Week 11: The Brout-Englert-Higgs mechanism

04 December 2017 10:48

The sigma model

Consider the following Lagrangian for two Dirac fields and four scalar fields $\sigma, \pi_1, \pi_2, \pi_3$

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \qquad \vec{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} \qquad \tau_i \equiv \text{Pauli matrices}$$

$$\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \vec{\pi}) \cdot (\partial^\mu \vec{\pi}) + \frac{1}{2} \partial_\mu \sigma \, \partial^\mu \sigma -$$
$$- g \bar{\psi} (\sigma - i \vec{\pi} \cdot \vec{\tau} \gamma_5) \psi - \frac{m^2}{2} (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2$$

We now reorganise the scalar fields as follows

$$\Sigma \equiv \sigma + i \vec{\tau} \cdot \vec{\pi} = \begin{pmatrix} \sigma + i\pi_3 & i\pi_1 + \pi_2 \\ i\pi_1 - \pi_2 & \sigma - i\pi_3 \end{pmatrix}$$

$$\sigma^2 + \vec{\pi}^2 = \frac{1}{2} \mathrm{Tr}(\Sigma^\dagger \Sigma)$$

We then obtain

$$\mathcal{L} = \bar{\psi}_L i \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i \gamma^\mu \partial_\mu \psi_R + \frac{1}{4} \mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma)$$
$$- \frac{\mu^2}{4} \mathrm{Tr}(\Sigma^\dagger \Sigma) - \frac{\lambda}{16} [\mathrm{Tr}(\Sigma^\dagger \Sigma)]^2 - g(\bar{\psi}_L \Sigma \psi_R + \bar{\psi}_R \Sigma^\dagger \psi_L)$$

This Lagrangian is invariant under two SU(2) transformations

$$\psi_L \to U_L \psi_L \qquad U_L = e^{-i \vec{\alpha}_L \cdot \frac{\vec{\tau}}{2}} \qquad \left(\tfrac{1}{2}, 0\right)$$

$$\psi_R \to U_R \psi_R \qquad U_R = e^{-i \vec{\alpha}_R \cdot \frac{\vec{\tau}}{2}} \qquad \left(0, \tfrac{1}{2}\right)$$

$$\Sigma \to U_L \Sigma U_R^\dagger \qquad \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

This symmetry gives two conserved currents J_L^μ and J_R^μ

Consider now the combinations

$$J^\mu_V = J^\mu_L + J^\mu_R \qquad\qquad J^\mu_V = J^\mu_L + J^\mu_R$$

$$U_L = e^{-i\vec{\alpha}_V \cdot \frac{\vec{\tau}}{2}} = U_R \equiv U_V \qquad U_L = e^{-i\vec{\alpha}_A \cdot \frac{\vec{\tau}}{2}} = U_R^\dagger \equiv U_A$$

$$\Sigma \to U_V \Sigma U_V^\dagger \qquad\qquad \Sigma \to U_A \Sigma U_A$$

Consider the potential

$$V(\sigma, \vec{\pi}) = \frac{m^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$

For $m^2 < 0$, this potential has a minimum for

$$\sigma^2 + \vec{\pi}^2 = -\frac{m^2}{\lambda} \equiv v^2$$

We now choose a specific vacuum configuration

$$\sigma = v \quad \vec{\pi} = 0$$

Let us now expand Σ around this configuration

$$\Sigma = v + h + i\vec{\tau}\cdot\vec{\pi} = v + \tilde{\Sigma}$$

$$\mathrm{Tr}(\Sigma^\dagger \Sigma) = 2v^2 + v\,\mathrm{Tr}(\tilde{\Sigma} + \tilde{\Sigma}^\dagger) + \mathrm{Tr}(\tilde{\Sigma}^\dagger \tilde{\Sigma})$$

The term $\mathrm{Tr}(\tilde{\Sigma} + \tilde{\Sigma}^\dagger)$ is invariant under $SU(2)_V$ but not under $SU(2)_A$. We have then spontaneously broken the symmetry as follows

$$SU(2)_L \times SU(2)_R \simeq SU(2)_V \times SU(2)_A \to SU(2)_V$$

The Lagrangian above describes the so-called <u>linear sigma model</u>
If the field h is very heavy, it still gives an effective theory for the pions, the <u>non-linear sigma model</u>

The resulting Lagrangian is

$$\mathcal{L} = \bar\psi i\gamma^\mu\partial_\mu\psi - g\left[\bar\psi_L(v+\tilde\Sigma)\psi_R + \bar\psi_R(v+\tilde\Sigma^\dagger)\psi_L\right] +$$

$$+\frac{1}{4}\mathrm{Tr}\left[\partial_\mu\tilde\Sigma^\dagger\partial^\mu\tilde\Sigma\right] + \frac{\lambda v^2}{4}\left[2v^2 + v\,\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger) + \mathrm{Tr}\,\tilde\Sigma^\dagger\tilde\Sigma\right] +$$

$$-\frac{\lambda}{16}\left[2v^2 + v\,\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger) + \mathrm{Tr}\,\tilde\Sigma^\dagger\tilde\Sigma\right]^2 =$$

$$= \bar\psi i\gamma^\mu\partial_\mu\psi - gv\bar\psi\psi - g\left[\bar\psi_L\tilde\Sigma\psi_R + \bar\psi_R\tilde\Sigma^\dagger\psi_L\right] +$$

$$+\frac{1}{4}\mathrm{Tr}\left[\partial_\mu\tilde\Sigma^\dagger\partial^\mu\tilde\Sigma\right] + \frac{\lambda v^4}{2} + \cancel{\frac{\lambda v^3}{4}\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger)} + \cancel{\frac{\lambda v^2}{4}\mathrm{Tr}(\tilde\Sigma^\dagger\tilde\Sigma)} +$$

$$-\frac{\lambda v^4}{4} - \frac{\lambda v^2}{16}\left(\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger)\right)^2 - \frac{\lambda}{16}\left[\mathrm{Tr}(\tilde\Sigma^\dagger\tilde\Sigma)\right]^2 +$$

$$-\cancel{\frac{\lambda v^3}{4}\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger)} - \cancel{\frac{\lambda v^2}{4}\mathrm{Tr}(\tilde\Sigma^\dagger\tilde\Sigma)} - \frac{\lambda v}{8}\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger)\,\mathrm{Tr}(\tilde\Sigma^\dagger\tilde\Sigma)$$

$$= \bar\psi i\gamma^\mu\partial_\mu\psi - gv\bar\psi\psi - g\left[\bar\psi_L\tilde\Sigma\psi_R + \bar\psi_R\tilde\Sigma^\dagger\psi_L\right] +$$

$$+\frac{1}{4}\mathrm{Tr}\left[\partial_\mu\tilde\Sigma^\dagger\partial^\mu\tilde\Sigma\right] - \frac{\lambda v^2}{16}\left[\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger)\right]^2 +$$

$$-\frac{\lambda v}{8}\mathrm{Tr}(\tilde\Sigma+\tilde\Sigma^\dagger)\,\mathrm{Tr}(\tilde\Sigma\tilde\Sigma^\dagger) - \frac{\lambda}{4}\left[\frac{1}{4}\mathrm{Tr}(\tilde\Sigma^\dagger\tilde\Sigma) - v^4\right] =$$

$$= \bar\psi i\gamma^\mu\partial_\mu\psi - gv\bar\psi\psi + \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}(2\lambda v^2)h^2$$

$$+\frac{1}{2}\partial_\mu\vec\pi\cdot\partial^\mu\vec\pi - g\bar\psi(h - i\vec\tau\cdot\vec\pi\gamma_5)\psi +$$

$$-\lambda v\,h(h^2+\vec\pi^2) - \frac{\lambda}{4}\left[(h^2+\vec\pi^2)^2 - v^4\right]$$

We notice the following features

i) The nucleons acquire a Dirac mass $m_D = gv$

ii) The field h has a mass squared $m_h^2 = 2\lambda v^2 > 0$

iii) The three $\vec\pi$ fields are massless $\Rightarrow$ Goldstone bosons, one for each generator of $SU(2)_A$

iv) Interaction of the pions with the nucleons

$$\mathcal{L} \supset ig\bar\psi\vec\tau\cdot\vec\pi\gamma_5\psi$$

This is invariant under $SU(2)_V$

The Higgs model of a spontaneously broken gauge symmetry

Consider scalar QED, described by the classical Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

$$D_\mu = \partial_\mu + ie A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$m^2 > 0$ spin-0 particle-antiparticle pair ϕ, ϕ^*
spin-1 massless particle A_μ

What particles do we get for $m^2 < 0$?
For $m^2 < 0$ the minimum of the scalar potential is at

$$\phi_0 = \sqrt{-\frac{m^2}{2\lambda}} = \frac{v}{\sqrt{2}} \Rightarrow \phi(x) = \frac{1}{\sqrt{2}}(v + h + i\chi)$$

$$D_\mu \phi = (\partial_\mu + ie A_\mu)\frac{1}{\sqrt{2}}(v + h + i\chi) =$$

$$= \frac{1}{\sqrt{2}}(\partial_\mu h - eA_\mu \chi) + \frac{i}{\sqrt{2}}(\partial_\mu \chi + eA_\mu (v+h))$$

$$(D_\mu \phi)^* = \frac{1}{\sqrt{2}}(\partial_\mu h - eA_\mu \chi) - \frac{i}{\sqrt{2}}(\partial_\mu \chi + eA_\mu (v+h))$$

$$(D_\mu \phi)^* (D^\mu \phi) = \frac{1}{2}(\partial_\mu h - eA_\mu \chi)(\partial^\mu h - eA^\mu \chi) +$$

$$+ \frac{1}{2}(\partial_\mu \chi + eA_\mu (v+h))(\partial^\mu \chi + eA^\mu (v+h)) =$$

$$= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi +$$

$$- eA_\mu [\chi \partial^\mu h - (v+h)\partial^\mu \chi] +$$

$$+ \frac{e^2}{2} A_\mu A^\mu [(v+h)^2 + \chi^2]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}(-2m^2) h^2 +$$

$$\underbrace{\frac{1}{2} \partial_\mu \chi \partial^\mu \chi + ev A_\mu \partial^\mu \chi + \frac{e^2 v^2}{2} A_\mu A^\mu}_{= \frac{e^2 v^2}{2}\left(A_\mu + \frac{1}{ev}\partial_\mu \chi\right)\left(A^\mu + \frac{1}{ev}\partial^\mu \chi\right)} + \dots$$

To obtain the actual degrees of freedom we perform a gauge transformation

$$A_\mu + \frac{1}{ev}\partial_\mu \chi \longrightarrow A_\mu' \qquad F_{\mu\nu}' = \partial_\mu A_\nu' - \partial_\nu A_\mu' = F_{\mu\nu}$$

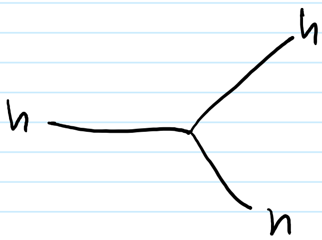
This gives

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}\underbrace{(-2m^2)}_{= m_h^2 > 0} - \frac{1}{4} F_{\mu\nu}' F'^{\mu\nu} + \frac{1}{2}\underbrace{e^2 v^2}_{= m_\gamma^2} A_\mu' A'^\mu$$

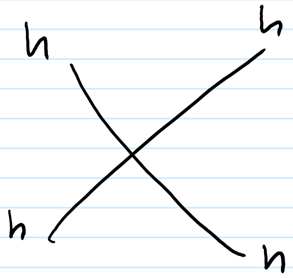
massive Higgs boson — massive photon

Degrees of freedom	scalar	vector
$m^2 > 0$	2	2 $\quad m_\gamma = 0$
$m^2 < 0$	1 $\quad m_h > 0$	3 $\quad m_\gamma > 0$

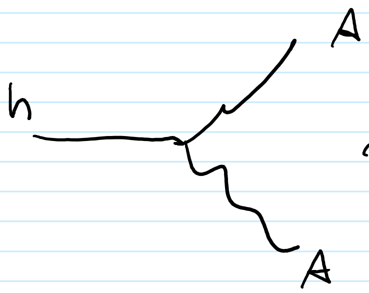
Feynman diagrams



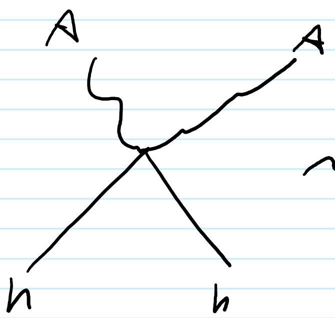
$\sim \lambda v$



$\sim \lambda$



$\sim e^2 v^2$



$\sim e^2$

Note: evading Goldstone theorem

In the presence of a gauge symmetry, to get from A_μ to the actual degrees of freedom we need to fix a gauge.

Axial gauge: $n^\mu A_\mu = 0$

This introduces a new vector n^μ in the theory, therefore

$$\rho^\mu(q) = (2\pi)^3 \sum_N \langle 0 | J^\mu(0) | N \rangle \langle N | \phi(x) | 0 \rangle e^{-ip_N x} \delta(q - p_N) =$$

$$= -i\left(q^\mu \rho_1(q^2, n\cdot q) + n^\mu \rho_2(q^2, n\cdot q) + c_3 n_\mu \delta^4(k)\right)$$

Covariant gauges, e.g. $\partial_\mu A^\mu = 0$ give rise to an indefinite metric, and this allows again to evade Goldstone's theorem

Summary

In the presence of a spontaneously broken gauge symmetry, there are no massless Goldstone bosons.

The corresponding degrees of freedom are "eaten" by the gauge bosons, which now become massive

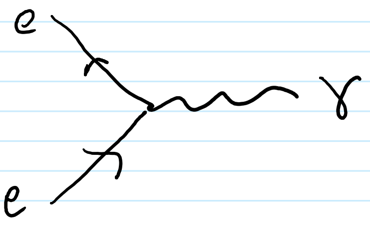
Each "would-be" Goldstone boson gives the extra longitudinal polarisation that characterises a massive vector field

This phenomenon is the Brout-Englert-Higgs mechanism

Standard Model for leptons

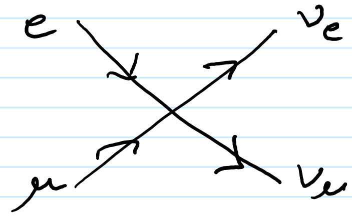
Low energy interactions

Electromagnetism : QED



$$\mathcal{L} \supset e\bar{\psi}\gamma^\mu A_\mu \psi$$

Weak decay : Fermi theory



$$\mathcal{L} \supset \frac{G_F}{\sqrt{2}}\left(\bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_{\nu_e}\right) \times \left(\bar{\psi}_\mu \gamma_\mu (1-\gamma_5)\psi_{\nu_\mu}\right) + h.c.$$

Problem: is it possible to write Fermi theory as the low energy version of a gauge theory?

Consider massless leptons and organise their left-handed components into SU(2) doublets

$$L_i = \left\{ \begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_L^\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_L^\tau \\ \tau_L \end{pmatrix} \right\} \qquad E_i = \{ e_R, \tau_R \mu_R, \}$$

Free massless Lagrangian (and no right-handed neutrino)

$$\mathcal{L} = i\bar{L}_i \gamma^\mu \partial_\mu L_i + i\bar{E}_i \gamma^\mu \partial_\mu E_i$$

Gauge global $SU(2) \times U(1)$ symmetry

$$L_i \to e^{-iY_L g' \frac{\alpha(x)}{2}}\, U(x) L_i \qquad U(x) \in SU(2)$$

$$E_i \to e^{-iY_E g' \frac{\alpha(x)}{2}}\, E_i \qquad \text{(singlet under } SU(2))$$

Gauge fields B_μ and W_μ

$$B_\mu \to B_\mu + \partial_\mu \alpha$$

$$W_\mu \to U(x) W_\mu U^\dagger(x) + i(\partial_\mu U(x)) U^\dagger(x)$$

Covariant derivatives

hypercharge

$$D_\mu = \partial_\mu + ig W_\mu + \frac{ig'}{2} Y B_\mu$$

Explicitly

Pauli matrices

$$D_\mu L_i = \partial_\mu + i g \frac{\tau^a}{2} W_\mu^a + i \frac{g'}{2} Y_L B_\mu$$

$$D_\mu E_i = \partial_\mu + i \frac{g'}{2} Y_E B_\mu$$

Explicit form of the interactions (for one lepton family)

$$\mathcal{L} = \left(\bar{\nu}_L \ \bar{e}_L \right) \left(i \gamma^\mu \left(\partial_\mu + i g \frac{\tau^a}{2} W_\mu^a + i \frac{g'}{2} Y_L B_\mu \right) \right) \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} +$$

$$+ \bar{e}_R \left(i \gamma^\mu \left(\partial_\mu + i \frac{g'}{2} Y_E B_\mu \right) \right) e_R =$$

$$= \bar{\nu}_L i \gamma^\mu \partial_\mu \nu_L + \bar{e}_L i \gamma^\mu \partial_\mu e_L + \bar{e}_R i \gamma^\mu \partial_\mu e_R +$$

$$- \frac{1}{2} \left(\bar{\nu}_L \ \bar{e}_L \right) \begin{pmatrix} g \not{W}_3 + g' Y_L \not{B} & g(\not{W}_1 - i \not{W}_2) \\ g(\not{W}_1 + i \not{W}_2) & -g \not{W}_3 + g' Y_L \not{B} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$- \frac{1}{2} \bar{e}_R \, g' Y_E \not{B} \, e_R$$

This Lagrangian is more conveniently rewritten if we introduce the weak isospin T_3

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix} \Big\} T_3 \qquad E = e_R \quad T_3 = 0$$

and the two charged fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$$

$$\mathcal{L} \supset - \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu W_\mu^+ e_L + h.c.$$ consistent with Fermi theory

$$- \sum_{i = \nu_L, e_L, e_R} \bar{\psi}_i \gamma^\mu \left(g T_{3,i} W_\mu^3 + \frac{g'}{2} Y_i B_\mu \right) \psi_i$$

From the form of the interaction, we see that neither B_μ nor W_μ^3 can be identified with the electromagnetic field.
We then construct the linear combinations

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3 \qquad \Rightarrow \qquad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W_\mu^3 \qquad\qquad W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$

Now we impose that the coupling of each lepton to the photon is what we get from QED

$$-eQ_i\bar{\Psi}_i\gamma^\mu A_\mu\Psi_i = -\bar{\Psi}_i\gamma^\mu\left(gT_{3,i}\sin\theta_W+\frac{g'}{2}Y_i\cos\theta_W\right)A_\mu\Psi_i$$

This gives the three equations

$$\nu_L: \quad \frac{g}{2}\sin\theta_W+\frac{g'}{2}Y_L\cos\theta_W = 0$$

$$e_L: \quad -\frac{g}{2}\sin\theta_W+\frac{g'}{2}Y_L\cos\theta_W = -e$$

$$e_R: \quad +\frac{g'}{2}Y_E\cos\theta_W = -e$$

Which gives

$$g'Y_L\cos\theta_W = -g\sin\theta_W$$

$$g'Y_L\cos\theta_W = -e \qquad \Rightarrow \qquad e = g\sin\theta_W, \quad Y_E = 2Y_L$$

$$\frac{g'}{2}Y_E\cos\theta_W = -e$$

Convention: $Y_L = -1 \Rightarrow Y_E = -2$

$$g'\cos\theta_W = g\sin\theta_W \Rightarrow \tan\theta_W = \frac{g'}{g}$$

$$e = g\sin\theta_W = g'\cos\theta_W$$

Note: with this convention we have a relation between electric charge, weak isospin and hypercharge

$$Q = T_3+\frac{Y}{2}$$

Prediction: interaction of leptons with a new neutral vector field, Z_μ, given by

$$\mathcal{L} \supset -\bar{\Psi}_i\gamma^\mu\left(gT_{3,i}\cos\theta_W-\frac{g'}{2}Y_i\sin\theta_W\right)Z_\mu\Psi_i$$

Exercise: obtain the coupling with the Z field of the electron and the neutrino

Masses for the vector bosons

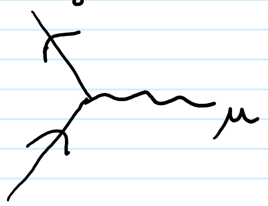
Masses for the vector bosons

The interaction with $W^{\pm}$ need to reproduce Fermi's theory of weak decay. Use left-handed projector to define

$$\nu_L = \frac{1-\gamma_5}{2}\psi_\nu \qquad e_L = \frac{1-\gamma_5}{2}\psi_e$$

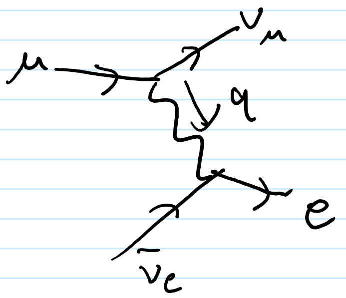
$$\mathcal{L} \supset -\frac{g}{2\sqrt{2}}\bar{\psi}_\nu \gamma^\mu W^+_\mu (1-\gamma_5)\psi_e + h.c.$$

This gives the Feynman diagram



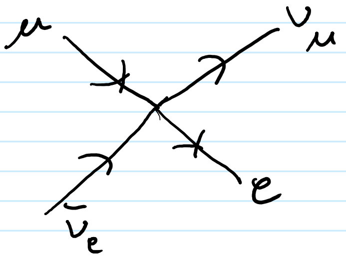
$$-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$$

Consider now muon decay mediated by a <u>massive</u> W



$$\left(-\frac{ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)\right)\times$$
$$\times\left(\frac{-i\eta_{\mu\nu}}{q^2-M_W^2}\right)\times\left(-\frac{ig}{2\sqrt{2}}\gamma^\nu(1-\gamma_5)\right)$$

If $q^2 \ll M_W^2$ (low energy), the diagram becomes



$$-i\frac{g^2}{8M_W^2}\left[\gamma^\mu(1-\gamma_5)\right]\left[\gamma_\mu(1-\gamma_5)\right]$$

But this is the result of Fermi's theory with

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8M_W^2\sin^2\theta_W}$$

This gives already a lower bound on M_W

$$M_W^2 \geq M_W^2\sin^2\theta_W = \frac{e^2}{4\sqrt{2}G_F} = \frac{\pi\alpha_{em}}{\sqrt{2}G_F}$$

$$\alpha_{em} = \frac{1}{137} \qquad G_F = 1.66\times10^{-5}\ \text{GeV}^{-2} \Rightarrow M_W \geq 37.3\ \text{GeV}$$

BEH mechanism in the Standard Model

We need to give mass to $W^\pm$, Z and leave A massless, which requires performing a spontaneous symmetry breaking

$$SU(2) \times U(1)_Y \to U(1)_{em}$$

We then introduce a scalar field

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

Under $SU(2) \times U(1)_Y$ the field ϕ transforms as

$$\phi(x) \to U(x)\, e^{-ig' Y \frac{\alpha(x)}{2}} \phi(x)$$

We consider the following potential for the classical field ϕ

$$V(\phi) = m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad m^2 < 0$$

This potential has infinitely many minima, for

$$\phi^\dagger \phi = -\frac{m^2}{2\lambda} \equiv \frac{v^2}{2}$$

Let us consider a vacuum configuration

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad |v_1|^2 + |v_2|^2 = v^2$$

We cannot pick up any ϕ_0, because the photon needs to stay massless $\Rightarrow$ ϕ_0 has to be invariant for $U(1)_{em}$

$$e^{ieQ\alpha} \phi_0 = \phi_0 \quad \Rightarrow \quad Q\phi_0 = 0$$

$$Q = T_3 + \frac{Y}{2} \mathbb{1} = \begin{pmatrix} \frac{1}{2} + \frac{Y}{2} & \\ & -\frac{1}{2} + \frac{Y}{2} \end{pmatrix}$$

$$Q\phi_0 = \frac{v}{\sqrt{2}} \begin{pmatrix} \left(\frac{1}{2} + \frac{Y}{2}\right) v_1 \\ \left(-\frac{1}{2} + \frac{Y}{2}\right) v_2 \end{pmatrix} = 0 \quad \Rightarrow \quad \begin{matrix} v_1 = 0 & |v_2| = v & Y = 1 \\ v_1 = |v| & v_2 = 0 & Y = -1 \end{matrix}$$

We adopt the choice $Y = 1$, and v_2 real

We now perform the following gauge transformation

$$\phi(x) \to U(x)\,\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \qquad U(x) = \frac{1}{\sqrt{\phi^\dagger\phi}}\begin{pmatrix} \phi_2 & -\phi_1 \\ \phi_1^* & \phi_2^+ \end{pmatrix} \in SU(2)$$

This amounts to a specific choice of the gauge, called the <u>unitary gauge</u>. In this gauge

$$D_\mu \phi = \left[\partial_\mu \mathbb{1} + i \begin{pmatrix} \frac{g}{2} W_\mu^3 + \frac{g'}{2} B_\mu & \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{g}{\sqrt{2}} W_\mu^- & -\frac{g}{2} W_\mu^3 + \frac{g'}{2} B_\mu \end{pmatrix}\right] \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \frac{g}{\sqrt{2}} W_\mu^+ \\ \frac{\sqrt{g^2+g'^2}}{2} Z_\mu \end{pmatrix} (v+h)$$

$$(D_\mu\phi)^\dagger (D^\mu \phi) = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) +$$

$$+ \left[\frac{g^2}{4} W_\mu^+ W^{\mu -} + \frac{g^2+g'^2}{8} Z_\mu Z^\mu\right) (h+v)^2$$

The W and Z bosons acquire masses

$$M_W^2 = \frac{1}{4} g^2 v^2 \qquad M_Z^2 = \frac{1}{4}(g^2+g'^2) v^2$$

$$\cos^2\theta_W = \frac{g^2}{g^2+g'^2} \Rightarrow M_W^2 = \cos^2\theta_W M_Z^2$$

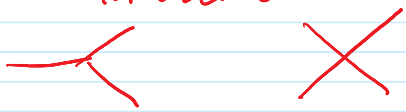
From the definition of the Fermi constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{2v^2} \Rightarrow v = \sqrt{\frac{1}{\sqrt{2}\, G_F}} = 246.22 \text{ GeV}$$

From the scalar potential we read the mass of the neutral scalar h, the Higgs boson

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \Rightarrow \phi^{*}\phi = \frac{(v+h)^2}{2}$$

$$\begin{aligned} V(\phi) &= m^2(\phi^{\dagger}\phi) + \lambda(\phi^{\dagger}\phi)^2 = \\ &= -\frac{\lambda v^2}{2}(v+h)^2 + \frac{\lambda}{4}(v+h)^4 = \\ &= \underbrace{-\frac{\lambda v^4}{4}}_{<0} + \frac{1}{2}\underbrace{(2\lambda v^2)}_{=-2m^2 = m_h^2 > 0} h^2 + \underbrace{\lambda v h^3 + \frac{\lambda}{4} h^4}_{\text{interactions}} \end{aligned}$$



Summary of gauge sector of SM

particle content: real scalar h, massive $\rightarrow m_H^2 = -2m^2 = 2\lambda v^2$

complex vector W_μ, massive $\rightarrow W^{\pm}$ $m_W^2 = \frac{1}{4}g^2 v^2$

real vector Z_μ, massive $\rightarrow Z$ $m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2$

real vector A_μ, massless $\rightarrow \gamma$

The Weinberg or weak mixing angle fixes the relation between the a-priori independent couplings g and g'

$$g \sin\theta_W = g' \cos\theta_W = e$$

$$m_W = m_Z \cos\theta_W$$