

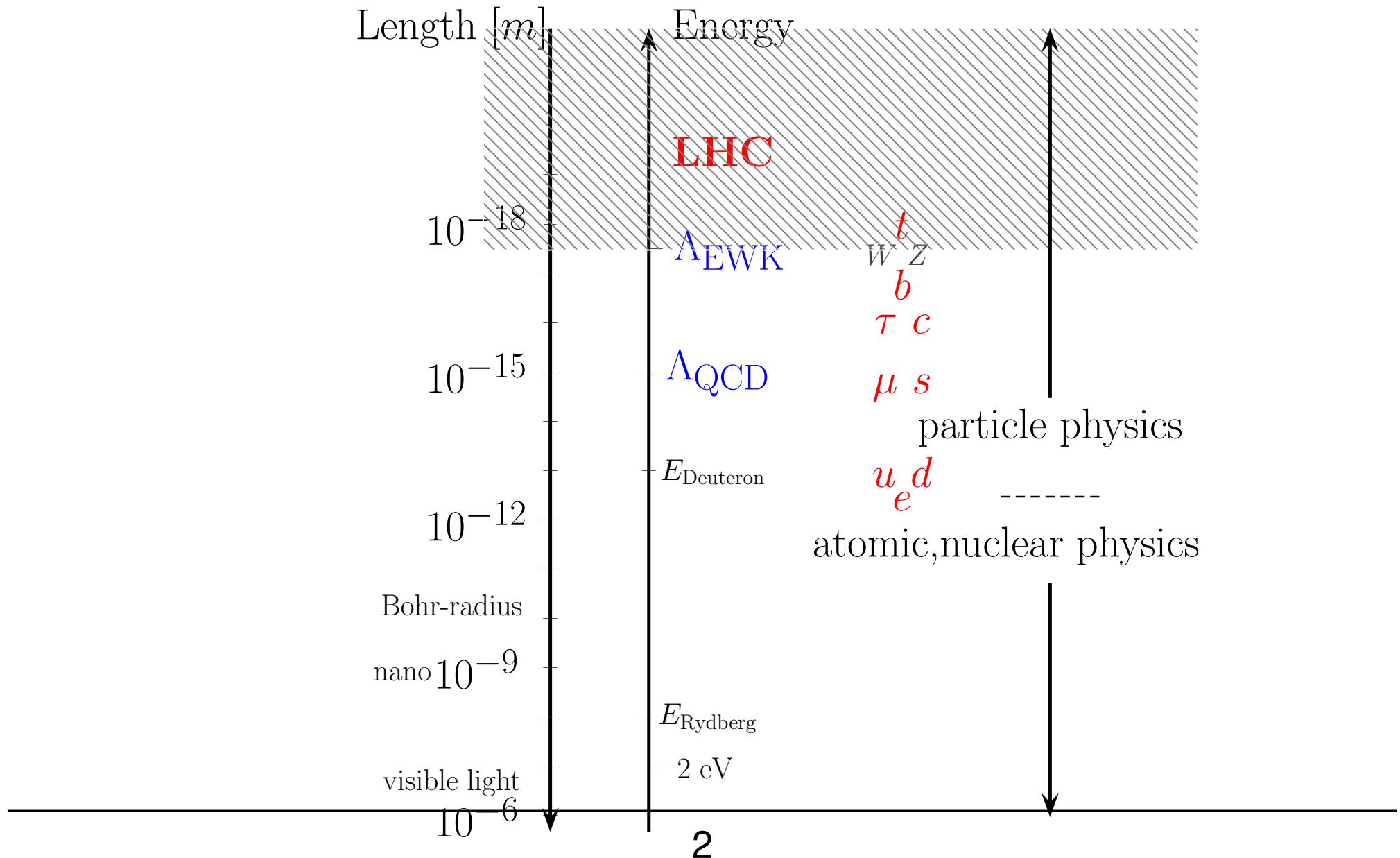
# Flavor Theory and

$B \rightarrow K^{(*)} \mu^+ \mu^-$  Implications

May 14, 2012

Gudrun Hiller, Dortmund

# Particle Physics Scales & the LHC



Known fundamental matter comes in generations  $\psi \rightarrow \psi_i$ ,  $i = 1, 2, 3$ .

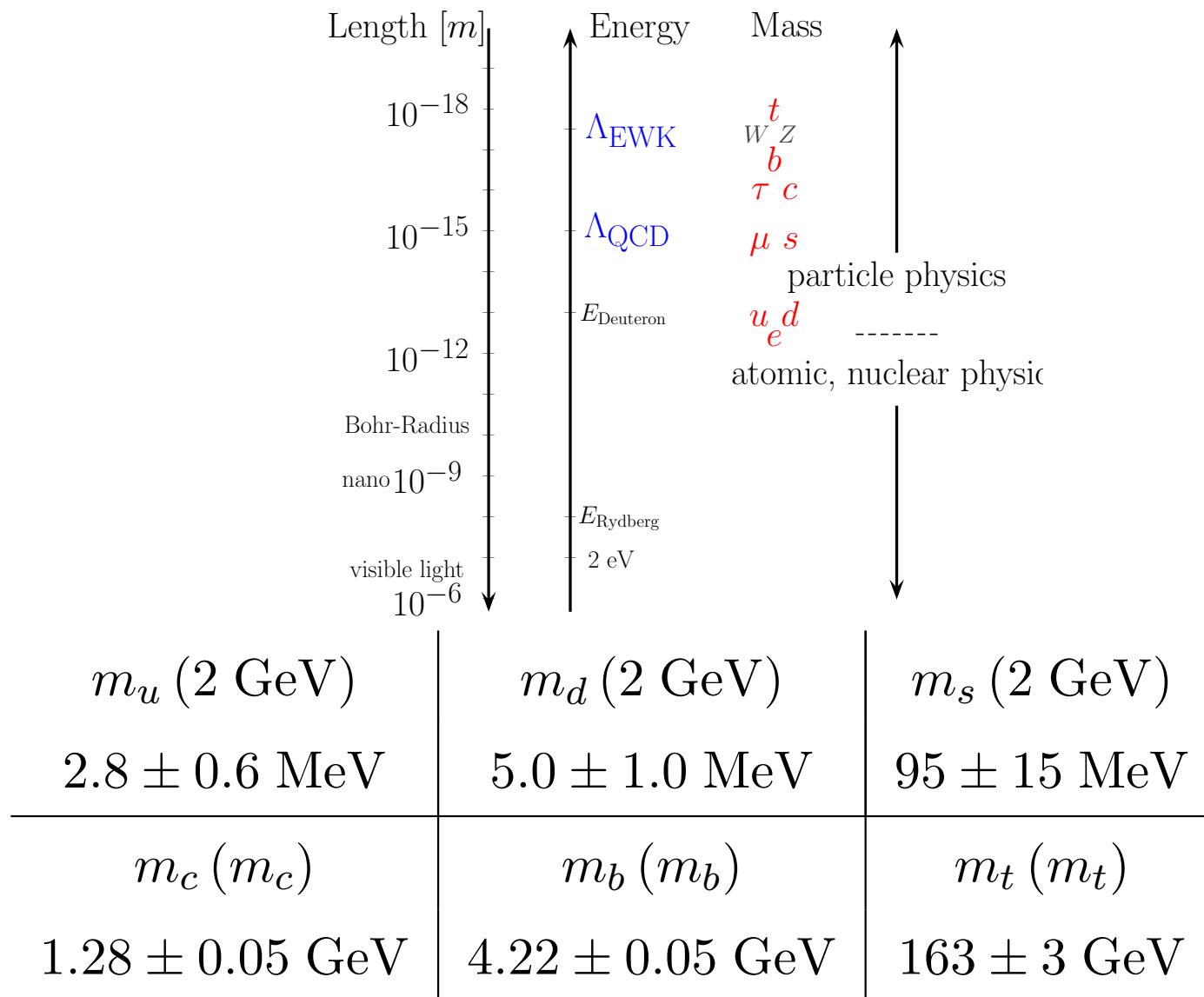
quarks: 
$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

leptons: 
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em}$$

The gauge interactions are generation-independent.

# Quark Spectrum



hierarchical! Spectrum spans five orders of magnitude.

Quarks mix and change flavor in weak interaction:

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}; \quad \lambda \simeq 0.2$$

$$\vartheta_{13} \sim \lambda^3 \ll \vartheta_{23} \sim \lambda^2 \ll \vartheta_{12} \sim \lambda \ll 1$$

hierarchical!

Large mixing angles for leptons (PMNS-Matrix):

$$\vartheta_{23} \sim 45^\circ, \vartheta_{12} \sim 35^\circ, \vartheta_{13} \sim O(10^\circ) \quad \text{all } O(1) - \text{anarchy?}$$

# CP is violated!.. together with Quark Flavor

---

Quark mixing matrix has 1 physical CP violating phase  $\delta_{CKM}$ .

(with 3 generations)



The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"



Photo: University of Chicago

Yoichiro Nambu

1/2 of the prize



Photo: KEK

Makoto Kobayashi

1/4 of the prize



Photo: Kyoto University

Toshihide Maskawa

1/4 of the prize

Kobayashi and Maskawa, Prog.Theor.Phys 49 (1973) 652

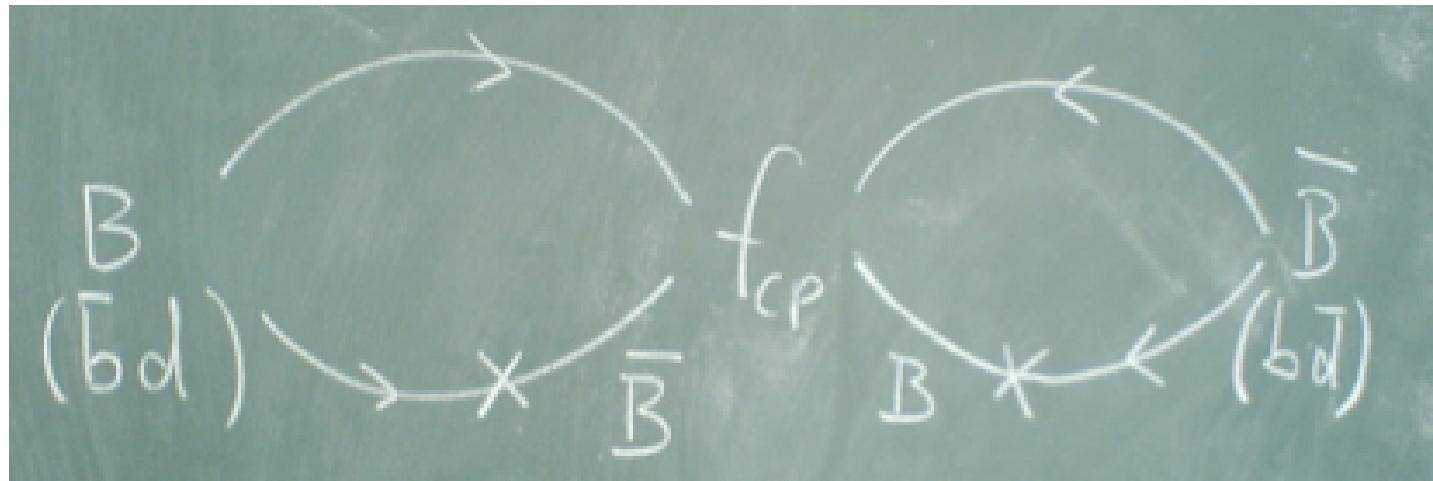
# CP is violated!.. together with Quark Flavor

---

Quark mixing matrix has 1 physical CP violating phase  $\delta_{CKM}$ .

Verified in  $B\bar{B}$  mixing

$$\sin 2\beta = 0.672 \pm 0.023 \text{ HFAG Aug 2010}$$



$\delta_{CKM}$  is large,  $O(1)!$

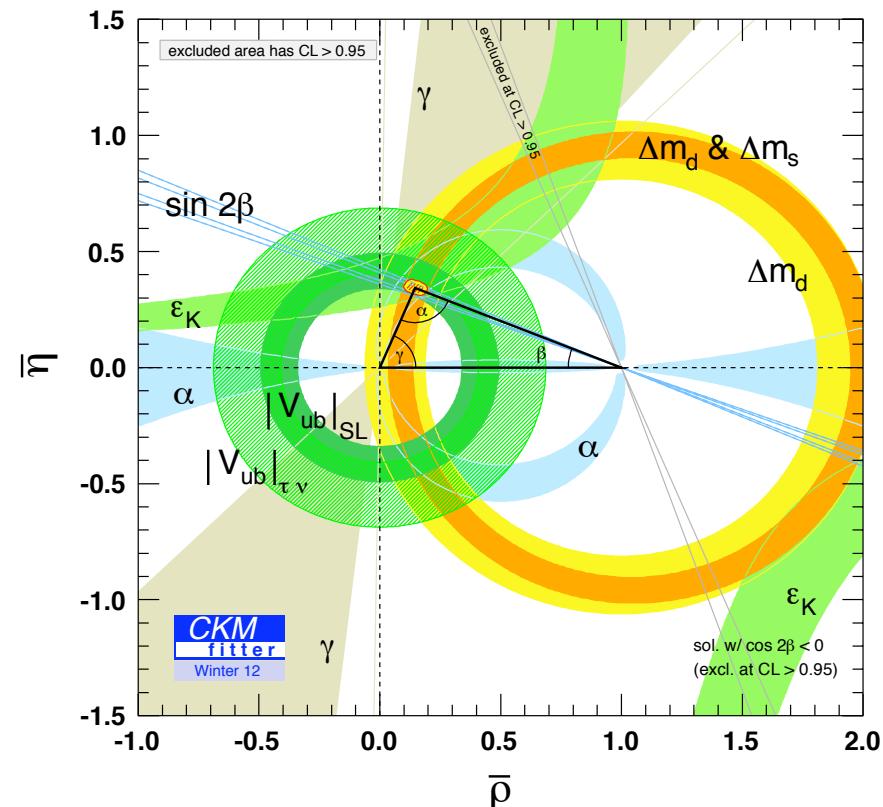
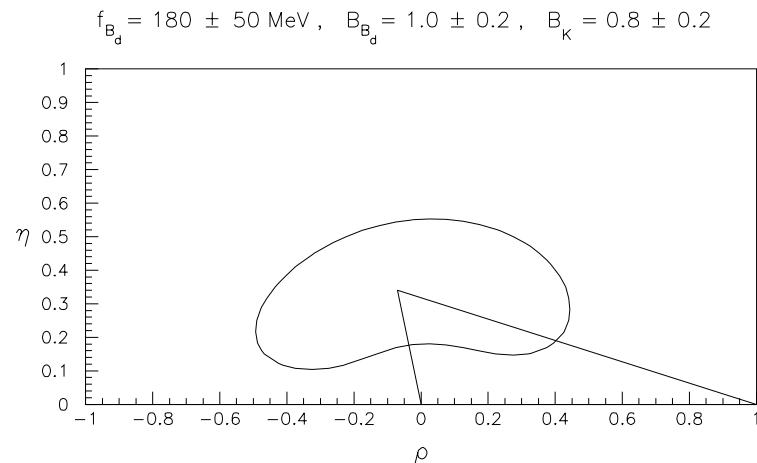
CPX also observed in  $B$ -decay  $A_{CP}(B \rightarrow K^\pm \pi^\mp) = -0.098 \pm 0.013$

HFAG Aug 2010

$$\Gamma(B \rightarrow K^+ \pi^-) \neq \Gamma(\bar{B} \rightarrow K^- \pi^+)$$

# SM Flavor and CP Violation/CKM 1995 vs today

The CKM-picture of flavor and CP violation is currently consistent with all – and quite different – laboratory observations, although some tensions exist.



$$V_{CKM} V_{CKM}^\dagger = 1$$

# Rare Processes observed Today

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Modulo "hints" all hadronic flavor changing data are currently ok with the SM within uncertainties.

Flavor changing neutral currents (FCNCs):

$s \rightarrow d: K^0 - \bar{K}^0, K \rightarrow \pi\nu\bar{\nu}$

$c \rightarrow u: D^0 - \bar{D}^0$  (first data on FCNC in up-sector)

dir. CPX **CDF,LHCb'11,12**

$b \rightarrow d: B^0 - \bar{B}^0, B \rightarrow \rho\gamma, b \rightarrow d\gamma, B \rightarrow \pi\mu\mu$  **LHCb'12**

$b \rightarrow s: B_s - \bar{B}_s, b \rightarrow s\gamma, B \rightarrow K_s\pi^0\gamma, b \rightarrow sll, B \rightarrow K^{(*)}ll$  (precision, angular observables available **Belle'09,CDF'11,BaBar,LHCb'12**),

$B_s \rightarrow \mu\mu$  (bound at SM-level  $< 4.5 \times 10^{-9}$  95 %CL **LHCb'12**)

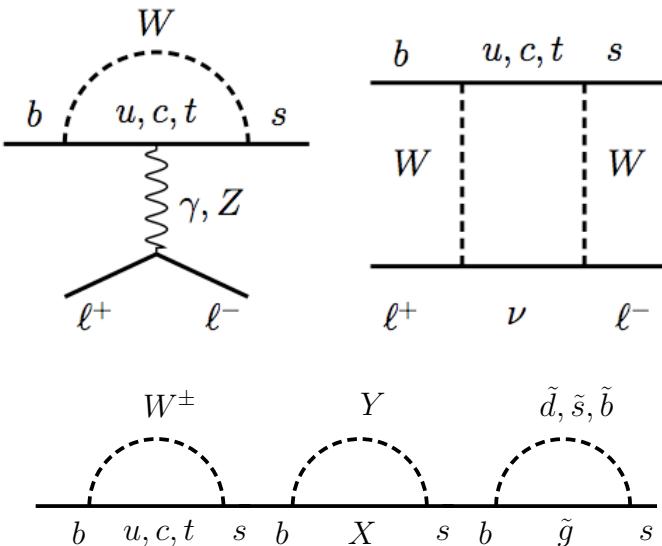
$t \rightarrow c, u$  and  $l \rightarrow l'$ : not observed

Precision tests of the SM with  $O(1000)$  plus events in  $B \rightarrow K^{(*)} \mu^+ \mu^-$

- – a lattice benchmark test opportunity
- – BSM/SUSY implications

# Exclusive semileptonic FCNC $b \rightarrow s\mu^+\mu^-$ decays

$$Br_{\text{SM}} \sim 10^{-6} - 10^{-7}$$



observed (at SM level):

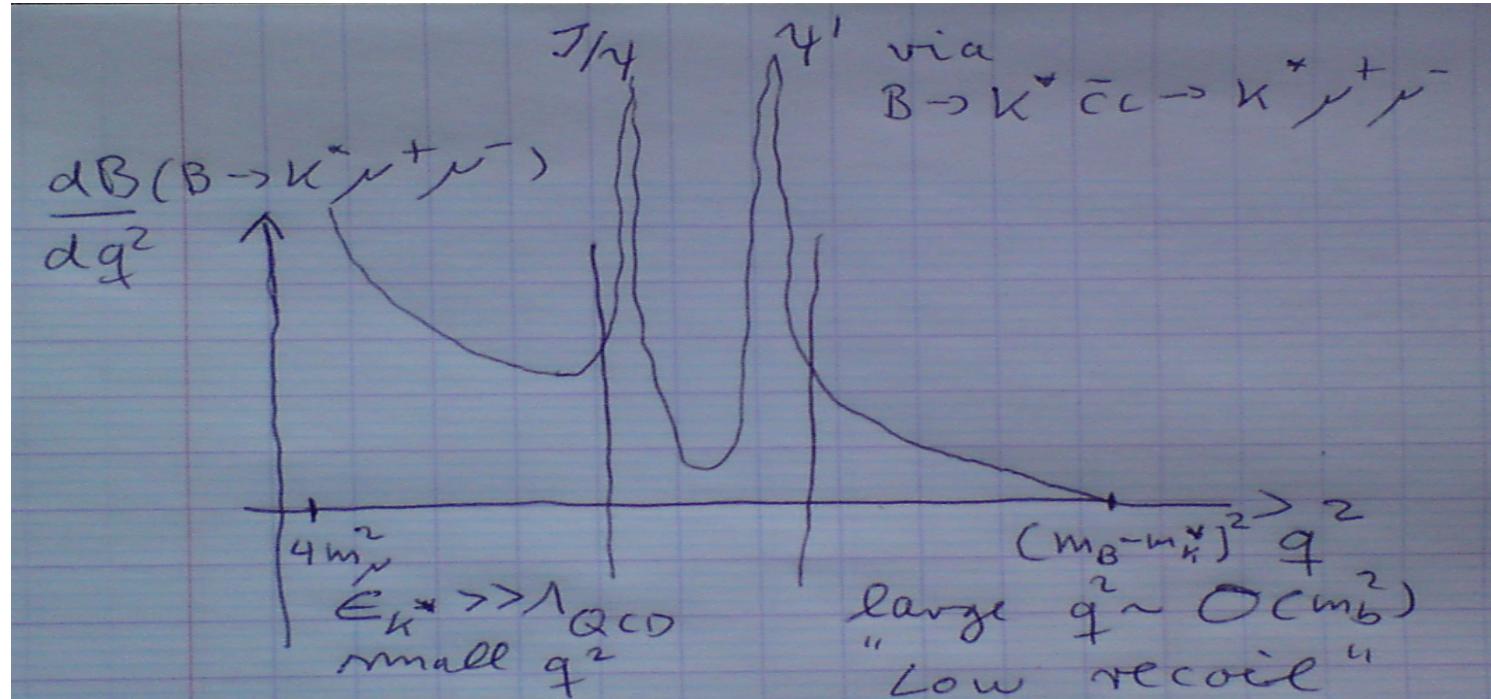
$B \rightarrow K^{(*)}\mu^+\mu^-$  BaBar, Belle, CDF  $6.8 \text{ fb}^{-1}$  and LHCb  $1 \text{ fb}^{-1}$  LHCb-CONF-2012-008

$B_s \rightarrow \Phi\mu^+\mu^-$  CDF 2011 [1101.1028 \[hep-ex\]](#) LHCb 2012 [LHCb-CONF-2012-008](#)

$\Lambda_b \rightarrow \Lambda\mu^+\mu^-$  CDF 2011 [1107.3753 \[hep-ex\]](#)

distributions measured. precision physics started.

# Dilepton Mass Spectra in $B \rightarrow K^* \mu^+ \mu^-$

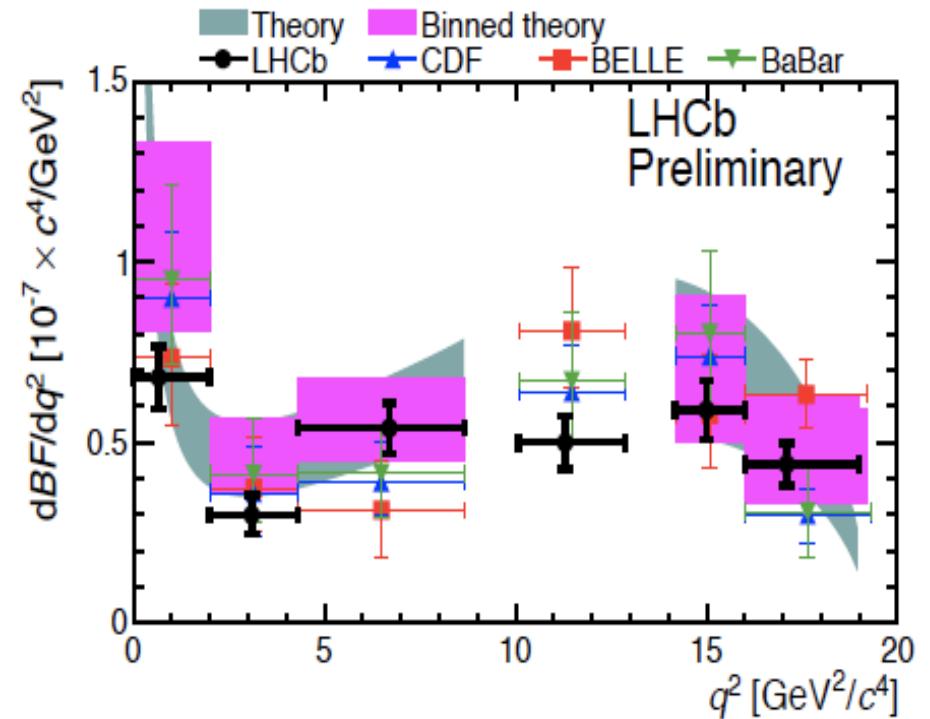
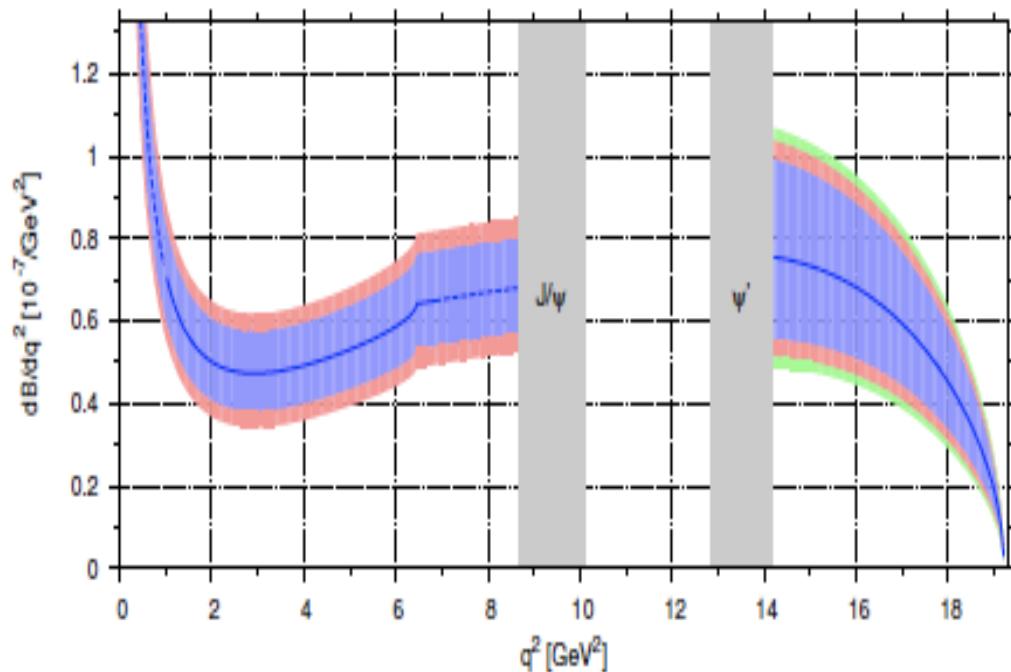


Different TH at **low  $q^2$**  QCDF; BBNS, Beneke, Feldmann, Seidel'01,04 and **high  $q^2$ /low recoil**

OPE in  $1/m_b$  Grinstein,Pirjol '04, Beylich, Buchalla,Feldmann'11; Low recoil  $B \rightarrow K^{(*)} \mu^+ \mu^-$  predictions/pheno

Bobeth,GH,vanDyk, Wacker '10,11 Binned data needed. New developments at low recoil in theory pheno+lattice greatly support exploitation of todays and tomorrow's data. E.g., Preliminary unquenched lattice  $B \rightarrow K^{(*)}$  form factors by Liu et al 1101.2726 [hep-ph].

# Dilepton Mass Spectra in $B \rightarrow K^* \mu^+ \mu^-$



left-hand Fig. from 1006.5013 [hep-ph] Blue band: form factor uncertainties, red:  $1/m_b$  right-hand Fig. from LHCb-CONF-2012-008

Biggest source of TH uncertainty: the  $B \rightarrow K^*$  form factors.

# Hadronic uncertainties $B \rightarrow K^*$ Form factors

---

$$\langle K^*(k, \epsilon) | \bar{s} \gamma_\mu b | B(p) \rangle = \frac{2 V(q^2)}{m_B + m_{K^*}} \epsilon_{\mu\rho\sigma\tau} \epsilon^{*\rho} p^\sigma k^\tau,$$

$$\langle K^*(k, \epsilon) | \bar{s} \gamma_\mu \gamma_5 b | B(p) \rangle =$$

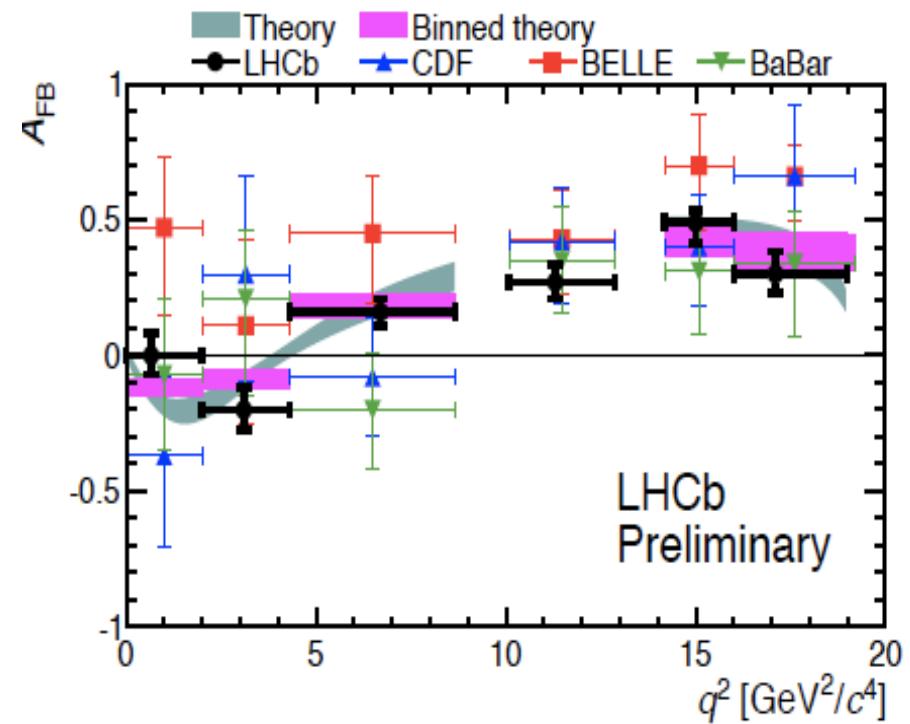
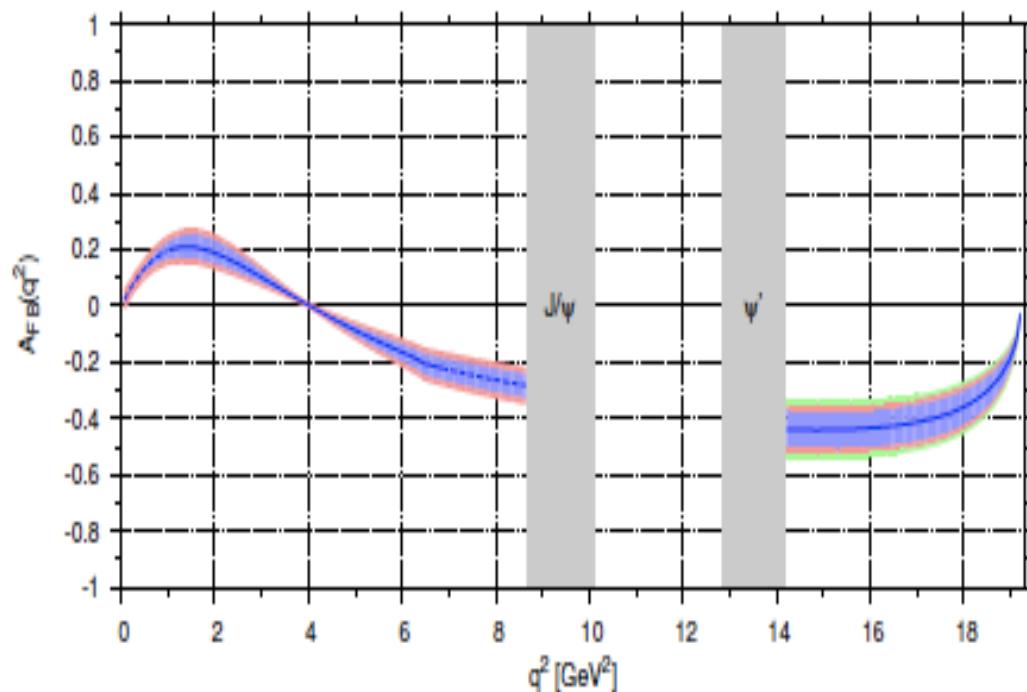
$$i \epsilon^{*\rho} \left[ 2 A_0(q^2) m_{K^*} \frac{q_\mu q_\rho}{q^2} + A_1(q^2) (m_B + m_{K^*}) (g_{\mu\rho} - \frac{q_\mu q_\rho}{q^2}) \right. \\ \left. - A_2(q^2) q_\rho \left( \frac{(p+k)_\mu}{m_B + m_{K^*}} - \frac{m_B - m_{K^*}}{q^2} (p-k)_\mu \right) \right]$$

plus tensor currents  $\langle K^*(k, \epsilon) | \bar{s} \sigma_{\mu\nu} b | B(p) \rangle$  with  $T_{1,2,3}$ .

Low recoil: improved isgur-wise relations, accessible to lattice QCD

$$T_1(q^2) = \kappa V(q^2), T_2(q^2) = \kappa A_1(q^2), \kappa = 1 - 2 \frac{\alpha_s}{3\pi} \ln \left( \frac{\mu}{m_b} \right) \simeq 1$$

# Forward-backward asymmetry $A_{\text{FB}}$ in $B \rightarrow K^* \mu^+ \mu^-$



left-hand Fig. from 1006.5013 [hep-ph] Blue band: form factor uncertainties, red:  $1/m_b$  right-hand Fig. from LHCb-CONF-2012-008

Sign of  $A_{\text{FB}}$  at large dilepton mass is SM-like. 0805.2525 [hep-ph]

Sign/zero of  $A_{\text{FB}}$  at low dilepton mass SM-like (LHCb'12)

$q_0^2|_{\text{LHCb}} = 4.9^{+1.1}_{-1.3} \text{ GeV}^2$  in SM:  $q_0^2|_{\text{SM}} = 4.0 \pm 0.3 \text{ GeV}^2$

# Opportunity: Angular Analysis $B \rightarrow V(\rightarrow PP)\mu^+\mu^-$

---

$$d\Gamma^4 \sim J dq^2 d\cos\Theta_l d\cos\Theta_{K^*} d\Phi \text{ hep-ph/9907386}$$

$$\begin{aligned} J(q^2, \theta_l, \theta_{K^*}, \phi) = & J_1^s \sin^2 \theta_{K^*} + J_1^c \cos^2 \theta_{K^*} + (J_2^s \sin^2 \theta_{K^*} + J_2^c \cos^2 \theta_{K^*}) \cos 2\theta_l \\ & + J_3 \sin^2 \theta_{K^*} \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_{K^*} \sin \theta_l \cos \phi \\ & + J_6 \sin^2 \theta_{K^*} \cos \theta_l + J_7 \sin 2\theta_{K^*} \sin \theta_l \sin \phi \\ & + J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_{K^*} \sin^2 \theta_l \sin 2\phi, \end{aligned} \quad (2.3)$$

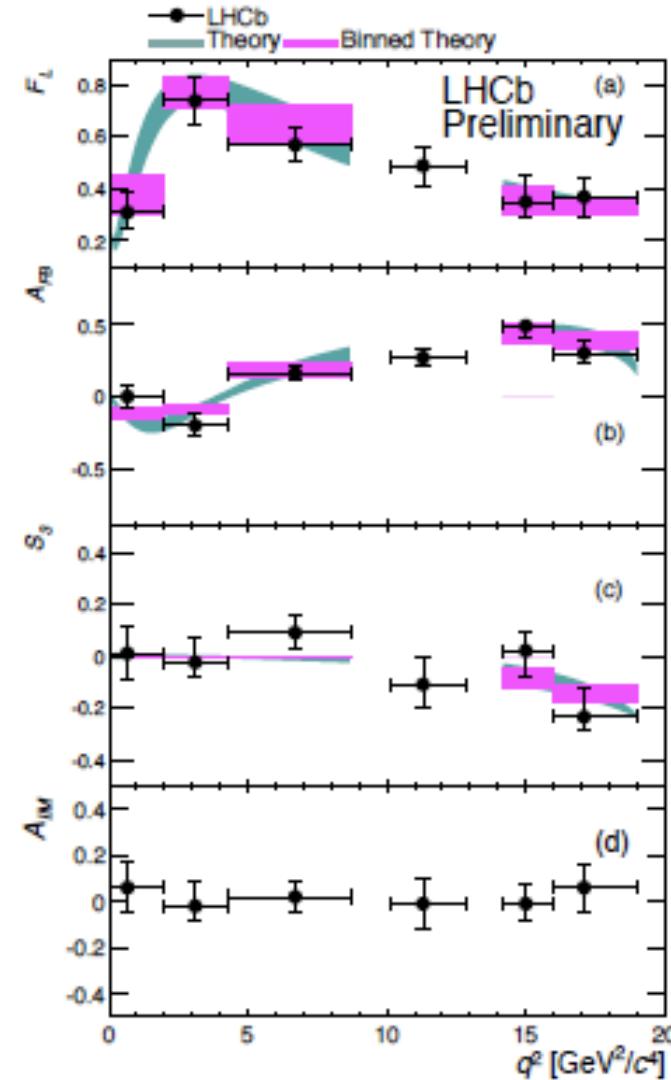
$\Theta_l$ : angle between  $l^-$  and  $\bar{B}$  in dilepton CMS (warning: different conventions in literature)

$\Theta_{K^*}$ : angle between K and  $\bar{B}$  in  $K^*$ -CMS

$\Phi$ : angle between normals of the  $K\pi$  and  $l^+l^-$  plane

## $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ Angular Analysis Results

- 4D fit to 3 angles and mass
- Larger data sample enables measurements of  $S_3$  and  $A_{IM}$
- Error bars include systematic uncertainties
- Data points at average  $q^2$  of signal candidates in data
- These are the most precise measurements to-date [preliminary]
- The results are consistent with the SM prediction [1]



# Extracting $B \rightarrow K^*$ form factors from data

At low hadronic recoil: OPE in  $1/m_b$  Grinstein, Pirjol '04, Beylich et al '10

$$A_i^{L,R} \propto C^{L,R} f_i + \mathcal{O}(\alpha_s \Lambda/m_b, C_7/C_9 1/m_b), \quad i = \perp, \parallel, 0 \quad \text{Bobeth et al '10}$$

$C^{L,R}$ : universal short-dist.-physics;  $C^{L,R} = (C_9^{\text{eff}} \mp C_{10}) + \kappa \frac{2\hat{m}_b}{\hat{s}} C_7^{\text{eff}}$

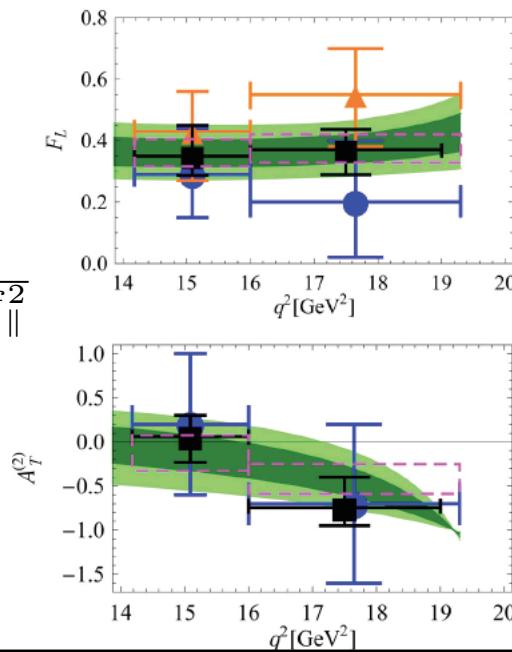
$f_i$ : generalized form factors:  $f_\perp \propto V, f_\parallel \propto A_1, f_0 \sim A_1, \lambda A_2$

$C^{L,R}$  drops

out in ratios:

$$F_L = \frac{|A_0^L|^2 + |A_0^R|^2}{\sum_{X=L,R} (|A_0^X|^2 + |A_\perp^X|^2 + |A_\parallel^X|^2)} = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}$$

$$A_T^{(2)} = \frac{|A_\perp^L|^2 + |A_\perp^R|^2 - |A_\parallel^L|^2 - |A_\parallel^R|^2}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}$$

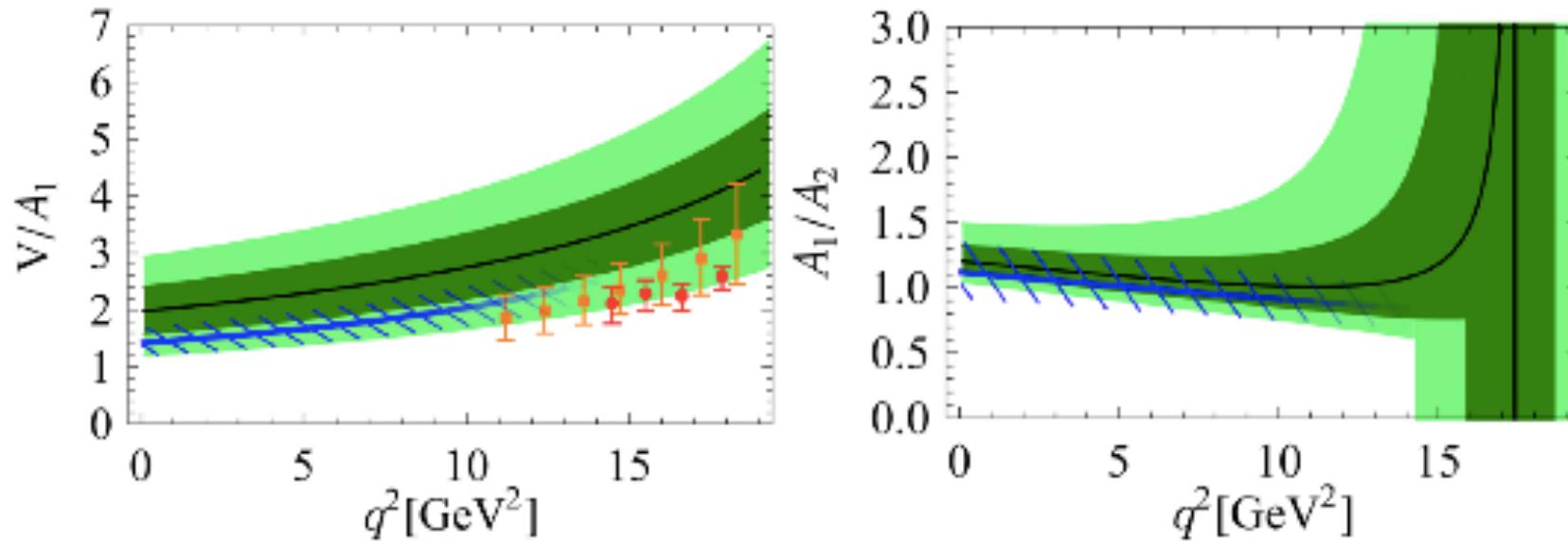


# Extracting $B \rightarrow K^*$ form factors from data

Series expansion  $z(t) \equiv z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$ ,

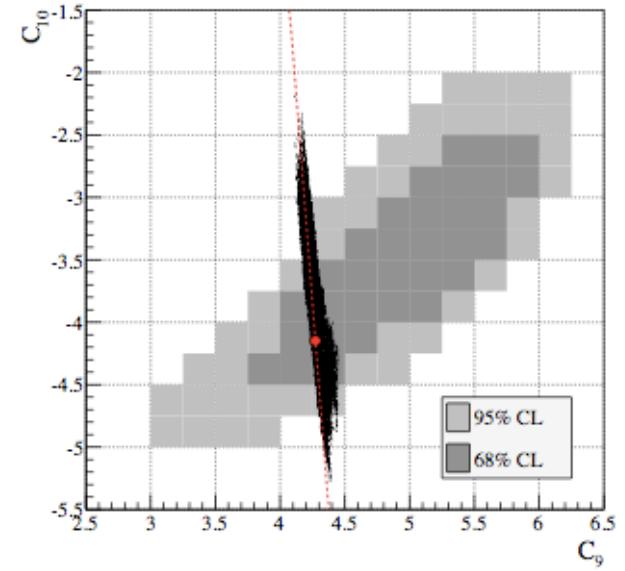
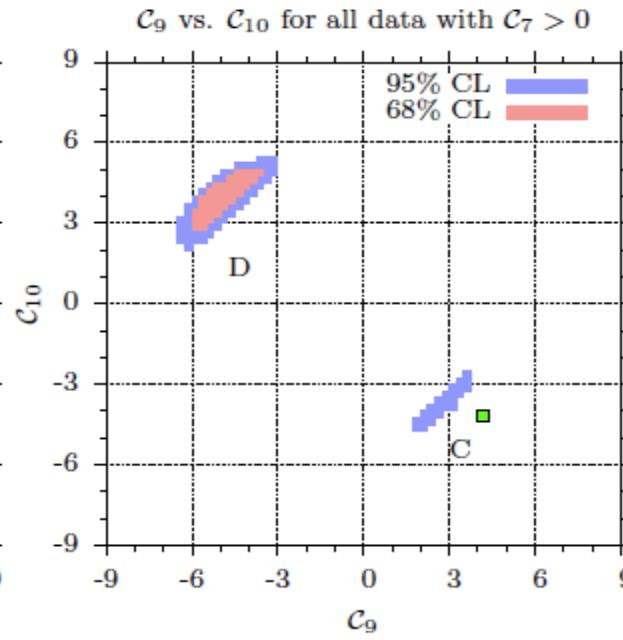
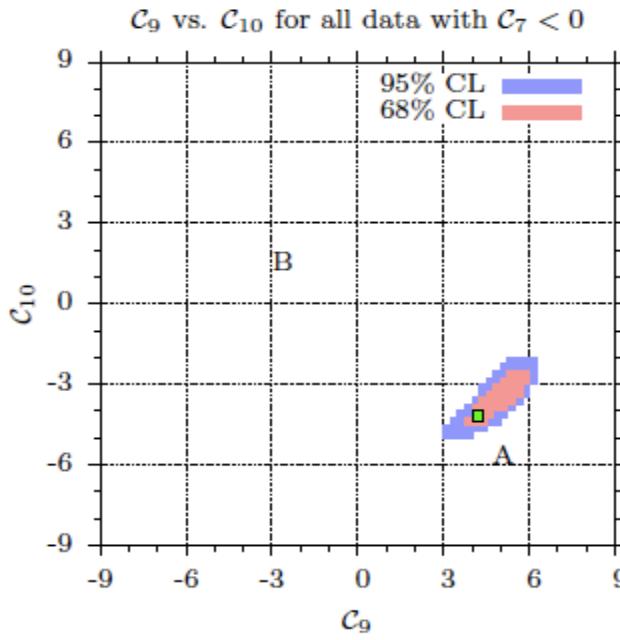
$$\hat{f}_i(t) = \frac{(\sqrt{-z(t, 0)})^m (\sqrt{z(t, t_-)})^l}{B(t) \varphi_f(t)} \sum_k \alpha_{i,k} z^k(t),$$

The best-fit results:  $\alpha_{\parallel}/\alpha_{\perp} = 0.43^{+0.11}_{-0.08}$ ,  $\alpha_0/\alpha_{\perp} = 0.15^{+0.03}_{-0.02}$



Yellow, red points; lattice QCD; blue bands: QCD sum rules Ball, Zwicky '05; green bands: 1, 2 $\sigma$  fit 1204.4444 [hep-ph]

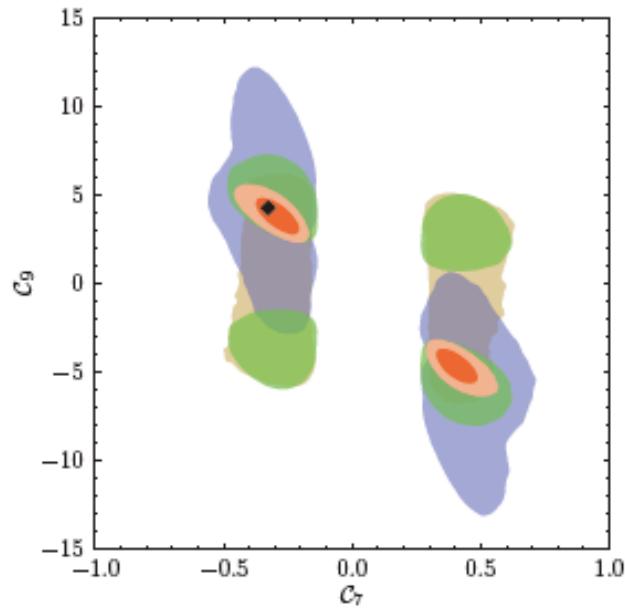
# Precision tests from global fits $\mathcal{L} \sim \sum C_i O_i$



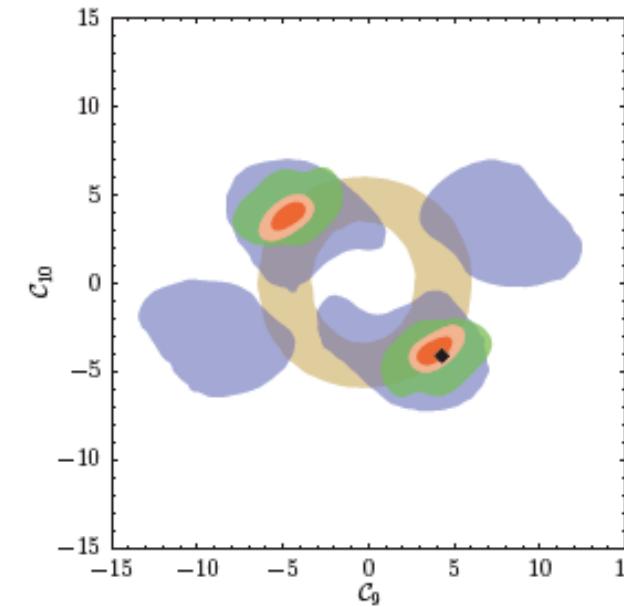
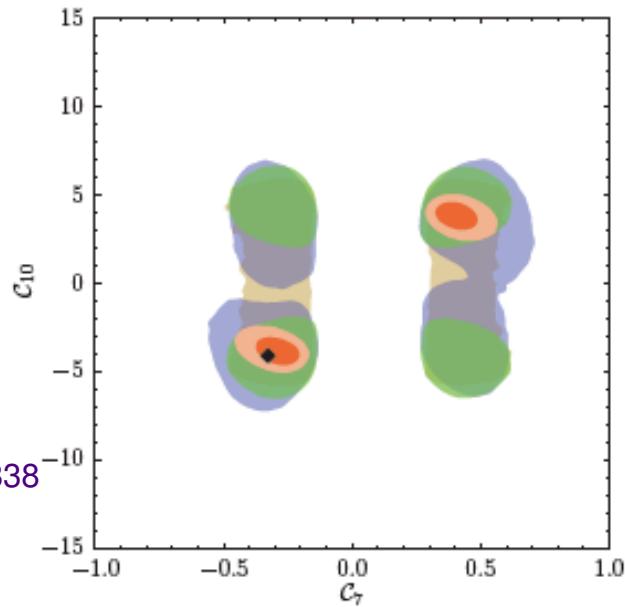
left: global analysis [1111.2558 \[hep-ph\]](#), also Altmannshofer et al '11 ; solution C is ruled out by AFB-zero. right: red dot: SM; grey areas: allowed by  $b \rightarrow s$  data; black points: SUSY model with squark flavor mixing [1205.1500 \[hep-ph\]](#)

flavor suppression with NP at  $\Lambda_{NP} = 1$  TeV:  $|\tilde{c}_{10}| < 5 \cdot 10^{-4} (4 \cdot 10^{-3})$   
 limit on scale iff no suppression  $\tilde{c}_{10} = 1$ :  $\Lambda_{NP} > 44$  TeV (16 TeV)

# Bayesian Fit to 2012 data



**Figure 1.** The marginalized 2-dimensional 95% credibility regions of the Wilson coefficients  $\mathcal{C}_{7,9,10}$  are shown when applying the  $B \rightarrow K^* \gamma$  constraints in combination with *i*) only low- and high- $q^2$  data from  $B \rightarrow K \ell \bar{\ell}$  [brown]; *ii*) only low- $q^2$  data from  $B \rightarrow K^* \ell \bar{\ell}$  [blue]; *iii*) only high- $q^2$  data from  $B \rightarrow K^* \ell \bar{\ell}$  [green]; and *iv*) all the data, including also  $B_s \rightarrow \bar{\mu} \mu$  [light red], showing as well the 68% credibility interval [red]. The SM values  $\mathcal{C}_{7,9,10}^{\text{SM}}$  are indicated by  $\blacklozenge$ .



Beaujean et al 1205.1838

Figure 5: SUSY spread of the  $A_{FB}(B \rightarrow K^* \mu^+ \mu^-)$  at low  $q^2$  in function of the lightest stop mass, for  $\tan \beta = 50$  (upper panel) and  $\tan \beta = 30$  (lower panel), in the left for  $A_0 = 0$  and in the right for  $A_0 = -1000$  GeV.

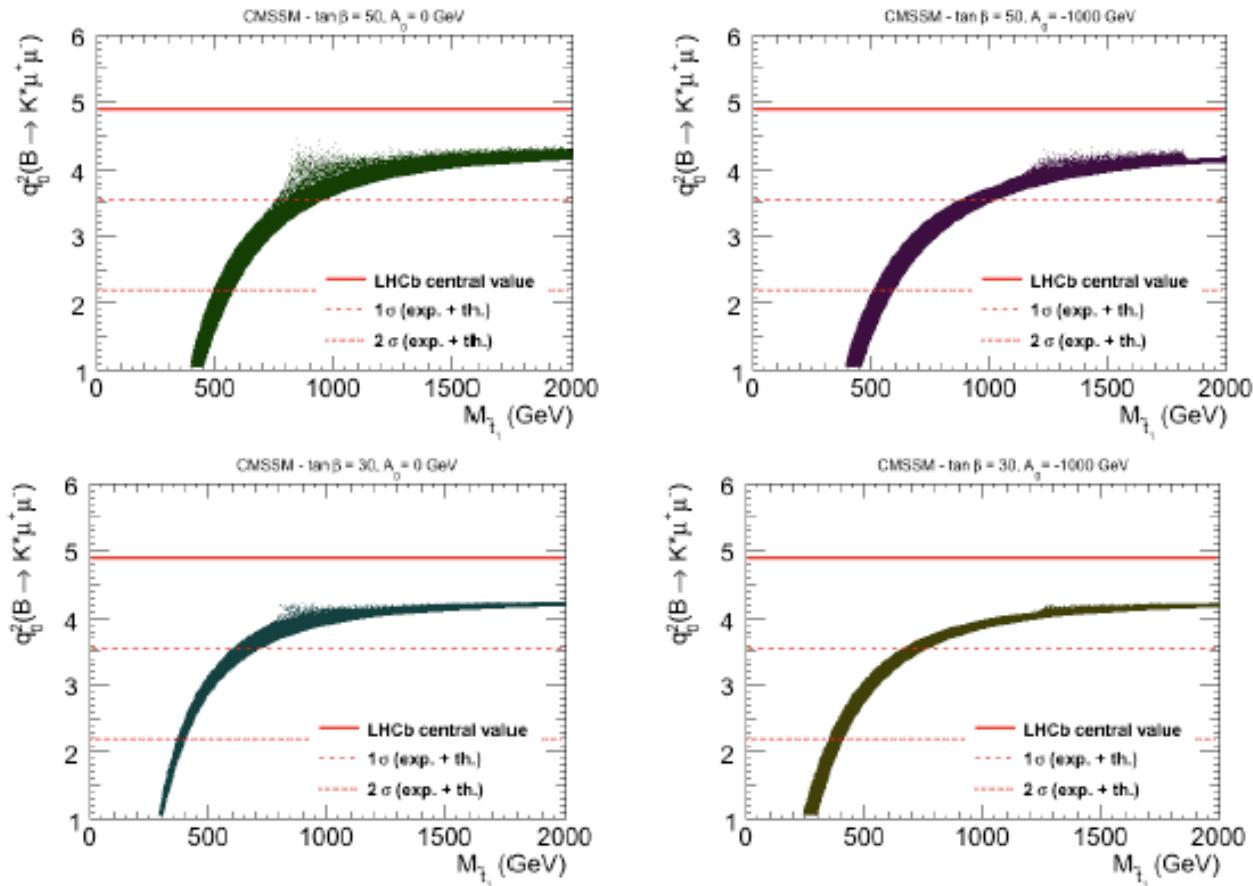


Figure 6: SUSY spread of the  $A_{FB}(B \rightarrow K^* \mu^+ \mu^-)$  zero-crossing, similar to Fig. 5.

Mahmoudi et al, 1205.1845  $q_0^2|_{\text{LHCb}} = 4.9^{+1.1}_{-1.3} \text{ GeV}^2$

## More Opportunities w. $B \rightarrow V(\rightarrow PP)\mu^+\mu^-$

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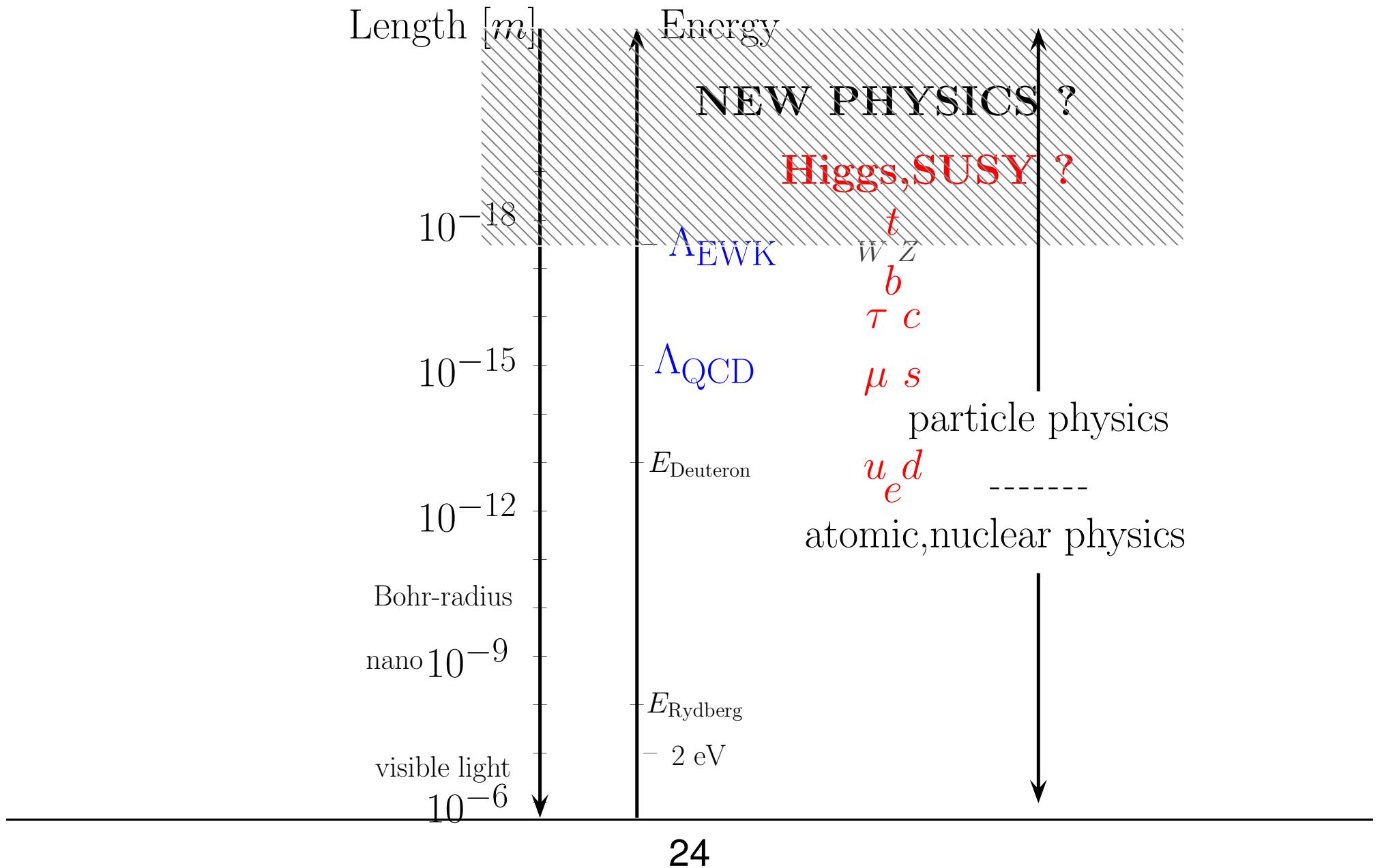
**CP-asymmetries** from angular distribution  $A_i \propto J_i - \bar{J}_i$ : SM: all doubly Cabibbo-suppressed and null tests of the SM.

$A_3, A_9$  vanish in SM by helicity conservation: sens. to RH currents  
 $A_3, A_9, (A_6)$  can be extracted from single-diff distribution in  $\Phi(\Theta_l)$   
 $A_{7,8,9}$  T-odd: not suppressed by small strong phases; O(1) with BSM  
 $A_{5,6,8,9}$  CP-odd: can be extracted without tagging from  $\Gamma + \bar{\Gamma}$ ;  
advantageous for  $B_s, \bar{B}_s \rightarrow (\Phi \rightarrow K^+K^-)\mu^+\mu^-$ .

**Low recoil region** allows to design (high- $q^2$ ) observables which are

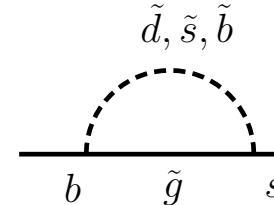
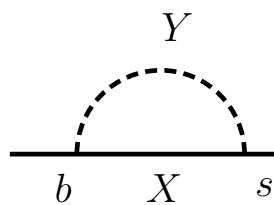
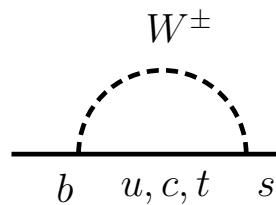
- independent of form factors ( $H_T^{(2,3)}, a_{\text{CP}}^i$ )
- independent of short-distance coefficients and test the form factors
- independent of either ones and test the theoretical low recoil framework  $H_T^{(1)} = 1, H_T^{(2)}/H_T^{(3)} = 1$

Bobeth et al 1006.5013, and 1105.0376



# Terascale Flavor facing todays FCNC Data

$$\mathcal{A}_{FCNC} \sim \underbrace{K_{ij} K_{kj}^*}_{mixing} \times \underbrace{\Delta m_{ik}^2}_{splitting(GIM)} \times \underbrace{(1/\Lambda_{NP})^n}_{decoupling}$$



With no suppression from flavor (mixing nor splitting) at 95 % C.L.:

	$K^0 \bar{K}^0$	$D^0 \bar{D}^0$	$B_d^0 \bar{B}_d^0$	$B_s^0 \bar{B}_s^0$
$\Lambda_{NP}$ [TeV]	$2 \cdot 10^5$	$5 \cdot 10^3$	$2 \cdot 10^3$	$3 \cdot 10^2$

Bona et al, 0707.0636 [hep-ph]

Connection to TeV-scale is lost, or TeV-scale flavor non-generic!

# Quark Flavor Masses and Mixings in SM

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Modulo "hints" all hadronic flavor changing data are currently ok with the SM within uncertainties.

6 masses, 3 angles and 1 phase:

$$Y_u \sim \begin{pmatrix} 10^{-5} & -0.002 & 0.008 + i 0.003 \\ 10^{-6} & 0.007 & -0.04 \\ 10^{-8} + i 10^{-7} & 0.0003 & 0.94 \end{pmatrix}$$
$$Y_d \sim \text{diag}(10^{-5}, 5 \cdot 10^{-4}, 0.025)$$

Very peculiar pattern.

- \* The Standard Model is a good description of microscopic processes up to energies of  $\mathcal{O}(100)$  GeV.
- \* The forthcoming searches at LHC and precision experiments will explore the Terascale. What are the flavor quantum numbers of new particles/SM partners ?
- \* Existing FCNC-data imply already strong constraints on the flavor structure of physics beyond the SM. These bounds will be tightened significantly.
- \* The observation of New Physics flavor couplings could point towards the origin of generational mixing and hierarchies, i.e., flavor.

- \* It's fun to have data!

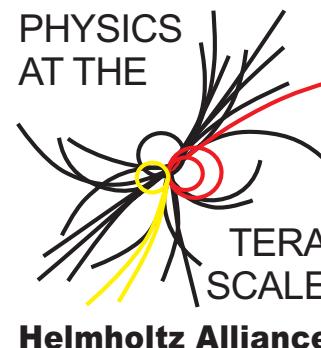
# Thanks for Flavor-Collaboration and Support to

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Arnd Behring, Christoph Bobeth, Christian Gross, Christian Hambrock, Yonit Hochberg, Yossi Nir, Danny van Dyk, Jong Soo Kim, Stefan Schacht, Henning Sedello, Christian Wacker



**bmb+f** - Förderschwerpunkt  
Elementarteilchenphysik  
Großgeräte der physikalischen  
Grundlagenforschung

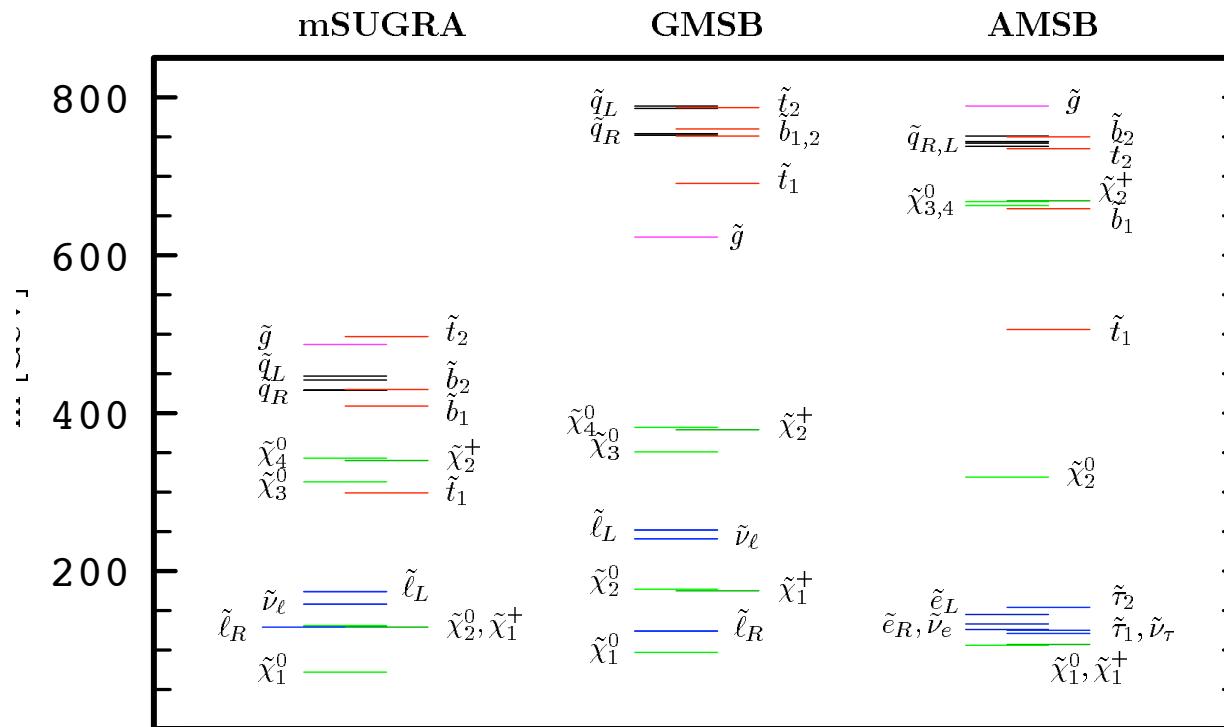


Abstract: Flavor physics studies have identified the standard model as the dominant source of quark flavor and CP violation (modulo "anomalies"). This has strong implications for the physics at the TeV scale. We briefly review the status of flavor physics and discuss how one could discover a breakdown of the standard model in flavor physics and understand flavor in and beyond the standard model. Emphasis is given on rare exclusive FCNC-decays into di-muons, which are well-suited for precision studies at the Large Hadron Collider (LHC) experiments.

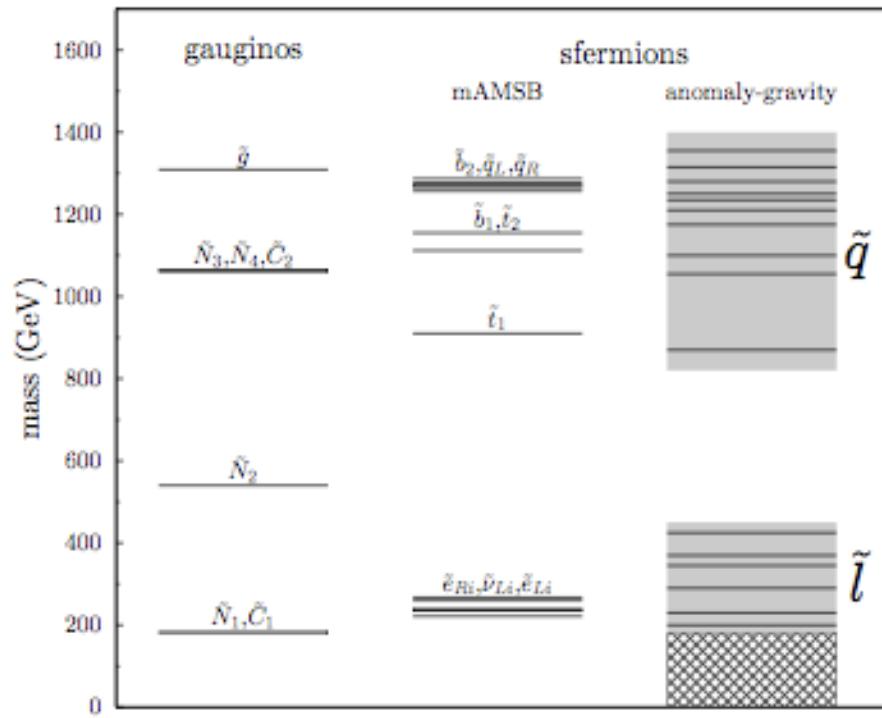
New TeV-sector; what's the flavor of the SM partners?

Is the flavored spectrum of  $\tilde{q}$  and  $\tilde{\ell}$  degenerate?

TESLA TDR Part III '01



or with large mass splitting?



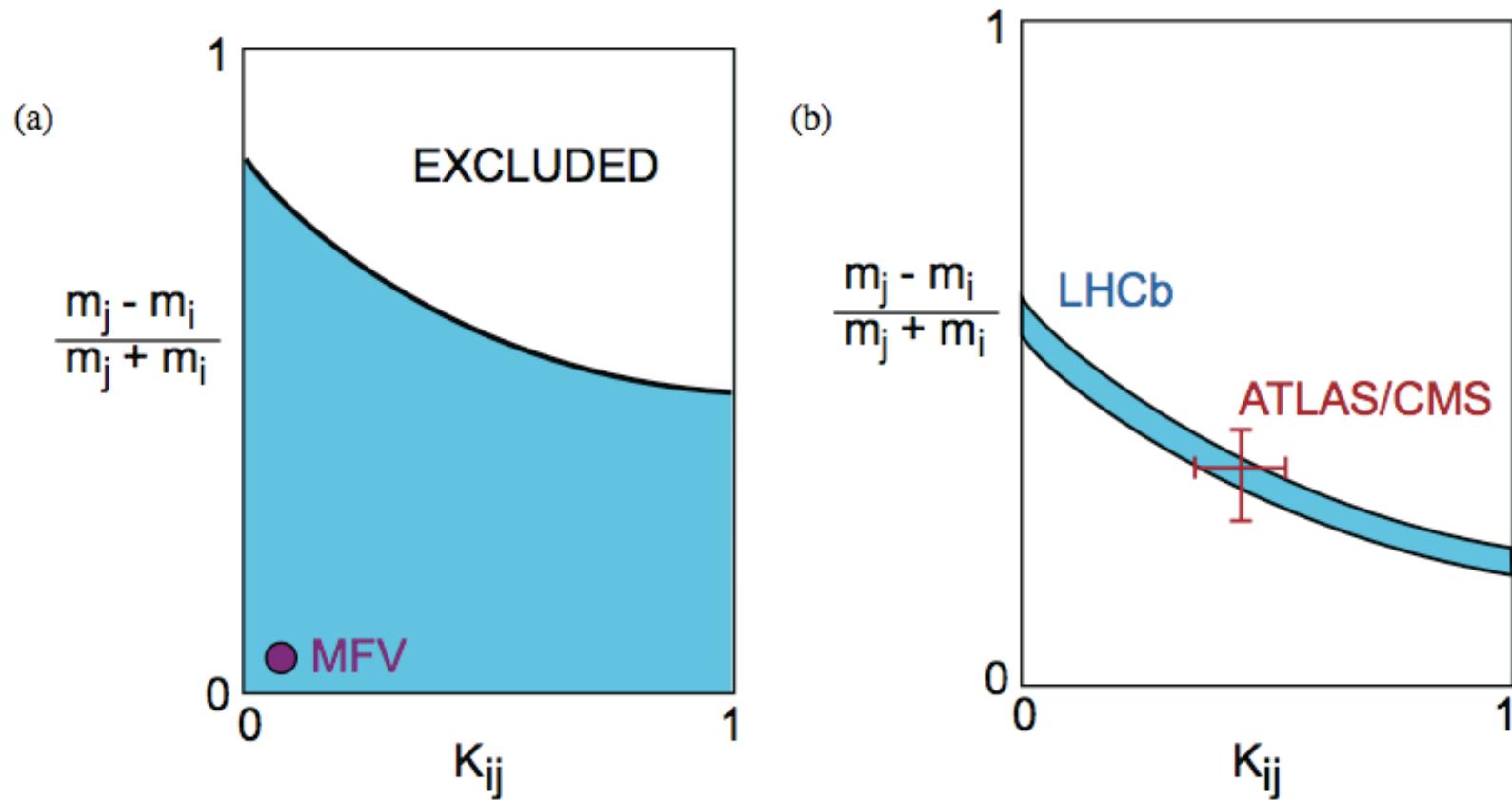
C.Gross and GH, 1101.5352 [hep-ph]

and how about flavor mixing and CP violation?

# Testing Flavor at the Terascale

FCNC loops probe product of flavor mixing  $K_{ij}$  and splitting  $\Delta m_{ij}^2$

$$\mathcal{A}_{FCNC} \sim K_{ij} K_{kj}^* \cdot \Delta m_{ik}^2$$

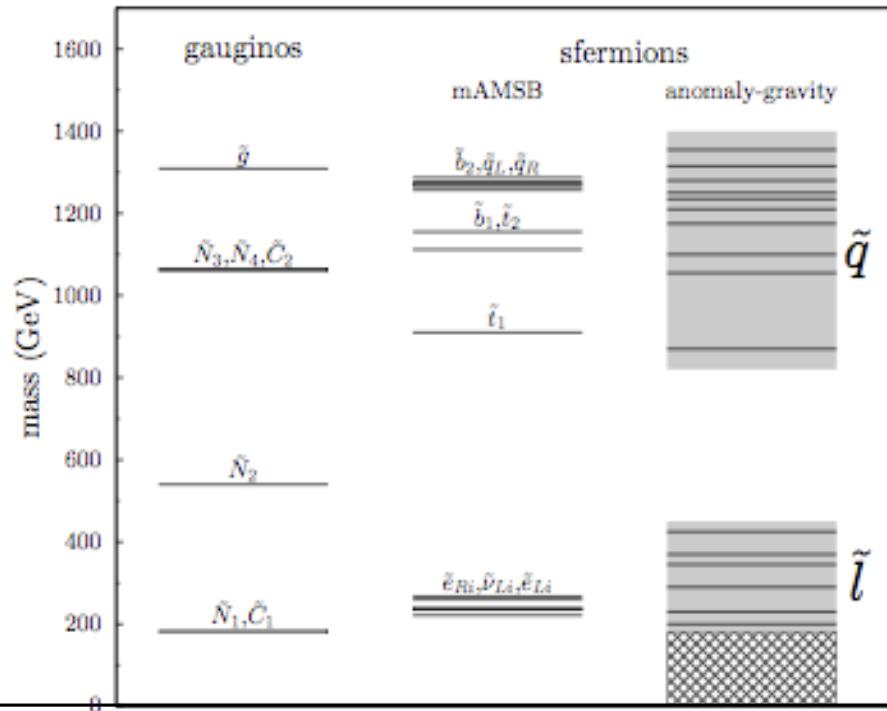


figs from Y.Nir 1010.2666 [hep-ph] schematic a) today b) hypothetical

# Collider-Flavor Physics (Examples)

Measuring the mass splitting  $\Delta m_{ij}$ :

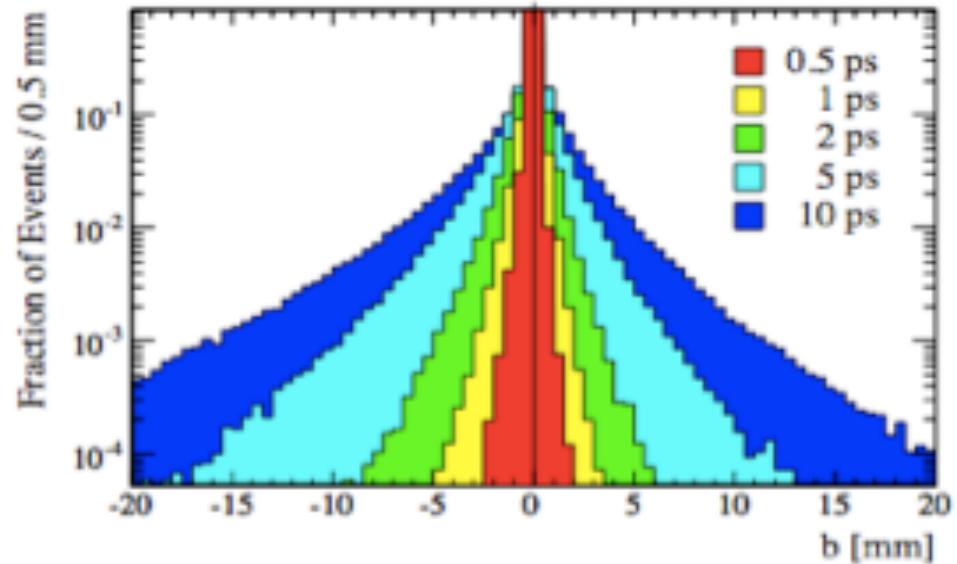
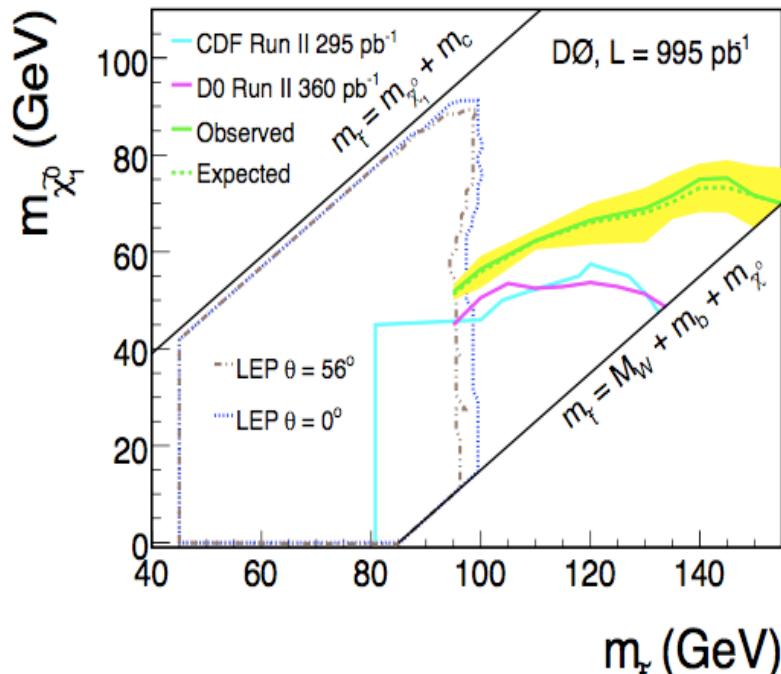
in cascades  $\chi_2^0 \rightarrow \chi_1^0 l^+ l^-$  and comparing  $e^+ e^-$  with  $\mu^+ \mu^-$  edges  
Allanach et al 0801.3666 [hep-ph] mSUGRA/CMSSM: splitting percent-permille; Hybrid anomaly-gravity:  $O(1)$  slepton splitting, uses alignment  $|K_{ij}| \ll 1$  to escape FCNC bounds 1101.5352 [hep-ph]



# Collider-Flavor Physics (Examples)

Measuring the flavor mixing  $K_{ij}$ : from decay length measurements with long lived stop decaying predominantly FCNC  $\tilde{t} \rightarrow c\chi_1^0$

left: 0803.2263 [hep-ex], right: GH, JS.Kim, H.Sedello 0910.2124 [hep-ph]



Light stops are produced with low BGD in association with like-sign tops  $pp \rightarrow \tilde{t}^*\tilde{t}^*tt, \tilde{t}\tilde{t}\overline{t}\overline{t}$  [Kraml, Raklev '05](#)

Up to 10 events with 1  $fb^{-1}$  (no detector effects, 14 TeV).

# Flavor Masses and Mixings in SM

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$$Y_u \sim \begin{pmatrix} 10^{-5} & -0.002 & 0.008 + i 0.003 \\ 10^{-6} & 0.007 & -0.04 \\ 10^{-8} + i 10^{-7} & 0.0003 & 0.94 \end{pmatrix}$$

$$Y_d \sim \text{diag} \left( 10^{-5}, 5 \cdot 10^{-4}, 0.025 \right) \left( \cdot \frac{\langle H_u \rangle}{\langle H_d \rangle} \right)$$

$$Y_e \sim \text{diag} \left( 10^{-6}, 6 \cdot 10^{-4}, 0.01 \right) \left( \cdot \frac{\langle H_u \rangle}{\langle H_d \rangle} \right)$$

Very peculiar pattern.

# Mismatch SM vs BSM Flavor

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$Y_u Y_u^\dagger, Y_u^\dagger Y_u, Y_d Y_d^\dagger, \dots$  (SM flavor)

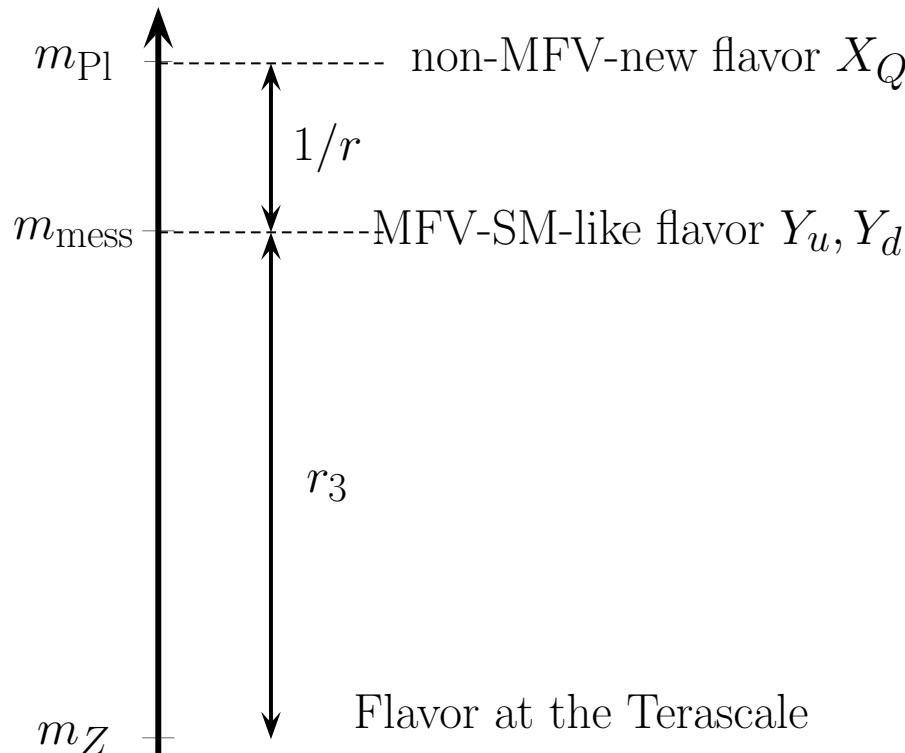
squark mass terms  $\mathcal{L} = \tilde{Q}_L^\dagger (M_{\tilde{Q}_L}^2)_{ij} \tilde{Q}_L j + \dots$  ( sflavor)

Could have common origin, e.g. Froggatt-Nielsen symmetries:

$$(Y_u)_{ij} \sim \epsilon^{Z_{uj} - Z_{qi}}, \quad (M_{\tilde{Q}_L}^2)_{ij} \sim \epsilon^{Z_{qj} - Z_{qi}}$$

or not, as in anarchy scenarios  $(M_{\tilde{Q}_L}^2)_{ij} \sim O(1)$ .

# Hybrid Gauge-Gravity Mediation



$$M_{\tilde{Q}_L}^2(m_Z) \sim \tilde{m}_{Q_L}^2 \cdot \underbrace{(r_3 \mathbf{1} + c_u Y_u Y_u^\dagger + c_d Y_d Y_d^\dagger)}_{SM-like\ flavor} + \underbrace{r X_{Q_L}}_{BSM\ flavor}$$

flavor observables probe off-diagonals:  $\sim r/r_3 X_{Q_L}$ .

observable signatures+ experimental program [GH,Hochberg,Nir 0812.0511, 1001.1513](#)