What I talk about when I talk about Inflation

Eugene A. Lim
w/ Josu Aurrekoetxea, Katy Clough and Raphael Flauger

arXiv: 1712.07352 **arXiv: 1608.04408** **arXiv: 190X.XXXX**

Is Inflation Paradigmatic?

Inflationary paradigm in trouble after Planck2013

Anna Ijjas[a,b], Paul J. Steinhardt[a,c,d,*], Abraham Loeb[a]

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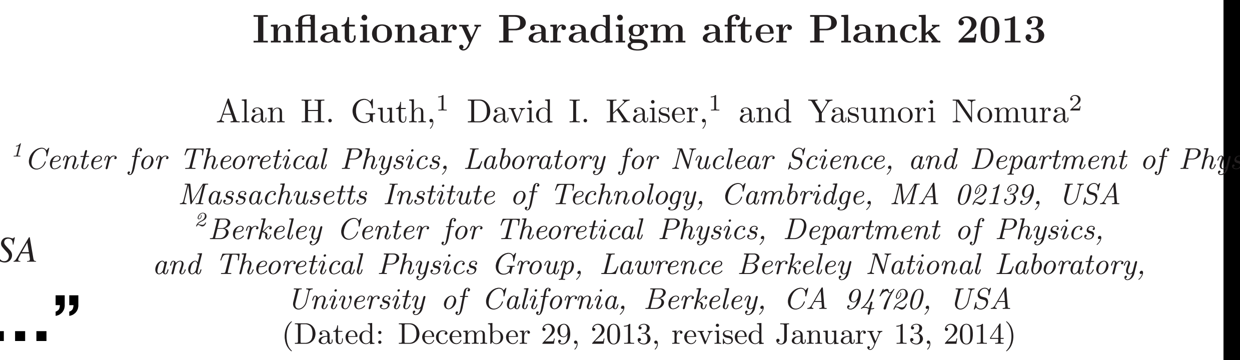
"…serious difficulties that cut to the core…"



Inflationary Paradigm after Planck 2013

Alan H. Guth,[1] David I. Kaiser,[1] and Yasunori Nomura[2]

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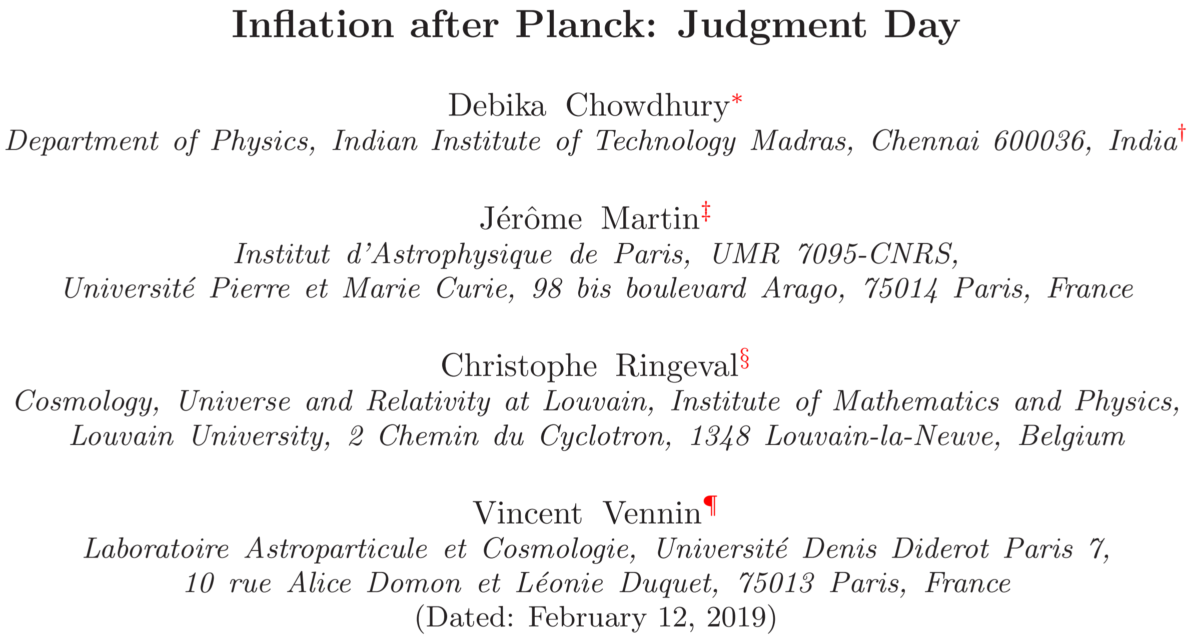
Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[2] *Berkeley Center for Theoretical Physics, Department of Physics,*
and Theoretical Physics Group, Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720, USA
(Dated: December 29, 2013, revised January 13, 2014)

"…on a stronger footing than ever."



Inflation after Planck: Judgment Day

Debika Chowdhury*
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Vincent Vennin¶
Laboratoire Astroparticule et Cosmologie, Université Denis Diderot Paris 7,
10 rue Alice Domon et Léonie Duquet, 75013 Paris, France
(Dated: February 12, 2019)

"…strengthened by Planck data."

SPACE

Cosmic Inflation Theory Faces Challenges

The latest astrophysical measurements, combined with theoretical problems, cast doubt on the long-cherished inflationary theory of the early cosmos and suggest we need new ideas

By Anna Ijjas, Paul J. Steinhardt, Abraham Loeb on February 1, 2017

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A *Scientific American* article about the theory of inflation prompted a reply from a group of 33 physicists, along with a response from the article's authors

Scientific American

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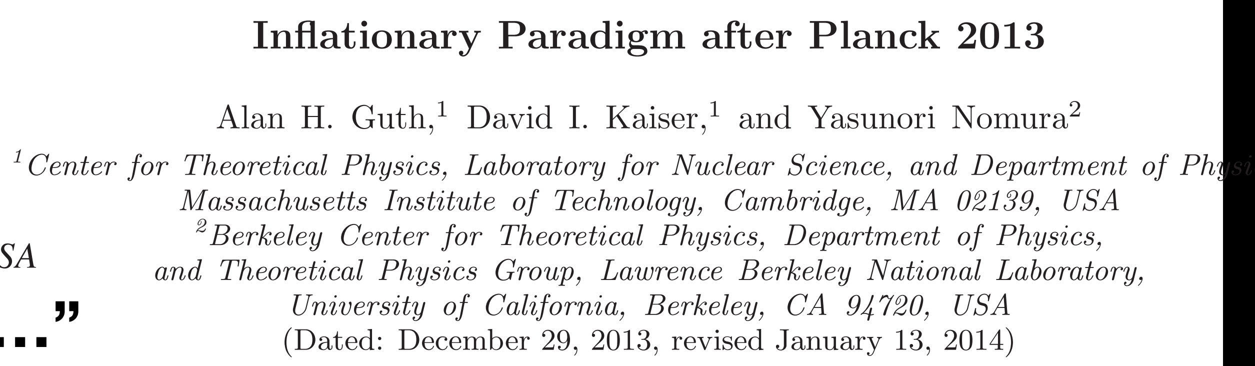

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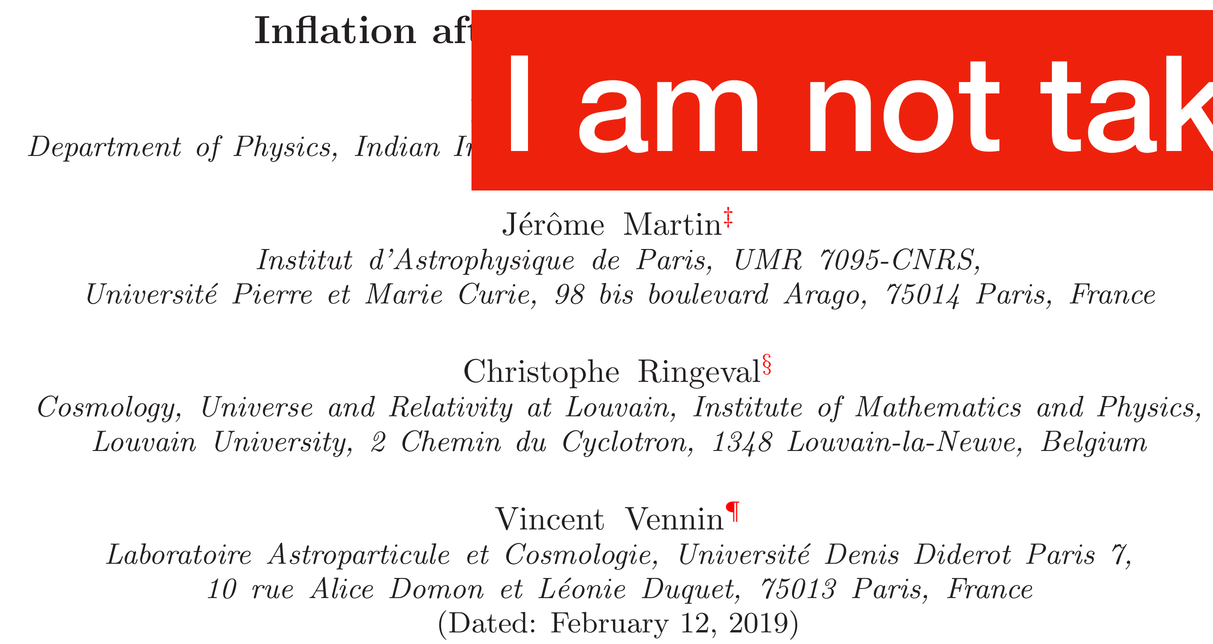
“…on a stronger footing than ever.”

I am not taking sides!



Inflation af

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Scientific American

A story in 2 parts

Part I

Conceptual Issues with Inflation

Part II

Exploring the Theory Space of Successful Inflation

Part I

Conceptual issues with Inflation

Our Universe is Special

Our Universe is Special

Usually framed as “Problems”

Our Universe is Special

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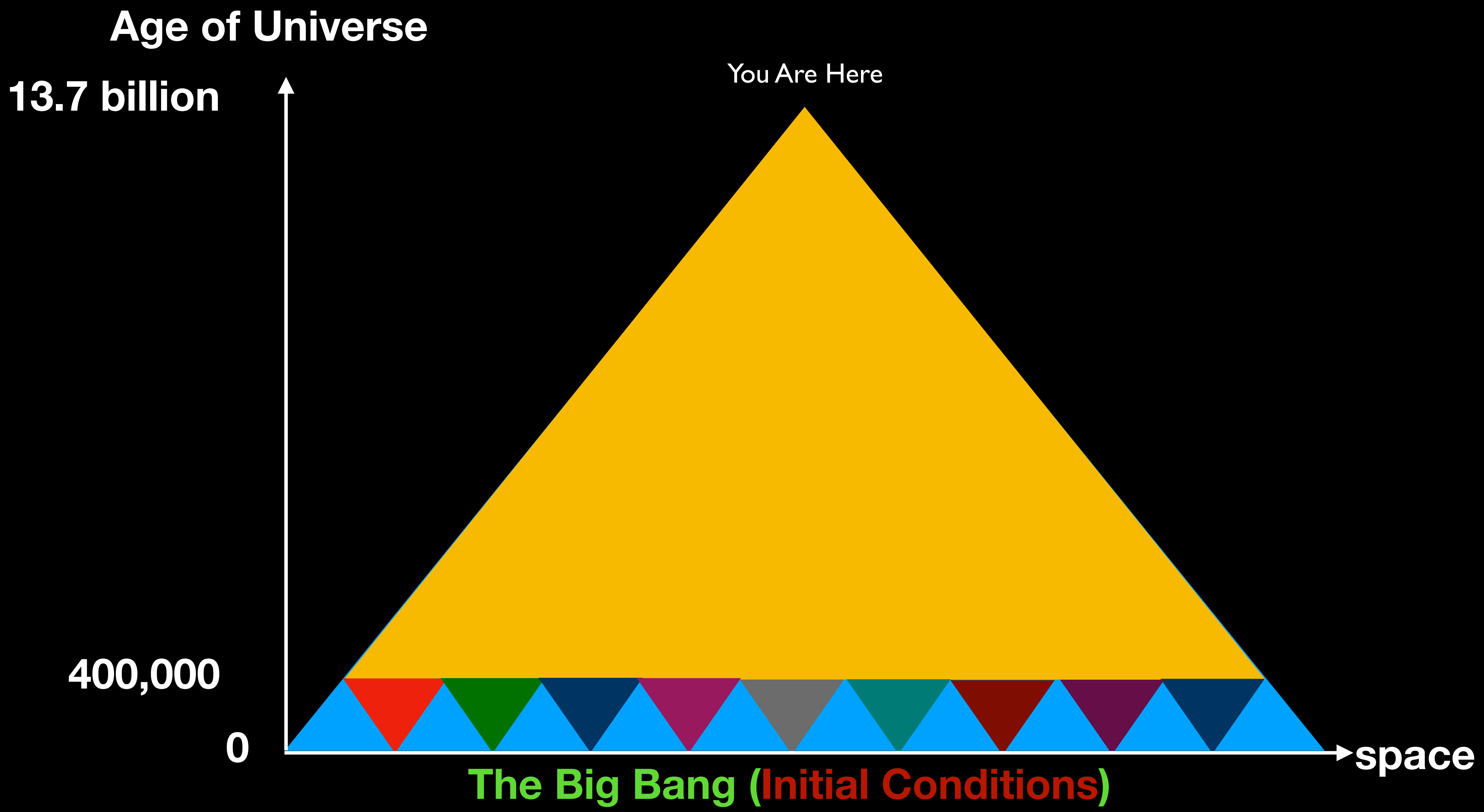
Horizon Problem

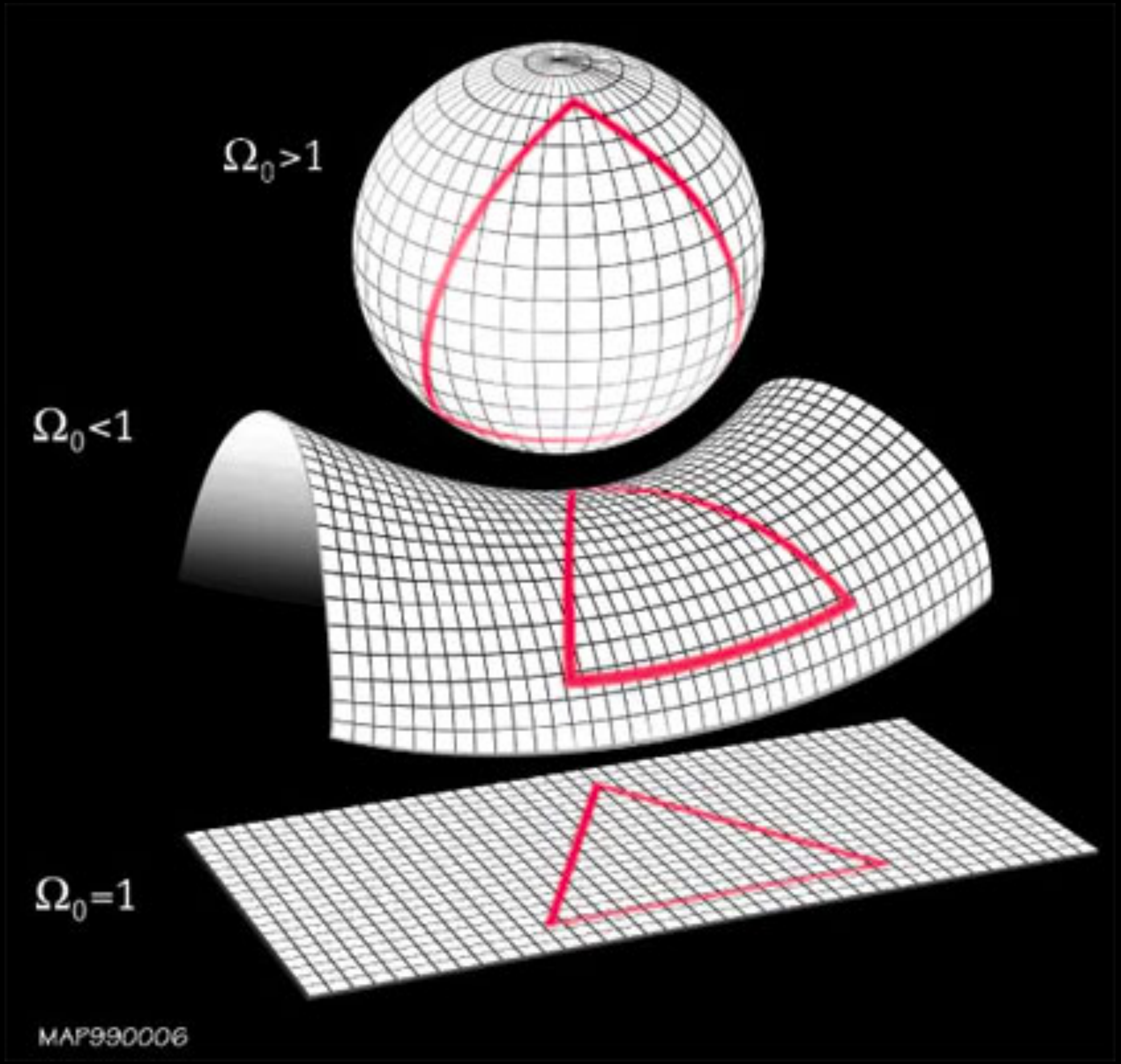


Our Universe is Special

Usually framed as “Problems”

Flatness Problem

$$|\Omega_K| < 0.01$$



Our Universe is Special

Why are they “Problems”?

Our Universe is Special

Why are they "Problems"?

A question of ***expectation*.**

We observed (realization) $|\Omega_K| < 0.01$

We expect $|\langle\Omega_K\rangle| \overset{?}{\gg} 0$

Our Universe is Special

Why are they “Problems”?

A question of ***expectation***.

We observed (realization) $|\Omega_K| < 0.01$

We expect $|\langle\Omega_K\rangle| \overset{?}{\gg} 0$

Actually, what do we really expect??

Expectation values need an ***ensemble*** to draw from.

Our Universe is Special

We can’t construct such an ensemble from observations (only one Universe).

No prior information -> *all possible initial states* are in play.

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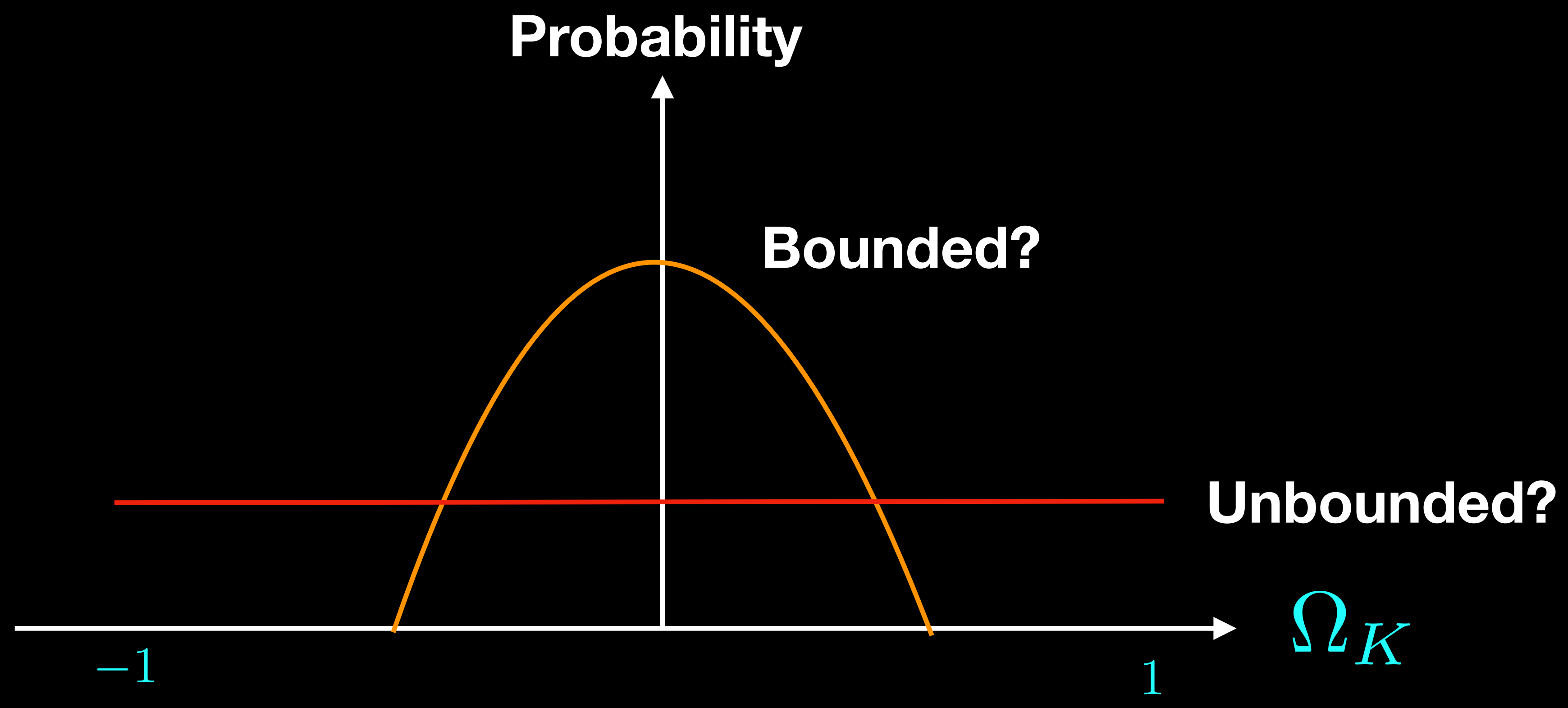
Probability Distribution of Ensemble



Initial Conditions or Dynamics?

The Big Bang (?)

Initial Conditions

Dynamics

Cosmological Expansion
Friedman-Robertson-Walker

Today

Observations

$\Omega_K < 0.01$

Homogenous

Initial Conditions or Dynamics?

The Big Bang (?)

Initial Conditions

Dynamics

Cosmological Expansion
Friedman-Robertson-Walker

Today

Observations

$\Omega_K < 0.01$

Homogenous

Can we explain Today by

Initial Conditions?
Dynamics?
Both?

Why is the Universe Special?

Option 1 : We got (un)lucky — “Fine-tuned”.

Option 1 : We got (un)lucky

The "Phase Space of All Possible Initial Conditions"

FIGURE 10. The Creator locating the tiny region of phase-space—one part in $10^{10^{123}}$—needed to produce a 10^{80}-baryon closed universe with a second law of thermodynamics in the form we know it.

Penrose 1989

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Why is the Universe Special?

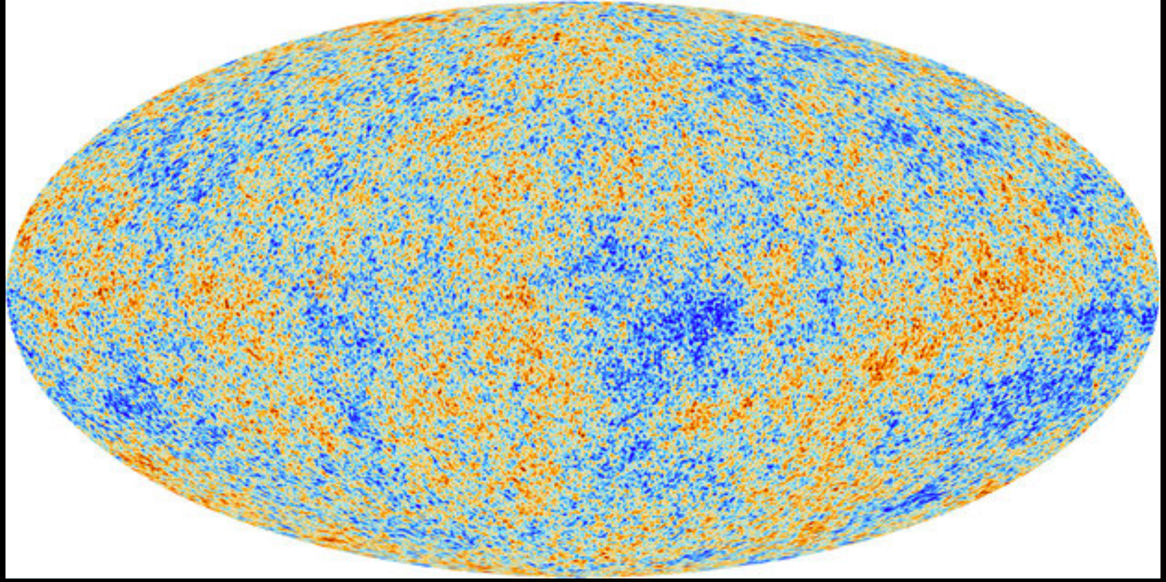
~~Option 1 : We got (un)lucky — “Fine-tuned”.~~

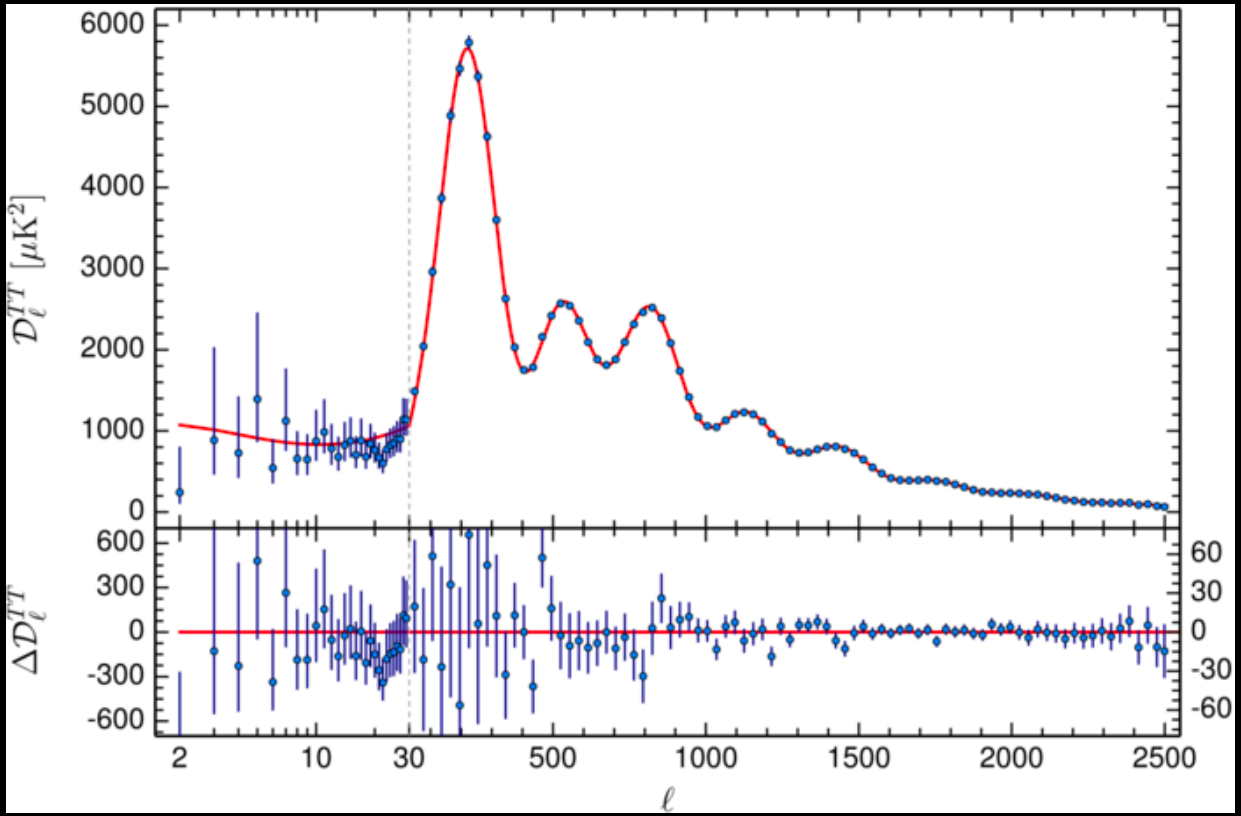
Option 2 : There is some unknown “Theory of Initial Conditions” that defines the ensemble and/or choose the initial state (Penrose, Wald, et al.)

Option 3 : Some dynamical process makes the initial conditions irrelevant (Inflation).

Option 3 : Inflation is a great success

Connecting gravity with quantum field theory, and makes confirmed predictions.





Straightforward framework, easy to modify, rich in pheno -> 10000++ papers since 1980.

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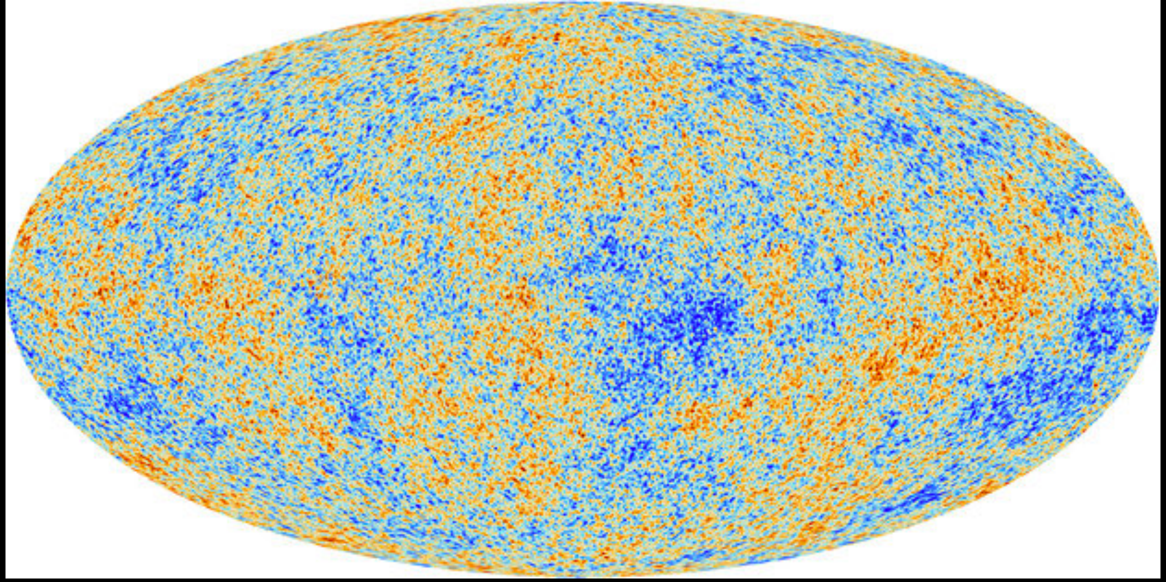

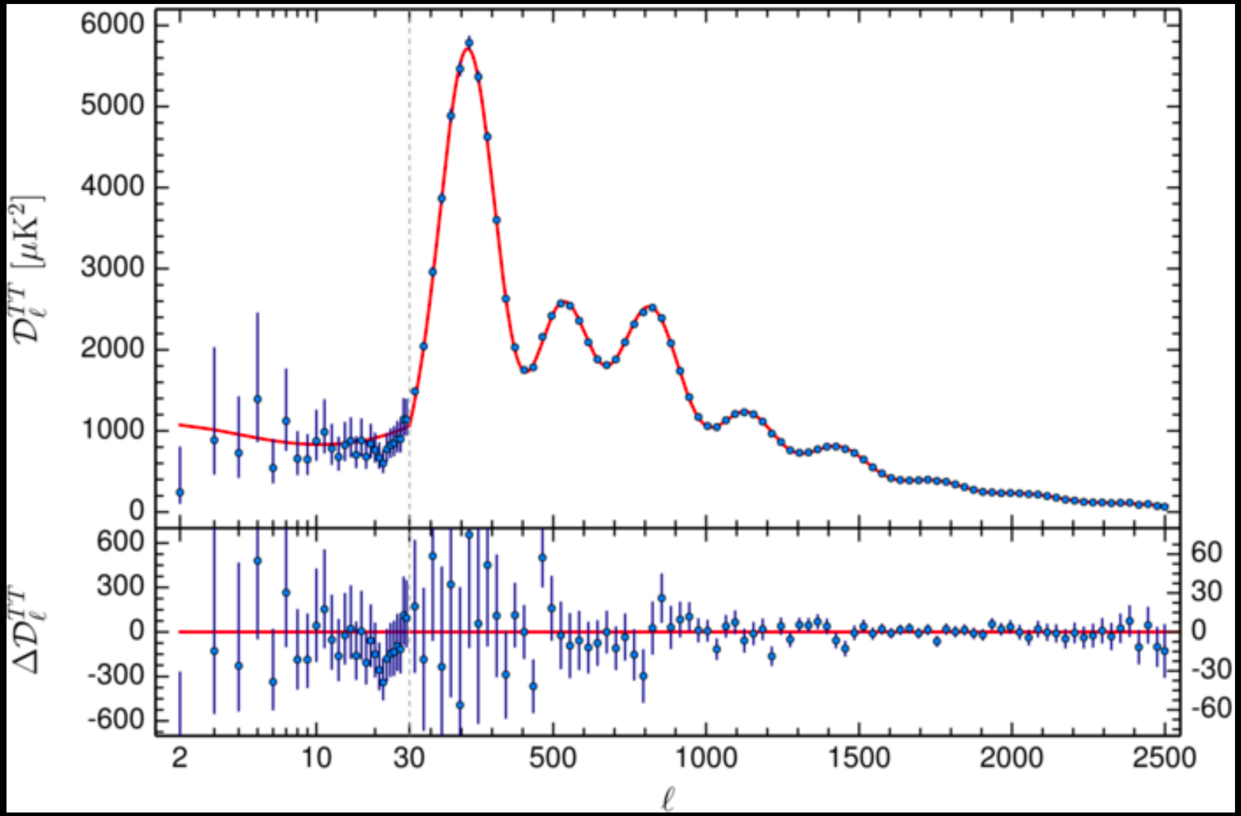

Straightforward framework, easy to modify, rich in pheno -> 10000++ papers since 1980.

"So easy! I got a paper out in 3 weeks. Would write again. A++++++"

a satisfied grad student

But does it work?

Option 3 : Some dynamical process makes the initial conditions irrelevant (Inflation).

?

There exist sick initial conditions which no model of inflation will inflate.

But does it work?

Option 3 : Some dynamical process makes the initial conditions irrelevant (Inflation).

?

There exist sick initial conditions which no model of inflation will inflate.

A Conceptual Retreat:

Option 3 : Inflation will begin for “generic” initial conditions.

What is “generic”? We have to deal with the ensemble again.

The Measure of all possible initial states

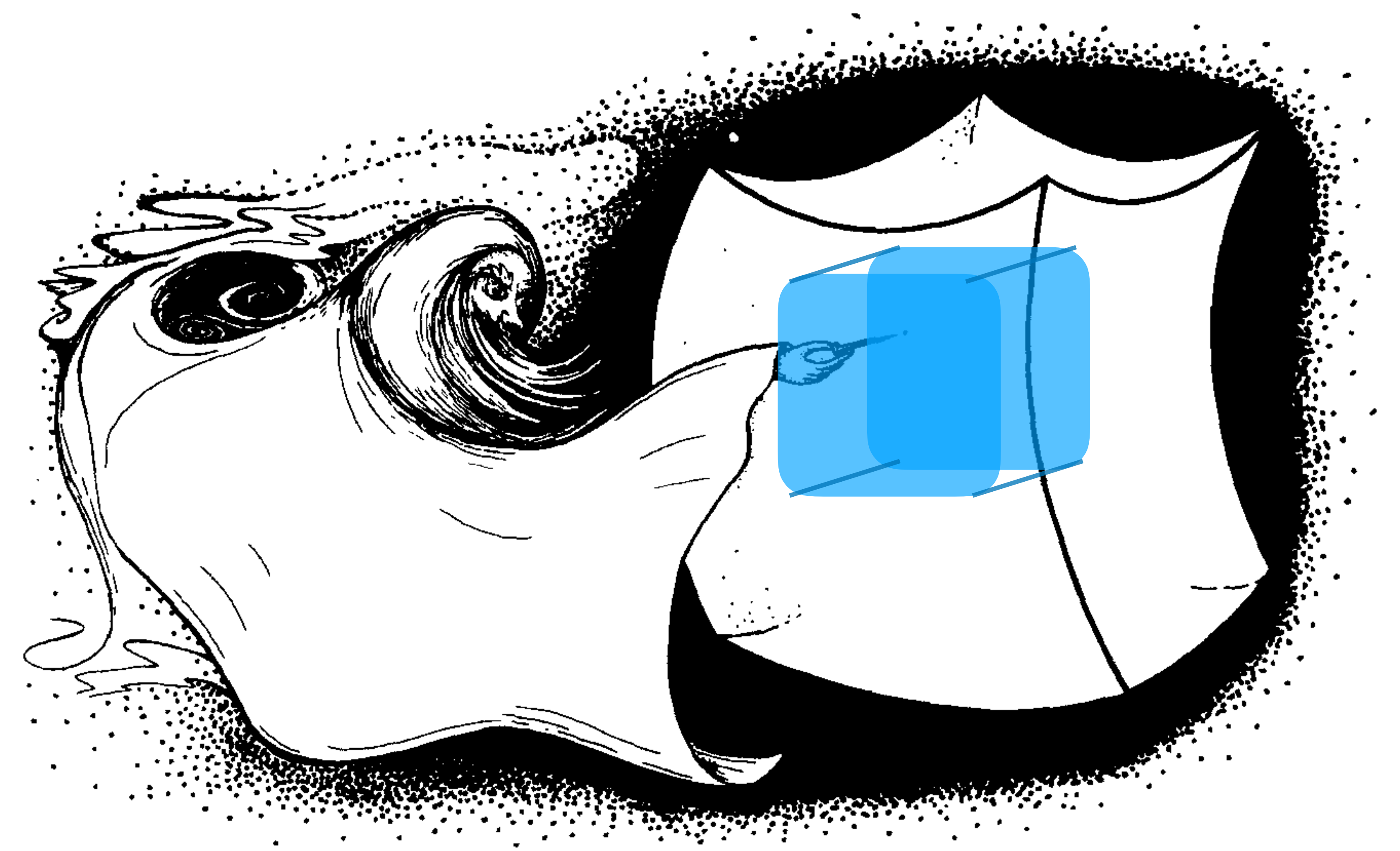

FIGURE 10. The Creator locating the tiny region of phase-space—one part in $10^{10^{123}}$—needed to produce a 10^{80}-baryon closed universe with a second law of thermodynamics in the form we know it.

How to *measure* the "big-ness/generic-ness" of this "successful inflation initial conditions space".

The Measure of all possible initial states

To get from all possible initial (micro)states -> an expectation of a macrostate, need a **Measure.**

$$\langle \Omega_K \rangle = \int F(p_i, q_i) \hat{\Omega}$$

all initial state space

Measure

The Measure of all possible initial states

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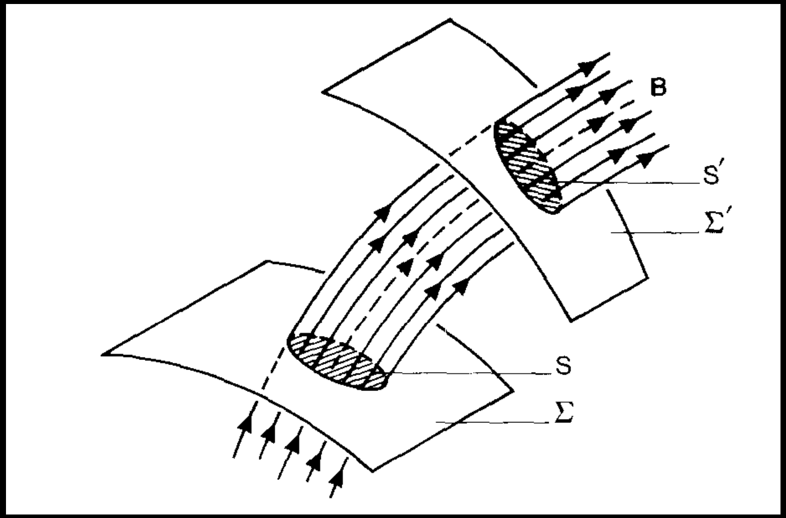
all initial state space

Measure

Liouville Measure : $\hat{\Omega} = \frac{(-1)^{n(n-1)/2}}{n!} \omega^n$ $\omega = \sum_i^n dp_i \wedge dq^i$

Conserved under time evolution:
initial conditions space =
trajectories space



An example : GHS Measure

Gibbons, Hawking, Stewart(1987)

Ignore everything, and consider only scale factor a and inflaton ϕ as canonical variables ("mini-superspace").

$$\hat{\Omega} = (dp_a \wedge da + dp_\phi \wedge d\phi)_{\text{Hamiltonian constraint}=0}$$

Q: what is the probability of Ω_K to result from inflation?

$$P(\Omega_K) = \frac{\hat{\Omega}(\Omega_k)}{\hat{\Omega}(\text{all states})}$$

An example : GHS Measure

Gibbons, Hawking, Stewart(1987)

$$\hat{\Omega}(\text{all states}) \propto \int \frac{1-\Omega_V-\frac{2}{3}\Omega_K}{|\Omega_K|^{5/2}(1-\Omega_V-\Omega_K)^{1/2}} d\Omega_K d\phi$$

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Carroll + Tam (2010) : flatness is *not* a problem since measure dominated by flat trajectories!

Gibbons + Turok (2008) : divergences are bad, regulate it!

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Measure on initial conditions is a critical open problem!!

Is Inflation a Theory?

“Can construct any theory/model to replicate any observations.”

"Theory is not falsifiable.”

Komatsu-Tesileanu Bayes Argument

from E. Komatsu’s Facebook Post (liked 106 times).

We Want	$P(\mathrm{Model}\vert\mathrm{Data})$	“Can we pin down the exact model of inflation given data”
We have	$P(\mathrm{Data}\vert\mathrm{Model})$	“Inflation occurs, can we determine how likely the data is given a model?”
Theory	$P(\mathrm{Model})$	Prior distribution of models.

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Theory	$P(\text{Model})$	Prior distribution of models.

Bayes' Theorem $$P(\text{Model}|\text{Data}) = \frac{P(\text{Data}|\text{Model})P(\text{Model})}{P(\text{Data})}$$

Normalizability (Measure!) $$\int d(\text{Model})P(\text{Model}) = 1$$

Komatsu-Tesileanu Bayes Argument

Bayes' Theorem

$$P(\mathrm{Model}|\mathrm{Data}) = \frac{P(\mathrm{Data}|\mathrm{Model})P(\mathrm{Model})}{P(\mathrm{Data})}$$

Komatsu-Tesileanu Bayes Argument

Analyze data to get this

Bayes' Theorem

$$P(\text{Model}|\text{Data}) = \frac{\boxed{P(\text{Data}|\text{Model})}\,P(\text{Model})}{P(\text{Data})}$$

Komatsu-Tesileanu Bayes Argument

Bayes' Theorem

$$P(\text{Model}|\text{Data}) = \frac{\overset{\text{Analyze data to get this}}{\boxed{P(\text{Data}|\text{Model})}}\,\overset{\text{Assume this exists}}{\boxed{P(\text{Model})}}}{P(\text{Data})}$$

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$$\overset{\text{Infer this}}{\boxed{P(\text{Model}|\text{Data})}} = \frac{\overset{\text{Analyze data to get this}}{\boxed{P(\text{Data}|\text{Model})}}\,\overset{\text{Assume this exists}}{\boxed{P(\text{Model})}}}{P(\text{Data})}$$

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"*Can construct any theory to replicate any observations.*"

-> Model space is infinite, so $P(\text{Model}) \to 0$

So is $P(\text{Model})$ finite or zero?

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“*Can construct any theory to replicate any observations.*”

-> Model space is infinite, so $P(\mathrm{Model}) \to 0$

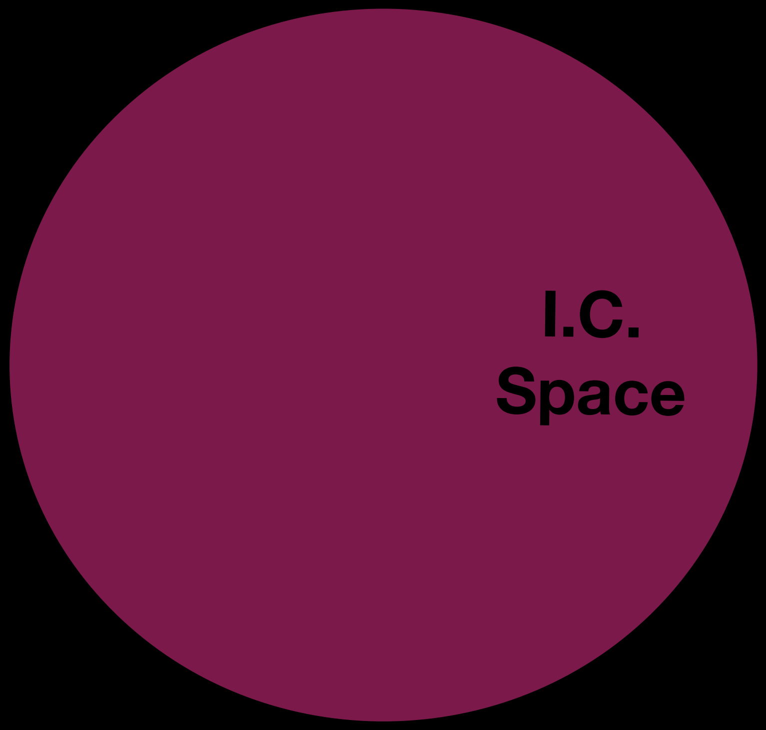
So is $P(\mathrm{Model})$ finite or zero?

Nobody knows how to compute $P(\mathrm{Model})$

“*What is the measure on the model space?*”

Initial Conditions Space *vs* Model Space

Initial Conditions Space : initial values of the variables of the Theory Space



Initial Conditions Space *vs* Model Space

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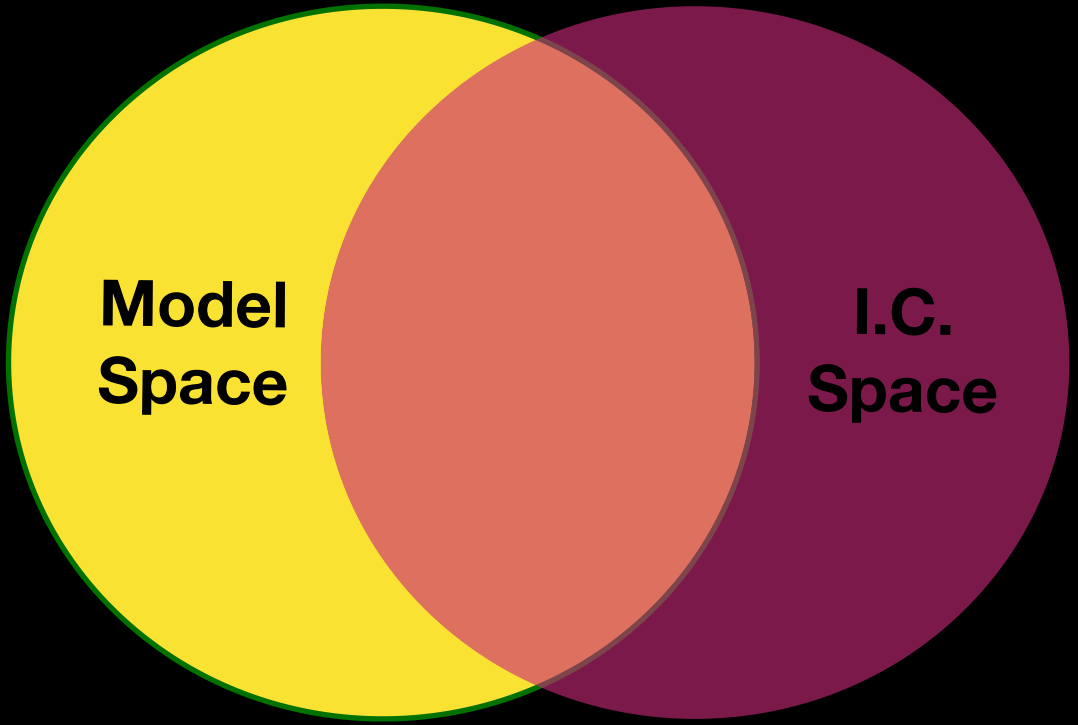
Model Space : different models/dynamics of inflation



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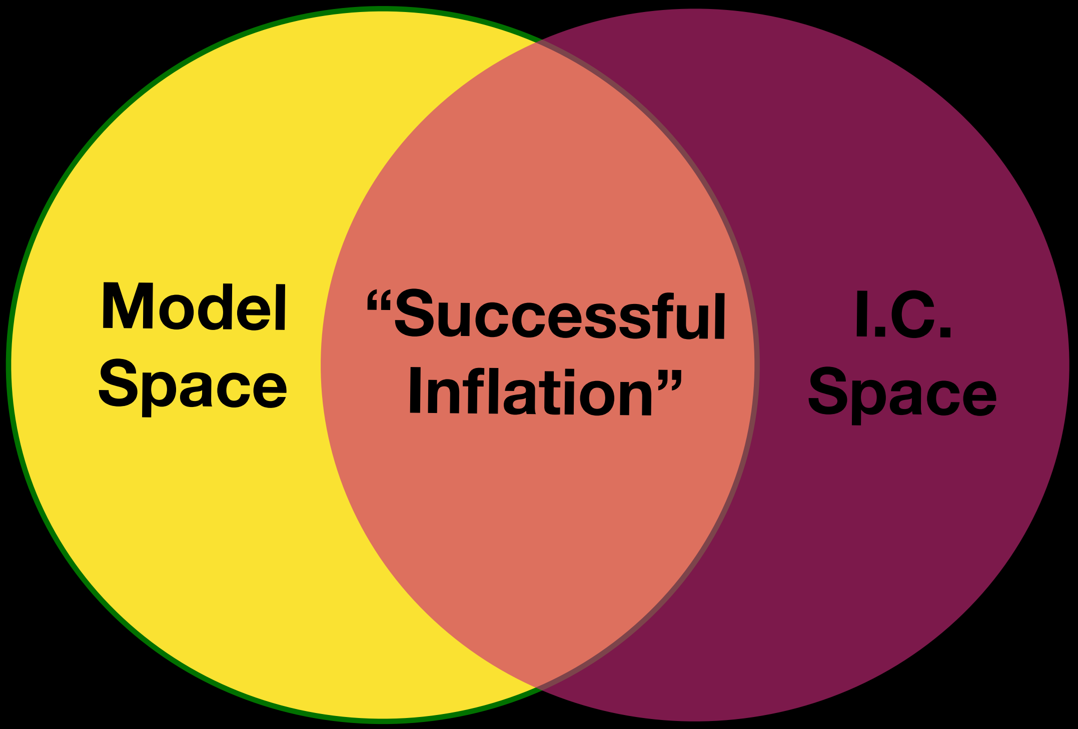

Model Space

“Successful Inflation”

I.C. Space

We need a joint measure on Initial conditions and Model space.

“Just need a small patch to inflate”

“Just need inflation to occur at a small patch, and the volume will exponentially increase and dominate all the non-inflated regions.”

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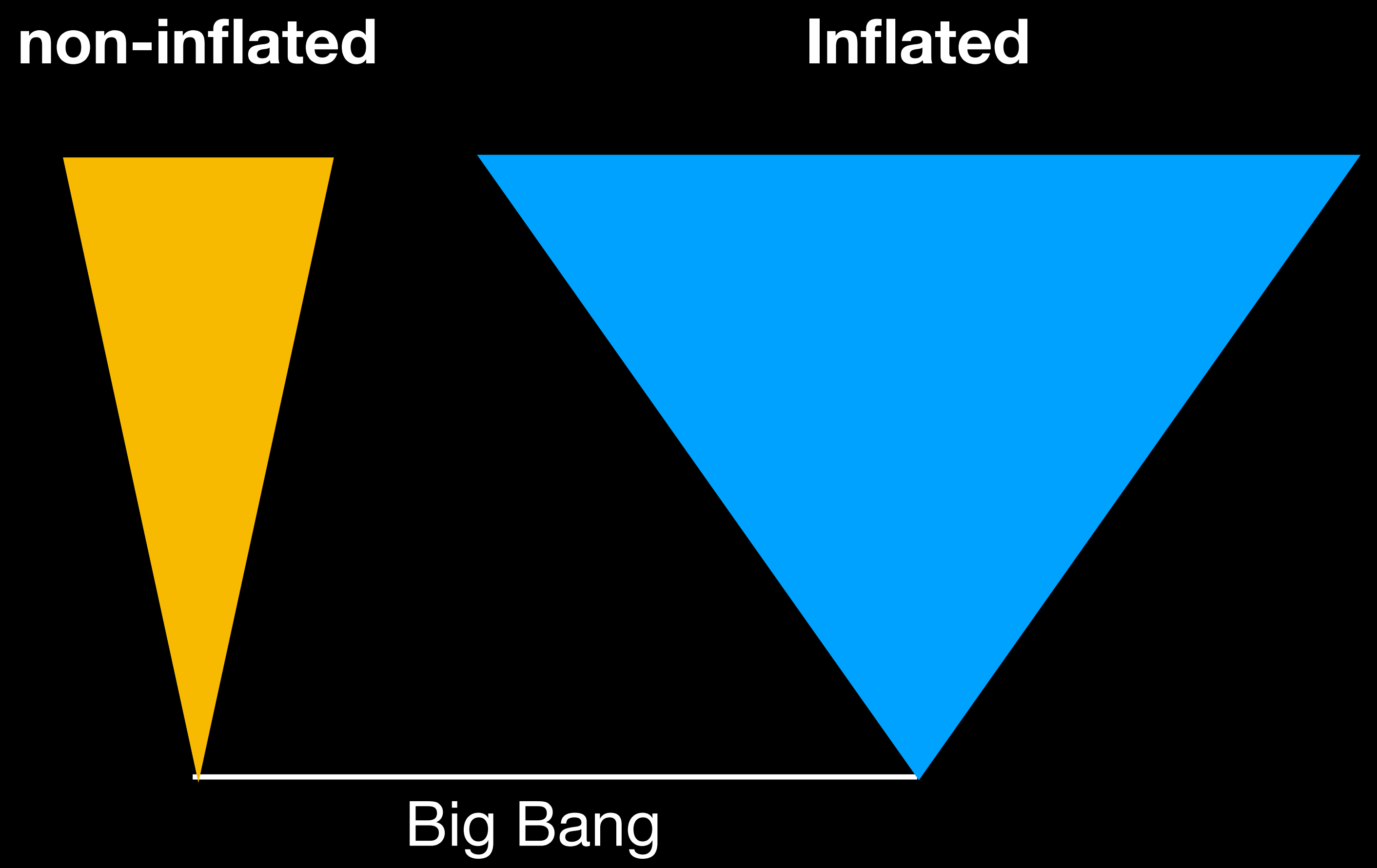

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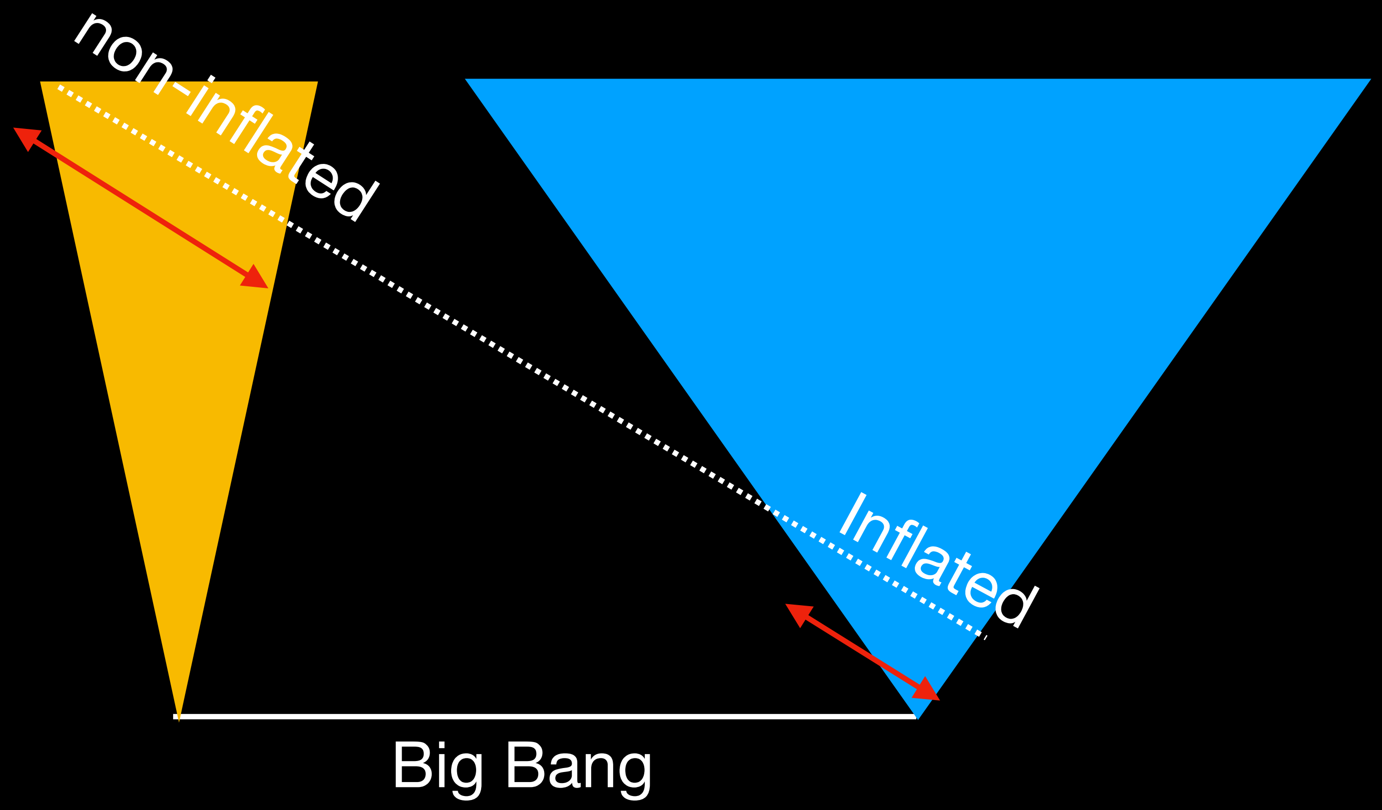
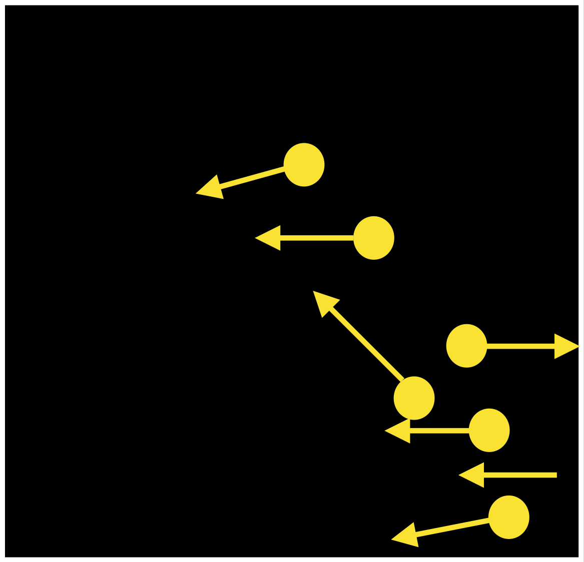


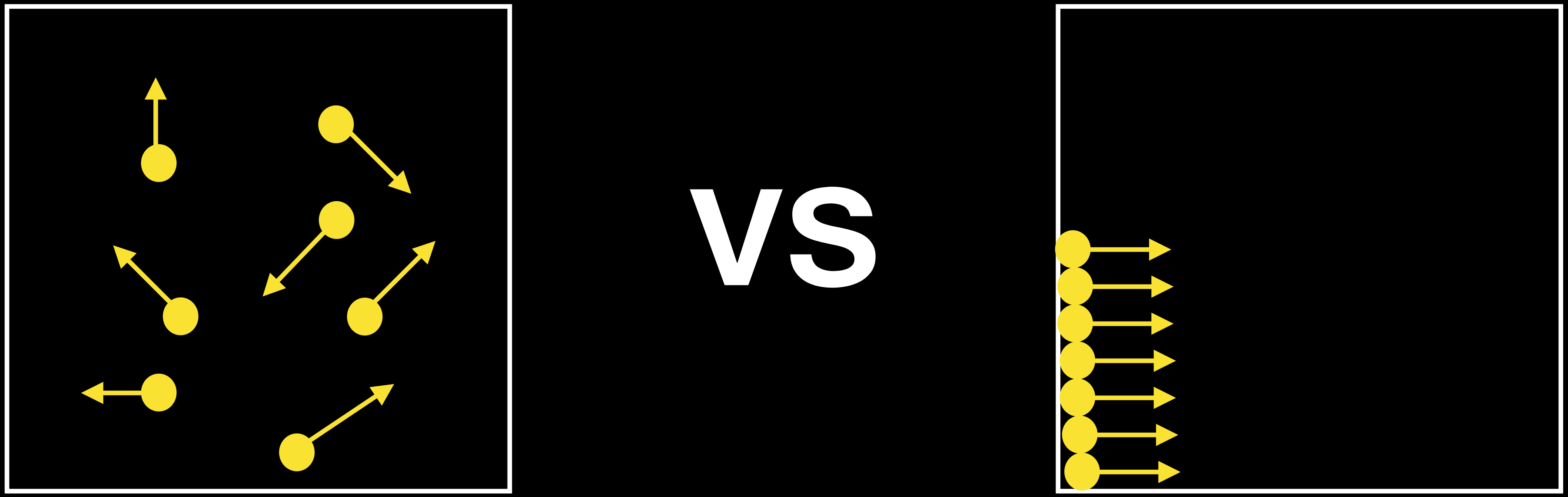
Gauge dependent : Need a Measure (again)!

Pop Quiz!

If you find a box like this on the street…



Which of these is more likely its original initial condition?



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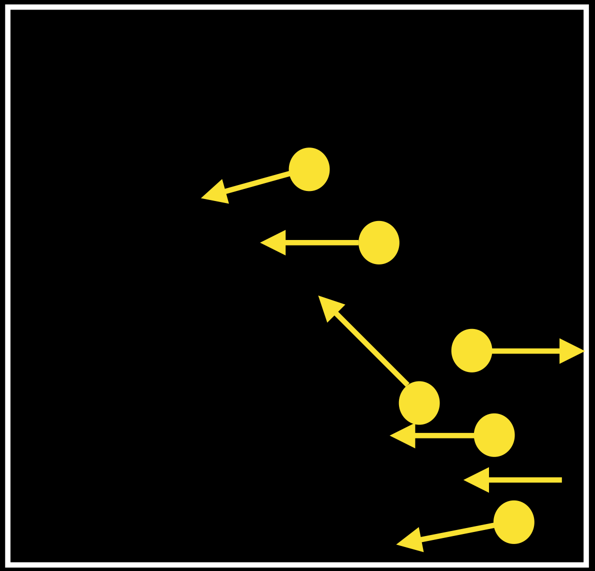

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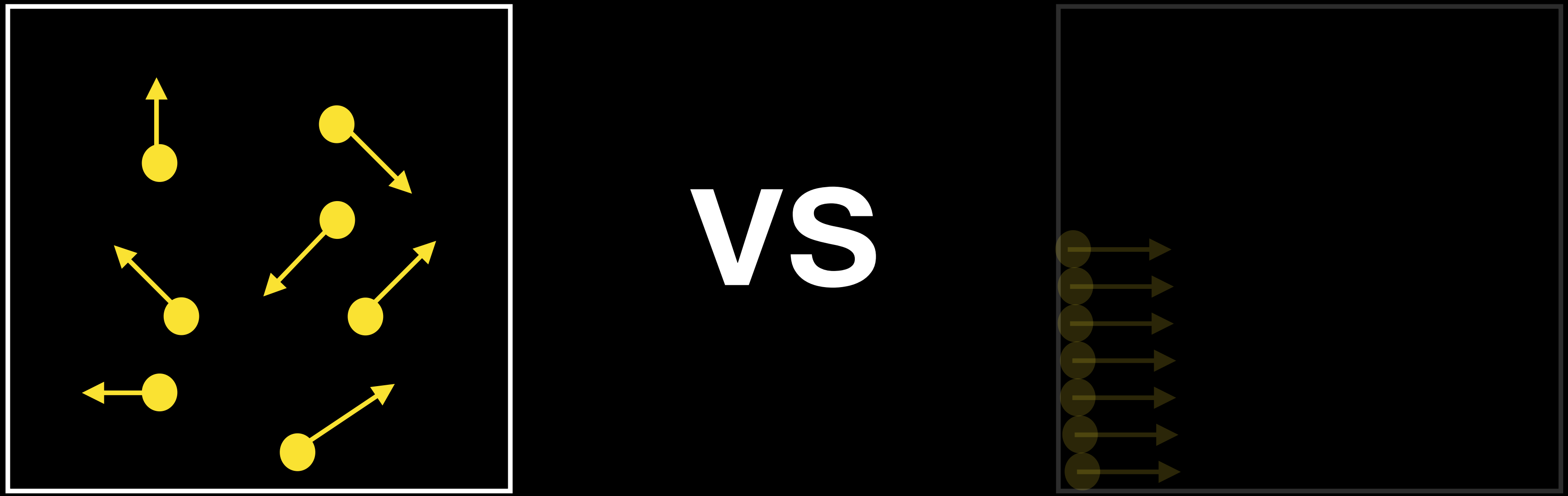
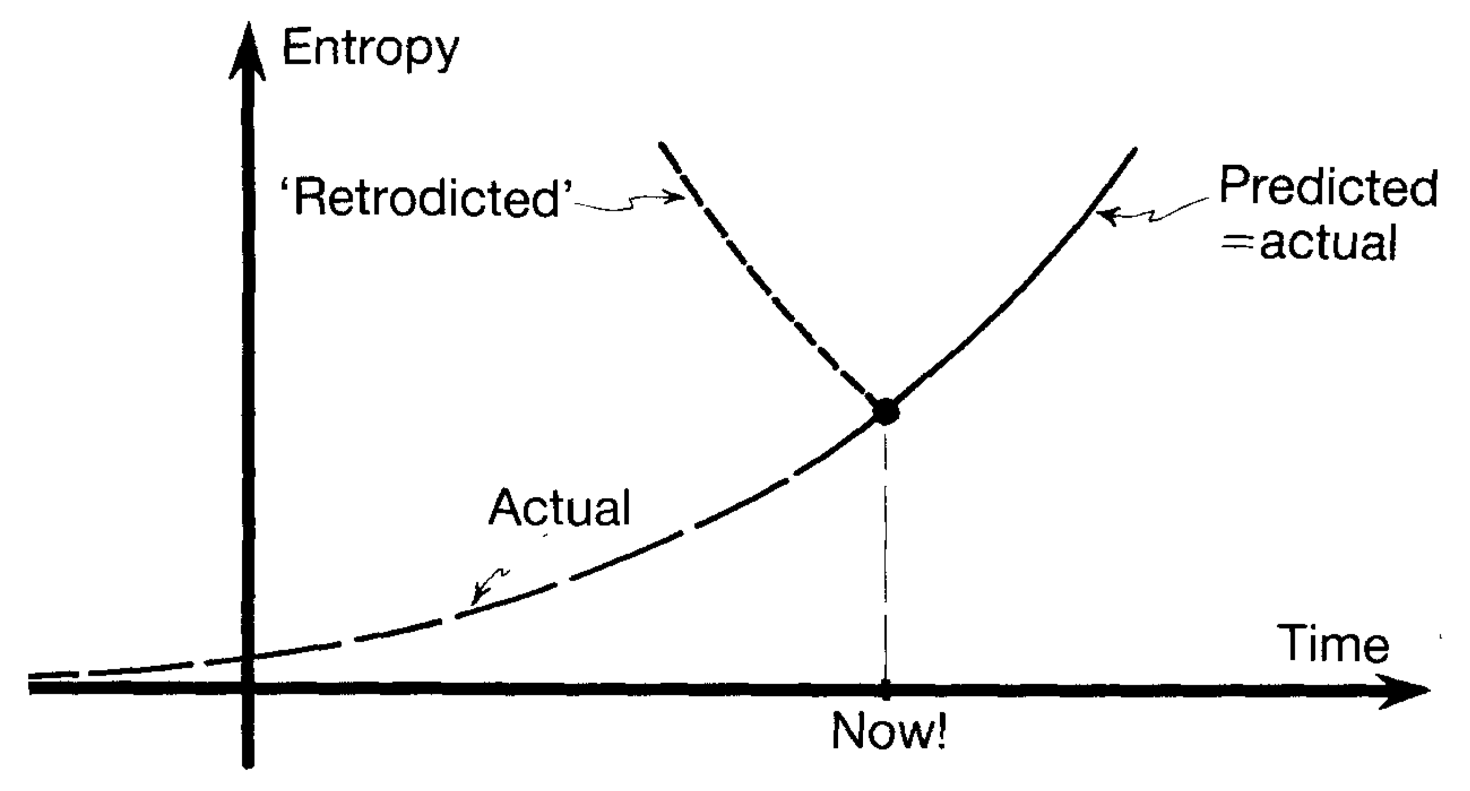


Microphysical laws are time-reversal invariant!

The Penrose Argument

Penrose 1989

“The past is low entropy by assumption.”



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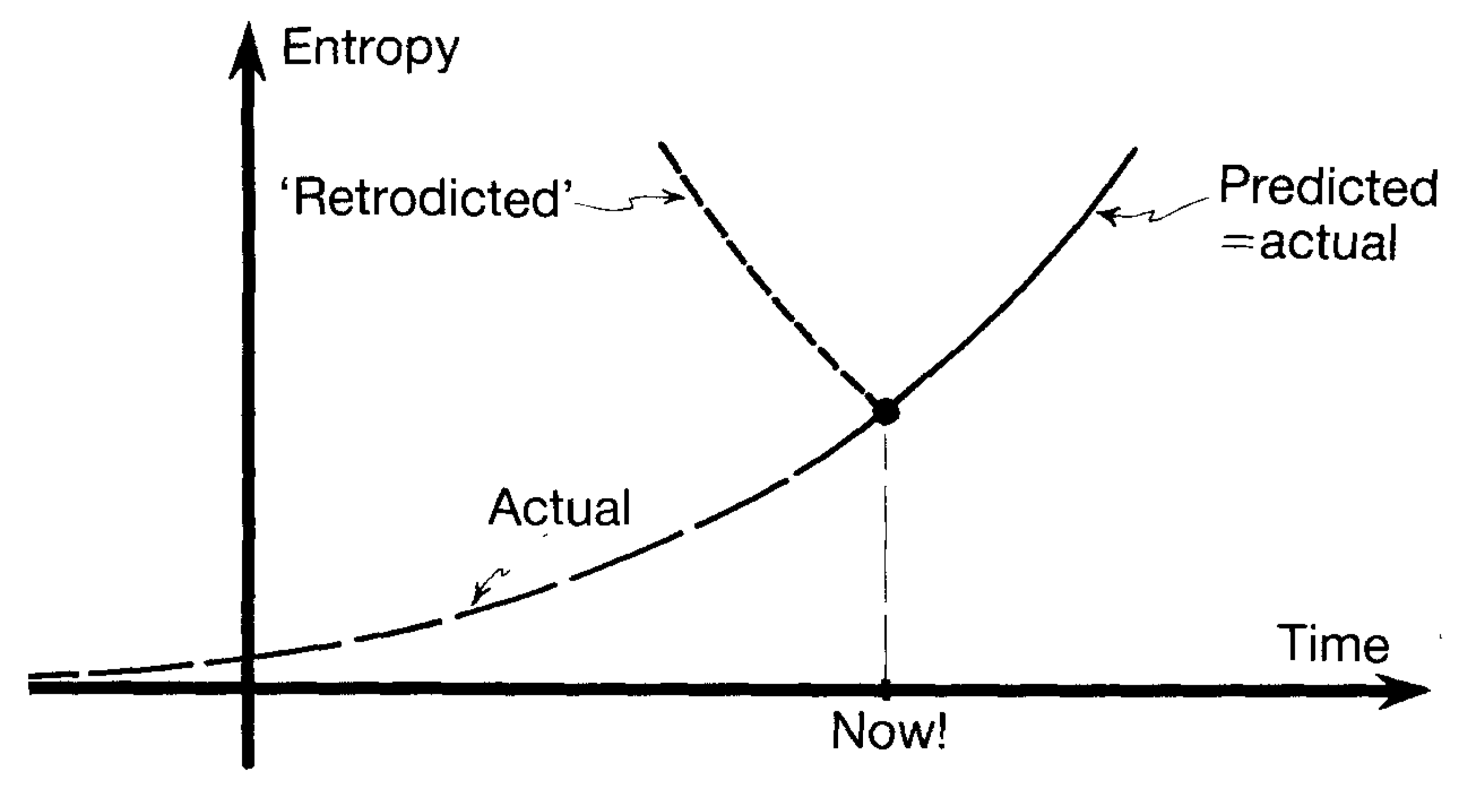

Penrose 1989

We need additional constraints/theory/assumptions *regardless* of Horizon/Flatness problems.

The Penrose Argument

Penrose 1989

Penrose :

Our challenge is *not* to assume the initial conditions are random (and hence generically high entropy) then solve it by inflation, but to explain why it is low entropy *in the first place*.

Option 2 : There is some unknown “Theory of Initial Conditions” that defines the ensemble and/or choose the initial state.

Part II

Exploring the space of Successful Inflation

Problems are Opportunities

My guess : something like inflation probably occurred, but it *we still need to explain the initial conditions.*

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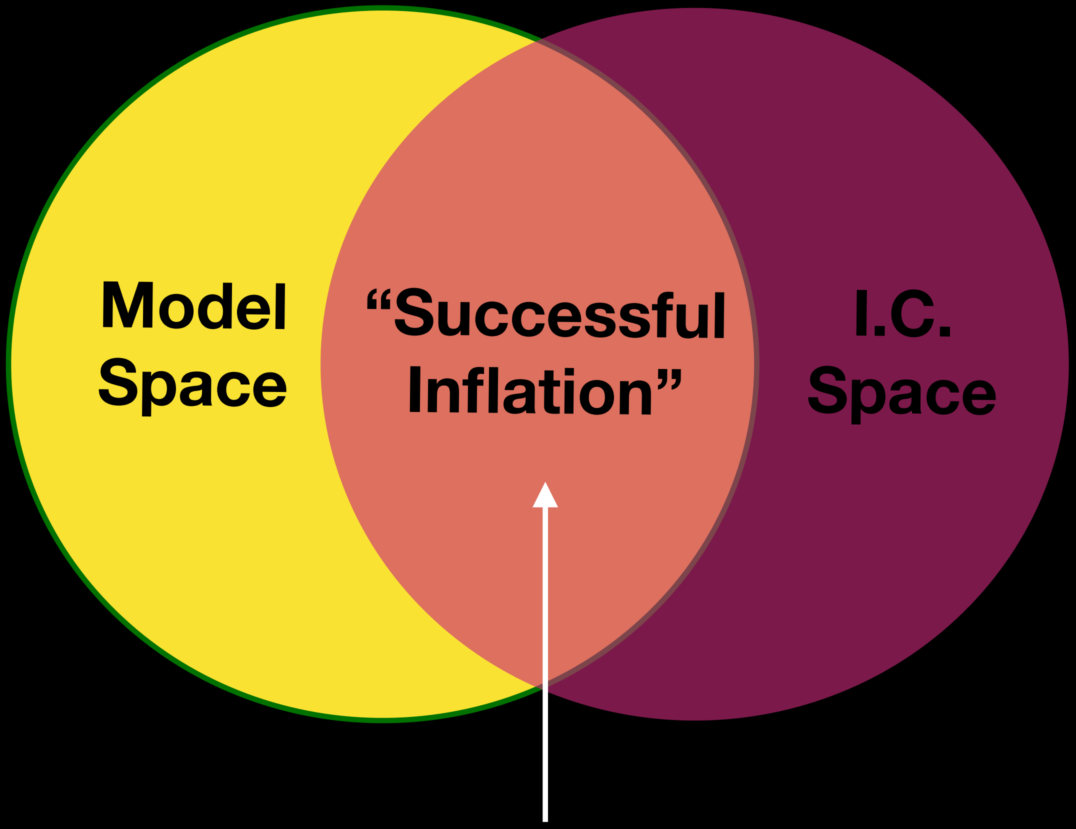

We should look here.

Problems are Opportunities

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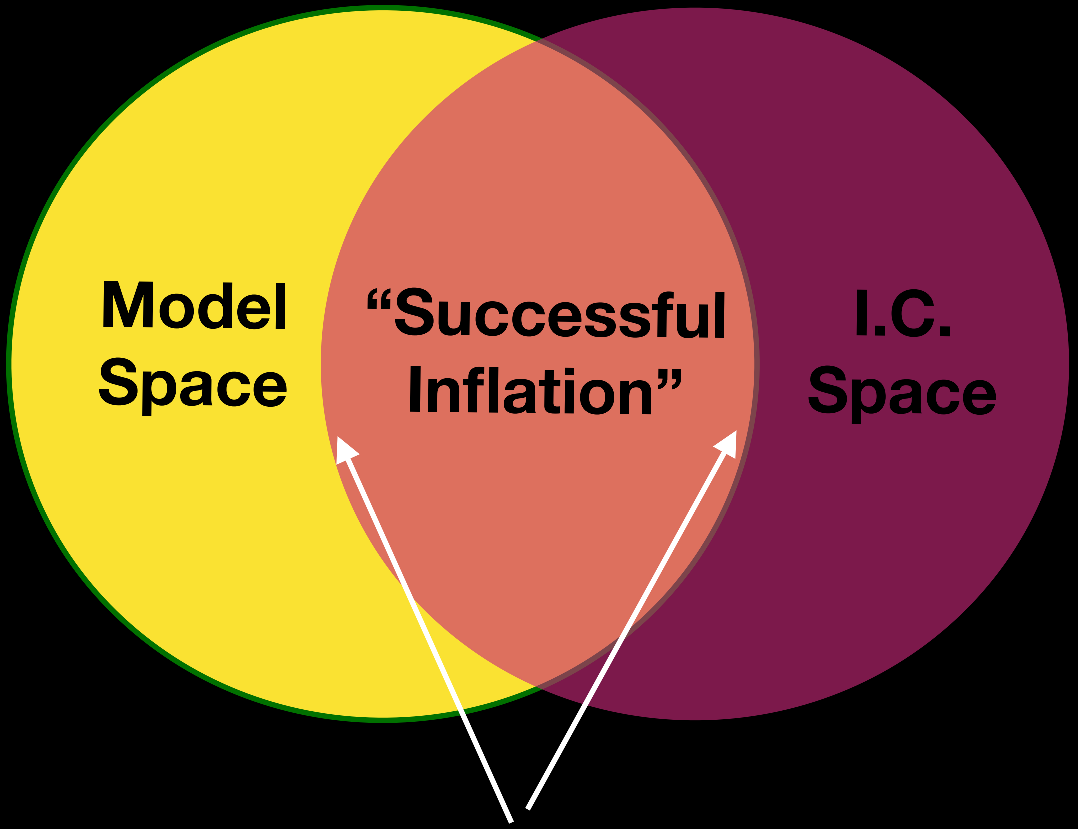

Actually, we should look here!

We should look for the ***existence of boundaries*** (and hence finiteness of measure).

Theory Testing Inflation

Step 1 : Pick an inflationary model (dynamics)

Step 2 : Choose initial conditions for this model

Do calculations (numerics/analytics), do we get >60 folds of inflation?

Theory Testing Inflation

Step 1 : Pick an inflationary model (dynamics)

Energy scale (high/low = “large field”/“small field”)
one field vs many fields
exact shape and couplings of the potential
many others….

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Energy scale (high/low = “large field”/“small field”)
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exact shape and couplings of the potential
many others….

Step 2 : Choose initial conditions for this model

Slow roll/fast roll?
Initially expanding/contracting?
Homogenous/Inhomogenous?
many others…

Do calculations (numerics/analytics), do we get >60 folds of inflation?

Theory Testing Inflation

Lots of work on *Homogenous* initial conditions.

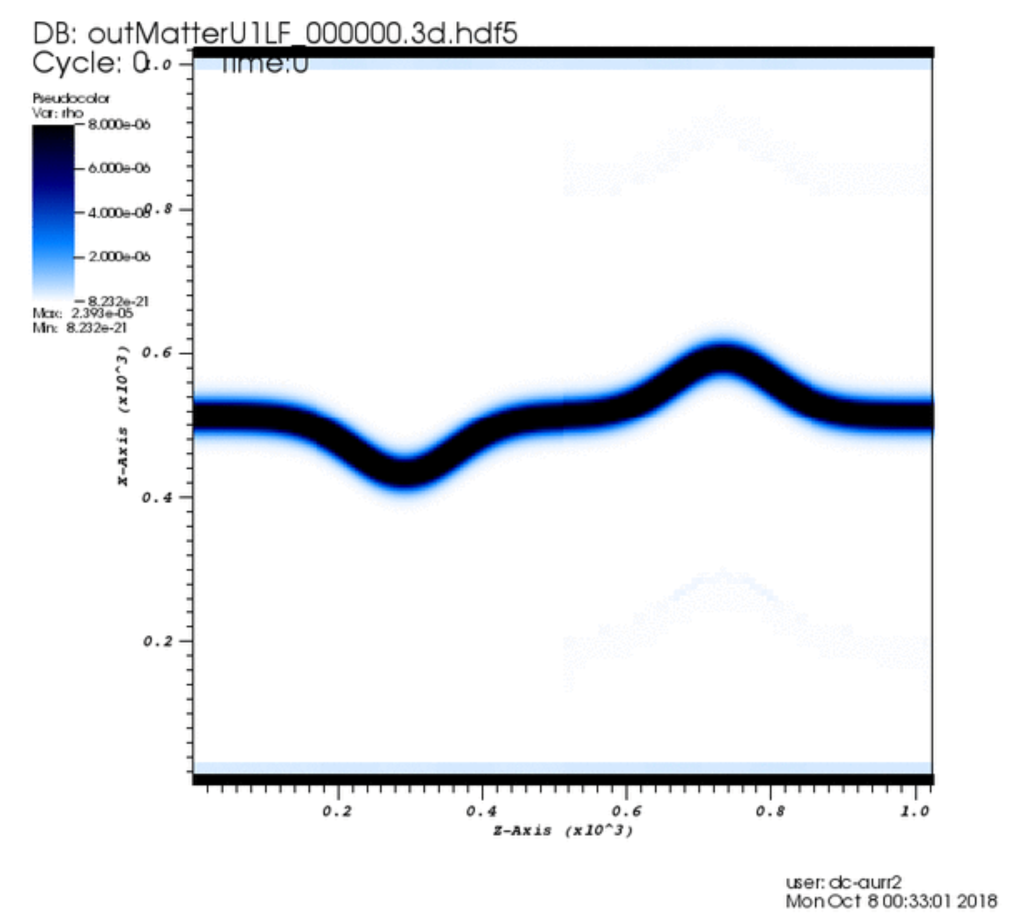
(Assumed Friedman-Robertson-Walker.)

Very little work on *Inhomogenous* initial conditions.

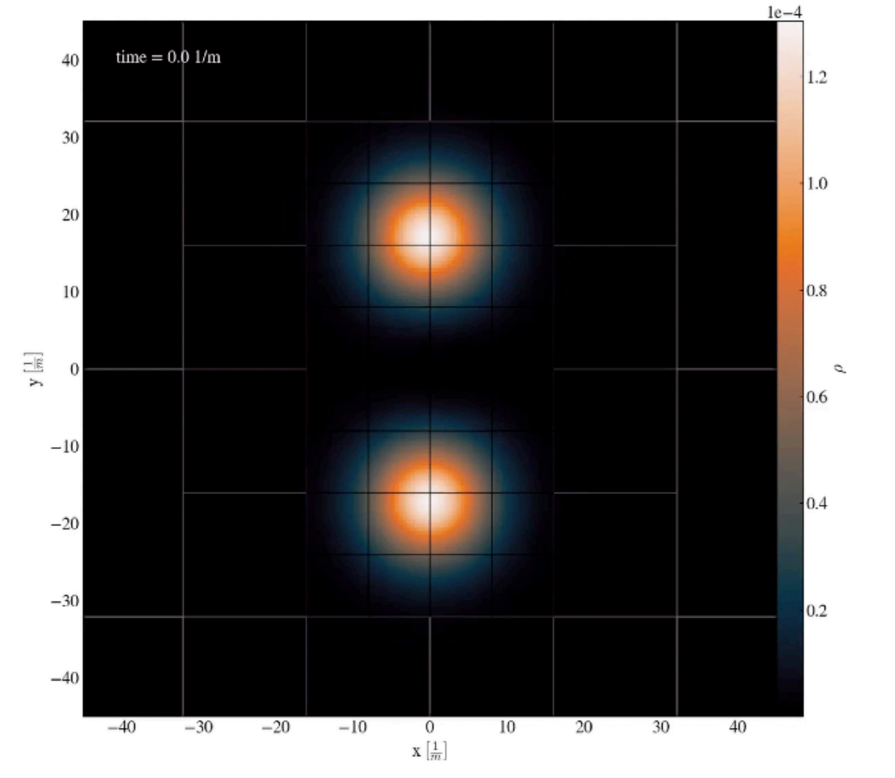
Hard! Need to solve full General Relavity equations : Numerical Relativity

GRCHOMBO

www.grchombo.org **Open source (BSD-3)**



GW from Cosmic Strings!



GW from collisions of Scalar Solitons!

Initial Conditions Space

Scalar/Matter Sector

$\phi(\mathbf{x})$ $\dot{\phi}(\mathbf{x})$

Initial Conditions Space

Gravity Sector

3-metric

$\gamma_{ij}(\mathbf{x})$

Extrinsic Curvature

$K_{ij}(\mathbf{x}) \sim \dot{\gamma}_{ij}$

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↓

Usually assumed conformal

$$\gamma_{ij} = \chi^{-2}\delta_{ij}$$

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Traceless **Trace**

$$A_{ij} + \frac{1}{3}\gamma_{ij}K$$

Tensor “GW” Modes

Expansion/ Contraction

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Tensor "GW" Modes **Expansion/ Contraction**

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$\phi(\mathbf{x})$ $\dot{\phi}(\mathbf{x})$

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

Freedom to specify $(\phi, \dot{\phi}, \gamma_{ij}, K_{ij})$
subject to Hamiltonian and 3 Momentum constraints

Initial Conditions Space

Scalar/Matter Sector

$\phi(\mathbf{x})$ $\quad \cancel{\dot{\phi}(\mathbf{x})}$

Gravity Sector

3-metric

$\gamma_{ij}(\mathbf{x})$

$\downarrow$

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Extrinsic Curvature

$K_{ij}(\mathbf{x}) \sim \dot{\gamma}_{ij}$

$\downarrow$

Traceless **Trace**

$$\cancel{A_{ij}} + \frac{1}{3}\gamma_{ij}K$$

$\downarrow$ $\downarrow$

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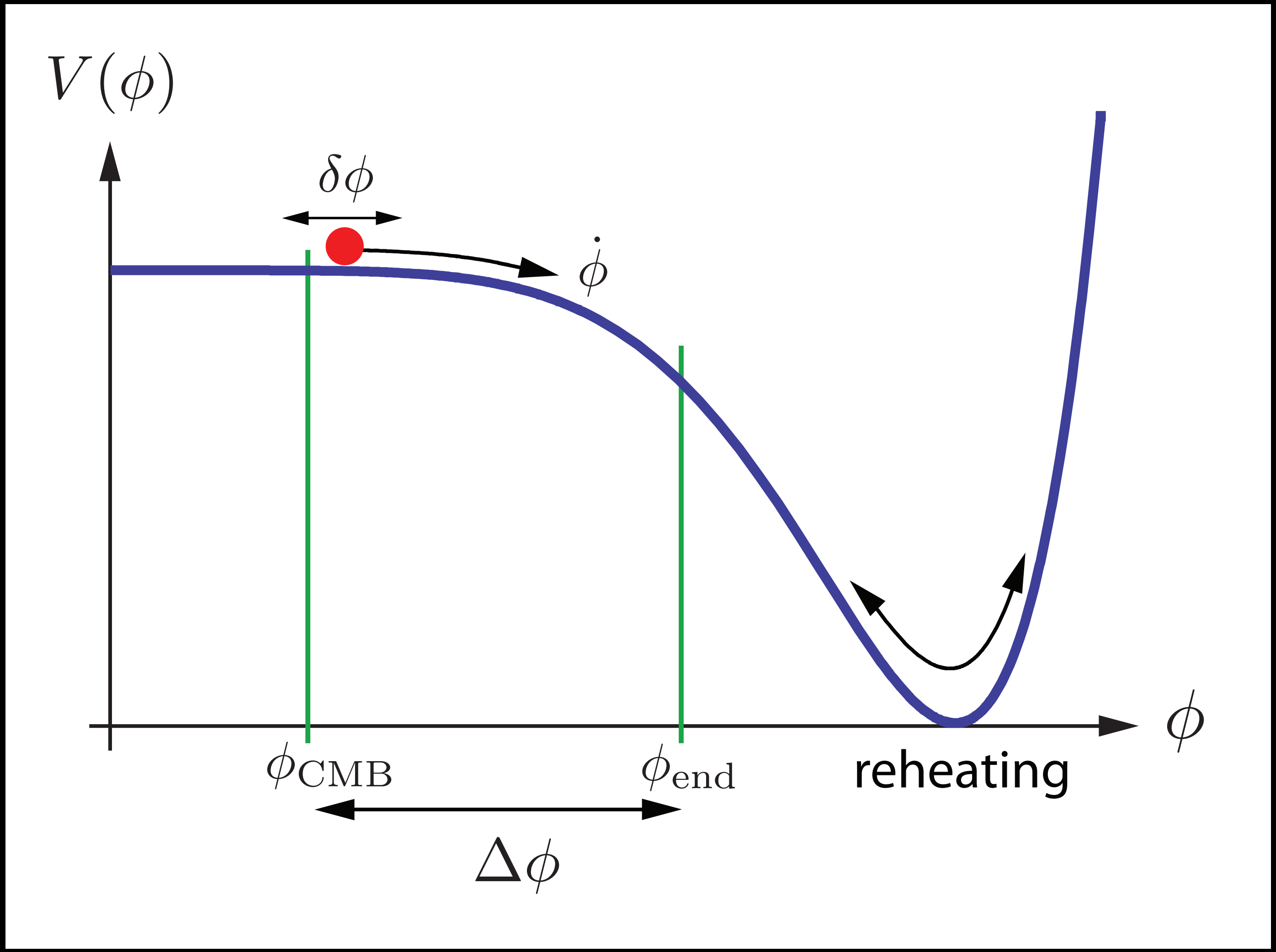
Model Space



Single Field models

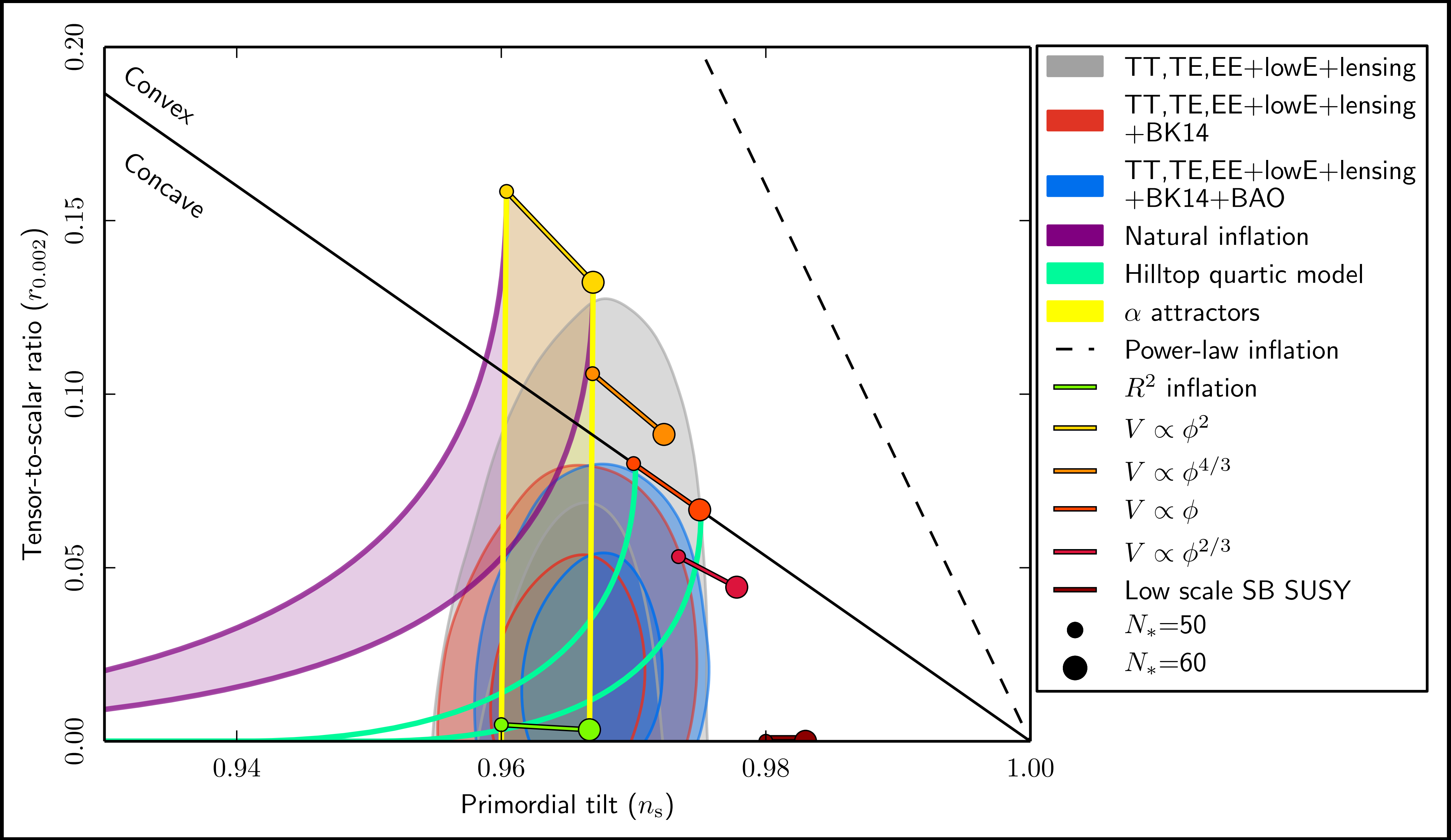
Slow Roll Plateau

End point (“reheating”)

Energy scale (High/low)

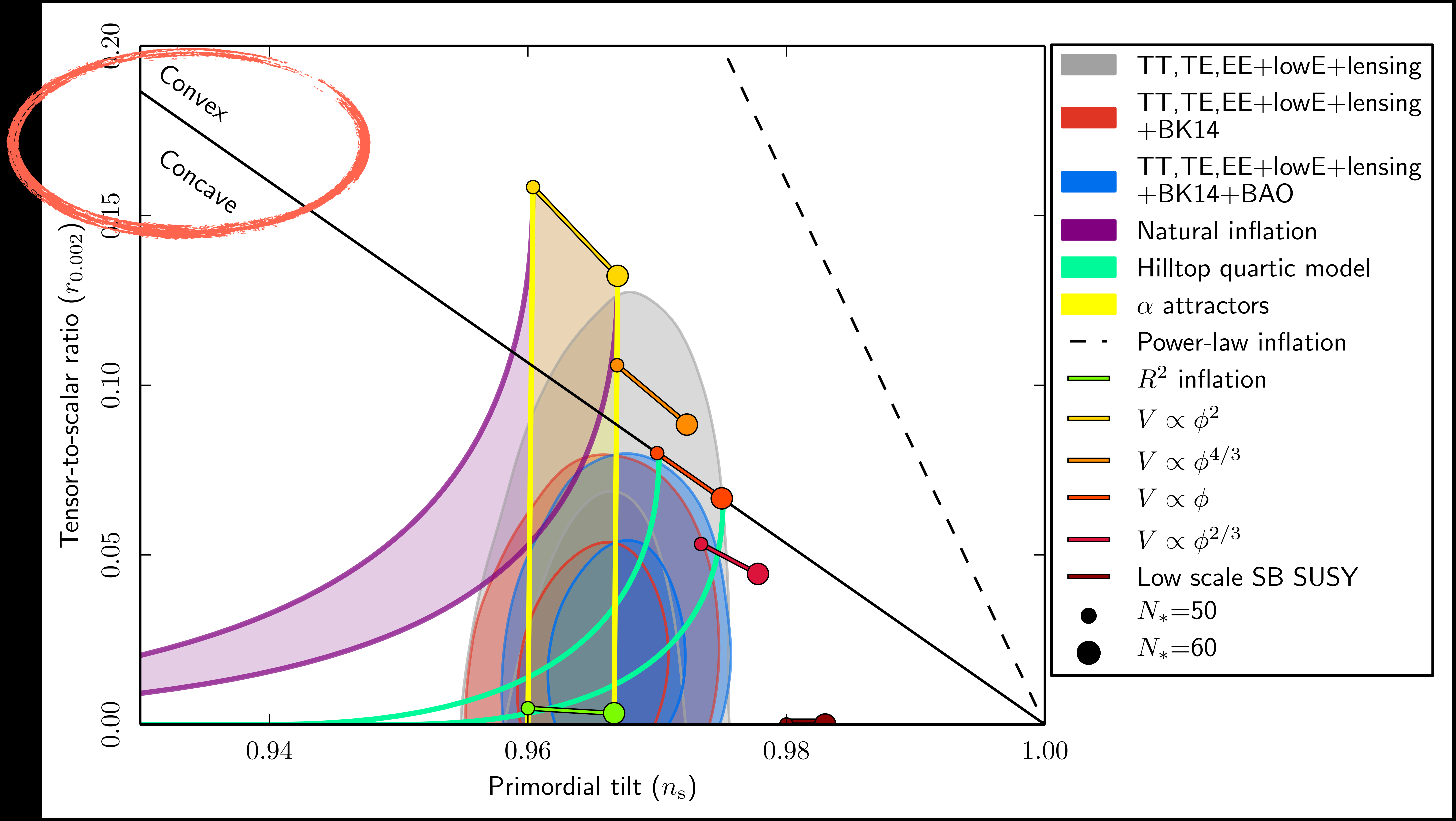
Shape (e.g. Concave/Convex)

Model Space



Planck 2018

Model Space

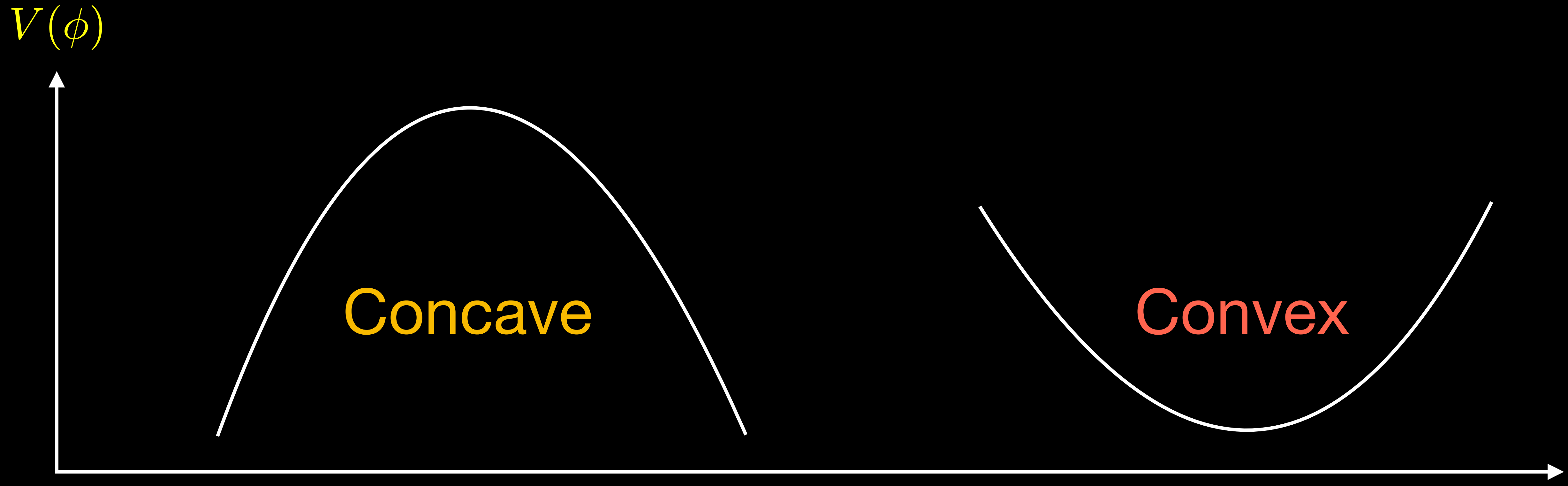


Planck 2018

Convex vs Concave

J. Aurrekoetxea, K. Clough, R. Flauger, E. Lim (WIP)

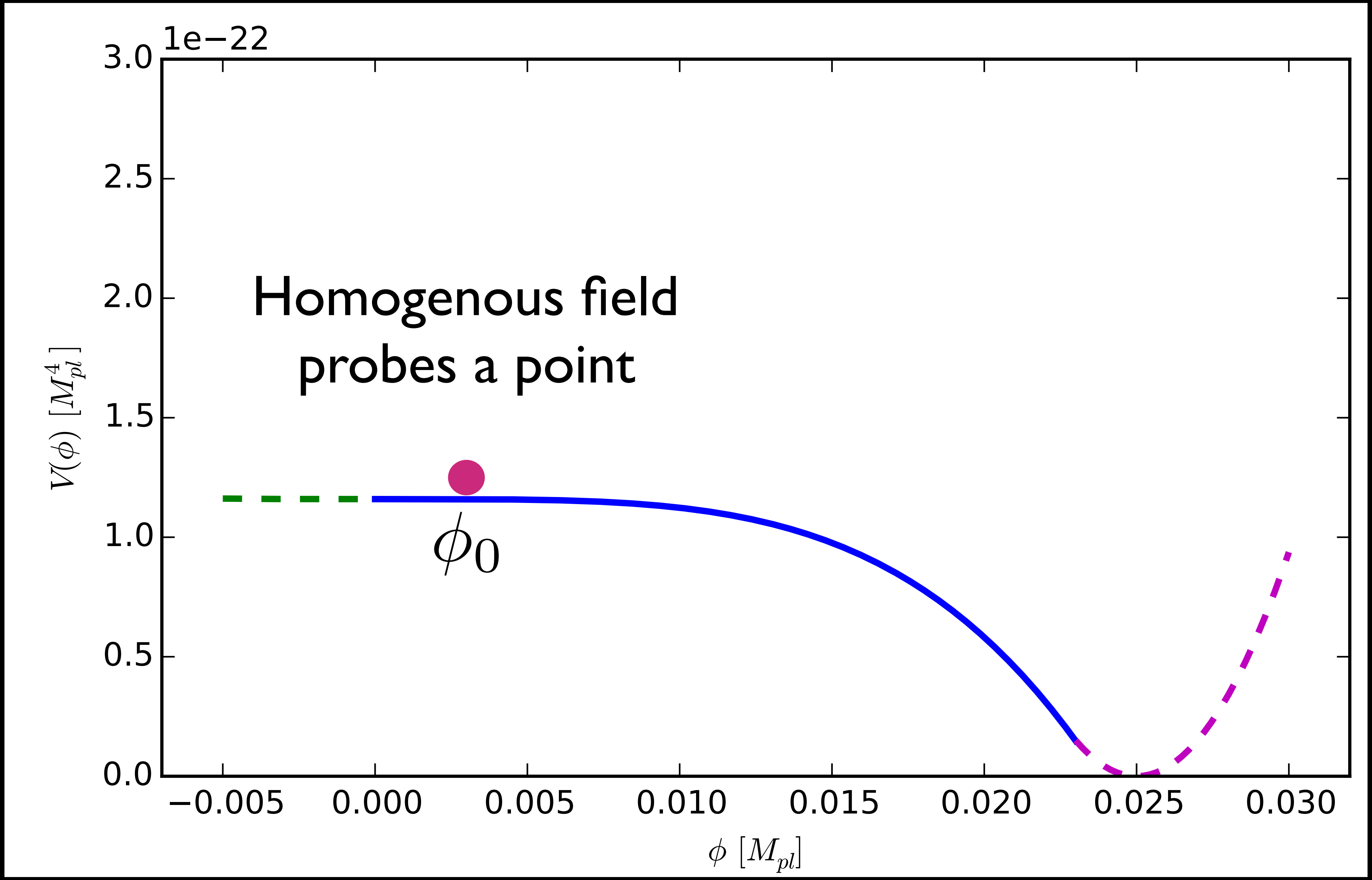
Inflationary Potentials



$$\frac{d^2V}{d\phi^2} < 0$$

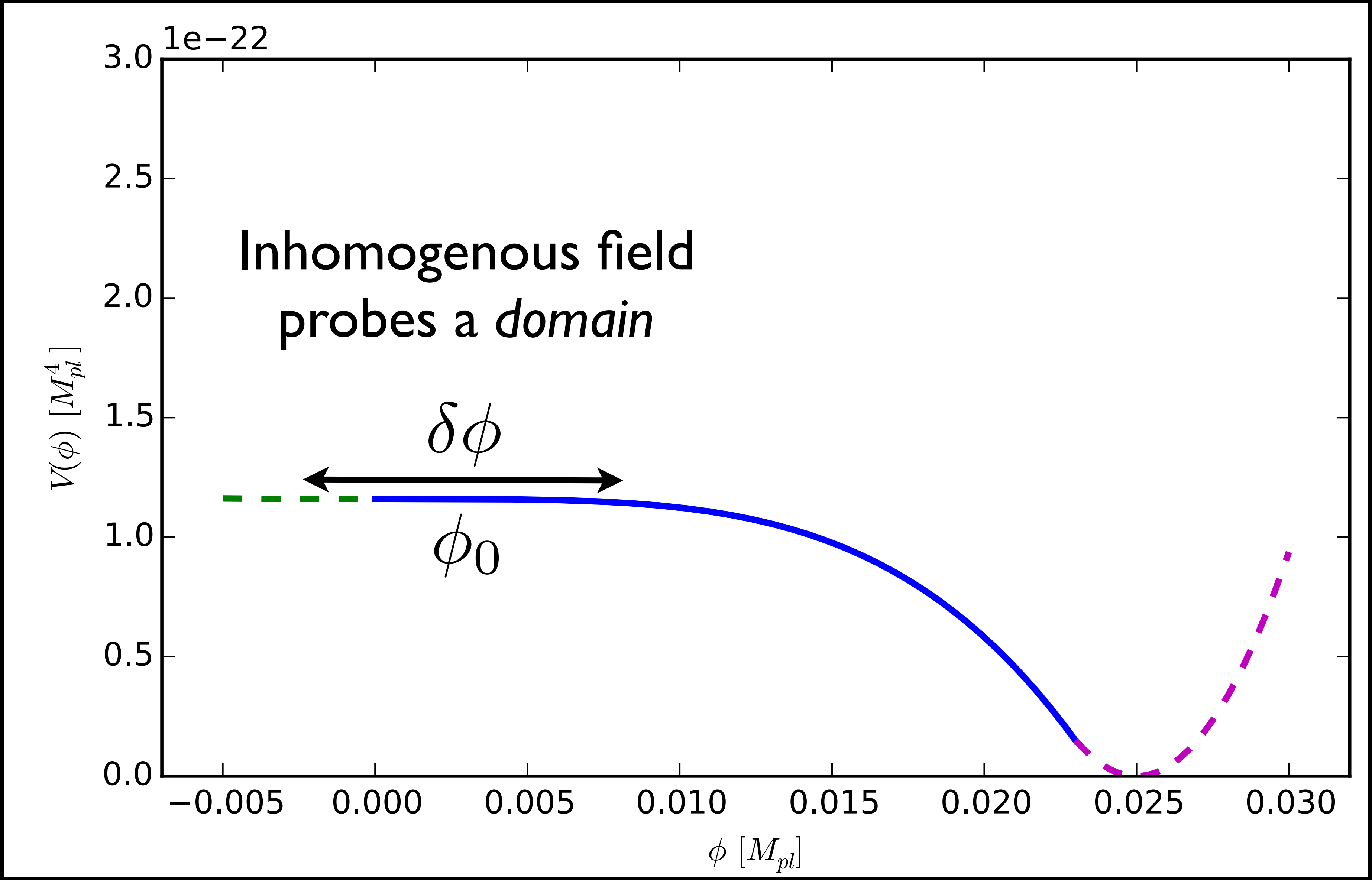
$$\frac{d^2V}{d\phi^2} > 0$$

Inhomogenous vs homogenous dynamics



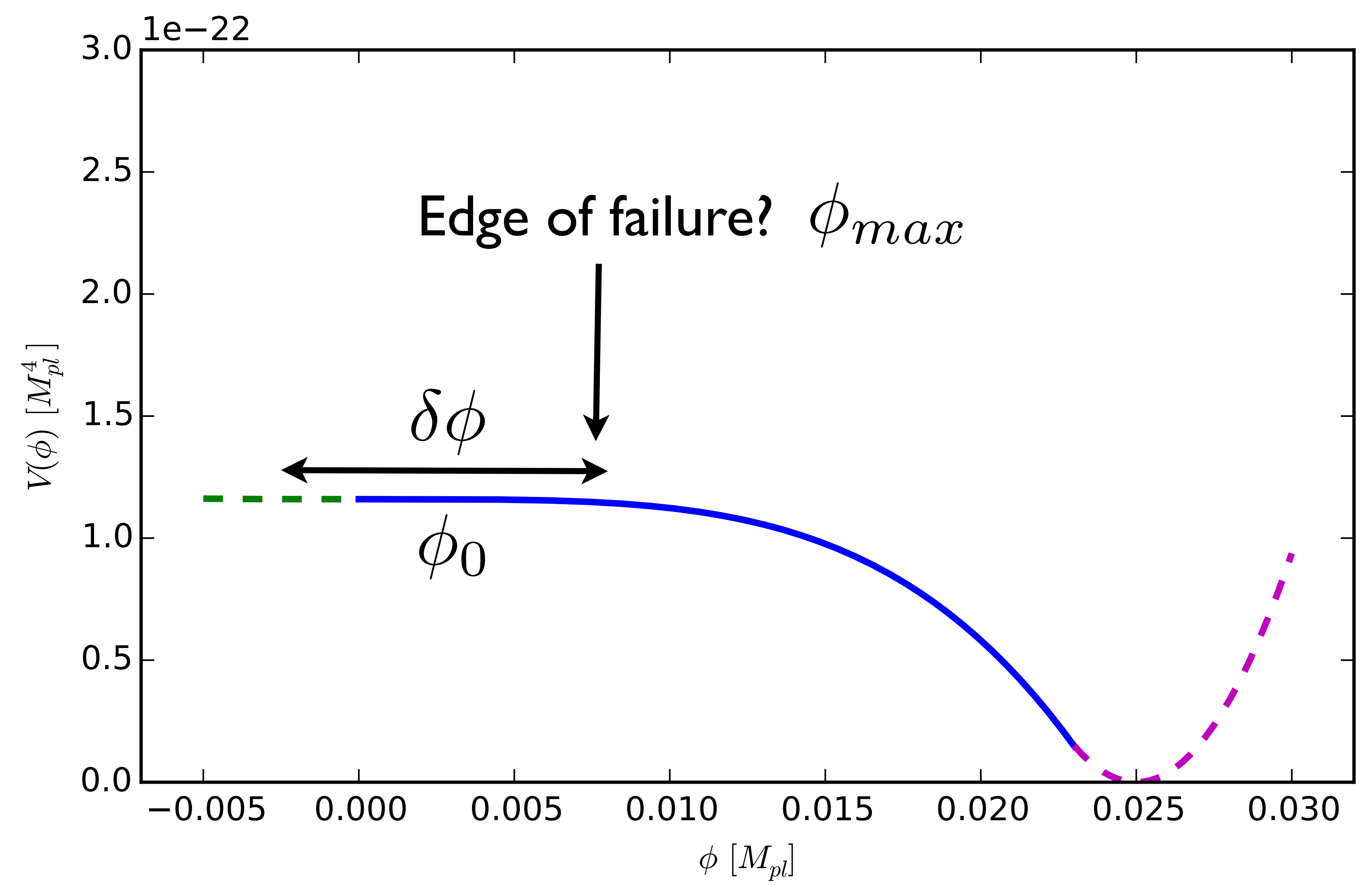
$$\phi(x, t_0) = \phi_0$$

Inhomogenous vs homogenous dynamics



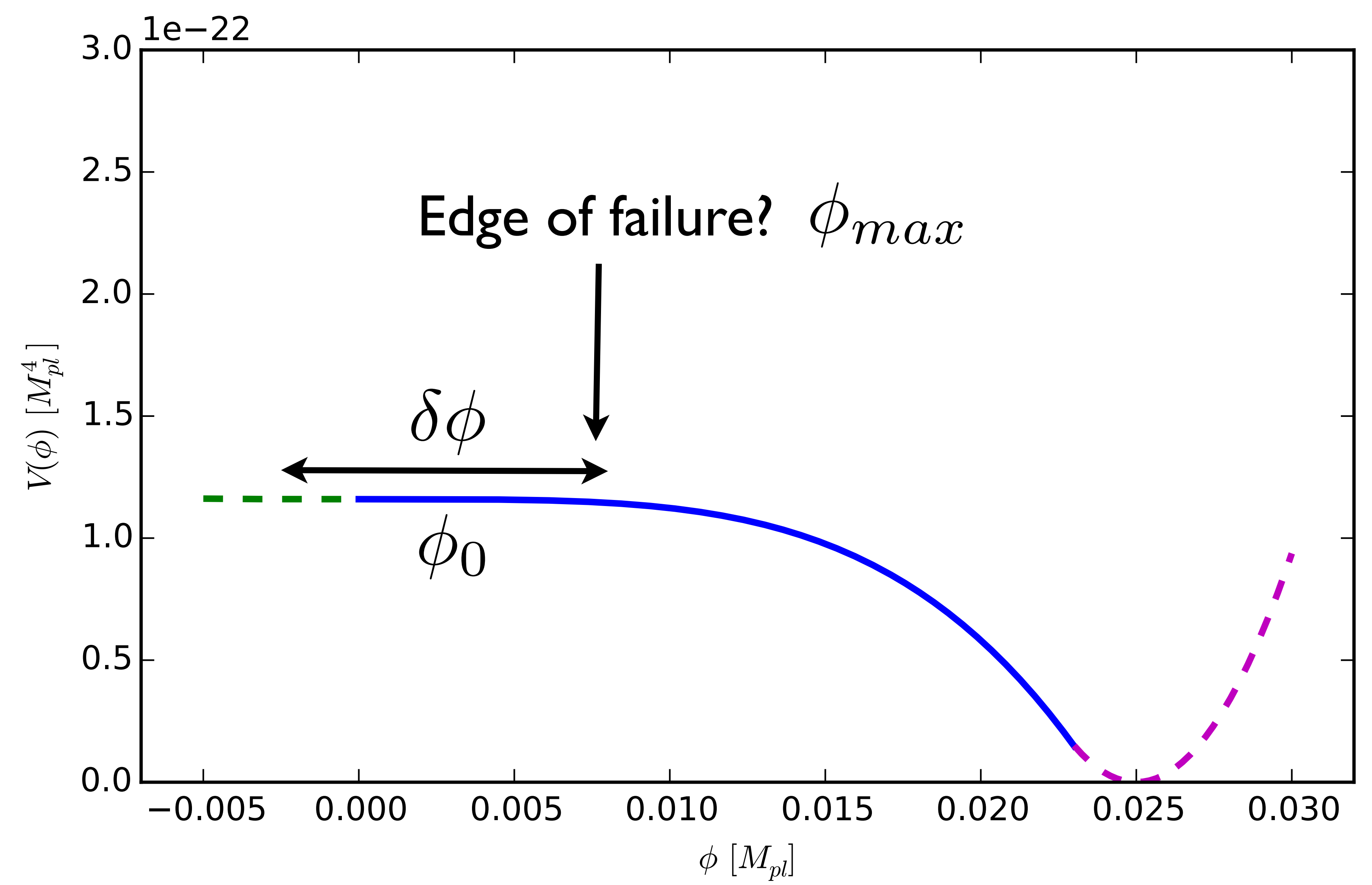
$$\phi(x, t_0) = \phi_0 + \delta\phi \sum_i \sin k_i x^i$$

Convex vs Concave



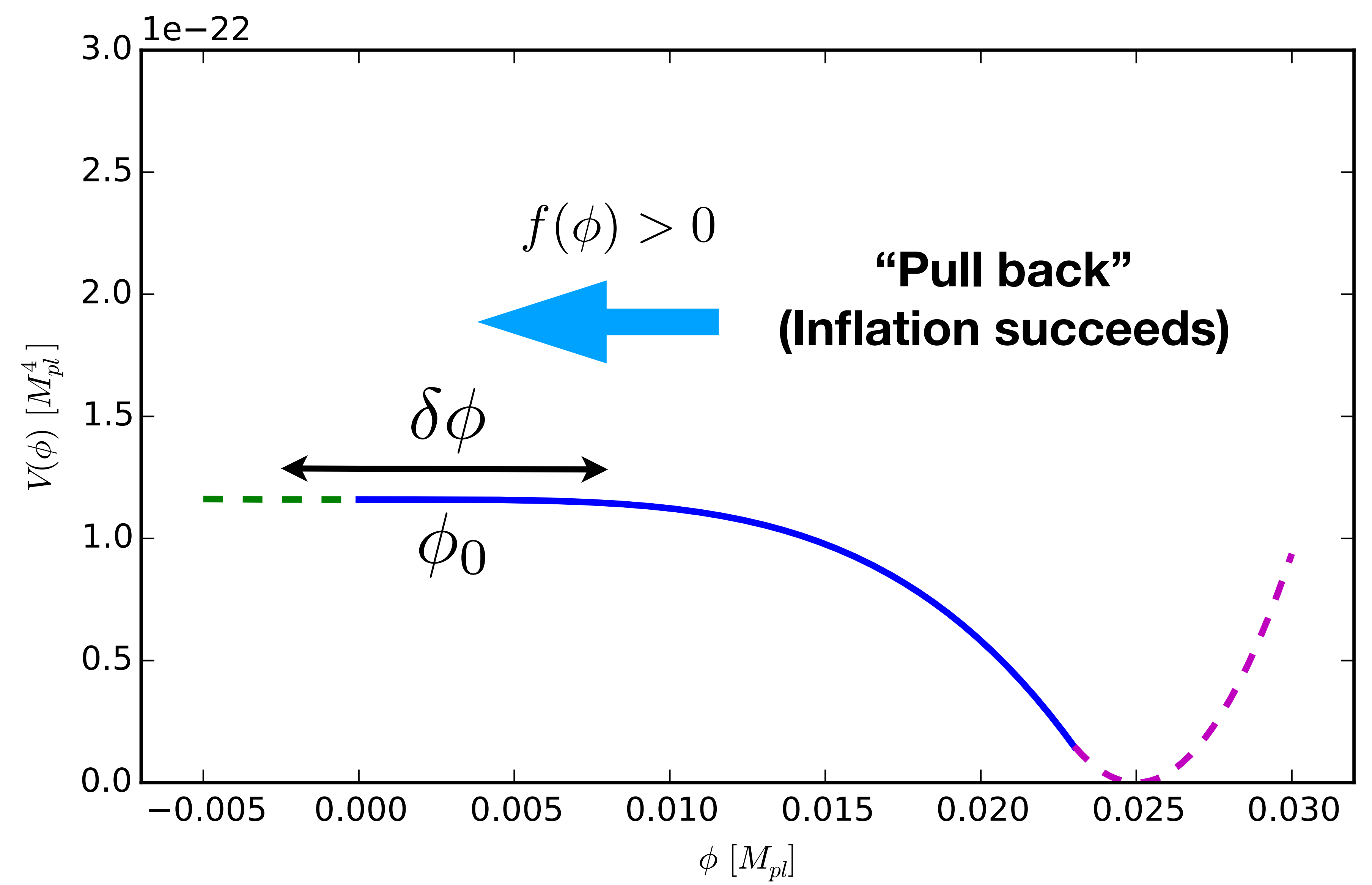
$$\cancel{-\ddot{\phi}} + \nabla^2\phi - \frac{dV}{d\phi} = 0 \Rightarrow \nabla^2\phi_{max} \overset{?}{=} \frac{dV(\phi_{max})}{d\phi}$$

Convex vs Concave



Define $$f(\phi) \equiv -\ddot{\phi} = -\nabla^2\phi + \frac{dV}{d\phi} = k^2\phi + \frac{dV}{d\phi}$$

Convex vs Concave



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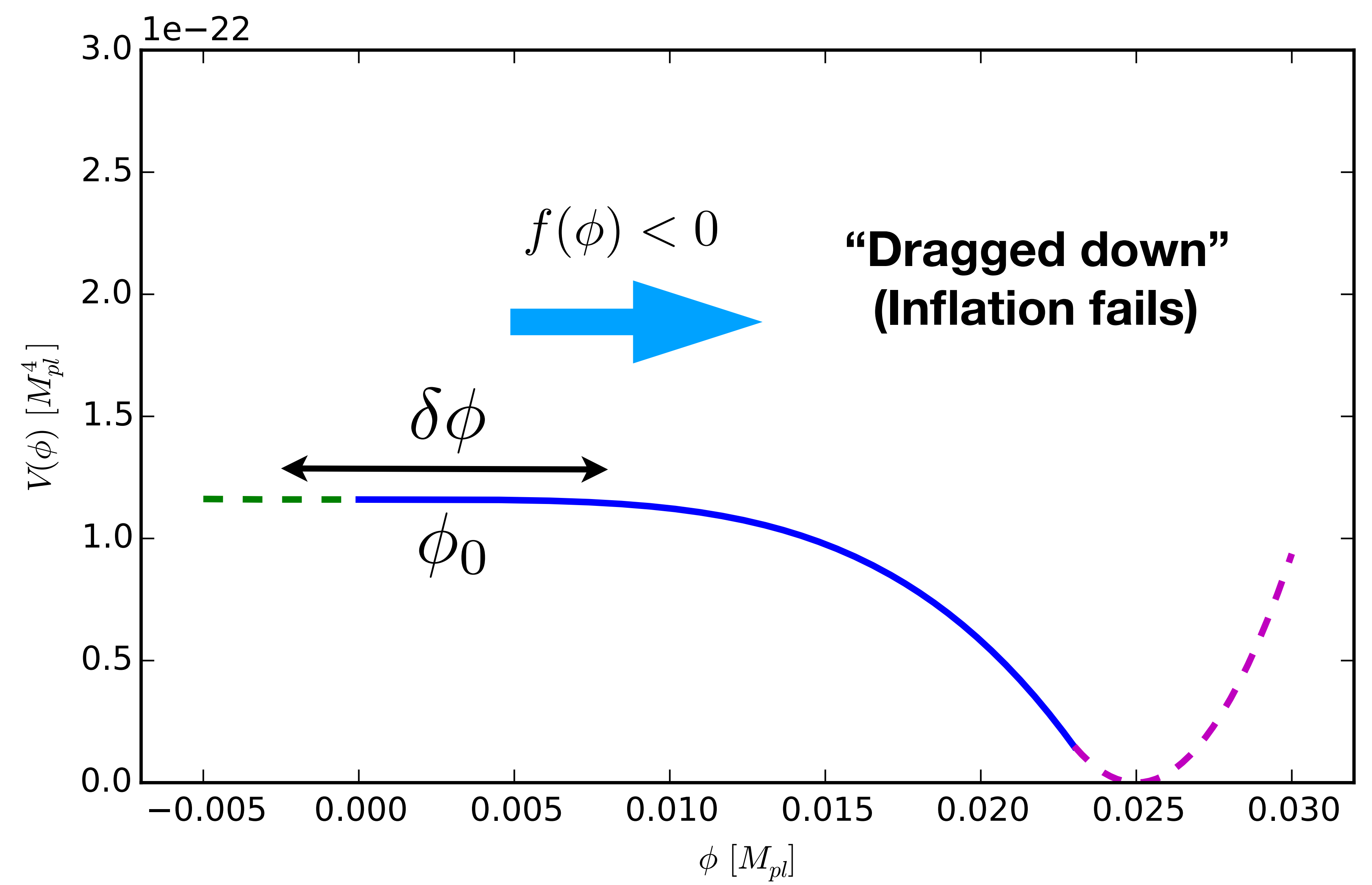
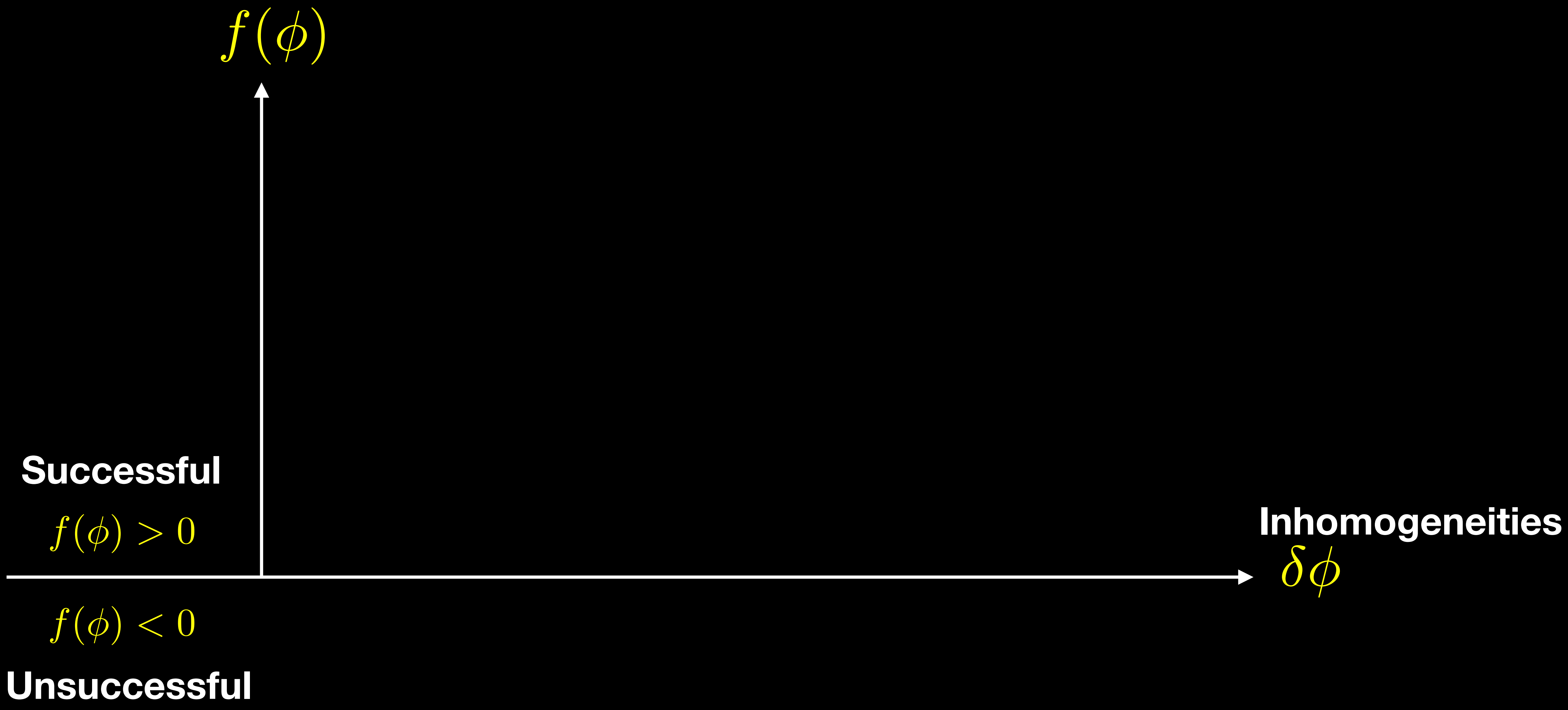


"Dragged down"
(Inflation fails)

Define $$f(\phi) \equiv -\ddot{\phi} = -\nabla^2\phi + \frac{dV}{d\phi} = k^2\phi + \frac{dV}{d\phi}$$

Convex vs Concave



$f(\phi)$

Successful

$f(\phi) > 0$

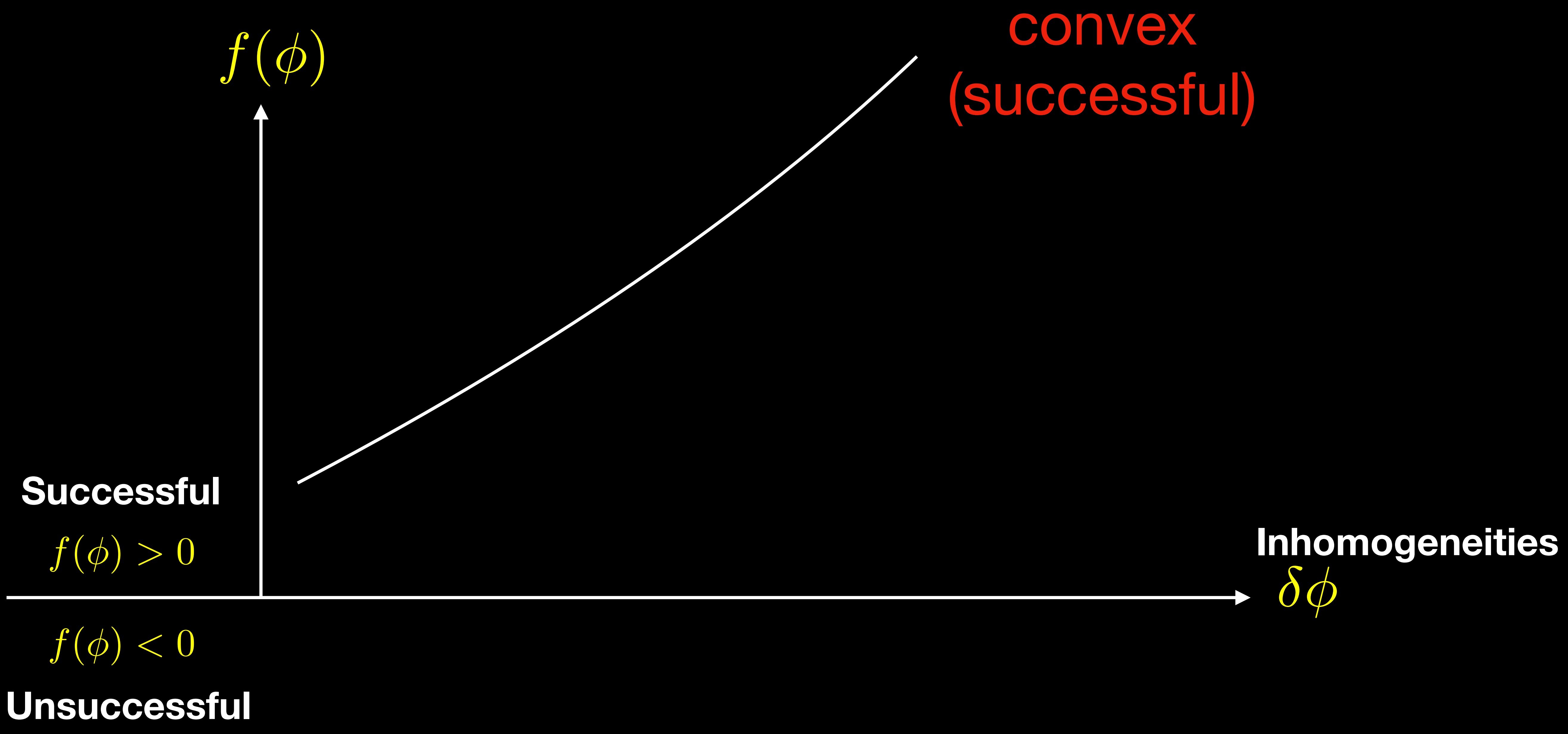
Inhomogeneities

$\delta\phi$

$f(\phi) < 0$

Unsuccessful

Convex vs Concave



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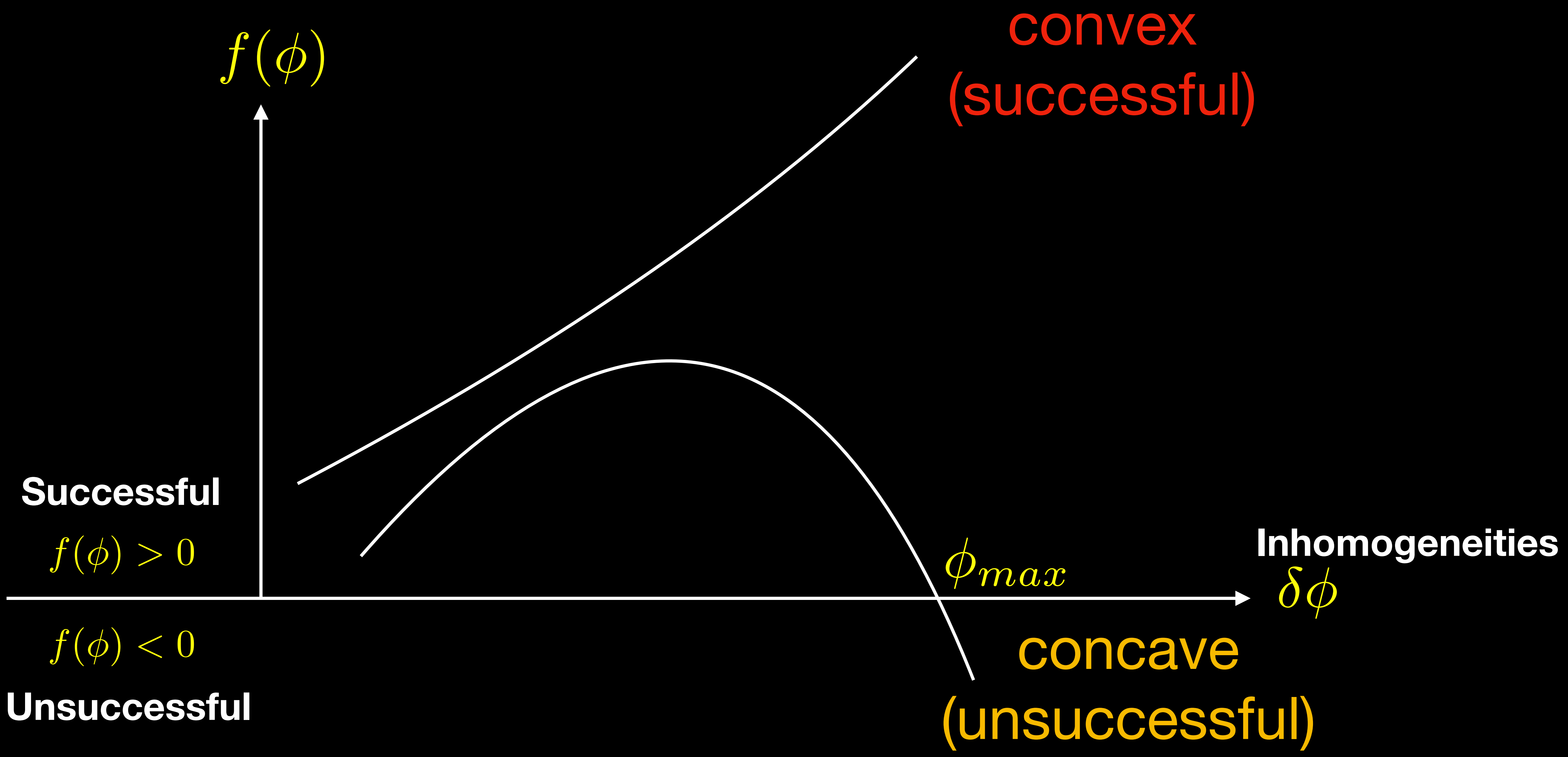

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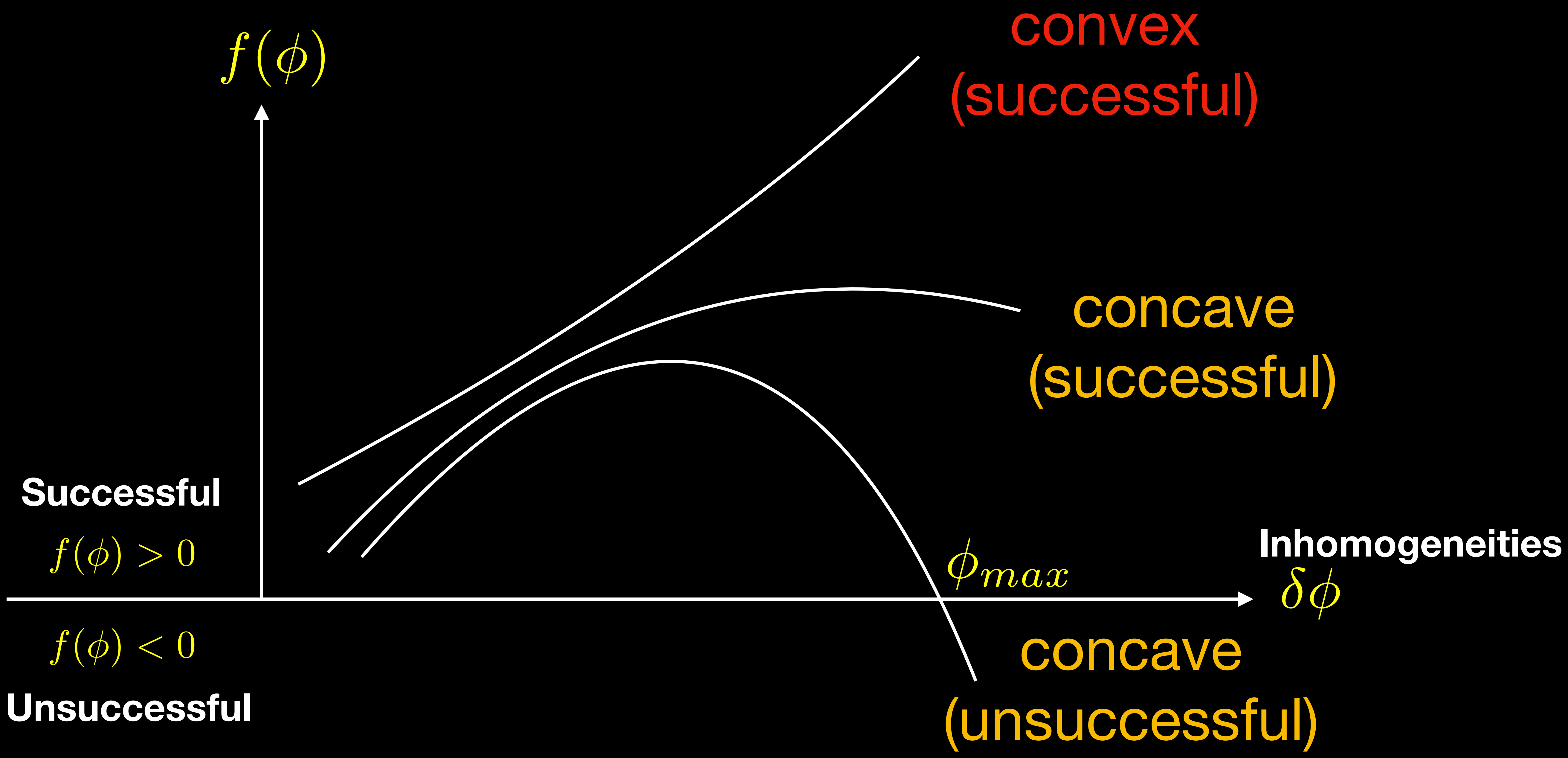


Convex vs Concave

A neat proof :

$$f(\phi_0, \delta\phi) = k^2 \delta\phi + \frac{dV}{d\phi} \qquad \phi = \phi_0 + \delta\phi$$

For $f(\phi_0, \delta\phi)$ to have a maxima,

$$\frac{\partial f}{\partial \delta\phi} = k^2 + \frac{d^2 V}{d\phi^2} = 0$$

Convex vs Concave

A neat proof :

$$f(\phi_0, \delta\phi) = k^2 \delta\phi + \frac{dV}{d\phi} \qquad \phi = \phi_0 + \delta\phi$$

For $f(\phi_0, \delta\phi)$ to have a maxima,

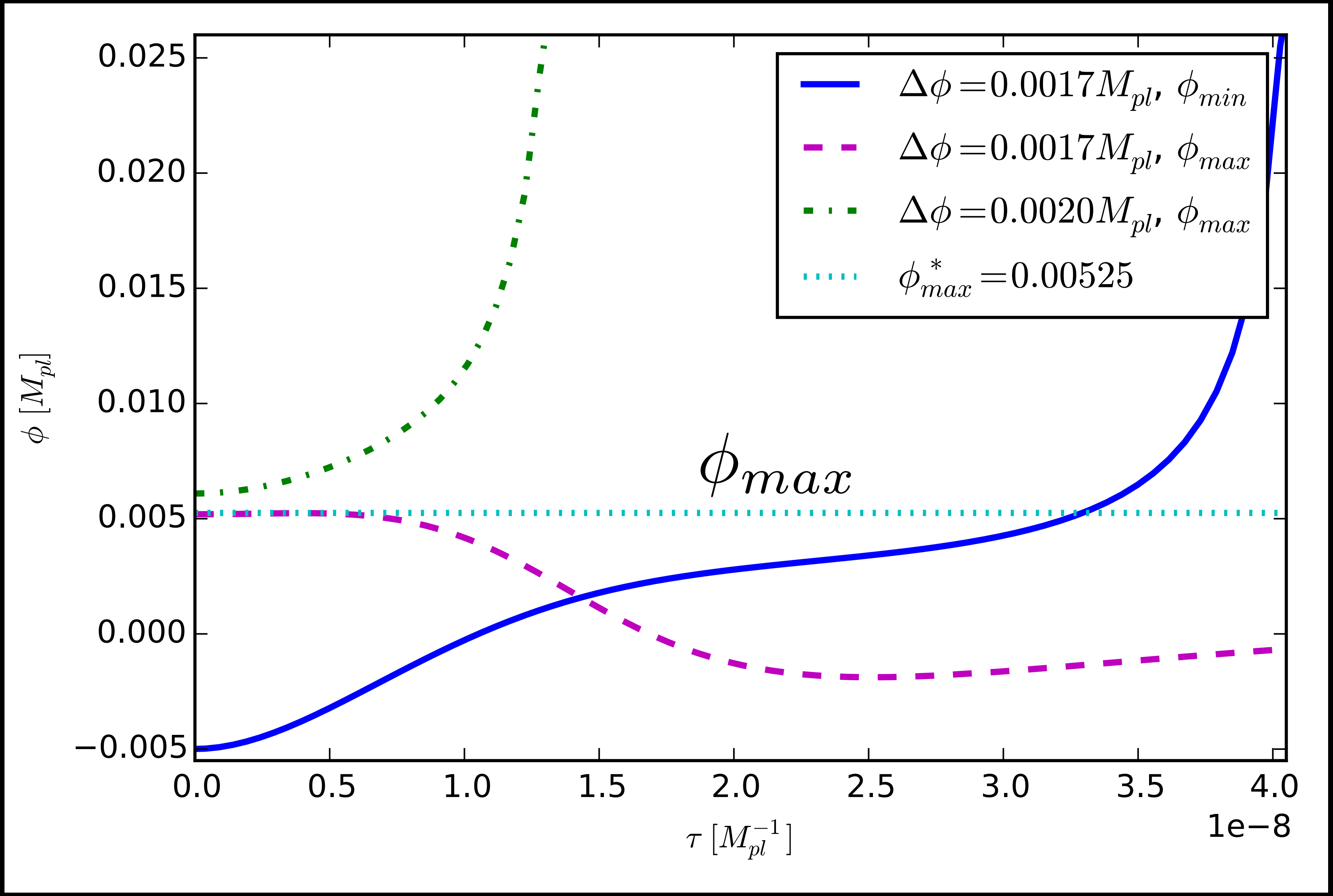
$$\frac{\partial f}{\partial \delta\phi} = k^2 + \frac{d^2V}{d\phi^2} = 0$$

Convex $\frac{d^2V}{d\phi^2} > 0$ no solution

Concave $\frac{d^2V}{d\phi^2} < 0$ solution may exists

Convex vs Concave

Check with full Numerical Relativity simulations



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For Quartic Hilltop $V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^4\right]$

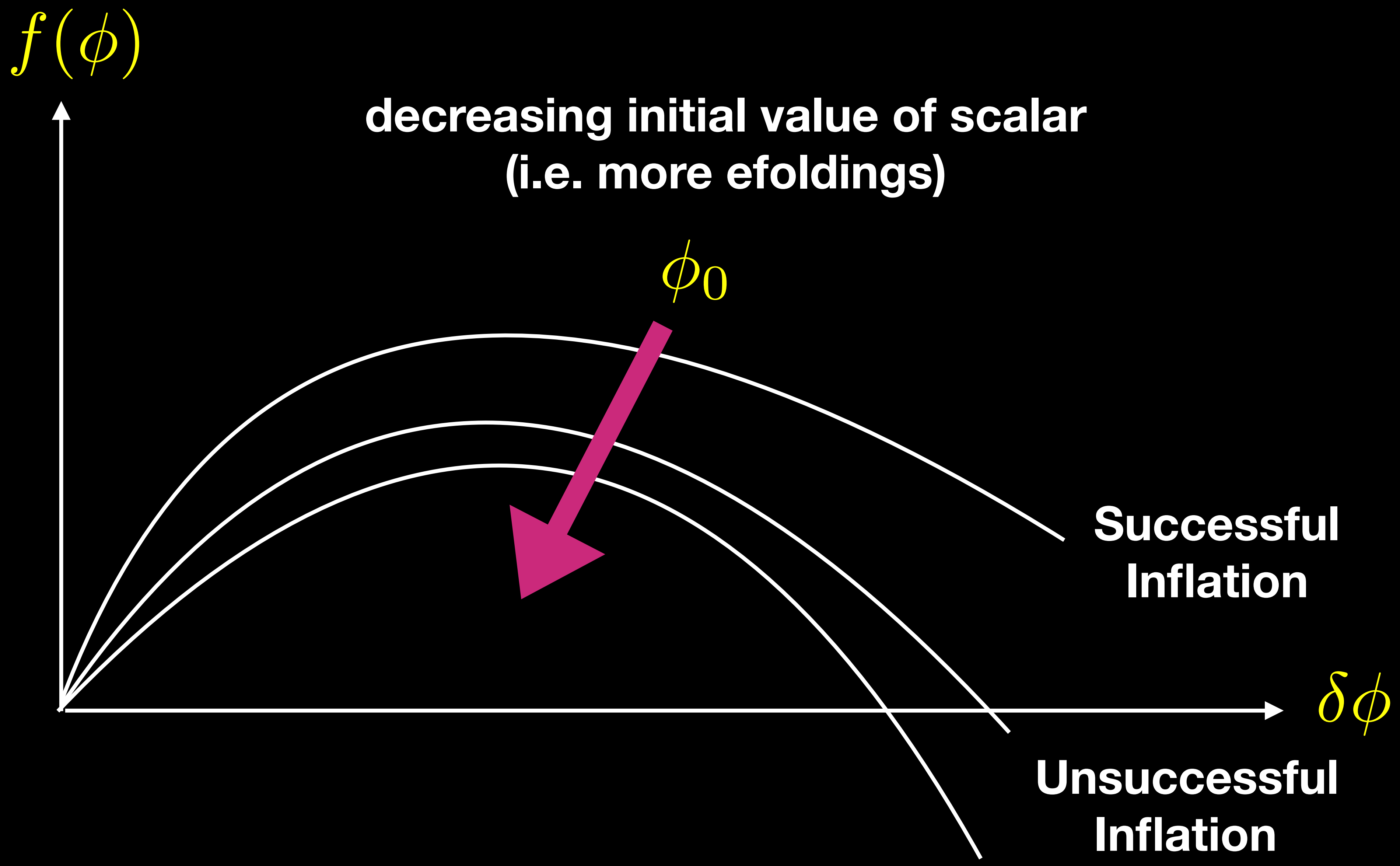
Joint Initial Conditions- Model Constraints

Since $\phi = \phi_0 + \delta\phi$ and $\delta\phi$ is bounded by end of the inflation, there exists $\phi_0 < \phi_0^*$ such that inflation will not fail to *any* value of scalar inhomogeneities.

$\phi_0^* \to N$ e-folds

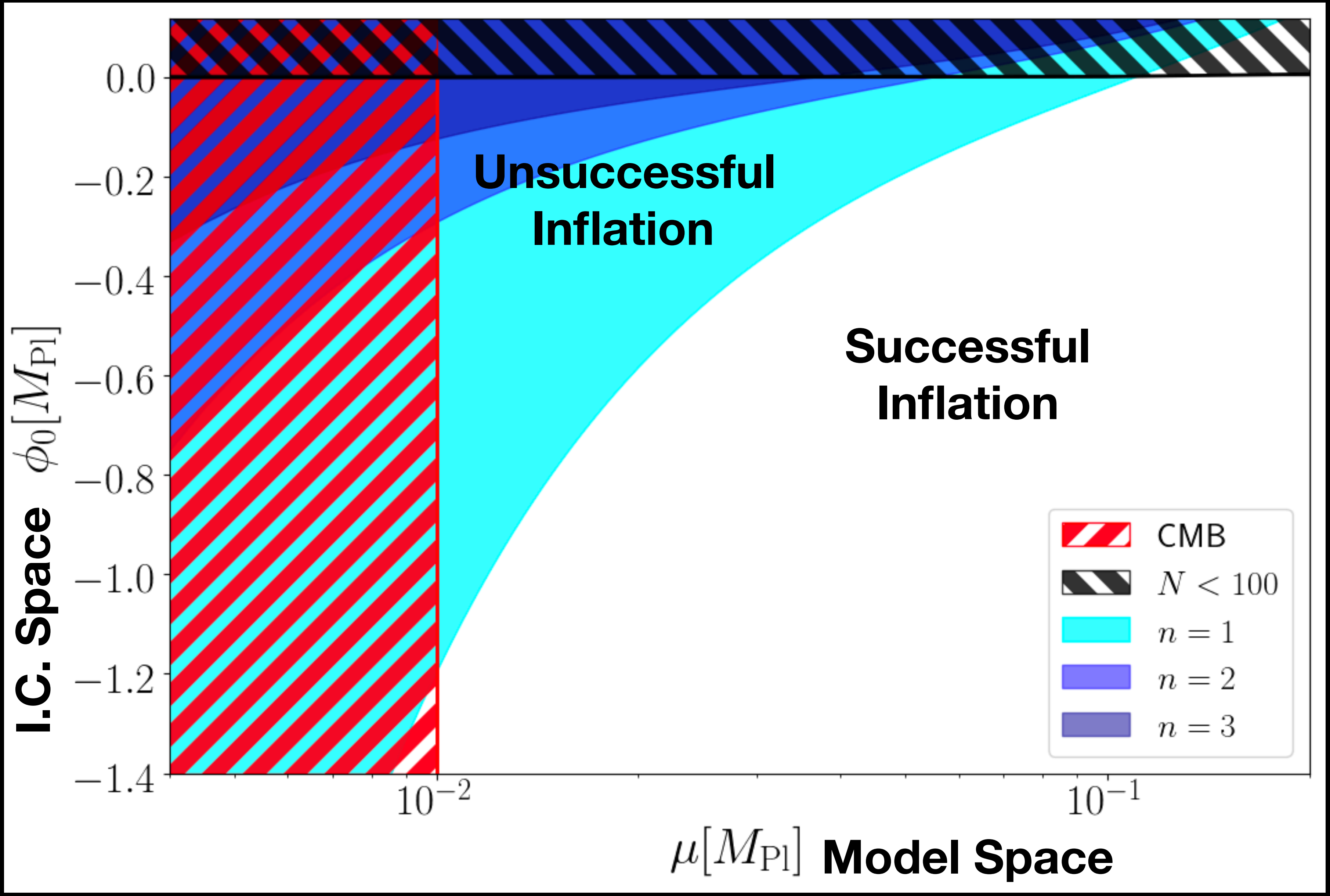
Thus we can guarantee inflation is robust if $\phi_0 < \phi_0^*$

Joint Initial Conditions-Model Constraints



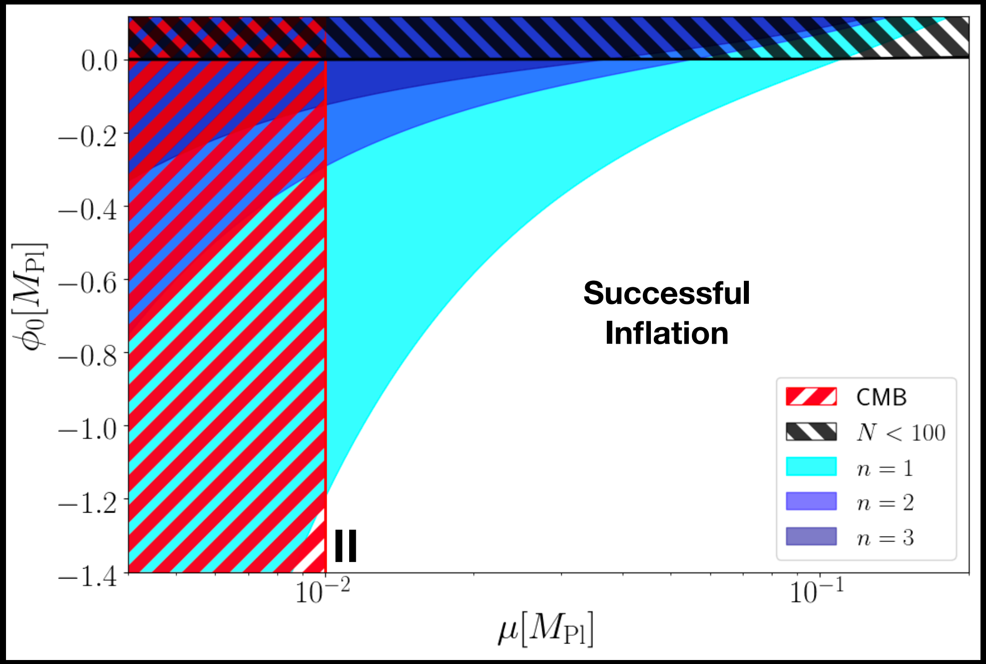
For Concave potentials, there exists a *model-dependent* initial condition ϕ_0 for successful inflation.

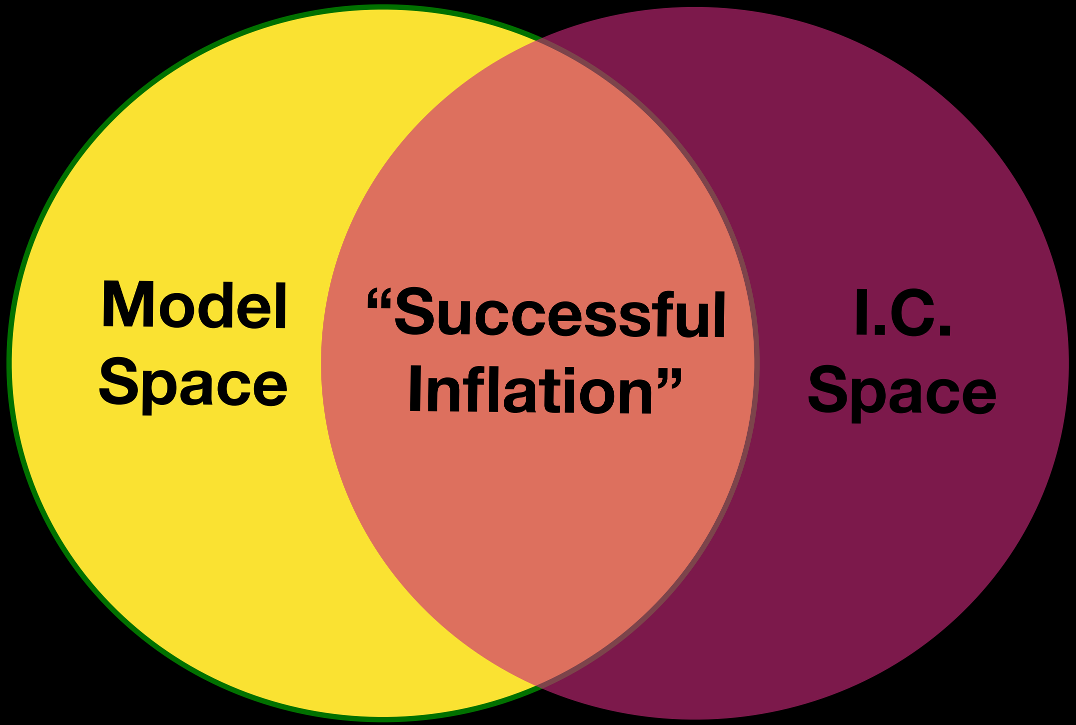
Joint Initial Conditions-Model Constraints



Cubic Hilltop $V(\phi) = \Lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^3\right]$

Joint Initial Conditions- Model Constraints





By probing the boundary, we can start to construct the distribution, and perhaps understand the measure of successful inflation.

Conclusions

- Inflation is designed to solve a question of expectations.
- We don't know what these "expectations" are — **"Measure Problem".**
- The measure/distribution encompasses both theory model space and initial condition space.
- Convex models are more robust than Concave models (Tension with Planck?)
- Demonstrate an exemplar on how to begin to construct the boundary of such a distribution.