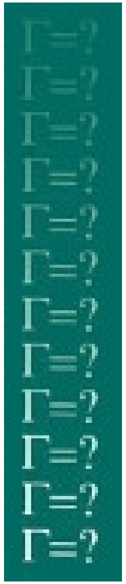
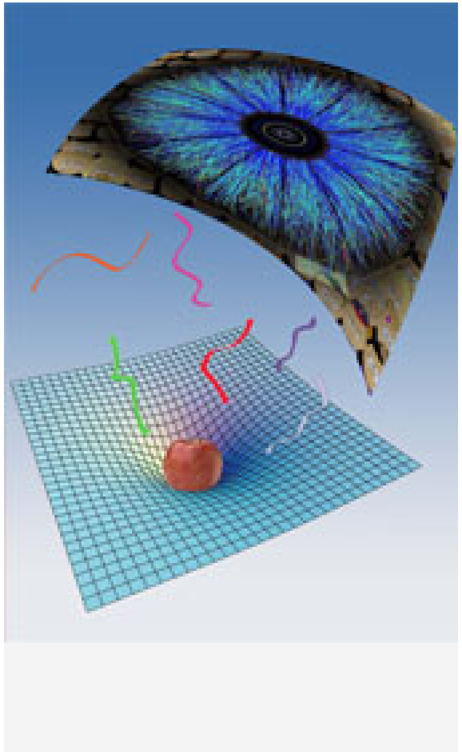
Super-renormalizable and Finite gravitational theories



Lesław Rachwał
(rachwal@fudan.edu.cn)
Fudan University
(Shanghai, China)



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Super-renormalizable and Finite gravitational theories

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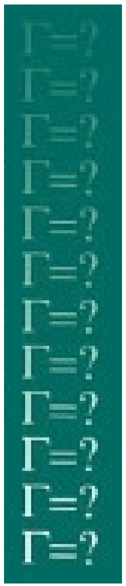
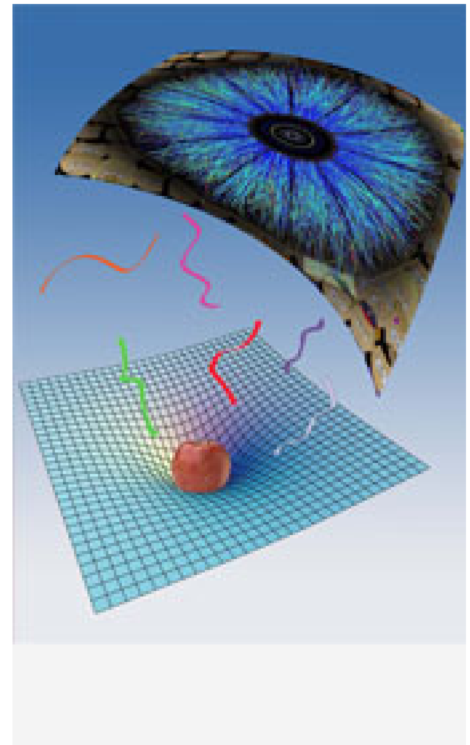
(Fudan University)





[based on arXiv: hep-th/1407.8036]
(in press in NPB)

In collaboration with
L. Modesto and S. Giaccari

Motivation

- Effective Field Theory of Quantum Gravity:
  - low energy QG is valid as QFT with a cutoff
  - despite non-renormalizability of E-H QG predictions are possible and calculable in EFT
- Fundamental Theory of Quantum Gravity:
  - defined without problems at any (high) energy scale
  - with higher derivatives
  - complete in UV regime
  - renormalizable or even finite in quantum realm
- Is it possible to construct?

Outline

- Quantum Gravity with Higher Derivative Action
- Renormalizability
- Analytic non-local Form-factors
- (1-loop) Super-renormalizability
- Killers of beta functions
- Finiteness
- Conclusions

E-H QG as EFT

- Quantum field theory of small low-energetic fluctuations of metric degrees of freedom around flat Minkowski spacetime (Donoghue et al '94-'00) $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- General covariance as a gauge symmetry on the linearized level
- Gravitons around flat spacetime are massless $\Leftarrow$ gauge symmetry
- Standard quantization using Faddeev-Popov trick for gauge theories
- Expansion of lagrangian in number of derivatives and in powers of graviton field $L_{grav} = \kappa^{-2} R(h) + o(R^2)$
- Feynman diagrams for gravitons interacting with matter

E-H QG as QFT

- Perturbative calculus with dimensionful coupling parameter (in *d=4*)

$$\kappa^2 = 16\pi G_N$$

- Pure gravity: 1-loop finite (no divergences)
- Gravity+matter: non-renormalizable
- E-H Gravity non-renormalizable

- The reason:
  - dimensionful coupling constant $M_{\mathrm{Pl}}^2 = \kappa^{-2}$
  - only two derivatives in the bare action, too fast UV propagation
  - metric fluctuations $h_{\mu\nu}$ are dimensionless, compare to gauge field

fluctuations A_μ

$$[h_{\mu\nu}] = E^0 \qquad [A_\mu] = E^1$$

Quest for Quantum Gravity

- QE-H: Non-renormalizable, but unitary
- Way out: Asymptotic Safety Scenario (Weinberg, Reuter, Niedermaier)

- Let's first quantize matter, put it on curved spacetime background, only later quantize gravitation (Shapiro)

- Observation:
  Counterterms needed to be added to the divergent matter effective action are of the type R^2 and C^2 (in $d=4$)

- Conclusions:
  - These counterterms contain higher derivatives of the bckg metric
  - We need quantum theory of gravity with higher derivatives

Higher Derivative Quantum Gravity (HDQG)

- Four derivative theory '77 Stelle

$$L = \lambda + \kappa^{-2} R + \omega_0^1 R^2 + \omega_0^2 R_{\mu\nu}^2$$

- First QG renormalizable in *d=4*
- Improved UV behavior for propagation of modes $\Pi \sim k^{-4}$
- Asymptotically Free in UV like YM
- Improvement of the E-H action in EFT for QG

- Classical Ostrogradsky instabilities
- Presence of massive ghost with negative residue
- Violation of Unitarity
- Rapid decay of Gravitational Vacuum

- Can not be viewed as a fundamental theory

Higher derivatives

$$L = \lambda + \kappa^{-2} R + \sum_{n=0}^{N} \omega_n^1 R \Box^n R + \sum_{n=0}^{N} \omega_n^2 R_{\mu\nu} \Box^n R^{\mu\nu}$$

- Behavior in UV improved even more
- Propagator is a polynomial of a degree *N*+2 in momentum *k*

$$\Pi \sim k^{-(4+2N)}$$

- Asymptotic Freedom for all couplings
- At low energy reduces effectively to E-H QG
- New ghost poles with oscillating sign of residues
- Unitarity problems still present!!!

Non-local form-factors

$$L = \lambda + \kappa^{-2} R + R F_1(\Box) R + R_{\mu\nu} F_2(\Box) R^{\mu\nu}$$

- Extension of the quadratic in curvature terms Tomboulis, Krasnikov

- The most general theory describing gravitons' propagation around flat spacetime

- Intrinsically non-local due to non-polynomial functions F_1 and F_2

- Example with one form-factor (multiplicative modification of the graviton propagator)

$$L = \lambda + \kappa^{-2} R + \kappa^{-2} G_{\mu\nu} \frac{e^{H(\Box/\Lambda^2)} - 1}{\Box} R^{\mu\nu}$$

Non-local form-factors

Requirements:

- Propagator has only first single poles with real masses (no tachyons) and with positive residues (no ghosts)
- In the spectrum only physical massless transverse graviton (spin 2)

Demands on a form-factor $e^{H(z)}$: $\quad z = \frac{\Box}{\Lambda^2}$

- is real and positive on the real axis and has no zeros on the complex plane, is analytic on the whole complex plane
- has proper asymptotics for large *z* (in UV) along and around real axis (nonpolynomial or polynomial with degree ≥1)

- Example:

$$H(z) = \frac{1}{2}\Gamma\left(0, p^2(z)\right) + \frac{1}{2}\gamma_E + \frac{1}{2}\log p^2(z)$$

Non-local form-factors

$$H(z)=\frac{1}{2}\Gamma(0,p^2(z))+\frac{1}{2}\gamma_E+\frac{1}{2}\log p^2(z)$$

- If *p(z)* is a polynomial then UV behavior is asymptotically polynomial, so asymptotically in UV HDQG
- But in *H(z)* there are no poles of *p(z)* due to analytic properties of *H(z)* !

- Unitarity of the theory secured at the perturbative level

- If degree of *p(z)* greater than zero, then theory is automatically multiplicatively renormalizable in *d=4*
- Define $$\deg p(z)=\gamma+1$$

Super-renormalizability

- Progator of modes in UV regime (asymptotics) $\Pi \sim k^{-(4+2\gamma)}$
- General *L*-loop integral is $\text{Integral} = \int (d^4 k)^L \frac{\text{vertices}}{\text{propagators}}$
- Superficial degree of divergence of *L*-loop graph Δ

$$\Delta = 4L + V[\text{vertex}] - I[\text{propagator}]$$

- Graviton field is dimensionless
- The same maximal number of derivatives in vertices as in propagators in UV $[\text{vertex}] = [\text{propagator}] = k^{4+2\gamma}$

- Topology of any graph $I = V + L - 1$

$$\Delta \le 4L - (L-1)(4+2\gamma) \qquad \Delta \le 4 - 2\gamma(L-1)$$

Super-renormalizability

In $d=4$ divergences are present:

- formally for $\gamma=-1$ at any loop order and Δ grows with growing $L \Rightarrow$ non-renormalizability of EH gravity
- for $\gamma=0$ at any loop order and $\Delta \leq 4 \Rightarrow$ renormalizability of R^2 gravity
- for $\gamma=1$ at loop order 1,2,3 $\Rightarrow$ 3-loop super-renormalizability
- for $\gamma=2$ at loop order 1,2 $\Rightarrow$ 2-loop super-renormalizability
- for $\gamma=3$ at loop order 1 $\Rightarrow$ 1-loop super-renormalizability

- Divergences remain only at 1-loop order for $\gamma \geq 3$

We achieved 1-loop super-renormalizability!

Finiteness

- No divergences at the quantum level
- Divergent part of the effective action ~ beta functions of the theory

$$\beta_i = 0$$

related to scale (conformal) invariance and FP of RG flow
- In 1-loop superrenormalizable theory perturbative contributions only at one loop only to four couplings $\lambda \quad \kappa^{-2} \quad \omega_0^1 \quad \omega_0^2$

$$L_{\text{div}} = \lambda + \kappa^{-2} R + \omega_0^1 R^2 + \omega_0^2 R_{\mu\nu}^2$$

- Contributions only from generally covariant terms, with 2γ+4 to 2γ (partial) derivatives on the metric

Finiteness

- Contributions to beta functions
- For cosmological constant $\qquad \beta_\lambda \sim \frac{\omega_{\gamma-2}}{\omega_\gamma}, \left(\frac{\omega_{\gamma-1}}{\omega_\gamma}\right)^2$
- For Planck constant $\qquad \beta_{\kappa^{-2}} \sim \frac{\omega_{\gamma-1}}{\omega_\gamma}, O(\mathrm{Riem}^3)$
- For quadratic in curvature terms $\qquad \beta_{\omega_0^{1,2}} \sim \frac{\omega_\gamma^1}{\omega_\gamma^2}, O(\mathrm{Riem}^3), O(\mathrm{Riem}^4)$

- Set to zero all $\omega_{\gamma-2}$ and $\omega_{\gamma-1}$ and terms cubic, quartic in curvature
- Add two killers of beta functions $\beta_{\omega_0^1}$ and $\beta_{\omega_0^2}$

Convergence in UV

- In UV regime $|z| \to +\infty \qquad e^{H(z)} \to e^{\gamma_E/2} |p(z)|$
- If we choose $p(z) = z^{\gamma+1}$

then the kinetic term $G_{\mu\nu} \frac{e^{H(z)}-1}{\Box} R^{\mu\nu} \to e^{\gamma_E/2} R_{\mu\nu} \frac{|z^{\gamma+1}|}{\Box} R^{\mu\nu} - e^{\gamma_E/2} R \frac{|z^{\gamma+1}|}{2\Box} R$

with identification $z = \frac{\Box}{\Lambda^2}$ and $\omega = \frac{e^{\gamma_E/2}}{\Lambda^{2(\gamma+1)}\kappa^2}$

$$\kappa^{-2} G_{\mu\nu} \frac{e^{H(z)}-1}{\Box} R^{\mu\nu} \to \omega R_{\mu\nu} \Box^{\gamma} R^{\mu\nu} - \frac{\omega}{2} R \Box^{\gamma} R$$

- No other terms in the kinetic part (besides $\kappa^{-2}R$) due to strong UV convergence

$$\forall n \in N \quad \lim_{z \to +\infty} \left(\frac{e^{H(z)}}{e^{\gamma_E/2} |z^{\gamma+1}|} - 1 \right) z^n = 0$$

Killers

- Quadratic in curvature ("kinetic") part of the Lagrangian

$$L = \omega_\gamma^1 R \Box^\gamma R + \omega_\gamma^2 R_{\mu\nu} \Box^\gamma R^{\mu\nu}$$

- One of the simplest choice

$$s_1 R^2 \Box^{\gamma-2} R^2 + s_2 R_{\mu\nu} R^{\mu\nu} \Box^{\gamma-2} R_{\rho\sigma} R^{\rho\sigma}$$

- Contribution to beta functions from killers $\beta_{\omega_0^{1,2}\,\mathrm{kill}} \sim \frac{s}{\omega_\gamma}$

Finiteness if $\beta_{\omega_0^{1,2}} + \beta_{\omega_0^{1,2}\,\mathrm{kill}} = 0$

- Contribution of killers to be computed using Barvinsky-Vilkovisky technology for traces of covariant operators on any background and in Dimensional Regularization (d=4-ε)

Computation

- 1-loop Quantum Effective Action $\Gamma = \frac{i}{2} \mathrm{Tr} \ln \hat{H}$ $\Gamma_{\mathrm{div}} = -\frac{1}{\varepsilon} \sum_i \beta_i X_i$
- Kinetic operator for quantum fluctuations on any curved background

$$H^{\mu\nu,\rho\sigma} = \frac{\delta^2 S}{\delta g_{\mu\nu} \delta g_{\rho\sigma}}$$

- Contribution from killers we need only to quadratic in curvature order
- In BV trace technology killers contribute only to *U* terms (with 2γ derivatives)

$$\mathrm{Tr} \ln \hat{H}_{K1} = s_1 \frac{i}{\varepsilon} \frac{24 R^2}{3 \omega_\gamma^1 + \omega_\gamma^2}$$

$$\mathrm{Tr} \ln \hat{H}_{K2} = s_2 \frac{i}{\varepsilon} \left(\frac{(-10 \omega_\gamma^1 + \omega_\gamma^2) R^2}{3 \omega_\gamma^2 (3 \omega_\gamma^1 + \omega_\gamma^2)} + \frac{2 (20 \omega_\gamma^1 + 7 \omega_\gamma^2) R_{\mu\nu}^2}{3 \omega_\gamma^2 (3 \omega_\gamma^1 + \omega_\gamma^2)} \right)$$

Finiteness

- Beta functions of quadratic in curvature couplings

$$\beta_{R^2} := a_1 s_1 + a_2 s_2 + c_1 \qquad \beta_{R^2_{\mu\nu}} := b_2 s_2 + c_2$$

- c_1 and c_2 are contributions from terms in “kinetic” part of Lagrangian

- Coefficients of killers required to kill beta functions

$$s_1 = \frac{-(3\omega_\gamma^1 + \omega_\gamma^2)(40\, c_1 \omega_\gamma^1 + 10\, c_2 \omega_\gamma^1 + 14\, c_1 \omega_\gamma^2 - c_2 \omega_\gamma^2)}{24\,(20\,\omega_\gamma^1 + 7\,\omega_\gamma^2)}$$

$$s_2 = \frac{-3\, c_2\, \omega_\gamma^2 (3\,\omega_\gamma^1 + \omega_\gamma^2)}{20\,\omega_\gamma^1 + 7\,\omega_\gamma^2}.$$

Finite Quantum Gravity

One of the simplest Lagrangian for finite QG theory in *d=4*

$$L=\lambda+\kappa^{-2}R+\kappa^{-2}G_{\mu\nu}\frac{e^{H(z)}-1}{\Box}R^{\mu\nu}+s_1R^2\Box R^2+s_2R_{\mu\nu}R^{\mu\nu}\Box R_{\rho\sigma}R^{\rho\sigma}+$$

$$+\sum_i c_i^{(3)}(R^3)_i+\sum_i c_i^{(4)}(R^4)_i+\sum_i c_i^{(5)}(R^5)_i \qquad z=\frac{\Box}{\Lambda^2}$$

with $\quad H(z)=\frac{1}{2}\Gamma(0,p^2(z))+\frac{1}{2}\gamma_E+\frac{1}{2}\log p^2(z) \quad p(z)=z^4 \quad \gamma=3$

Lagrangian in UV

$$L_{\mathrm{UV}}=\lambda+\kappa^{-2}R+\omega R_{\mu\nu}\Box^3R^{\mu\nu}-\frac{\omega}{2}R\Box^3R+s_1R^2\Box R^2+s_2R_{\mu\nu}^2\Box R_{\rho\sigma}^2+$$

$$+\sum_i c_i^{(3)}(R^3)_i+\sum_i c_i^{(4)}(R^4)_i+\sum_i c_i^{(5)}(R^5)_i \qquad \omega=\frac{e^{\gamma_E/2}}{\Lambda^8\kappa^2}$$

Conclusions

- E-H QG is valid non-renormalizable EFT below Planck scale
- HDQG is renormalizable, can be made even 1-loop super-renormalizable, has massive ghosts
- Nonlocality in formfactors solves unitarity problems, HDQG revival !!
- Still possible polynomial behaviors for propagation asymptotically in UV
- Divergences only at one-loop order
- Perturbative finiteness obtained by adding killers
- Easy generalizations to higher dimensions and for higher curvature terms in the action

Conclusions

Finite Quantum Gravity Exists!!!

Thank you
for attention!