On the Maximal Strength of a First-Order Electroweak Phase Transition and its Gravitational Wave Signal

Marek Lewicki

University of Sussex, 22 Oct 2018

Based on:
John Ellis, ML, José Miguel No arXiv:1809.08242

Baryogenesis

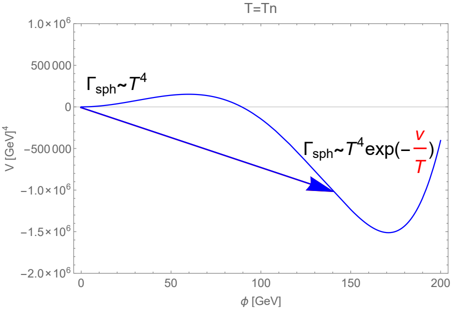
Generating Baryon asymmetry requires:

- C and CP violation
  ✓ present in SM quark sector
  (needs enhancement... not a part of this talk though)
- Baryon number violation
  ✓ $SU(2)$ sphalerons present in SM
- Departure from thermal equilibrium
  I order phase transition $\longrightarrow$ BSM needed

A. D. Sakharov 67'

Baryon number violation

- $SU(2)$ sphalerons violate baryon number



- In thermal equilibrium $SU(2)$ sphalerons wash out the baryon asymmetry.
  $\rightarrow$They have to be decoupled after the phase transition
- This leads to the famous bound:

$$\frac{v}{T} \gtrsim 1$$

Shaposhnikov 85' 86' 87'

Simple example

- Standard Model with an $|H|^6/\Lambda^2$ interaction
- We consider the following potential

$$V(H) = -m^2|H|^2 + \lambda|H|^4 + \frac{1}{\Lambda^2}|H|^6$$

- the tree-level potential reads

$$V(h)^{\text{tree}} = -\frac{m^2}{2}h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8}\frac{h^6}{\Lambda^2}$$

- We use the observed mass of the Higgs boson and the measured Higgs vev

$$v = 246\ \text{GeV}, \quad m_h = 125\ \text{GeV}$$

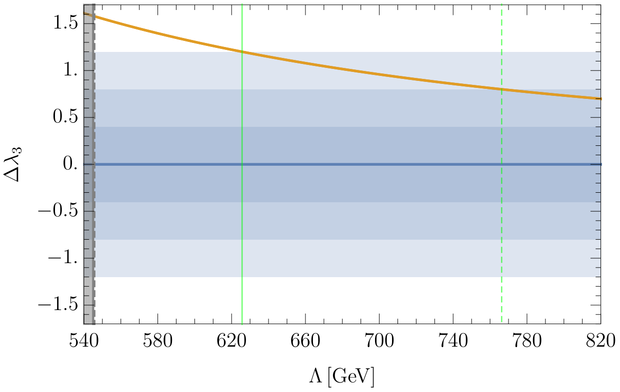
in the renormalisation conditions

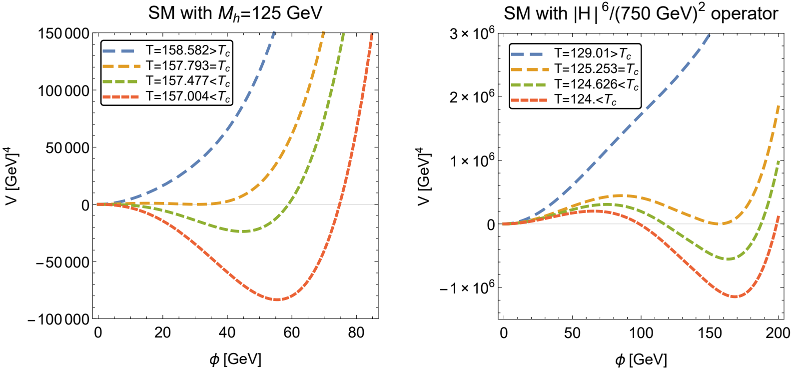
$$V'(h=v) = 0, \quad V''(h=v) = m_h^2$$

Collider probes

- The triple Higgs coupling is modified

$$\lambda_3 = \frac{1}{6}\frac{\partial^3 V}{\partial h^3}\bigg|_{h=v} = \frac{m_h^2}{2v} + \frac{v^3}{\Lambda^2} = \lambda_3^{\rm SM} + \frac{v^3}{\Lambda^2}$$





If $M_h < 85 GeV$ in SM we would have a I order phase transition

Kajantie, Laine, Rummukainen, Shaposhnikov 97'

phase transition dynamics

Scalar sphaleron: static field configuration passing the barrier (excited through thermal fluctuations)

- decay rate

$$\Gamma(T) \approx T^4 \exp\left(-\frac{S_3(T)}{T}\right),$$

- $\mathcal{O}(3)$ symmetric action

$$S_3(T) = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V(\phi, T)\right].$$

- EOM $\rightarrow$ bubble profile

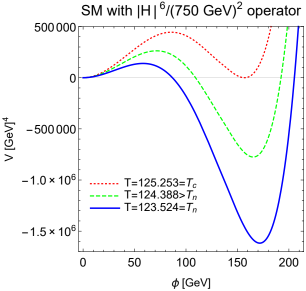
$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{\partial V(\phi,T)}{\partial \phi} = 0,$$

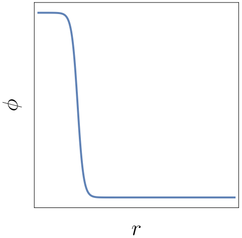
$$\phi(r \rightarrow \infty) = 0 \quad \text{and} \quad \dot{\phi}(r = 0) = 0.$$

- nucleation temperature

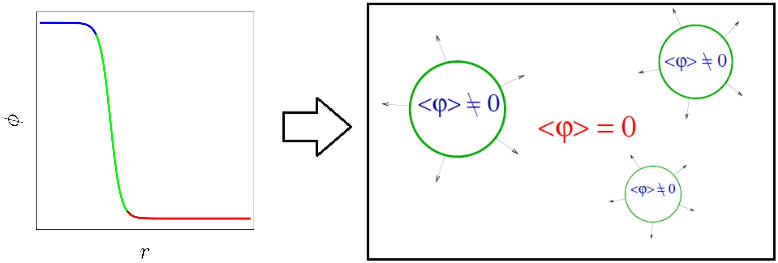
$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

Linde 81' 83'





Electroweak phase transition



Morrissey 12'

Gravitational waves

- Gravitational waves can be described by two parameters characterising the phase transition

$$\alpha \approx \left. \frac{\Delta V - T\frac{d\Delta V}{dT}}{\rho_R} \right|_{T=T_*}, \quad \Delta V = V_f - V_t$$

$$\Gamma \propto e^{-\frac{S_3(T)}{T}} = e^{\beta\,(t-t_0)} \Longrightarrow \frac{\beta}{H} = T\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\bigg|_{T=T_*}$$

- Signals are produced by three main mechanisms:
  - collisions of bubble walls: $\Omega_{col} \propto \left(\frac{\alpha}{\alpha+1}\right)^2 \left(\frac{\beta}{H}\right)^{-2}$

    Kamionkowski ‘93, Huber ‘08, Hindmarsh ‘18,
  - sound waves: $\Omega_{sw} \propto \left(\frac{\alpha}{\alpha+1}\right)^2 \left(\frac{\beta}{H}\right)^{-1}$

    Hindmarsh ‘13 ‘15 ‘17
  - turbulence $\Omega_{turb} \propto \left(\frac{\alpha}{\alpha+1}\right)^{\frac{3}{2}} \left(\frac{\beta}{H}\right)^{-1}$

    Caprini ‘09
- The frequency of the signal changes as $f \propto \frac{\beta}{H}$

- In presence of strong suprcooling vacuum energy cannot be neglected

$$\alpha \approx \frac{\rho_V}{\rho_R} \gtrsim 1 \implies H^2 = \frac{1}{3M_p^2}\left(\frac{\pi^2}{30} g_* T^4 + \Delta V(T=0)\right)$$

- Affects nucleation temperature

$$\int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma(T)}{H(T)^4} = 1$$

- Finding the percolation temperature involves the probability of remaining in the false vacuum

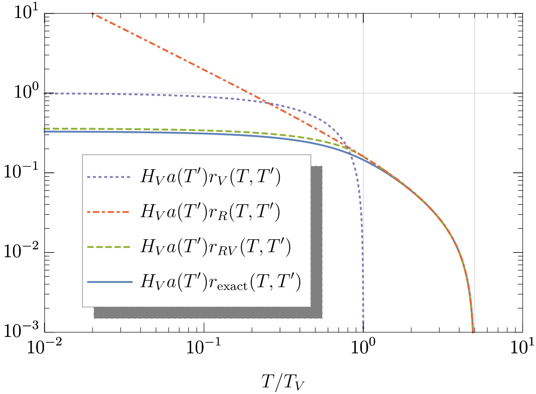
$$P(t) = e^{-I(t)}, \quad I(t) = \frac{4\pi}{3} \int_{t_c}^{t} dt' \, \Gamma(t') \, a(t')^3 \, r(t,t')^3$$

$$P(t_p) \lesssim 0.7 \implies I(t_p) = 0.34$$

Bubble growth

- comoving coordinate size of a bubble nucleated at t' and growing until t

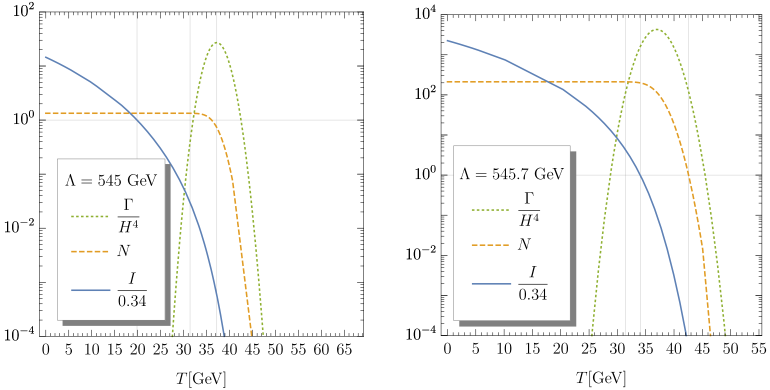
$$a(t')r(t,t') = a(t') \int_{t'}^{t} \frac{v_w \, d\tilde{t}}{a(\tilde{t})}$$



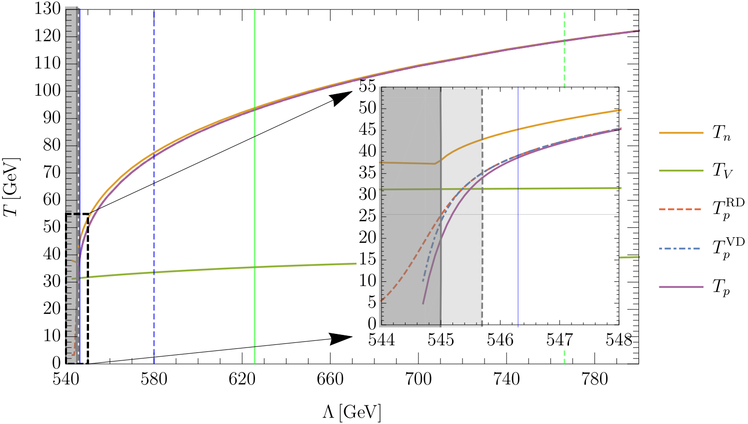
- For successful percolation the physical volume of false vacuum $\mathcal{V}_{\text{false}} \propto a(t)^3 P(t)$ has to decrease

$$\frac{1}{\mathcal{V}_{\text{false}}}\frac{d\mathcal{V}_{\text{false}}}{dt} = 3H(t) - \frac{dI(t)}{dt} = H(T)\left(3 + T\,\frac{dI(T)}{dT}\right) < 0$$

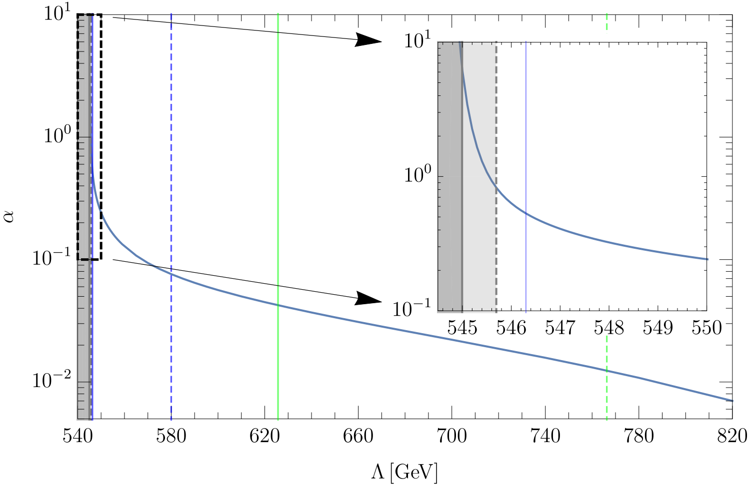
Turner '1992



Percolation temperature



Strength of the transition



relevant length scale for GWs

- The mean bubble separation R_*:

$$n_B = (R_*)^{-3} = \int_{t_c}^{t_p} dt' \, \frac{a(t')^3}{a(t_p)^3} \, \Gamma(t') P(t')$$

- Size of bubbles carrying the most energy

1. The physical size at time t of bubbles nucleated time t':

$$R(t, t') = a(t) \, r(t, t') \, .$$

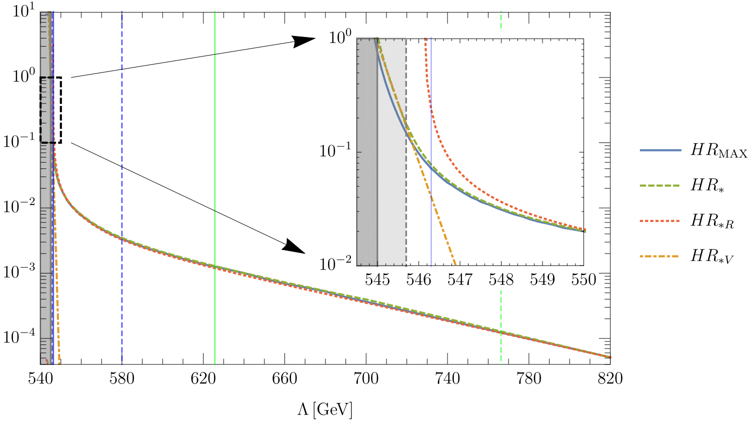
2. The distribution of bubble sizes at temperature T:

$$\frac{dn}{dR}(t, R) = -\frac{dt'}{dR} \frac{a(t'(R))^3}{a(t)^3} \Gamma(t'(R)) P(t'(R)) \, .$$

3. Dominant contribution to the GWs comes from the bubbles that contain the largest fraction of the energy budget

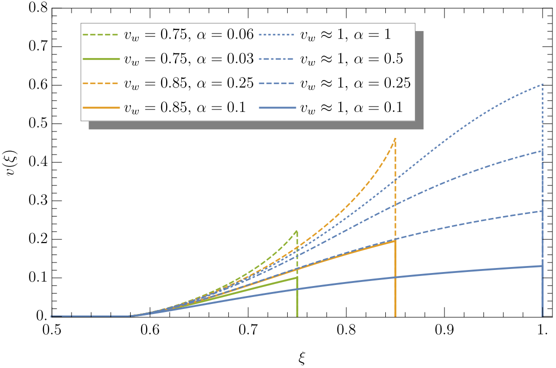
$$\mathcal{E}_B(t, R) \equiv R^3 \, \frac{dn}{dR}(t, R) \Longrightarrow R_{\mathrm{MAX}}$$

Relevant scale for GWs



- To make contact with more standard terminology

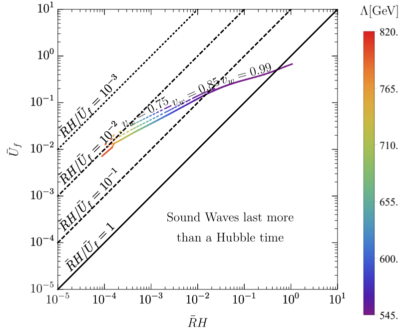
$$\frac{\beta}{H} \Longrightarrow \frac{(8\pi)^{\frac{1}{3}}}{HR}$$

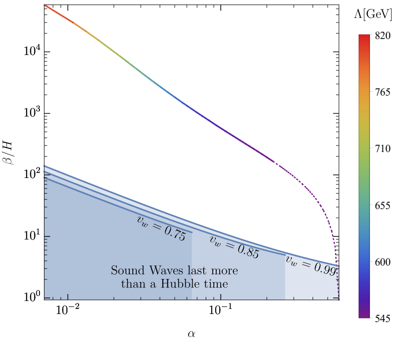


- Root-mean-square four-velocity of the plasma reads

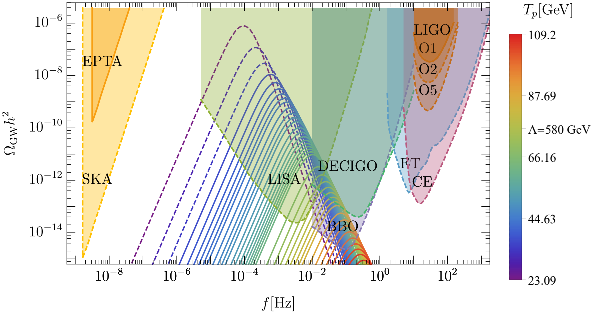
$$\bar{U}_f^2 = \frac{3}{v_w^3} \int_{c_s}^{v_w} \xi^2 \frac{v^2}{1 - v^2} d\xi \, .$$

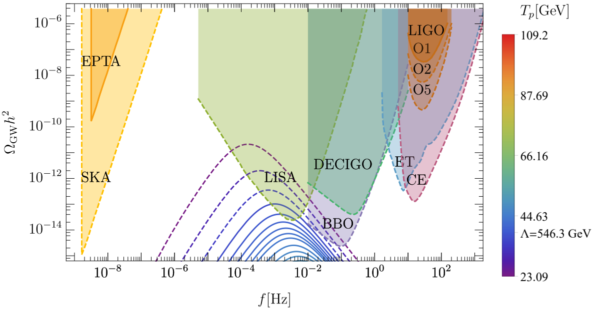
Espinosa '10 Hindmarsh '15 '17





- The amplitude of the GW signal will probably be reduced by a factor $\sim H\bar{R}/\bar{U}_f$


T_p[GeV]
109.2
87.69
Λ=580 GeV
66.16
44.63
23.09
$\Omega_{\rm GW}h^2$
f[Hz]
EPTA
SKA
LISA
DECIGO
BBO
LIGO
O1
O2
O5
ET
CE
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}


T_p[GeV]
109.2
87.69
66.16
44.63
Λ=546.3 GeV
23.09
$\Omega_{\rm GW}h^2$
f[Hz]
EPTA
SKA
LISA
DECIGO
BBO
LIGO
O1
O2
O5
ET
CE
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}

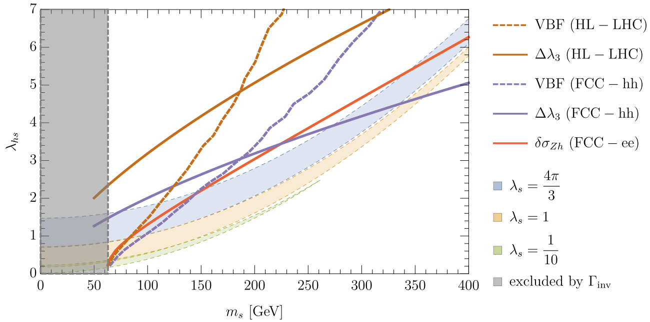
Model 2: Neutral singlet

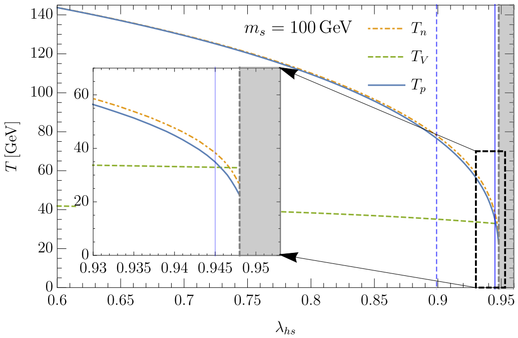
- We add an additional singlet scalar to SM

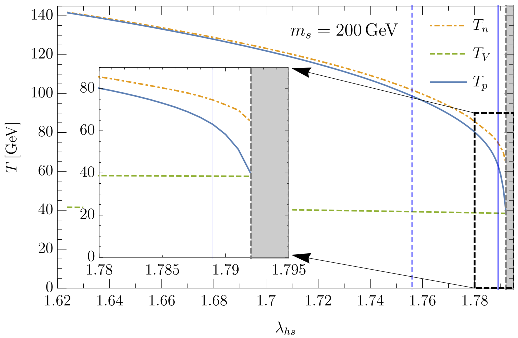
$$V^{\text{tree}}(H,s) = -\mu_h^2\,|H|^2 + \lambda_h\,|H|^4 + \frac{\lambda_{hs}}{2}\,|H|^2\,s^2 + \frac{\mu_s^2}{2}s^2 + \frac{\lambda_s}{4}s^4$$

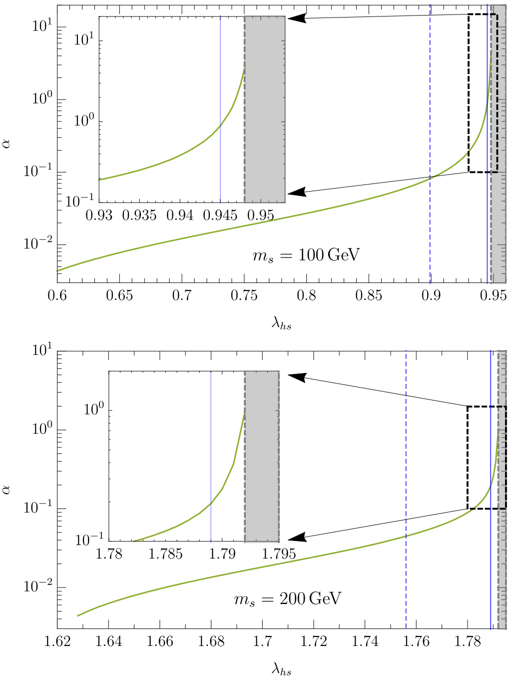
- Singlets physical mass

$$m_s{}^2 = \mu_s^2 + \lambda_{hs}\,v^2/2$$

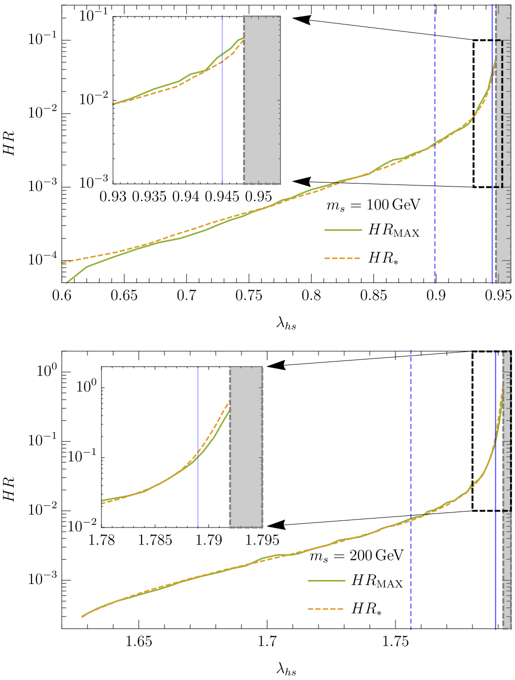



$m_s = 100\,\mathrm{GeV}$
T_n
T_V
T_p
$T\,[\mathrm{GeV}]$
λ_{hs}
0
20
40
60
80
100
120
140
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
0.93
0.935
0.94
0.945
0.95

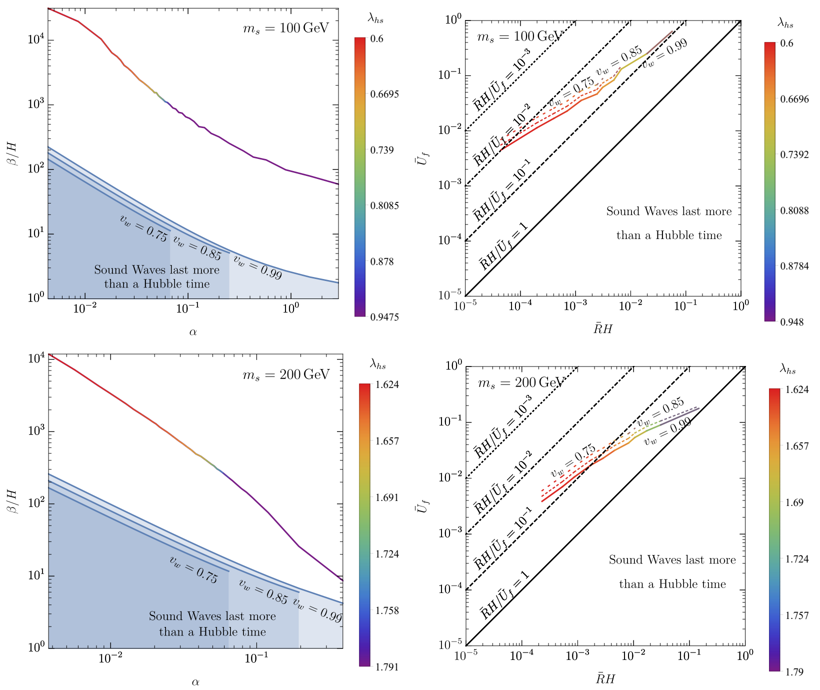

$m_s = 200\,\mathrm{GeV}$
T_n
T_V
T_p
$T\,[\mathrm{GeV}]$
λ_{hs}
0
20
40
60
80
100
120
140
1.62
1.64
1.66
1.68
1.7
1.72
1.74
1.76
1.78
1.785
1.79
1.795



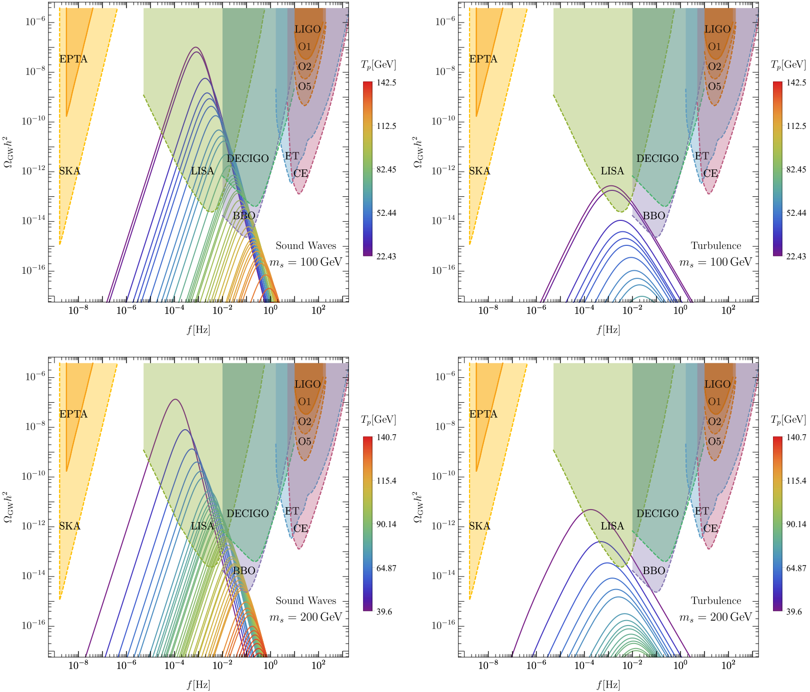
α
λ_{hs}
$m_s = 100\,\mathrm{GeV}$
$m_s = 200\,\mathrm{GeV}$



HR
λ_hs
m_s = 100 GeV
HR_MAX
HR_*
m_s = 200 GeV
HR_MAX
HR_*



$m_s = 100\,\mathrm{GeV}$
β/H
α
$v_w = 0.75$
$v_w = 0.85$
$v_w = 0.99$
Sound Waves last more than a Hubble time
λ_{hs}
0.6
0.6695
0.739
0.8085
0.878
0.9475
$m_s = 100$ GeV
$\bar{U}_f$
$\bar{R}H$
$\bar{R}H/\bar{U}_f = 10^{-3}$
$\bar{R}H/\bar{U}_f = 10^{-2}$
$\bar{R}H/\bar{U}_f = 10^{-1}$
$\bar{R}H/\bar{U}_f = 1$
$v_w = 0.75$
$v_w = 0.85$
$v_w = 0.99$
Sound Waves last more than a Hubble time
λ_{hs}
0.6
0.6696
0.7392
0.8088
0.8784
0.948
$m_s = 200\,\mathrm{GeV}$
β/H
α
$v_w = 0.75$
$v_w = 0.85$
$v_w = 0.99$
Sound Waves last more than a Hubble time
λ_{hs}
1.624
1.657
1.691
1.724
1.758
1.791
$m_s = 200$ GeV
$\bar{U}_f$
$\bar{R}H$
$\bar{R}H/\bar{U}_f = 10^{-3}$
$\bar{R}H/\bar{U}_f = 10^{-2}$
$\bar{R}H/\bar{U}_f = 10^{-1}$
$\bar{R}H/\bar{U}_f = 1$
$v_w = 0.75$
$v_w = 0.85$
$v_w = 0.99$
Sound Waves last more than a Hubble time
λ_{hs}
1.624
1.657
1.69
1.724
1.757
1.79



EPTA
SKA
LISA
DECIGO
BBO
LIGO
O1
O2
O5
ET
CE
Sound Waves
$m_s = 100\,\mathrm{GeV}$
Turbulence
$m_s = 100\,\mathrm{GeV}$
Sound Waves
$m_s = 200\,\mathrm{GeV}$
Turbulence
$m_s = 200\,\mathrm{GeV}$
$T_p[\mathrm{GeV}]$
142.5
112.5
82.45
52.44
22.43
140.7
115.4
90.14
64.87
39.6
$\Omega_{\mathrm{GW}}h^2$
$f[\mathrm{Hz}]$

Conclusions

- For PTs with strong supercooling ($\alpha \gtrsim 1$) energy of the unstable false vacuum has to be taken into account. It affects dynamics of the transition and if vacuum domination lasts for a significant amount of time can jeopardize the successful completion of the phase transition.
- Condition for GW production by sound waves to be *long-lasting* (approximately a Hubble time) is generally not fulfilled. Because of this the sound wave GW signal could be weakened, with turbulence setting in earlier, resulting in a smaller overall GW signal as compared to current literature predictions.
- After supercooled PT the universe reheats to $T_r \approx T_V$ which is the temperature relevant for redshifting of GWs and which sets a bounds the peak frequency of the GW signal from the phase transition to be $f \gtrsim 10^{-4}$ Hz.