Higgs ξ-inflation for the 125–126 GeV Higgs

(Based on arXiv:1306.6931)

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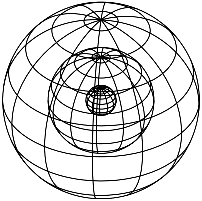


Seminar, University of Sussex

November 4, 2013

Outline

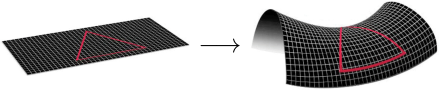
Part I

Inflation from a particle physics perspective

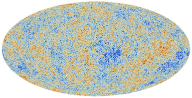
What is inflation?

- A (supposed) period of accelerating expansion of the early universe
- Before the radiation-dominated era, the scalar potential dominated the energy density of the universe
  $\hookrightarrow$ space grows exponentially

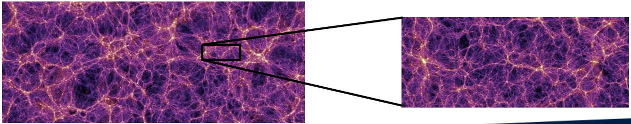
- Proposed to explain
  - Flatness problem



  - Horizon problem



- Produces a nearly scale-invariant spectrum of density fluctuations



Achieving inflation

- Let scalar field ϕ be the "inflaton"

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

- Need a slowly rolling field for inflation

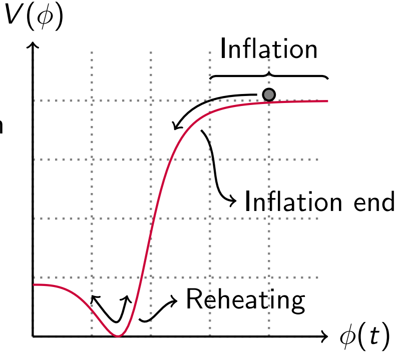
$$\dot{\phi}^2 \ll V(\phi), \qquad \ddot{\phi} \ll 3H\dot{\phi}$$

- In terms of the slow roll parameters

$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta \equiv M_{\rm Pl}^2\left(\frac{V''}{V}\right),$$

inflation occurs so long as

$$\boxed{\epsilon < 1, \quad |\eta| < 1 \qquad \Longleftrightarrow \qquad \text{Inflation}}$$



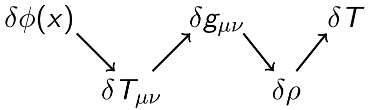
- After inflation, ϕ oscillates about the minimum and its kinetic energy is converted into SM particles $\longleftarrow$ Reheating

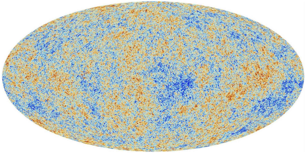
Matching observations

- The amount of inflation described by $N =$ number of times space expands by a factor of e (e-folds)

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{M_{\mathrm{Pl}}} \int_{\phi_f}^{\phi_i} \frac{d\phi}{\sqrt{2\epsilon}}$$

- Precise constraints come from measuring primordial density fluctuations via the cosmic microwave background (CMB)





Quantum fluctuations exist at all scales during inflation and produce a nearly scale-invariant spectrum

Matching observations

- The predicted temperature anisotropies from inflation are

$$\left(\frac{\delta T}{T}\right)^2_\ell = \frac{16\pi}{45 M_{\rm Pl}^4}\frac{V(\phi_\ell)}{\epsilon(\phi_\ell)}, \quad \phi_\ell = \left\{ \begin{array}{l} \text{field value when scale } \ell \text{ leaves} \\ \text{horizon during inflation} \end{array} \right.$$

- Measurement of $\delta T/T$ for $\ell \simeq 3000$ Mpc gives

$$\frac{V(\phi_*)}{\epsilon(\phi_*)} \simeq 5.6 \times 10^{-7} M_{\rm Pl}^4, \quad \phi_* = \left\{ \begin{array}{l} \text{field value } N_* \simeq 50\text{–}60 \text{ e-folds} \\ \text{before end of inflation} \end{array} \right.$$

- Two other parameters are particularly useful for constraining inflationary models

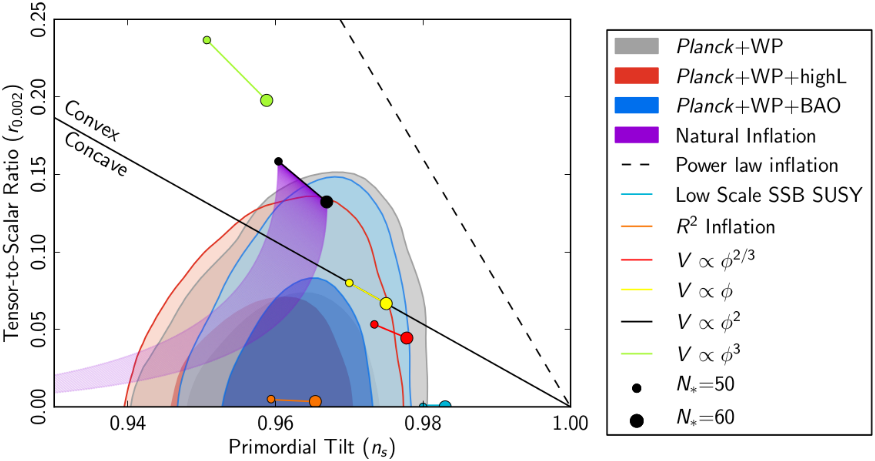
$n_s = 1 - 6\epsilon + 2\eta$ — Spectral index, deviation from scale-invariance

$r = 16\epsilon$ — Tensor-to-scalar ratio, size of $g_{\mu\nu}$ perturbations

As above, $n_s(\phi)$ and $r(\phi)$ are evaluated at ϕ_*

Matching observations

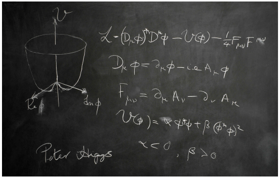
- Many inflationary models have been constructed

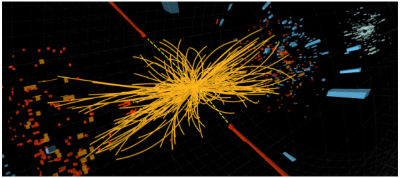


(Planck collaboration, 2013)

Candidates for the inflaton

- Identity of inflaton still unknown; often assumed to be a new weakly-coupled scalar field, but ...





... we have just discovered our first fundamental scalar field

- Can the Higgs boson be the inflaton?
  - ▷ h^4 chaotic inflation (Linde, 1983) $\longleftarrow$ experimentally disfavoured ✗
  - ▷ Quasi-flat SM potential (Isidori et al., 2008) $\longleftarrow$ too few e-folds ✗
  - ▷ False vacuum inflation (Masina & Notari, 2012) $\longleftarrow$ needs second scalar ✗
  - ▷ New Higgs inflation (Germani & Kehagias, 2010) $\longleftarrow$ new scale $M < M_{\rm Pl}$ **?**
  - ▷ Higgs ξ-inflation (Bezrukov & Shaposhnikov, 2008) $\longleftarrow$ unitarity issues **?**

Part II

Higgs ξ-inflation: Tree level

Model definition

- Higgs ξ-inflation is based on a non-minimal coupling of the Higgs doublet to the Ricci scalar

$$\mathcal{L} = -\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \xi H^\dagger H \mathcal{R} + \mathcal{L}_{\rm SM}$$

This is the only local, gauge-invariant interaction with dimension ≤ 4

- For computing tree-level predictions, use unitary gauge $H = \frac{1}{\sqrt{2}}\binom{0}{h}$
  $\hookrightarrow$ Jordan frame action

$$S_J = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm Pl}^2}{2}\left(1 + \frac{\xi h^2}{M_{\rm Pl}^2}\right)\mathcal{R} + (\partial_\mu h)^2 - \frac{\lambda}{4}h^4\right]$$

- Slow-roll calculations require a minimal gravity sector
  $\hookrightarrow$ Conformal transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_{\rm Pl}^2}$$

Model definition

- The resulting Einstein frame action is

$$\boxed{\Omega^2 = 1 + \xi h^2/M_{\rm Pl}^2}$$

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_{\rm Pl}^2}{2}\tilde{\mathcal{R}} + \frac{1}{2}\left(\frac{\Omega^2 + 6\xi^2 h^2/M_{\rm Pl}^2}{\Omega^4}\right)(\partial_\mu h)^2 - \frac{\lambda h^4}{4\Omega^4}\right]$$

- Define a new scalar field χ with canonical kinetic term

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_{\rm Pl}^2}{\Omega^4}}$$

- The Einstein frame action is then

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_{\rm Pl}^2}{2}\tilde{\mathcal{R}} + \frac{1}{2}(\partial_\mu\chi)^2 - U(\chi)\right], \quad U(\chi) = \frac{\lambda[h(\chi)]^4}{4\Omega^4}$$

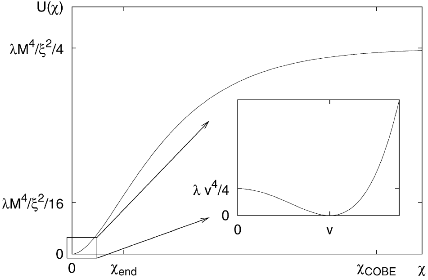
- $U(\chi)$ flattens for $h(\chi) \gtrsim M_{\rm Pl}/\sqrt{\xi}$ $\longleftarrow$ Inflationary region

Model predictions

- Can perform slow-roll calculations with $U(\chi)$

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{U'}{U}\right)^2 \simeq \frac{4M_{\rm Pl}^4}{3\xi^2 h^4}$$

$$\eta = M_{\rm Pl}^2\frac{U''}{U} \simeq \frac{4M_{\rm Pl}^4}{3\xi^2 h^4}\left(1 - \frac{\xi h^2}{M_{\rm Pl}^2}\right)$$



(Bezrukov & Shaposhnikov, 2008)

- Inflation ends at $h_{\rm end} \simeq 1.07 M_{\rm Pl}/\sqrt{\xi}$ when $\epsilon \simeq 1$
- Working backwards, $N = 59$ e-folds produced at $h_* \simeq 9.14 M_{\rm Pl}/\sqrt{\xi}$
- The spectral index and tensor-to-scalar ratio for Higgs ξ-inflation are

$$n_s(\chi_*) \simeq 0.967, \qquad r(\chi_*) \simeq 0.0031$$

Model predictions

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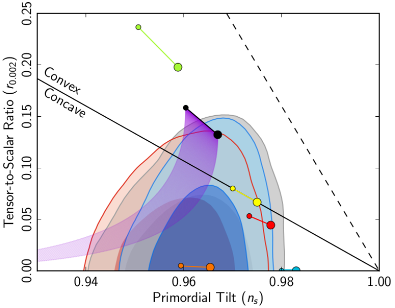

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Perturbative unitarity breakdown

- Generating the observed primordial density fluctuations requires

$$\frac{U(\chi_*)}{\epsilon(\chi_*)} \simeq 5.6 \times 10^{-7} M_{\mathrm{Pl}}^4 \implies \boxed{\xi \simeq 48000\sqrt{\lambda} \simeq 17000}$$

- Such a large value $\xi \sim 10^4$ creates a problem:
  (1) Perturbative unitarity breaks down at $M_{\mathrm{Pl}}/\xi \ll M_{\mathrm{Pl}}/\sqrt{\xi}$

$$\xi h^2 \mathcal{R} \longrightarrow \frac{\xi\sqrt{M_{\mathrm{Pl}}^2 + \xi h^2}}{M_{\mathrm{Pl}}^2 + \xi h^2 + 6\xi^2 h^2} \hat{h}^2 \Box \hat{g} \simeq \frac{\xi}{M_{\mathrm{Pl}}} \hat{h}^2 \Box \hat{g} \qquad \text{for } h \simeq 0$$

  (2) New physics entering at $\Lambda = M_{\mathrm{Pl}}/\xi$ is naively expected to affect the potential in an uncontrollable way
  (3) Self-consistency of Higgs ξ-inflation is questionable

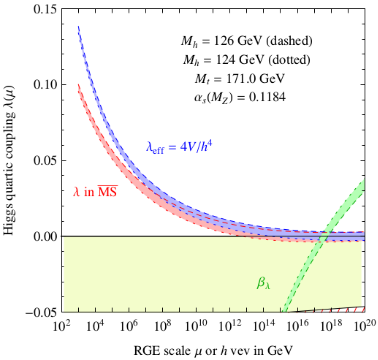
- Proponents argue that the scale of perturbative unitarity breakdown depends on h (it is larger during inflation) and so does not spoil the inflationary predictions
  $\hookrightarrow$ Assumes scale of new physics is background field-dependent

Perturbative unitarity breakdown

- For $M_h \simeq 125$–126 GeV, λ can run to very small values near the Planck scale (where inflation occurs)
- How does this affect the predictions for Higgs ξ-inflation? We expect

$$\xi \simeq 48000\sqrt{\lambda} \ll 17000$$

  Can this address the perturbative unitarity breakdown problem?



(Degrassi et al., 2012)

- Actually, M_t must be about 2–3σ below its central value for Higgs ξ-inflation to be possible
  - Glass half empty: Model disfavoured at 2–3σ
  - Glass half full: Special region within 2–3σ of measurements
- Need to go beyond the tree level

Part III

Radiative corrections

Renormalization group equations

- The most important higher-loop effect is the running of λ
- Need the RG equations for Higgs ξ-inflation

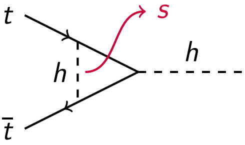
$$\pi_h = \frac{\partial \mathcal{L}_E}{\partial \dot{h}} = \sqrt{-\tilde{g}} \left(\frac{d\chi}{dh}\right)^2 \tilde{g}^{0\nu} \tilde{\partial}_\nu h \quad \xrightarrow{\textit{Jordan}} \quad \Omega^2 \sqrt{-g} \left(\frac{d\chi}{dh}\right)^2 \dot{h}$$

Canonical commutation relation gives

$$[h, \pi_h] = i\hbar\delta^3(\vec{x} - \vec{y}) \quad \Longrightarrow \quad [h, \dot{h}] = s(h) i\hbar\delta^3(\vec{x} - \vec{y})$$

where $s(h) = \frac{1+\xi h^2/M_{\rm Pl}^2}{1+(1+6\xi)\xi h^2/M_{\rm Pl}^2} \simeq \frac{1}{1+6\xi}$ for large h

- The *physical* Higgs propagators are suppressed during inflation, but not those of the Nambu-Goldstone bosons



Renormalization group equations

- Two slightly different ways of dealing with the suppressed Higgs propagators
  (1) Insert a factor s for each off-shell Higgs in the RG equations of the SM (De Simone, Hertzberg & Wilczek, 2009)
  (2) View the effect as a suppression of the Higgs coupling to SM fields. For $\xi \gg 1$, use the RG equations from the chiral electroweak theory (Bezrukov & Shaposhnikov, 2009)
- Have tried to reconcile the differences, but have been unable to reproduce the results from method (2) using Feynman diagrams
- For this analysis, use the two-loop RG equations derived using method (1) with leading three-loop corrections to λ, y_t and γ
- Note the running of ξ is given by

$$\beta_\xi = \left(\xi + \frac{1}{6}\right) \frac{\beta_{m^2}}{m^2}, \qquad \xi(M_{\rm Pl}/\xi_0) = \xi_0$$

Effective potential

- The SM effective potential must also be modified by the suppressed Higgs propagators $\longleftarrow$ result seems to be frame-dependent

Renormalization prescription II (Jordan frame)

- Perturb the tree-level SM potential about the background value, compute the masses of the perturbations, then transform to the Einstein frame

$$M_h^2 = 3s\lambda h^2, \quad M_G^2 = \lambda h^2, \quad M_W^2 = \frac{g^2 h^2}{4}, \quad M_Z^2 = \frac{(g^2 + g'^2)h^2}{4}, \quad \ldots$$

The higher-loop corrections take the usual Coleman-Weinberg form with these modified particle masses

$$U(\chi) = \frac{\lambda h^4}{4\Omega^4} + \frac{1}{16\pi^2}\left[\frac{M_h^4}{4\Omega^4}\left(\ln\frac{M_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3M_G^4}{4\Omega^4}\left(\ln\frac{M_G^2}{\mu^2} - \frac{3}{2}\right) + \ldots\right]$$

- Choose the renormalization scale $\mu = h$ to minimize the log terms

Effective potential

- The Einstein frame prescription is similar but we perform the conformal transformation before computing the particle masses

Renormalization prescription I (Einstein frame)

- Transform the tree-level SM potential to the Einstein frame, perturb it about the background value, and compute the masses of the perturbations

$$U_0 = \frac{\lambda h^4}{4\Omega^4} \implies M_h^2 = \frac{3s\lambda h^2}{\Omega^4}\left(\frac{1-\frac{\xi h^2}{M_{\rm Pl}^2}}{1+\frac{\xi h^2}{M_{\rm Pl}^2}}\right), \quad M_G^2 = \frac{\lambda h^2}{\Omega^4}, \quad M_W^2 = \frac{g^2 h^2}{4\Omega^2}, \ldots$$

$$U(\chi) = \frac{\lambda h^4}{4\Omega^4} + \frac{1}{16\pi^2}\left[\frac{M_h^4}{4}\left(\ln\frac{M_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3M_G^4}{4}\left(\ln\frac{M_G^2}{\mu^2} - \frac{3}{2}\right) + \ldots\right]$$

- There is an additional suppression of M_h^2 and M_G^2 in this prescription
- Choose the renormalization scale $\mu = h/\Omega$ to minimize the log terms

Effective potential

- The additional suppression of M_h^2 and M_G^2 makes little numerical difference to the effective potential since the contributions of these masses are already small ($\lambda \ll 1$)
- The most important difference between the Einstein and Jordan frame renormalization prescriptions is the functional dependence $\mu(h)$

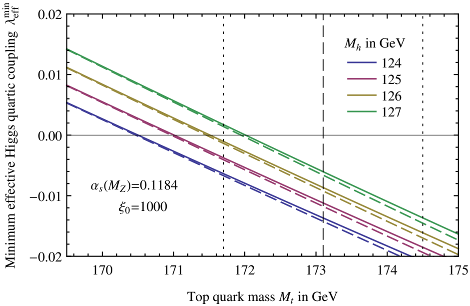
$$\mu = \begin{cases} \frac{h}{\sqrt{1+\xi h^2/M_{\rm Pl}^2}} & \text{Prescription I (Einstein frame)} \\ h & \text{Prescription II (Jordan frame)} \end{cases}$$

  Can have a large impact on the effective potential during inflation!
- For this analysis, consider both prescriptions and use the two-loop effective potential with the appropriately modified particle masses
- Moreover, define the effective Higgs self-coupling $\lambda_{\rm eff}(\mu)$ through

$$U(\chi) \equiv \frac{\lambda_{\rm eff}(\mu) h^4}{4\Omega^4}$$

Two-loop inflationary predictions

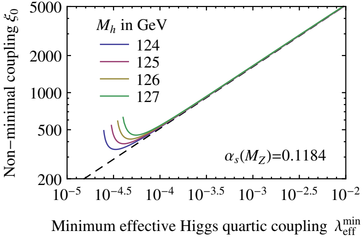
- Use the RG equations to run the initial values of M_h, M_t, α_s, ... up to the inflationary scale and use the two-loop effective potential
- To explore $\lambda_{\text{eff}}(\mu) \ll 1$, replace M_t with $\lambda_{\text{eff}}^{\text{min}} \equiv \min\{\lambda_{\text{eff}}(\mu)\}$
  $\hookrightarrow$ fine-tuning
- For a fixed M_h, $\lambda_{\text{eff}}^{\text{min}}$ and α_s,
  1. Choose ξ_0. Adjust M_t to give the desired $\lambda_{\text{eff}}^{\text{min}}$
  2. Determine U/ϵ at $N = 59$ e-folds before the end of inflation
  3. Repeat the steps above until the correct U/ϵ normalization is obtained
  4. Compute the predictions for n_s and r

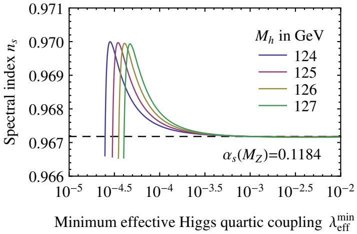


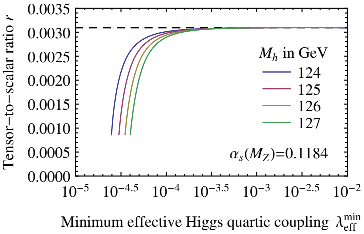
- The numerical results are presented as a function of $\lambda_{\text{eff}}^{\text{min}}$

Results for prescription I

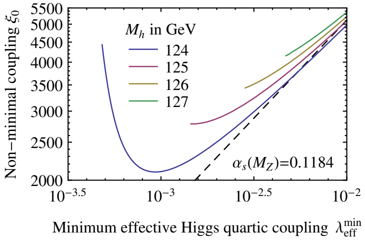
- Non-minimal coupling can be as small as $\xi \sim 400$ for $\lambda_{\text{eff}}^{\text{min}} \sim 10^{-4.4}$
  - $\hookrightarrow$ still too large to address the perturbative unitarity problem
- Need larger ξ for $\lambda_{\text{eff}}^{\text{min}} < 10^{-4.4}$
- Eventually the effective potential develops a second minimum that spoils Higgs ξ-inflation
- Predictions for n_s and r remain within the 1σ region ↴



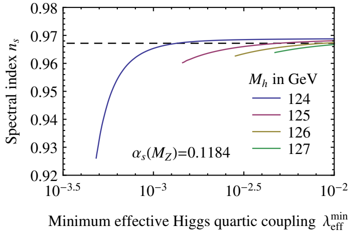


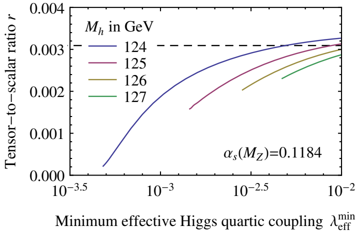


Results for prescription II



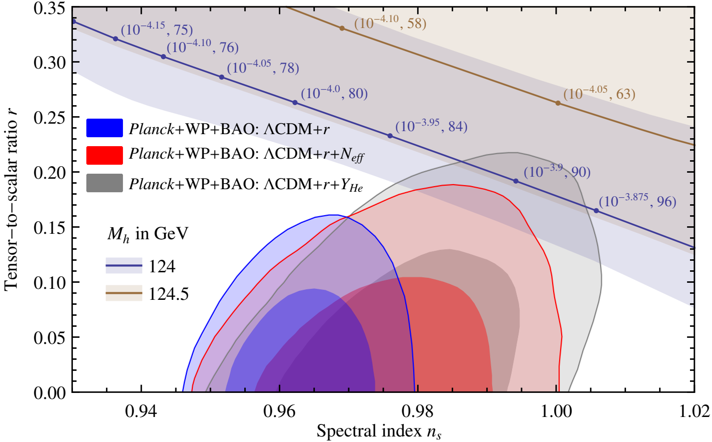
- Two regions for prescription II: large and small $\lambda_{\text{eff}}^{\text{min}}$ regions
- The large $\lambda_{\text{eff}}^{\text{min}}$ region behaves similarly to prescription I but the running of $\lambda_{\text{eff}}(\mu)$ more important
- Can only have ξ as small as about $\xi \sim 2000$–4000 before the potential develops a second minimum
- Predictions for n_s and r show more variation, still within 1σ ↴





Results for prescription II

- The small $\lambda_{\text{eff}}^{\text{min}}$ region is very different from the other results



- Allows $\xi \sim 90$ at 2–3σ with an observable level of r, but there is still a perturbative unitarity problem

Part IV

Conclusions

Conclusions

- Higgs ξ-inflation is one of the few remaining inflationary models that does not require scalar fields in addition to those in the SM
- The breakdown of perturbative unitarity at $M_{\rm Pl}/\xi$ (below the scale of inflation) has long been a potential problem for this model
- We have investigated whether the recently measured Higgs mass, for which $\lambda_{\rm eff}(\mu) \ll 1$ near the Planck scale, can address this problem

	μ	$\lambda_{\rm eff}^{\rm min}$	ξ	n_s	r
Prescription I	$\frac{h}{\sqrt{1+\xi h^2/M_{\rm Pl}^2}}$	$\gtrsim 10^{-4.6}$	$\gtrsim 400$	0.97	$\lesssim 0.003$
Prescription II	h	$\gtrsim 10^{-3.3}$	$\gtrsim 2000$	0.96–0.97	$\lesssim 0.003$
		$\sim 10^{-3.9}$	~ 90	0.97–1.00	0.15–0.25

- The perturbative unitarity problem remains but small $\lambda_{\rm eff}^{\rm min}$ allows a new region of Higgs ξ-inflation with observable tensor-to-scalar ratio

Thank you for your attention!