Symmetry in Particle Physics, Problem Sheet 10

1. Consider the following Lagrangian for a real scalar field ϕ and a Dirac spinor ψ:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) + i\bar{\psi}\gamma^\mu \partial_\mu \psi - g\phi\bar{\psi}\psi\,, \qquad V(\phi) \equiv \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4\,, \quad \lambda > 0\,,$$

   (a) Show that the Lagrangian is invariant under the transformation

$$\phi \to -\phi\,, \qquad \psi \to i\gamma^5\psi\,.$$

   (b) Ground state configurations for a scalar field are those who have the minimun energy. As such, they have no kinetic energy and minimise the potential. Find the ground state configurations ϕ_0 for $\mu^2 \geq 0$. What particles (spin and masses) does the Lagrangian describe?
   Assume that $\mu^2 < 0$ and show that the ground state configurations obey

$$\phi_0 = \pm v\,, \quad \text{with } v \equiv \sqrt{-\frac{m^2}{\lambda}}\,.$$

   Expand the scalar field ϕ around the vacuum configuration v as follows:

$$\phi(x) = v + h(x)\,,$$

   where $h(x)$ is a new scalar field. Rewrite the Lagrangian in terms of $h(x)$. What particles (spin and masses) does the theory describe in this case?

2. Consider the following Lagrangian for a complex scalar field ϕ and a Dirac field ψ:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \partial_\mu\phi^*\partial^\mu\phi - V(\phi^*\phi) - g(\phi\,\bar{\psi}_R\,\psi_L + \phi^*\,\bar{\psi}_L\,\psi_R)\,,$$

   where ψ_L and ψ_R are the left- and right-handed components of ψ, defined by

$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi\,, \qquad \psi_R = \frac{1}{2}(1+\gamma^5)\psi\,,$$

   and

$$V(\phi^*\phi) = \mu^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2$$

   with $\lambda > 0$. The above Lagrangian is invariant under the global chiral transformation

$$\psi_R \to e^{i\alpha}\psi_R\,, \qquad \psi_L \to e^{-i\alpha}\psi_L\,, \qquad \phi \to e^{2i\alpha}\phi$$

   (a) Consider the case $\mu^2 \geq 0$. Show that the ground state of the theory corresponds to the field configuration $\phi = 0$. What particles (masses and spin) does this theory describe?

(b) Consider now the case $\mu^2 < 0$. What are the field configurations corresponding to the ground state?

(c) What particles (spin and masses) does the theory describe in the case $\mu^2 < 0$? Hint. Write vacuum configurations ϕ_0 in the form $\phi_0 = e^{i\alpha_0} v$, with $v \geq 0$. Choose one of them, and expand the field around ϕ_0, as follows

$$\phi(x) = \frac{e^{i\alpha_0}}{\sqrt{2}} (v + h(x) + i\chi(x)) \, .$$