\mathrm{L}} \psi^b_{\mathrm{R}}$
- 2 pseudo-scalar bosons $\sim \bar{\psi}^a_{\mathrm{L/R}} \gamma_4 \psi^b_{\mathrm{L/R}}$
- 2 vector bosons $\sim \bar{\psi}^a_{\mathrm{L/R}} \gamma_\mu \psi^b_{\mathrm{L/R}}$
- 1 pseudo-vector boson $\sim \bar{\psi}^a_{\mathrm{L}} i \gamma_\mu \gamma_4 \psi^b_{\mathrm{R}}$

$\Rightarrow$ in general: competing order parameters!

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$\Rightarrow$ in general: competing order parameters!

- Focus on Lorentz-invariant and parity-conserving condensation channel: scalar i.a.
- $N_{\mathrm{R}} = 1$ and $N_{\mathrm{L}} \geq 1$: similar to SM with $N_{\mathrm{L}} = 2$

$\Rightarrow$ Yukawa action

$$S_{\mathsf{Yuk}} = \int \mathrm{d}^3 x \Big\{ \frac{1}{2\lambda} \phi^{a\dagger} \phi^a + \bar{\psi}^a_{\mathrm{L}} \mathrm{i} \partial\!\!\!/ \psi^a_{\mathrm{L}} + \bar{\psi}_{\mathrm{R}} \mathrm{i} \partial\!\!\!/ \psi_{\mathrm{R}} + \phi^{a\dagger} \bar{\psi}_{\mathrm{R}} \psi^a_{\mathrm{L}} - \phi^a \bar{\psi}^a_{\mathrm{L}} \psi_{\mathrm{R}} \Big\}$$

Truncation

NLO derivative expansion

$$\begin{aligned}\Gamma_k \quad = \quad & \int \mathrm{d}^3x \Big\{ Z_{\mathrm{L},k}\bar{\psi}^a_{\mathrm{L}} \mathrm{i}\partial\!\!\!/ \psi^a_{\mathrm{L}} + Z_{\mathrm{R},k}\bar{\psi}_{\mathrm{R}} \mathrm{i}\partial\!\!\!/ \psi_{\mathrm{R}} + Z_{\phi,k}\left(\partial_\mu \phi^{a\dagger}\right)\left(\partial^\mu \phi^a\right) + U_k(\phi^{a\dagger}\phi^a) \\ & + \bar{h}_k \bar{\psi}_{\mathrm{R}} \phi^{a\dagger} \psi^a_{\mathrm{L}} - \bar{h}_k \bar{\psi}^a_{\mathrm{L}} \phi^a \psi_{\mathrm{R}} \Big\}.\end{aligned}$$

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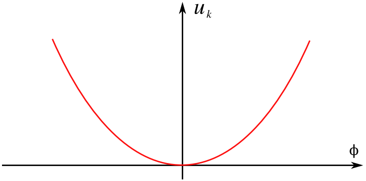
For the symmetric regime (SYM), we expand the effective potential around zero field,

$$u_k = m_k^2 \tilde{\rho} + \frac{\lambda_{2,k}}{2!}\tilde{\rho}^2 + \frac{\lambda_{3,k}}{3!}\tilde{\rho}^3 + \ldots$$

$$m^2,\ \lambda_{n_{\max}} > 0.$$



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$$+ \bar{h}_k \bar{\psi}_{\mathrm{R}} \phi^{a\dagger} \psi^a_{\mathrm{L}} - \bar{h}_k \bar{\psi}^a_{\mathrm{L}} \phi^a \psi_{\mathrm{R}} \Big\}.$$

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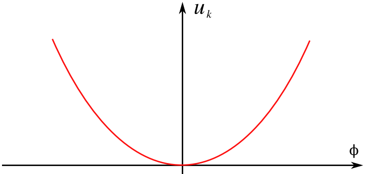
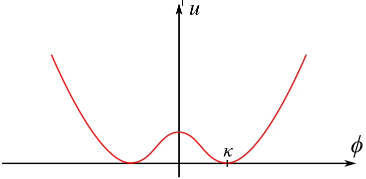


For the SSB regime, the minimum of u_k is $\kappa_k := \tilde{\rho}_{\min} > 0$,

$$u_k = \frac{\lambda_{2,k}}{2!}(\tilde{\rho} - \kappa_k)^2 + \frac{\lambda_{3,k}}{3!}(\tilde{\rho} - \kappa_k)^3 + \dots$$
$$\kappa,\ \lambda_{n_{\max}},\ \lambda_2 > 0.$$



Critical exponents in SYM and SSB

SYM: β-function for Yukawa coupling

$$\partial_t h^2 = (\eta_\phi + \eta_L + \eta_R - 1) h^2$$

- Sum rule as a condition for an interacting FP (similar to QEG)

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N_{L}	h_*^2	m_*^2	λ_2^*	η_ϕ^*	η_{L}^*	η_{R}^*	ν	ω
1	4.496	0.326	5.099	0.716	0.142	0.142	1.132	0.786
2	3.364	0.104	3.643	0.512	0.162	0.325	1.100	0.809

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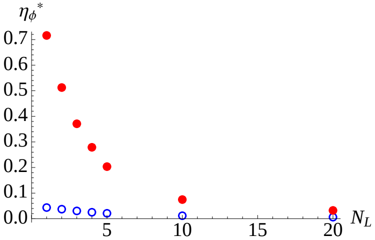
SSB: fixed-point needs balancing of fermion and boson fluctuations.

N_{L}	h_*^2	κ_*	λ_2^*	η_ϕ^*	η_{L}^*	η_{R}^*	ν	ω
3	2.718	0.009	2.967	0.371	0.154	0.487	0.883	0.675
4	2.713	0.042	2.954	0.279	0.125	0.637	1.043	0.678
5	2.519	0.079	2.717	0.204	0.100	0.746	1.124	0.715
10	1.452	0.256	1.506	0.075	0.046	0.913	1.092	0.872
100	0.148	3.301	0.149	0.006	0.004	0.993	1.008	0.989

Critical exponents in SYM and SSB

- Universal FP values and critical exponents as a function of N_{L}:



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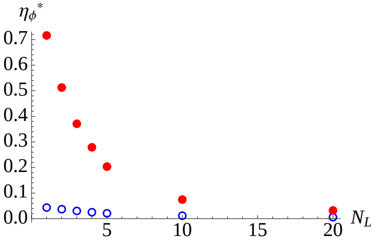

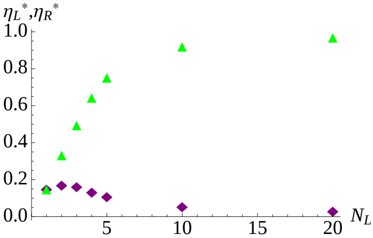

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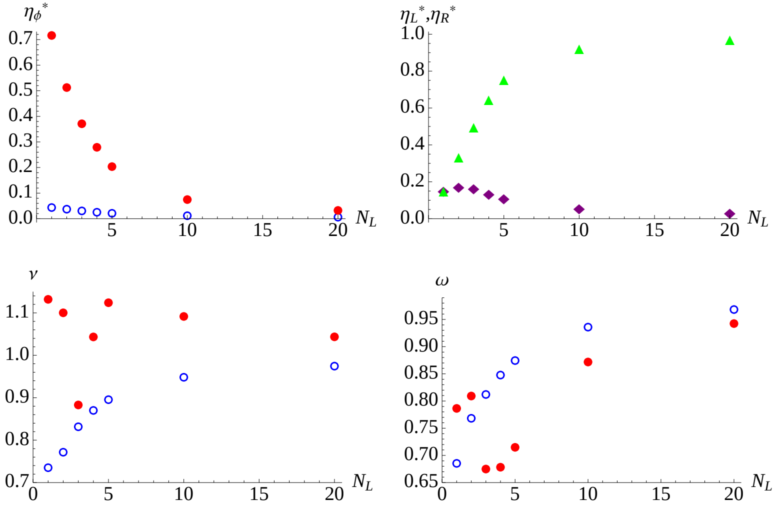

Conclusions

CHIRAL YUKAWA SYSTEMS IN d=4:

- Revealed a possible AS mechanism for the standard model with high predictivity
- Massless Goldstone and fermion fluctuations not present in the standard model (Destabilization at NLO)
- Include $SU(N_{\mathrm{L}})$ gauge bosons (work in progress with H. Gies and S. Rechenberger)

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CHIRAL YUKAWA SYSTEMS IN d=3:

- Different FP mechanisms in SYM and SSB
- Benchmark values for critical exponents in new universality classes for strongly correlated chiral fermions
- Test of convergence: $\eta \lesssim 1$, check with $O(N)$-model, (in principle NNLO required)
- Take into account other condensation channels - competing order parameters

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- Revealed a possible AS mechanism for the standard model with high predictivity
- Massless Goldstone and fermion fluctuations not present in the standard model (Destabilization at NLO)
- Include $SU(N_{\mathrm{L}})$ gauge bosons (work in progress with H. Gies and S. Rechenberger)

CHIRAL YUKAWA SYSTEMS IN d=3:

- Different FP mechanisms in SYM and SSB
- Benchmark values for critical exponents in new universality classes for strongly correlated chiral fermions
- Test of convergence: $\eta \lesssim 1$, check with $O(N)$-model, (in principle NNLO required)
- Take into account other condensation channels - competing order parameters

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