Dirac neutrinos from flavor symmetry

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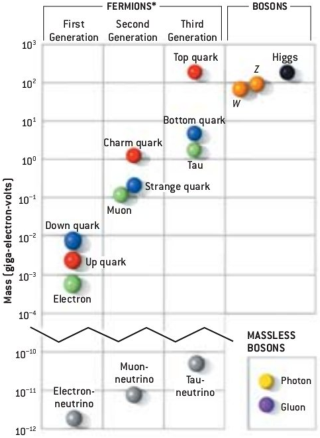
Matter content in the Standard Model (SM):



Refreshing the Flavor Problem:

- $m_\nu \neq 0$, neutrinos DO have mass and is pretty small:

$$\Delta m^2_{21} = 7.59^{+0.20}_{-0.18} \times 10^{-5}\mathrm{eV}^2 \text{ and } |\Delta m^2_{31}| = 2.5^{+0.09}_{-0.16} \times 10^{-3}\mathrm{eV}^2.$$

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- what is the **neutrino nature** (Majorana Vs Dirac)?,
- why 3 families?

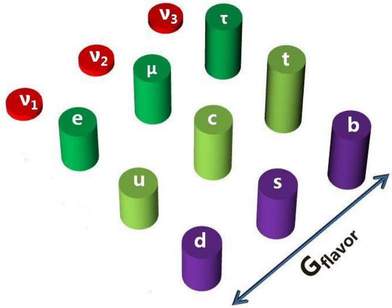
...

Then:

We need to go *beyond* the SM.
One possibility is consider an additional symmetry, $SM \times \mathcal{G}$, with which:

- is justified the presence of **other fields** (RH-neutrinos, H_n, flavons, etc.) and
- is provided some structure to the fermion mass matrices.

A good way for explaining fermion masses and mixing angles is using relations among the families, **horizontal symmetries**, $\mathcal{G} = \mathcal{G}_{\mathcal{F}}$.



where

1) $\Lambda_F >> \Lambda_{EW}$. Effective theories.
2) $\Lambda_F \sim \Lambda_{EW}$. Renormalizable models.

Flavor Symmetries

The additional symmetry group $\mathcal{G}_F$ could be:

- *Continuous*: $U(1)$[1], $U(2)$;
- *Discrete*: Z_n, Q_4[2], D_6[3], $A_{4,5}$[4], $S_{3,4}$[5], T'[6]

[1] Froggat and Nielsen...
[2] Frigerio, Hagedorn, Aranda, C.B., Rojas, Ramos...
[3] Babu...
[4] Altarelli, Feruglio, Merlo, Ma, Tanimoto, Valle...
[5] Meloni, Mondragon, A. and M. , Morisi, Peinado...
[6] Aranda, Chen, Frampton, Merlo,...

What we want to do?

We want to focus on the possibility that Majorana mass terms are not allowed from the flavor symmetry, i. e. neutrinos are Dirac fermions

Let us consider one right-handed neutrino (N_R) per generation and that these fields are accommodated in a 3-dimensional irreducible representation of certain $\mathcal{G}_F$. Then, we have that in order to forbid any Majorana operator, e.g. $N_R^T N_R$,

- ★ $N_R \cong \mathcal{R}$ such that $\mathcal{R} \otimes \mathcal{R}$ is not invariant $\forall \mathcal{R} \in \mathcal{G}_F$.

We found that ★-requirement is fulfilled by some non-Abelian groups.

non-Abelian groups[7]

- $\Delta(3N^2)$ for $N \geq 3$: these groups contain nine singlets and $(N^2-3)/3$ triplets for $N = 3\mathbb{Z}$. Otherwise, for $N \neq 3\mathbb{Z}$, they have three singlets and $(N^2-1)/3$ triplets.

[7] H. Ishimori et al., Prog. Theor. Phys.Suppl. 183, 1 (2010)

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Why $N \geq 3$?:

i) For N=1, $\Delta(3)$ is isomorphic to Z_3. This group is **discarded** because it is Abelian.

ii) For N=2, $\Delta(12)$ is isomorphic to A_4 but $\mathbf{3} \otimes \mathbf{3} \supset \mathbf{1}$, then is also **discarded**.

iii) For N=3, $\Delta(27)$ has two 3-dimensional ($\mathbf{3}_1$ and $\mathbf{3}_2$) and nine 1-dimensional irreps ($\mathbf{1}_i$ with $i = 1, ..., 9$). In this group $\mathbf{3}_i \otimes \mathbf{3}_i = \mathbf{3}_j \oplus \mathbf{3}_j \oplus \mathbf{3}_j$ then is a **candidate for creating a model of Dirac neutrinos**.

[7] H. Ishimori et al., Prog. Theor. Phys.Suppl. 183, 1 (2010)

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- $\Sigma(3N^3)$ for $N \geq 3$: the set of groups with $N(N^2+8)/3$ conjugacy classes, $3N$ singlets and $N(N^2-1)/3$ triplets.

[7] H. Ishimori et al., Prog. Theor. Phys.Suppl. 183, 1 (2010)

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- T_N for $N = 7, 13, 19, 31, 43, 49$: these groups have 3 singlets and $(N-1)/3$ three-dimensional irreducible representations.

[7] H. Ishimori et al., Prog. Theor. Phys.Suppl. 183, 1 (2010)

The model:

	$\overline{L}$	ℓ_{1R}	ℓ_{2R}	ℓ_{3R}	N_R	H
$SU(2)_L$	**2**	**1**	**1**	**1**	**1**	**2**
$\Delta(27)$	**3**	**1**	$\mathbf{1}'$	$\mathbf{1}''$	**3**	$\mathbf{3}'$

Table: Matter assignments of the model.

The most general invariant Lagrangian for leptons is written as

$$\mathcal{L}_\ell = \sum_{i=1}^{3} Y_i^\ell \bar{L} \ell_{iR} H + Y^\nu \bar{L} N_R \tilde{H} + h.c.,$$

where we use the compact notation $H = (H_1, H_2, H_3)$ and $\tilde{H} = (\tilde{H}_1, \tilde{H}_2, \tilde{H}_3)$ with $\tilde{H}_i \equiv i\sigma_2 H^*$.

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and mass matrices, after EWSB, are:

$$\begin{aligned} M_\nu &= \begin{pmatrix} a v_1 & b v_3 & c v_2 \\ c v_3 & a v_2 & b v_1 \\ b v_2 & c v_1 & a v_3 \end{pmatrix} \\ M_\ell &= \begin{pmatrix} Y_1^\ell v_1 & Y_2^\ell v_1 & Y_3^\ell v_1 \\ Y_1^\ell v_2 & \omega Y_2^\ell v_2 & \omega^2 Y_3^\ell v_2 \\ Y_1^\ell v_3 & \omega^2 Y_2^\ell v_3 & \omega Y_3^\ell v_3 \end{pmatrix} \end{aligned} \tag{1}$$

where $\omega = e^{2\pi i/3}$ and v_i are Higgs scalar vevs.

The vev aligment[7] $\langle H \rangle = v(1,1,1)$ turns out to be *natural* in $\Delta(27)$ and taking it into account the mass matrices get the following form,

$$\begin{aligned} M_\nu &= v \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \\ M_\ell &= v \begin{pmatrix} Y_1^\ell & Y_2^\ell & Y_3^\ell \\ Y_1^\ell & \omega Y_2^\ell & \omega^2 Y_3^\ell \\ Y_1^\ell & \omega^2 Y_2^\ell & \omega Y_3^\ell \end{pmatrix} \end{aligned} \tag{2}$$

[7]This minimum is the global one, for more details see, C. Nishi, Phys. Rev. D 88, 033010 (2013),[1306.0877].

Warning

The lepton mixing matrix is,

$$U = U_\ell^\dagger U_\nu \tag{3}$$

where U_ℓ and U_ν are those which diagonalize

$$M_\ell M_\ell^\dagger \quad \text{and} \quad M_\nu M_\nu^\dagger, \tag{4}$$

respectively. But when the vev alignment is $\langle H \rangle = v(1,1,1)$ we have that,

$$U_\ell = U_\nu = U_\omega \equiv \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \tag{5}$$

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and then

$$U = U_\ell^\dagger U_\nu = \mathbb{I}. \quad \text{This is not compatible with data!} \tag{6}$$

We found that if we consider small deviations of the vev alignment, e.i. $\langle H \rangle = \hat{v}(1+\epsilon_1, 1+\epsilon_2, 1)^T$, the mass matrices become

$$M_\nu = \hat{v}\begin{pmatrix} a(1+\epsilon_1) & b & c(1+\epsilon_2) \\ c & a(1+\epsilon_2) & b(1+\epsilon_1) \\ b(1+\epsilon_2) & c(1+\epsilon_1) & a \end{pmatrix} \tag{7}$$

$$M_\ell = \hat{v}\begin{pmatrix} Y_1^\ell(1+\epsilon_1) & Y_2^\ell(1+\epsilon_1) & Y_3^\ell(1+\epsilon_1) \\ Y_1^\ell(1+\epsilon_2) & \omega Y_2^\ell(1+\epsilon_2) & \omega^2 Y_3^\ell(1+\epsilon_2) \\ Y_1^\ell & \omega^2 Y_2^\ell & \omega Y_3^\ell \end{pmatrix}.$$

and the lepton mixing matrix is not the identity,

$$U = U_\ell^\dagger U_\nu \neq \mathbb{I}\,. \tag{8}$$

Then, we rewrote the parameters (a, b, c, Y_i^ℓ) in terms of the neutrino mass splittings (Δm_{21}^2 and $|\Delta m_{31}^2|$) and charge lepton masses, deviated the vev up to 30% and selected those solutions which satisfy the global fits[8] for the mixing angles at 3σ,

$$0.017 < \sin^2\theta_{13} < 0.033$$

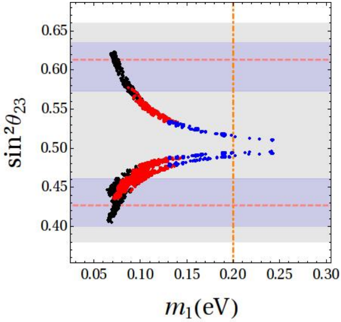
$$0.36(0.37) < \sin^2\theta_{23} < 0.68(0.67)\ \text{NH(IH)}$$

$$0.27 < \sin^2\theta_{12} < 0.37.$$

[8] D. Forero, M. Tortola and J. W. F.Valle, Phys.Rev. D86, 073012 (2012).

NH case

We got a correlation between the atmospheric angle and the lightest neutrino mass.



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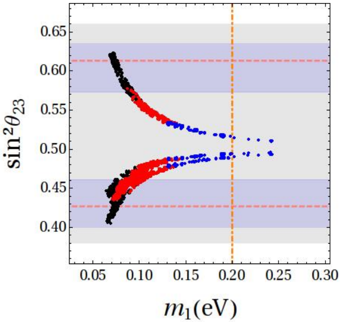

Figure: NH case: the horizontal dotted lines represent the best fit values, the blue and gray horizontal bands are the 1σ and 2σ allowed ranges, respectively. The blue, red and black points are model expectations corresponding to vev deviations of 10%, 20% and 30% respectively. The vertical dot-dashed line indicates KATRIN's sensitivity.

IH case

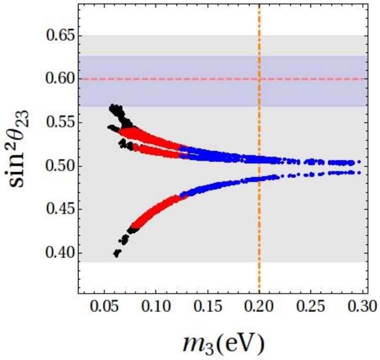

Figure: IH case: note that in this case a 30% vev deviation is not enough to reach the best fit value of θ_{23}.

We also checked that the model is not ruled out by the decay $\mu \to e\gamma$. The experimental bound[9] set by MEG is of the order $\mathcal{O} \sim 10^{-13}$.

Cases	$\mathrm{Br}^{\mathrm{th}}(\mu \to e\gamma)$	m_{ν_1} (eV)	$\sin^2\theta_{23}$
i)	1.98×10^{-14}	0.2399	0.4956
ii)	1.74×10^{-14}	0.0930	0.4615
iii)	1.65×10^{-14}	0.0762	0.6107

Theoretical branching ratios for the process $\mu \to e\gamma$ for three different cases corresponding to three different sets of (ϵ_1,ϵ_2), m_{ν_1}, and $\sin^2\theta_{23}$.

[9]MEG Collaboration, J. Adam et al.,1303.0754

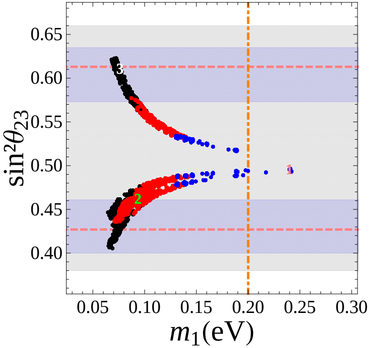

Figure: This plot is for NH in our model and $i = 1, 2, 3$ are the corresponding branching ratios for those sets in last Table.

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- **Thank you.**

BACKUP

The scalar potential[10] invariant under Standard Model and $\Delta(27)$,

$$V_{\Delta(27)} = \mu^2 \left[H_1^\dagger H_1 + H_2^\dagger H_2 + H_3^\dagger H_3\right] + \lambda_0 \left[H_1^\dagger H_1 + H_2^\dagger H_2 + H_3^\dagger H_3\right]^2$$

$$+ \lambda_1 \left[(H_1^\dagger H_1)^2 + (H_2^\dagger H_2)^2 + (H_3^\dagger H_3)^2\right]$$

$$- \lambda_1 \left[(H_1^\dagger H_1)(H_2^\dagger H_2) + (H_2^\dagger H_2)(H_3^\dagger H_3) + (H_3^\dagger H_3)(H_1^\dagger H_1)\right]$$

$$+ \lambda_2 \left[\left|H_1^\dagger H_1\right|^2 + \left|H_2^\dagger H_2\right|^2 + \left|H_3^\dagger H_3\right|^2\right]$$

$$+ \lambda_3 \left[(H_1^\dagger H_2)(H_1^\dagger H_3) + (H_2^\dagger H_3)(H_2^\dagger H_1) + (H_3^\dagger H_1)(H_3^\dagger H_2)\right] + h.c.$$

[10] G. Branco, J. Gerard and W. Grimus, Phys.Lett. B136, 383 (1984)

On the other hand, to this scalar potential we add soft breaking terms (SBTs) so as to induce a small deviation from the vev alignment (1,1,1) [11], namely

$$V' = V_{\Delta(27)} + V_{SB} \tag{9}$$

where

$$V_{SB} = \mu_{12}^2 H_1^\dagger H_2 + \mu_{13}^2 H_1^\dagger H_3 + \mu_{23}^2 H_2^\dagger H_3 + h.c. \tag{10}$$

[11] This leads to a correlation between the atmospheric angle and the lightest neutrino mass as showed

Cases	m_{h_1}	m_{h_2}	m_{h_3}	m_{A_2}	m_{A_3}	$m_{H_2^\pm}$	$m_{H_3^\pm}$
i)	124.68	431.64	458.96	434.60	462.34	300.14	338.65
ii)	124.83	254.94	291.34	405.28	414.29	240.56	255.36
iii)	124.89	274.21	337.97	398.77	402.60	292.562	304.14

Cases	λ_0	λ_1	λ_2	λ_3
i)	1.69	1.88	-1.89	-1.40
ii)	0.78	-0.39	1.71	-1.83
iii)	1.99	-1.12	0.53	-1.90