**Symmetry in Particle Physics, Problem Sheet 8**

1. Consider two *quantum* free scalar fields $\phi_i(x)$, $i = 1, 2$, given by

$$\phi_i(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E_{\vec{p}}} \left( a_i(\vec{p}) e^{-ip\cdot x} + a_i^\dagger(\vec{p}) e^{ip\cdot x} \right) , \quad E_{\vec{p}} = \sqrt{p^2 + m^2} ,$$

as well as the conserved current

$$J^\mu = (\partial^\mu \phi_1)\phi_2 - (\partial^\mu \phi_2)\phi_1 .$$

   (a) Compute the conserved charge $Q$ corresponding to the current $J^\mu$ in terms of creation and annihilation operators.

   Hint. Creation and annihilation operators for different fields commute.

   (b) Consider the field

$$\phi(x) = \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x)) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E_{\vec{p}}} \left( a(\vec{p}) e^{-ip\cdot x} + b^\dagger(\vec{p}) e^{ip\cdot x} \right) .$$

   Show that the one-particle states created by $a^\dagger(\vec{p})$ and $b^\dagger(\vec{p})$ are eigenstates of the charge operator $Q$. What are the corresponding eigenvalues?

2. Consider the following Lagrangian for a classical real *classical* vector field $A^\mu$:

$$\mathcal{L} = \frac{c_1}{2} F^{\mu\nu} F_{\mu\nu} + \frac{c_2}{2} (\partial_\mu A^\mu)^2 ,$$

where $c_1, c_2$ are real parameters and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$.

   (a) Compute the equations of motions for the field $A^\mu$.

   (b) Show that $\Box(\partial_\mu A^\mu) = 0$.

   (c) Consider the gauge transformation

$$A^\mu \to A^\mu + \partial^\mu \alpha .$$

   What is the condition on $\alpha$ such that $\mathcal{L}$ is gauge invariant?

   (d) Let us fix now $c_2 = 0$. Compute the Hamiltonian density $\mathcal{H}$ and show that the kinetic energy is positive if and only if $c_1 < 0$.