Week 10: Spontaneous symmetry breaking

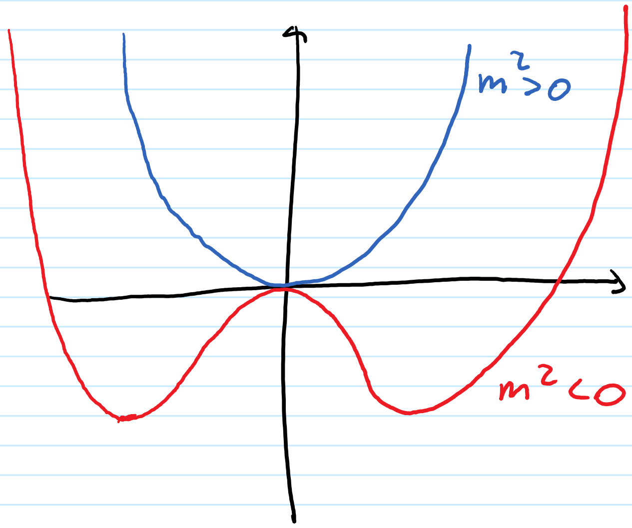
27 November 2017 10:02

Discrete symmetries

Let us consider the following Lagrangian for a real scalar field ϕ

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4}\phi^4 = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi)$$

The potential $V(\phi)$ has two different behaviours according to the sign of m^2



For $m^2>0$, $\phi=0$ corresponds to the minimum of the potential. Small perturbations around $\phi=0$ lead to a theory for scalar particles with positive squared mass

For $m^2<0$, we cannot expand the field ϕ around $\phi=0$, because any small perturbation will drive us away from that point. This leads to a theory with tachyons

In order to quantise such a theory perturbatively, we need to expand the field around either of the two minima of the potential, located at

$$\phi(x) = \phi_0 = \pm v \qquad v = \sqrt{\frac{-m^2}{\lambda}}$$

For instance, let us expand $\phi(x)$ around $\phi_0 = +v$

$$\phi(x) = v + h(x)$$

Substituting into the Lagrangian gives

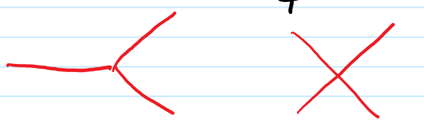
$$\mathcal{L} = \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2} m^2 (v+h)^2 - \frac{\lambda}{4}(v+h)^4 =$$

$$= \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2} m^2 (v^2 + 2hv + h^2) +$$

$$- \frac{\lambda}{4}\left(v^4 + 4hv^3 + 6h^2v^2 + 4h^3 v + h^4\right) =$$

$$= \frac{1}{2}\partial_\mu h\, \partial^\mu h - \Big\{ \frac{1}{2}\Big(\underbrace{m^2 v^2 + \frac{\lambda}{2} v^4}_{= -\frac{\lambda}{4} v^4 < 0}\Big) + (\underbrace{m^2 + \lambda v^2}_{=0}) v h +$$

$$+ \frac{1}{2}(\underbrace{m^2 + 3\lambda v^2}_{m_h^2 = -2m^2 > 0}) h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 \Big\} \equiv$$



$$\equiv \frac{1}{2}\partial_\mu h\, \partial^\mu h - V(h)$$

$$V(h) = \frac{1}{2} m_h^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 + \text{constant term}$$

$$m_h^2 = -2m^2 = 2\lambda v^2 > 0 \quad \text{well-defined perturbative theory!}$$

The original Lagrangian had a $\mathbb{Z}_2$ symmetry $\phi \to -\phi$

The new Lagrangian is not symmetric under $h \to -h$ because of the presence of the h^3 term. The presence of this term follows from the fact that we have expanded around a non-symmetric configuration

However, the original $\mathbb{Z}_2$ symmetry is still there, but it is realised <u>non linearly</u>

$$h \to -h - 2v$$

This phenomenon is known as <u>spontaneous symmetry breaking</u>

Spontaneous symmetry breaking : a theory has a symmetry (e.g. stemming from a classical Lagrangian), whose corresponding operator U does not leave the ground ~~state~~ invariant

$$U|0\rangle \neq |0\rangle$$

In the case of spontaneous symmetry breaking one or more scalar fields acquire a vacuum expectation value (VEV)

i) Only scalar fields can acquire a VEV. In fact, suppose we have fields $\Phi_a(x)$ transforming under Poincaré as

$$U(\Lambda,a)\,\Phi_a(x)\,U^{-1}(\Lambda,a) = S_{ab}(\Lambda^{-1})\,\Phi_b(\Lambda x + a)$$

Since $U(\Lambda,a)|0\rangle = |0\rangle$, then

$$\langle 0|\,\Phi_a(x)\,|0\rangle = \langle 0|e^{iPx}\,\Phi_a(0)\,e^{-iPx}|0\rangle =$$

$$= \langle 0|\,\Phi_a(0)\,|0\rangle = \langle 0|\,U(\Lambda)\,\Phi_a(0)\,U^{-1}(\Lambda)|0\rangle =$$

$$= S_{ab}(\Lambda^{-1})\,\langle 0|\,\Phi_b(0)\,|0\rangle \Rightarrow S_{ab}(\Lambda^{-1}) = \delta_{ab}$$

Hence $\Phi_a(x)$ are scalar fields

Note: the proof works also if the vacuum has finite energy E_0

ii) If $|0\rangle$ is symmetric, then $\langle 0|\,\phi_a(x)|0\rangle = 0$ for any scalar field. In fact, suppose we have a symmetry S realised on a scalar field $\phi(x)$ as follows

$$U_S\,\phi_a(x)\,U_S^{\dagger} = \Pi^S_{ab}\,\phi_b(x) \quad \text{and} \quad U_S|0\rangle = |0\rangle$$

then

$$\langle 0|\,\phi_a(0)\,|0\rangle = \langle 0|\,U_S\,\phi_a(0)\,U_S^{\dagger}|0\rangle = \Pi^S_{ab}\,\langle 0|\,\phi_b(x)|0\rangle$$

If $\Pi^S_{ab} \neq \delta_{ab}$, this implies $\langle 0|\,\phi_a(x)|0\rangle = 0$

Example: $\mathbb{Z}_2$ symmetry

We have a "parity" operator P such that

$$P\phi(x)P^\dagger = -\phi(x)$$

We have two vacua $|0,+\rangle$ and $|0,-\rangle$ such that

$$\langle 0,\pm|\phi(x)|0,\pm\rangle = \pm v \iff P|0,\pm\rangle = |0,\mp\rangle$$

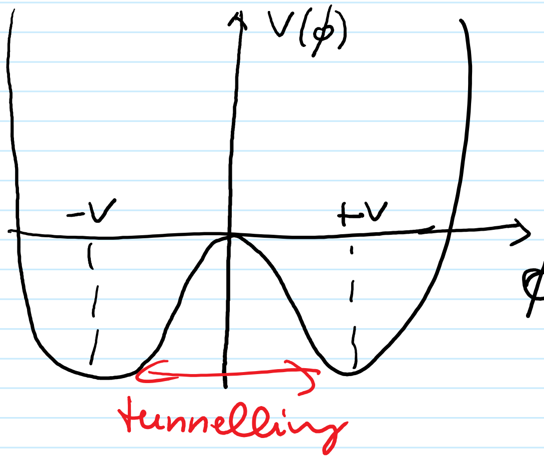
The parity operator transforms one vacuum into the other.
If $\mathbb{Z}_2$ is a symmetry of the theory, then $[P,H]=0$
$\Rightarrow$ $|0,+\rangle$ and $|0,-\rangle$ are <u>degenerate vacua</u>

In fact, let $H|0,+\rangle = E_+|0,+\rangle$, then

$$H|0,-\rangle = HP|0,+\rangle = \Big(PH|0,+\rangle + \underbrace{[H,P]}_{=0}\Big)|0,+\rangle =$$

$$= PH|0,+\rangle = E_+ P|0,+\rangle = E_+|0,-\rangle$$

In ordinary quantum mechanics, the true ground state is
$|\Omega\rangle = \frac{1}{\sqrt{2}}\left(|0,+\rangle + |0,-\rangle\right)$ such that $P|\Omega\rangle = |\Omega\rangle$



Even if the system starts with a non-symmetric state, say $|0,+\rangle$, quantum tunnelling will bring the system into the symmetric state $|\Omega\rangle$

Tunnelling occurs because $\langle 0,-|H|0,+\rangle \neq 0$

In quantum field theory, if $|u\rangle$ and $|v\rangle$ are two degenerate ground states, then $\langle v|H|u\rangle = 0$

$\Rightarrow$ no tunnelling, for small perturbations the system fluctuates around any of the degenerate vacua

In fact, suppose we select $|u\rangle$ and $|v\rangle$ among a set of degenerate vacua $|w\rangle$

For any couple of hermitian operators $A(x)$ and $B(x)$, we consider the following expectation value

$$\langle u|A(\vec{x})\,B(0)|v\rangle = \sum_w \langle u|A(0)|w\rangle\langle w|B(0)|v\rangle + \\ + \sum_N \langle u|A(0)|N\rangle\langle N|B(0)|v\rangle\, e^{-i\vec{p}_N\cdot\vec{x}}$$

For $|\vec{x}|\to\infty$ we can neglect the fast oscillating contribution due to multi-particle states $|N\rangle$

Similarly

$$\langle u|B(\vec{x})A(0)|v\rangle \xrightarrow{|\vec{x}|\to\infty} \sum_w \langle u|B(0)|w\rangle\langle w|A(0)|v\rangle$$

But, due to causality $\langle u|\left[A(\vec{x}),B(0)\right]|v\rangle = 0$

This implies that, as matrices, $\langle u|A(0)|v\rangle$ and $\langle u|B(0)|v\rangle$ commute

$\Rightarrow$ simultaneous diagonalisation

$$\langle u|A(0)|v\rangle = a_v\,\delta_{uv} \quad \text{and} \quad \langle u|B(0)|v\rangle = b_v\,\delta_{uv}$$

In this basis, if the Hamiltonian is constructed out of products of such operators

$$\langle u|H|v\rangle = E_0\,\delta_{uv} \Rightarrow \text{no tunnelling!}$$

Spontaneous breaking of a global continuous symmetry

Consider a complex scalar field, with the Lagrangian

$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \qquad \lambda > 0$$

The Lagrangian has a global $U(1)$ symmetry

We look for vacuum solutions, that minimise the Hamiltonian. Since the kinetic energy is positive definite, such solutions need to have $\partial_\mu \phi = 0 \Rightarrow \phi = \text{constant}$

Any of these solutions minimise the potential

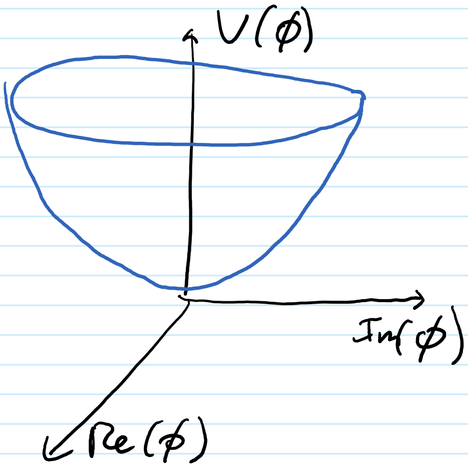
$$V(\phi) = m^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

Let ϕ_0 be any of such solutions. Then, if the theory is perturbative

$$\phi(x) = \phi_0 + \sqrt{Z_\phi} \int (dp) \left(a(\vec{p}) e^{-ipx} + b^+(\vec{p}) e^{ipx} \right) + O(\lambda)$$

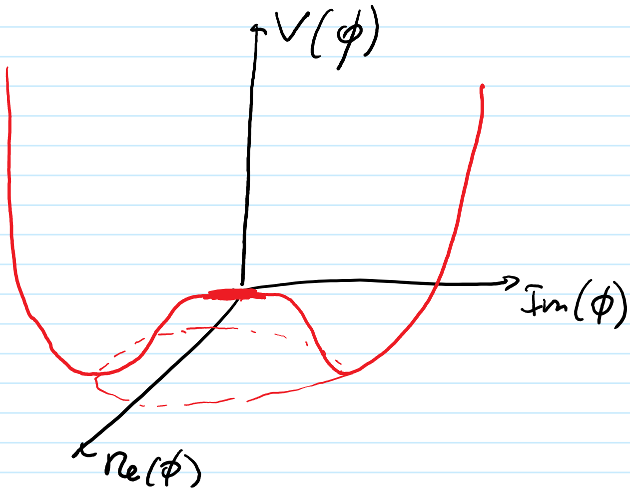
This means that the configuration ϕ_0 corresponds to a vacuum $|0\rangle$ such that $\langle 0 | \phi(x) | 0 \rangle = \phi_0$

$m^2 > 0$



One global minim $\phi_0 = 0$

$m^2 < 0$



Infinitely many global minima

$$m^2 + 2\lambda \phi_0^* \phi_0 = 0$$

$$|\phi_0| = \sqrt{\frac{-m^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

Spontaneous symmetry breaking

In order to perturbatively quantise this theory for $m^2 < 0$, we expand the field ϕ around any of its vacuum configurations

$$\phi_0 = \frac{v}{\sqrt{2}} e^{i\alpha_0} \qquad \phi(x) = \frac{e^{i\alpha_0}}{\sqrt{2}}\left(v + h(x) + i\chi(x)\right)$$

real fields

The Lagrangian becomes

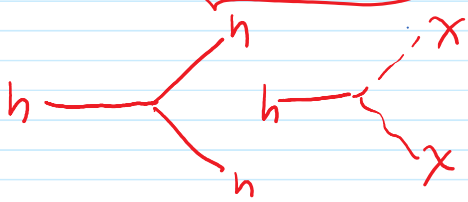
$$\mathcal{L} = \frac{1}{2}\partial_\mu(h - i\chi)\,\partial^\mu(h + i\chi) - \frac{1}{2}m^2\left(v^2 + 2vh + h^2 + \chi^2\right) +$$
$$- \frac{\lambda}{4}\left(v^4 + 4hv^3 + v^2(6h^2 + 2\chi^2) + 4vh(h^2+\chi^2) + (h^2+\chi^2)^2\right)$$

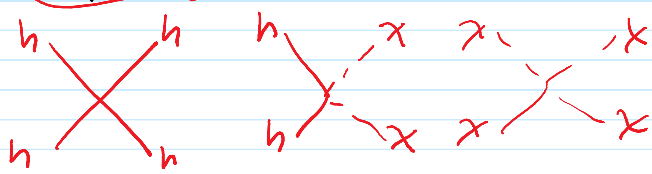
$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) +$$

$$- \Big\{ \underbrace{\frac{1}{2}\left(m^2 v^2 + \frac{\lambda}{2}v^4\right)}_{= -\frac{\lambda v^2}{4} < 0} + \underbrace{(m^2 + \lambda v^2)}_{=0} vh \qquad (m^2 = -\lambda v^2) \qquad +$$

$$+ \frac{1}{2}\underbrace{(m^2 + 3\lambda v^2)}_{= m_h^2 = -2m^2 > 0} h^2 + \frac{1}{2}\underbrace{(m^2 + \lambda v^2)}_{= m_\chi^2 = 0!} \chi^2 +$$

$$- \underbrace{\lambda v h (h^2 + \chi^2)}\qquad \underbrace{- \frac{\lambda}{4}(h^2+\chi^2)^2}$$





$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - V(h,\chi)$$

$$V(h,\chi) = \frac{1}{2}m_h^2 h^2 - \lambda v h (h^2+\chi^2) - \frac{\lambda}{4}(h^2+\chi^2)^2 + \text{constant}$$

Two scalars with tree-level squared masses

$m_h^2 = -2m^2 = 2\lambda v^2 > 0$
$m_\chi^2 = 0$ massless!

The original Lagrangian is still invariant under U(1) global transformations of the field $\phi(x)$

$$\phi(x) \to e^{i\alpha}\phi(x) \simeq \phi(x) + i\alpha\,\phi(x)$$

In the current parameterisation

$$i\alpha\,\phi(x) = i\alpha\,\frac{e^{i\alpha_0}}{\sqrt{2}}\left(v+h+i\chi\right) = \frac{e^{i\alpha_0}}{\sqrt{2}}\left(-\alpha\chi + i\alpha(v+\chi)\right)$$

$$h \to h - \alpha\chi \qquad \chi \to \chi + \alpha h + \alpha v$$

On the massless field, the symmetry is <u>non-linearly realised</u>

One can obtain the masses of the particles by expanding the potential around ϕ_0 (simpler using real fields)

$$\phi(x) = \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right) \qquad \phi_0 = \frac{v}{\sqrt{2}}\left(\cos\alpha_0 + i\sin\alpha_0\right)$$

$$V(\phi) = \frac{\lambda}{2}\left[-v^2(\phi_1^2+\phi_2^2) + \frac{1}{2}(\phi_1^2+\phi_2^2)^2\right] \simeq$$

$$\simeq V(\phi_0) + \frac{1}{2}\left(\phi_1-\phi_{0,1},\ \phi_2-\phi_{0,2}\right) M^2 \begin{pmatrix}\phi_1-\phi_{0,1}\\ \phi_2-\phi_{0,2}\end{pmatrix}$$

$$M^2 = \left.\frac{\partial^2 V}{\partial\phi_1\,\partial\phi_2}\right|_{\phi=\phi_0} = 2\lambda v^2\begin{pmatrix}\cos^2\alpha_0 & \sin\alpha_0\cos\alpha_0\\ \sin\alpha_0\cos\alpha_0 & \sin^2\alpha_0\end{pmatrix}$$

$\det M^2 = 0 \implies$ one zero eigenvalue $\Rightarrow$ massless particle

<u>Goldstone theorem</u>

If a theory is invariant under a continuous global symmetry transformation, then either

i) the ground state (vacuum) is invariant as well

ii) or there exists a particle without spin and zero mass

For each generator of a broken continuous global symmetry, there exists a spinless massless particle, the <u>Goldstone boson</u>

Heuristic proof for perturbative scalar field theory

Consider a classical Lagrangian for scalar fields ϕ^i

$$\mathcal{L} = \underset{\text{kinetic}}{T(\phi^i)} - \underset{\text{potential}}{V(\phi^i)}$$

Under a global transformation (in infinitesimal form)

$$\phi^a \to \phi^a + \alpha_a T^a{}_b \phi^b \quad \text{and} \quad \delta\mathcal{L} = 0$$

The vacuum of the theory corresponds to field configurations ϕ_0^a such that

$$T(\phi_0^a) = 0 \quad \text{and} \quad \left.\frac{\partial V(\phi^i)}{\partial \phi^a}\right|_{\phi^a = \phi_0^a} = 0$$

If the vacuum state is not invariant, then

$$\alpha_a T^a{}_b \phi_0^b \neq 0$$

Choose a particular ground-state configuration and expand the potential about i

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^a (\phi - \phi_0)^b \underbrace{\left.\frac{\partial^2 V}{\partial\phi_a \partial\phi_b}\right|_{\phi=\phi_0}}_{= m^2_{ab} \text{ (mass matrix)}} + \dots$$

Eigenvalues of $m^2_{ab} \Rightarrow$ particle masses

Invariance of $V \Rightarrow 0 = \delta V = V(\phi) - V(\phi_0) = \frac{1}{2}(\alpha_a T^a_b \phi_0^b) m^2_{bc} (\alpha_c T^c_d \phi_0^d)$

This means that $\alpha_a T^a_b \phi_0^b$ is a non-vanishing eigenvector of m^2_{ab} with eigenvalue $= 0$

Goldstone theorem in QFT

A continuous global symmetry transformation

$$U = \exp(i\alpha_a T^a)$$

gives rise to conserved Noether currents J^μ_a, i.e. $\partial_\mu J^\mu_a = 0$
The corresponding charges Q^a are time-independent

$$Q_a = \int d^3x\, J^0_a(x) \qquad \frac{dQ_a}{dt} = i[H, Q_a] = 0$$

If a vacuum $|0\rangle$ is not invariant under this symmetry,

$$e^{i\alpha_a Q_a}|0\rangle \Rightarrow Q_a|0\rangle \neq 0$$

we have a family of degenerate vacua

$$|\alpha_a\rangle = e^{i\alpha_a Q_a}|0\rangle$$

In fact, if $H|0\rangle = E_0|0\rangle$, we have

$$H|\alpha_a\rangle = H e^{i\alpha_a Q_a}|0\rangle = e^{i\alpha_a Q_a} H|0\rangle = E_0|\alpha_a\rangle$$

Since $Q_a|0\rangle \neq 0$, then Q_a produces an excitation of the vacuum that has zero momentum

$$\vec{P}\, Q_a|0\rangle = [\vec{P}, Q_a]|0\rangle = 0$$

$$H Q_a|0\rangle = [H, Q_a]|0\rangle + Q_a H|0\rangle = E_0 Q_a|0\rangle$$

$Q_a|0\rangle$, at zero momentum, has no extra energy on top of the vacuum $\Rightarrow$ massless particle
To make the argument more rigorous, we consider scalar fields $\phi_i(x)$ such that

$$\langle 0|\phi_i(x)|0\rangle \neq 0$$

$$e^{i\alpha_a Q_a}\phi_i(x)e^{-i\alpha_a Q_a} = \left(e^{i\alpha_a T^a}\right)_{ij}\phi_j(x)$$

For any generator Q_a of a broken symmetry, we have

$$[Q_a, \phi_i(x)] = T^a_{ij}\,\phi_j(x)$$

Step 1: write the commutator of a current and a field in terms of a spectral density $\rho(q^2)$

$$\langle 0|[J_a^\mu(x),\phi_i(y)]|0\rangle = \langle 0|[J_a^\mu(x-y),\phi_i(0)]|0\rangle =$$

$$= \langle 0|J_a^\mu(x-y),\phi_i(0)|0\rangle - \langle 0|\phi_i(0)J_a^\mu(x-y)|0\rangle$$

Inserting a complete set of state, and setting $E_0=0$, gives

$$\langle 0|J_a^\mu(x)\phi_i(0)|0\rangle = \sum_N \langle 0|J_a^\mu(0)|N\rangle\langle N|\phi_i(0)|0\rangle e^{-ip_N\cdot x} =$$

$$= \int \frac{d^4q}{(2\pi)^3}\,\tilde{\rho}^\mu_{ai}(q)\,e^{-iqx}$$

$$\tilde{\rho}^\mu_{ai}(q) = (2\pi)^3\sum_N \langle 0|J_a^\mu(0)|N\rangle\langle N|\phi_i(0)|0\rangle\,\delta^4(p_N-q)$$

From Lorentz invariance, and the positivity of energy

$$\tilde{\rho}^\mu_{ai}(q) = -iq^\mu\rho_{ai}(q^2)\,\Theta(q_0)$$

Assume $J_a^\mu(x)$ and $\phi_i(x)$ to be hermitian, then

$$\langle 0|\phi_i(0)J_a^\mu(x)|0\rangle = \left(\langle 0|J_a^\mu(x)\phi_i(0)|0\rangle\right)^* =$$

$$= \int\frac{d^4q}{(2\pi)^3}\left(iq^\mu\rho^*_{ai}(q^2)\,\Theta(q_0)\right)e^{iqx}$$

This gives

$$\langle 0|[J_a^\mu(x),\phi_i(0)]|0\rangle = \int\frac{d^4q}{(2\pi)^3}(-iq^\mu)\left(\rho_{ai}(q^2)e^{-iqx}+\rho^*_{ai}(q^2)e^{iqx}\right)\Theta(q_0) =$$

$$= \partial^\mu\int\frac{d^4q}{(2\pi)^3}\left(\rho_{ai}(q^2)e^{-iqx}-\rho^*_{ai}(q^2)e^{iqx}\right)\Theta(q_0)$$

Step 2: use causality to show that $\rho^*_{ai}(q^2)=\rho_{ai}(q^2)$

For $x^2<0$ we change integration variable, so that $x=(0,\vec{x})$

$$\int\frac{d^4q}{(2\pi)^3}\left(\rho_{ai}(q^2)e^{-iqx}-\rho^*_{ai}(q^2)e^{iqx}\right)\Theta(q_0) =$$

causality

$$= \int\frac{d^4q}{(2\pi)^3}\left(\rho_{ai}(q^2)-\rho^*_{ai}(q^2)\right)e^{i\vec{q}\cdot\vec{x}}\,\Theta(q_0) = 0 \Rightarrow \rho_{ai}(q^2)=\rho^*_{ai}(q^2)$$

Summary of steps 1 and 2

$$\langle 0|[J_a^\mu(x), \phi_i(0)]|0\rangle = \partial^\mu \int_0^\infty d\mu^2 \, \rho_{ai}(\mu^2) \, i\Delta(x-y, \mu^2)$$

$$i\Delta(x,\mu^2) = \int \frac{d^4q}{(2\pi)^3} \left(e^{-iqx} - e^{iqx}\right) \Theta(q_0) \, \delta(q^2-\mu^2)$$

Step 3: use current conservation to show that $q^2 \rho_a(q^2) = 0$

$$0 = \langle 0|[\partial_\mu J_a^\mu(x), \phi_i(0)]|0\rangle =$$

$$= \Box \int_0^\infty d\mu^2 \, \rho_{ai}(\mu^2) \, \Delta(x,\mu^2)$$

But $\Delta(x,\mu^2)$ satisfies Klein-Gordon equation

$$(\Box + m^2)\Delta(x,\mu^2) = 0$$

This gives

$$\int_0^\infty d\mu^2 \mu^2 \rho_{ai}(\mu^2) \Delta(x,\mu^2) = 0 \Rightarrow \mu^2 \rho_{ai}(\mu^2) = 0$$

Specialise our result to

$$\langle 0|[J_a^0(x), \phi_i(y)]|0\rangle\Big|_{x_0=y_0} = \int_0^\infty d\mu^2 \rho_{ai}(\mu^2) \frac{\partial}{\partial x_0} i\Delta(x-y,\mu^2)\Big|_{x_0=y_0}$$

Recall properties of $\Delta(x-y,\mu^2)$

$$\frac{\partial}{\partial x^0}\Delta(x-y),\mu^2)\Big|_{x_0=y_0} = -\delta^3(\vec{x}-\vec{y})$$

This gives

$$\langle 0|[J_a^0(x), \phi_i(y)]|0\rangle\Big|_{x_0=y_0} = -i\,\delta^3(\vec{x}-\vec{y}) \int_0^\infty d\mu^2 \rho_{ai}(\mu^2)$$

<u>Step 4</u>: impose spontaneous symmetry breaking to show that $\rho_{ai}(q^2) = N_a \delta(q^2)$

$$\int d^3\vec{x} \, \langle 0 | [J^0_a(x), \phi_i(y)] | 0 \rangle \Big|_{x_0 = y_0} = \langle 0 | [Q_a, \phi_i(y)] | 0 \rangle =$$

$$= T^a_{ij} \langle 0 | \phi_{ij}(y) | 0 \rangle = T^a_{ij} \langle 0 | \phi_{ij}(0) | 0 \rangle$$

From step 3, we also know that

$$\int d^3\vec{x} \, \langle 0 | J^0_a(x), \phi_i(y) | 0 \rangle = -i \int_0^\infty d\mu^2 \rho_{ai}(\mu^2)$$

If no spontaneous symmetry breaking $\Rightarrow \rho_{ai}(\mu^2)$

In the case of spontaneous symmetry breaking, we have

$$T^a_{ij} \langle 0 | \phi_j(0) | 0 \rangle = -i \int_0^\infty d\mu^2 \rho_{ai}(\mu^2)$$

with the constraint $\mu^2 \rho_{ai}(\mu^2) = 0$, this implies

$$\rho_{ai}(\mu^2) = i\delta(\mu^2) T^a_{ij} \langle 0 | \phi_j(0) | 0 \rangle$$

There is one <u>massless particle</u> in the spectrum for each broken generator T^a!

<u>Step 5</u>: use the definition of $\rho_{ai}(q^2)$ to show that these massless particles are bosons with the same internal quantum numbers of J^0

$\tilde{\rho}_{ai}(q)$ takes contribution only on one-particle states

$$-iq_0 \rho_{ai}(q^2) = \int \frac{d^3\vec{p}}{2|\vec{p}|} \langle 0 | J^0_a(0) | \vec{p} \rangle \langle \vec{p} | \phi_i(0) | 0 \rangle \delta^4(q-p)$$

$U(\Lambda)\phi_i(0)|0\rangle = \phi_i(0)|0\rangle \Rightarrow |\vec{p}\rangle$ gives a boson of spin 0

Suppose $J^0_a(0)$ has some internal quantum number

$J^0_a(0)|0\rangle$ creates a state with the same quantum numbers of $J^0_a(0)$ (e.g. parity) $\Rightarrow |\vec{p}\rangle$ has the same quantum numbers as $J^0_a(0)$