Ultraviolet Divergences in Maximal Supersymmetric Theories

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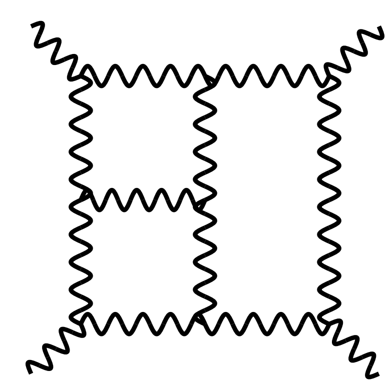
G. Bossard, P.S. Howe & K.S.S., arXiv 0901.4661 & 0908.3883 [hep-th]

Ultraviolet Divergences in Gravity

- Simple power counting in gravity and supergravity theories leads to a naïve degree of divergence

$$\Delta = (D-2)L + 2$$

in D spacetime dimensions. So, for D=4, L=3, one expects $\Delta = 8$. In dimensional regularization, only logarithmic divergences are seen ($\frac{1}{\epsilon}$ poles, $\epsilon = D - 4$), so 8 powers of momentum would have to come out onto the external lines of such a diagram.



- Local supersymmetry implies that the pure curvature part of such a D=4, 3-loop divergence candidate must be built from the square of the Bel-Robinson tensor

Deser, Kay & K.S.S

$$\int \sqrt{-g}\, T_{\mu\nu\rho\sigma} T^{\mu\nu\rho\sigma} \,, \qquad T_{\mu\nu\rho\sigma} = R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta} R_{\rho\alpha\sigma\beta} + {}^{*}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\, {}^{*}R_{\rho\alpha\sigma\beta}$$

- This is directly related to the α'^3 corrections in the superstring effective action, except that in the string context such contributions occur with finite coefficients. The question remains whether such string theory contributions develop poles in $(\alpha')^{-1}$ as one takes the zero-slope limit $\alpha' \to 0$ and how this bears on the ultraviolet properties of the corresponding field theory.

Berkovits; Green, Russo & Vanhove

- The consequences of supersymmetry for the ultraviolet structure are not restricted to the requirement that counterterms be supersymmetric invariants.

- There exist more powerful "non-renormalization theorems," the most famous of which excludes infinite renormalization within D=4, N=1 supersymmetry of chiral invariants, given in N=1 superspace by integrals over half the superspace:

$$\int d^2\theta W(\phi(x,\theta,\bar{\theta}))\ , \quad \bar{D}\phi = 0$$

- Key tools in proving non-renormalization theorems are superspace formulations and the background field .

- For example, the Wess-Zumino model in N=1, D=4 supersymmetry is formulated in terms of a chiral superfield $\phi(x,\theta,\bar{\theta})$: $\bar{D}\phi = 0$; $\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\alpha}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}$

- In the background field method, one splits the superfield into "background" and "quantum" parts,

$$\phi = \varphi + Q$$

background quantum

- The chiral constraint on $Q(x,\theta,\bar{\theta})$ can be solved by introducing a "prepotential": $Q = \bar{D}^2 X \qquad (\bar{D}^3 \equiv 0)$

- Although the Wess-Zumino action requires chiral superspace integrals $I = \int d^4x\, d^4\theta\, \bar{\phi}\phi + Re \int d^4x\, d^2\theta\, \phi^3$ when written in terms of the total field ϕ, the parts involving the quantum field Q appearing inside loop diagrams can be re-written as $\int d^4x\, d^4\theta = \int d^4x\, d^2\theta d^2\bar{\theta}$ full superspace integrals using the "integration=differentiation" property of Berezin integrals.

- Upon expanding into background and quantum parts, one finds that the chiral interaction terms can be rewritten as full superspace integrals, e.g.

$$\int d^4x\, d^2\theta Q^2\varphi = \int d^4x\, d^4\theta\, X\bar{D}^2X\varphi$$

- Thus all counterterms written using the background field φ must be writable as full-superspace integrals.

- The key points in the non-renormalization theorems are the requirement that allowed counterterms be written as full $\int d^{4M}\theta$ superspace integrals for the linearly realized M-extended supersymmety, while integrands must be written using a clearly defined set of basic objects and the integrated counterterms also have to satisfy all applicable gauge symmetries and must also be locally constructed (*i.e.* written without such operators as $\Box^{-1}$).

- So, in D=4, N=1 supersymmetry, full superspace integrals like $\int d^4x d^4\theta f(\phi,\bar{\phi})$ (or "D terms") are allowed for divergence structures, but chiral integrals like $\int d^4x d^2\theta\, g(\phi)$ (or "F terms") are not.

- The strength of a given supersymmetric non-renormalization theorem depends on the extent of linearly realizable, or "off-shell" supersymmetry. This is the extent of supersymmetry for which the algebra can close without use of the equations of motion.

- Knowing the extent of this off-shell supersymmetry is tricky, and may involve formulations (*e.g.* harmonic superspace) with infinite numbers of auxiliary fields.

Galperin, Ivanov, Kalitsin, Ogievetsky & Sokatchev

- For maximal N=4 Super Yang-Mills and maximal N=8 supergravity, the linearly realizable supersymmetry has been known since the 1980's to be at least half the full supersymmetry of the theory.

- The full extent of a theory's supersymmetry, even though it may be non-linear, also restricts the infinities since the *leading* counterterms have to be invariant under the original unrenormalized supersymmetry transformations.

- Assuming that 1/2 supersymmetry is linearly realizable and requiring gauge and supersymmetry invariances, predictions were derived for the first divergent loop orders in maximal (N=4 ↔ 16 supercharge) SYM and (N=8 ↔ 32 sc.) SUGRA:

Howe, K.S.S & Townsend

Max. SYM first divergences, assuming half SUSY off-shell (8 supercharges)

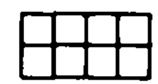
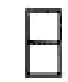
Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	4	∞
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	F^4	finite

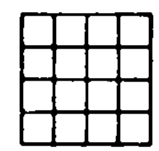
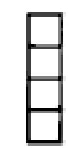
Max. SUGRA first divergences, assuming half SUSY off-shell (16 supercharges)

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	2	3
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^4 R^4$	$\partial^6 R^4$	R^4	R^4

- When written in terms of the full "on-shell" supersymmetry, the F^4 super Yang-Mills and the R^4 supergravity candidates have similar "1/2 BPS structure". In their D=4 incarnations, they are

Howe, K.S.S. & Townsend
Kallosh

$$\Delta I_{SYM} = \int (d^4\theta d^4\bar{\theta})_{105}\, \mathrm{tr}(\phi^4)_{105}$$  ***105*** ϕ_{ij}  ***6*** of SU(4)

$$\Delta I_{SG} = \int (d^8\theta d^8\bar{\theta})_{232848} (W^4)_{232848}$$  ***232848*** W_{ijkl}  ***70*** of SU(8)

- However, it now seems that such counterterm analysis in terms of BPS degree is incomplete. The calculational front has recently progressed remarkably.

Unitarity-based calculations

Bern, Carrasco, Dixon, Dunbar, Johansson, Kosower, Perelstein, Roiban, Rozowsky et al.

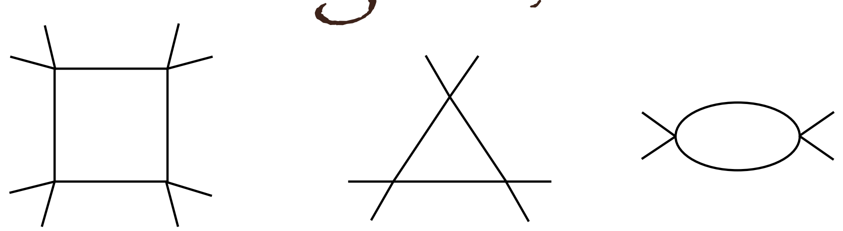
- Within the last decade, there have been significant advances in the computation of loop corrections in quantum field theory.
- These developments include the organization of amplitudes into a new kind of perturbation theory starting with maximal helicity violating amplitudes (MHV), then next-to-MHV (NMHV), *etc.*
- They also incorporate a specific use of dimensional regularization together with a clever use of unitarity cutting rules.

- Normally, one thinks of unitarity relations, such as the optical theorem, as giving information only about the imaginary parts of amplitudes. However, if one keeps all orders in an expansion in $\epsilon = D - 4$, then loop integrals like $\int d^{(4+\epsilon)}p$ require integrands to have an additional momentum dependence $f(s) \to f(s)s^{-\epsilon/2}$, where s is a momentum invariant. Then, since $s^{-\epsilon/2} = 1 - (\epsilon/2)\ln(s) + \ldots$ and $\ln(s) = \ln(|s|) + i\pi\Theta(s)$, one can learn about the real parts of an amplitude by retaining imaginary terms at order ϵ .

- This gives rise to a procedure for the *cut construction* of higher-loop diagrams.

- Another key element in the unitarity-based analysis of amplitudes is the Brown-Feynman-Passarino-Veltman procedure for the reduction of Feynman-diagram propagators, replacing numerator factors like $2k \cdot p$ where $p^2 = 0$ by $(k+p)^2 - k^2$ and then canceling corresponding denominators.

- This procedure can yield a variety of irreducible configurations in the reduced diagram, including boxes, triangles and bubbles.



- Important simplifications occur if one can show there are ultimately no bubbles or triangles in the reduced amplitude.

- A key link between maximal supergravity and maximal SYM is the Kawai-Lewellen-Tye (KLT) relation between open- and closed-string amplitudes. These give rise to tree-level relations between field-theoretic max. SUGRA and max. SYM field-theory amplitudes, *e.g.*

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)$$

- Combining this with the unitarity-based calculations, in which all amplitudes are ultimately reduced to integrals over products of tree amplitudes, one has a way to obtain higher-loop supergravity amplitudes from SYM amplitudes.

- Another remarkable aspect of the unitarity-based methods is the simplification of vertices. The off-shell 3-graviton vertex has the form (with about 100 terms)

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1, k_2, k_3) =$$

$$\begin{aligned}
&\text{sym}[-\tfrac{1}{2}P_3(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1 \cdot k_2 \eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\
&\quad + P_6(k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) \\
&\quad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) \\
&\quad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1 \cdot k_2 \eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$

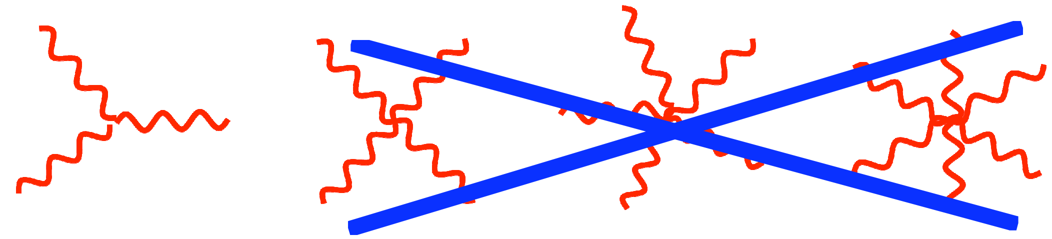
- Putting this vertex on-shell for tree diagrams simplifies it to

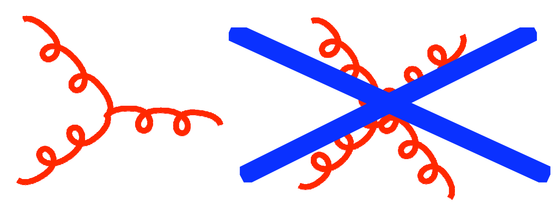
$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic}) \times (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

which is just the square of the colour-stripped version of the SYM amplitude,

$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

in agreement with KLT.

- More remarable still is the fact that on-shell tree amplitudes can be built entirely using 3-point vertices: contributions from the infinite numbers of higher-point vertices all cancel out: 

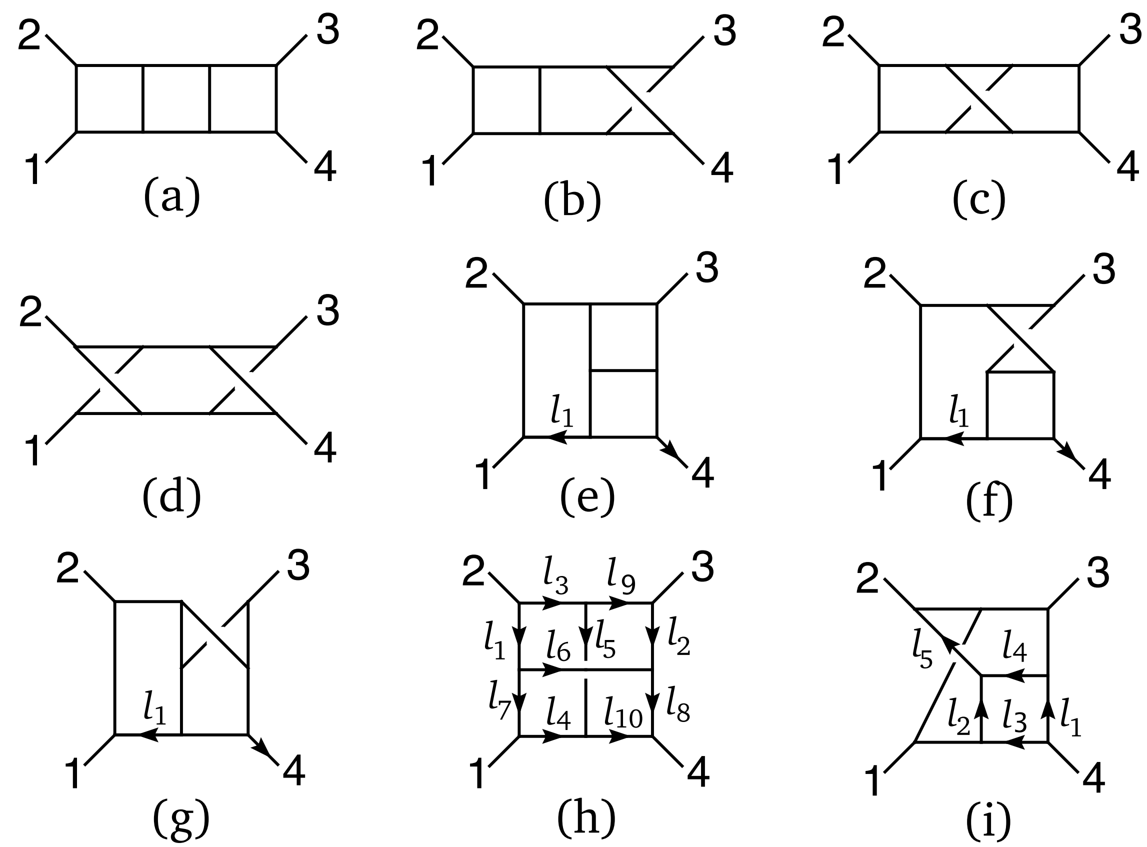
- This is also a reflection of a simplification in SYM, where the 4-point vertex does not contribute to on-shell tree amplitude calculations: 

- Using these on-shell simplfications inside general tree diagrams requires also a technique of making complex shifts of external momenta. $p_1^\mu(z) = p_1^\mu - zq^\mu \ , \quad p_n^\mu(z) = p_n^\mu + zq^\mu$

$q^2 = 0 \ , \quad p \cdot q = 0 \ , \quad (p_i^\mu(z))^2 = 0$ Britto, Cachazo, Feng & Witten; Badger, Glover, Khoze & Svrcek

- An important development was the completion of the 3-loop max. supergravity calculation: Bern, Carrasco, Dixon, Johansson, Kosower & Roiban.

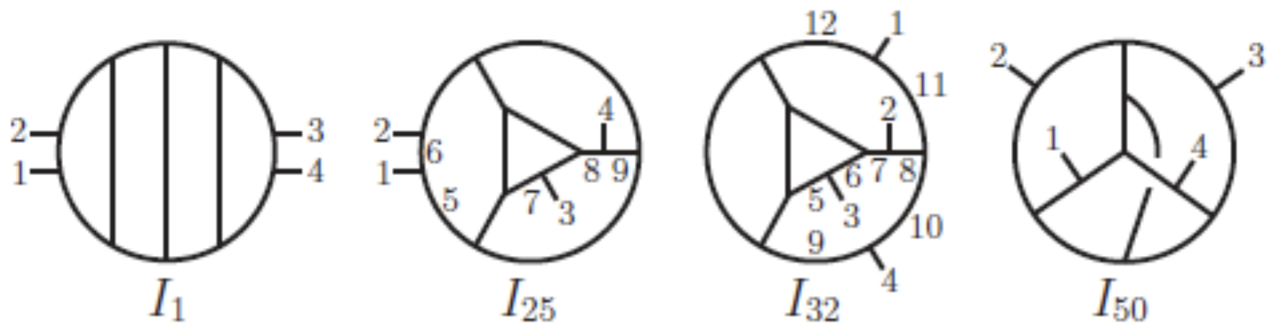
Normal Feynman diagram calculation of these would involve about 10^{20} terms



- Diagrams (a-g) can be evaluated using iterated two-particle cuts, but diagrams (h) & (i) cannot. The result is *finite* at L=3 in D=4. A surprising result is that the finite parts have an unexpected six powers of momentum that come out onto the external lines, giving a $\partial^6 R^4$ leading effective action correction.

Moreover, the 4-loop calculation has also now been done (May 2009).

Bern, Carrasco, Dixon, Johansson & Roiban



+ 46 more topologies

- Result: $M_4^{4\text{-loop}} = \left(\frac{\kappa}{2}\right)^{10} stu M_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{50} c_i I_i$ is also ultraviolet finite in D=4 *and* in D=5 (rather unexpected).
- Additional consequence: two bottles of wine have been lost in a bet to Zvi Bern.

Current calculation status

Bern, Carrasco, Dixon, Johansson & Roiban

- The development of the unitarity methods of calculation have led to surprising cancellations at the 3- and 4-loop orders, yielding the following (minimal) anticipations for the super Yang-Mills and supergravity divergence onsets:

Max. SYM first divergences, current lowest possible orders.

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	6?	∞
BPS degree	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	$\partial^2 F^4$	finite

Max. supergravity first divergences, current lowest possible orders.

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6?	5?
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	$\frac{1}{4}$
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^{12} R^4$	$\partial^4 R^4$

- Moreover, there is a strange circumstance in maximal super Yang-Mills theory for L=3, D=6. Although the 1/4 BPS $\mathrm{tr}\,\partial^2 F^4$ divergence structure occurs as expected at the 3-loop order, the 1/4 BPS $(\mathrm{tr}\,\partial F^2)(\mathrm{tr}\,\partial F^2)$ *double-trace* structure *fails* to occur in D=6.

- Nonetheless, both the single-trace and double-trace structures *do* show up as expected in the L=2, D=7 theory.

- This appears to be an echo of the situation in max. supergravity, where the 1/8 BPS $\partial^6 R^4$ counterterm occurs in D=6, L=3, but does *not* occur in D=5, L=4.

Algebraic Renormalization

Dixon; Howe, Lindstrom & White; Piguet & Sorella; Hennaux; Stora; Baulieu & Bossard

- Another approach to analyzing the divergences in supersymmetric gauge theories, using the *full* supersymmetry, begins with the Callan-Symanzik equation for the renormalization of the Lagrangian when considered as an operator insertion, governing, *e.g.*, mixing with the half-BPS operator $S^{(4)} = \mathrm{tr}(F^4)$. Letting the classical action be $S^{(2)}$, the C-Z equation in dimension D is $\mu\frac{\partial}{\partial\mu}[S^{(2)}\cdot\Gamma] = (4-D)[S^{(2)}\cdot\Gamma] + \gamma_{(4)}g^{2n_{(4)}}[S^{(4)}\cdot\Gamma] + \cdots,$ where $n_{(4)} = 4,\ 2,\ 1$ for $D = 5,\ 6,\ 8$.

- From this, one learns that $(n_{(4)}-1)\beta_{(4)} = \gamma_{(4)}$ so the beta function for the $S^{(4)} = \mathrm{tr}(F^4)$ operator is determined by the anomalous dimension $\gamma_{(4)}$.

- Combining the supersymmetry generator with a commuting spinor parameter to make a scalar operator $Q = \bar{\varepsilon}Q$, the expression of SUSY invariance for a D-form density in D-dimensions is $Q\mathcal{L}_D + d\mathcal{L}_{D-1} = 0$. Combining this with the SUSY algebra $Q^2 = -i(\bar{\varepsilon}\gamma^\mu\varepsilon)\partial_\mu$ and using the Poincaré Lemma, one finds $i_{i(\bar{\varepsilon}\gamma\varepsilon)}\mathcal{L}_D + S_{(Q)|\Sigma}\mathcal{L}_{D-1} + d\mathcal{L}_{D-2} = 0$.
- Hence, one can consider cocycles of the extended nilpotent differential $d + S_{(Q)|\Sigma} + i_{i(\bar{\varepsilon}\gamma\varepsilon)}$ acting on formal form-sums $\mathcal{L}_D + \mathcal{L}_{D-1} + \mathcal{L}_{D-2} + \cdots$.
- The supersymmetry Ward identities then imply that the whole cocycle must be renormalized in a coherent way. In order for an operator like $S^{(4)}$ to mix with the classical action $S^{(2)}$, their cocycles need to have the same structure.

Ectoplasm

Gates, Grisaru, Knut-Whelau, & Siegel
Berkovits and Howe
Bossard, Howe & KSS

- The construction of supersymmetric invariants is isomorphic to the construction of cohomologically nontrivial closed forms in superspace:
  $I = \int_{M_0} \sigma^* \mathcal{L}_D$ is invariant (where σ^* is a pull-back to the "body" subspace M_0) if $\mathcal{L}_D$ is a closed form in superspace, and is nonvanishing if $\mathcal{L}_D$ is nontrivial.
- Revisit the BRST formalism, but now include all gauge symmetries (in particular including spatial diffeomorphisms) in the nilpotent BRST operator s. The invariance condition for $\mathcal{L}_D$ is $s\mathcal{L}_D + d_0\mathcal{L}_{D-1} = 0$ where d_0 is the usual bosonic exterior derivative. Since $s^2 = 0$ and s anticommutes with d_0, one obtains $s\mathcal{L}_{D-1} + d_0\mathcal{L}_{D-2} = 0$.

- So the cohomological problem reappears in BRST guise, but with the commuting spinor ε replaced by the commuting supersymmetry ghost. One needs to study the cohomology of the nilpotent operator $\delta = s + d_0$, whose cochains $\mathcal{L}_{D-q,q}$ are (D-q) forms with ghost number q, *i.e.* (D-q) forms with q spinor indices. The spinor indices are totally symmetric since the supersymmetry ghost is commuting.

- For gauge-invariant supersymmetric integrands, this establishes an isomorphism between the cohomology of closed forms in superspace (aka "ectoplasm") and the constuction of BRST invariant counterterms.

Superspace cohomology

Bonora, Pasti & Tonin

- Flat superspace has a standard basis of invariant 1-forms

$$E^a = dx^a - \frac{i}{2} d\theta^\alpha (\Gamma^a)_{\alpha\beta}\theta^\beta$$
$$E^\alpha = d\theta^\alpha$$

dual to which are the superspace covariant derivatives (∂_a, D_α)

- There is a natural bi-grading of superspace forms into even and odd parts: $\Omega^n = \oplus_{n=p+q}\Omega^{p,q}$

- Correspondingly, the flat superspace exterior derivative splits into three parts with bi-gradings (1,0), (0,1) & (-1,2):

$$d = d_0(1,0) + d_1(0,1) + t_0(-1,2)$$

bosonic der. fermionic der. torsion

$$d_0 \leftrightarrow \partial_\mu \qquad d_1 \leftrightarrow D_\alpha$$

where for a (p,q) form in flat superspace, one has

$$(t_o\omega)_{a_2\cdots a_p\beta_1\cdots\beta_{q+2}} \sim (\Gamma^{a_1})_{(\beta_1\beta_2}\omega_{a_1\cdots a_p\beta_3\cdots\beta_{q+2})}$$

- The nilpotence of the total exterior derivative d implies the relations

$$\begin{aligned} t_0^2 &= 0 \\ t_0 d_1 + d_1 t_0 &= 0 \\ d_1^2 + t_0 d_0 + d_0 t_0 &= 0 \end{aligned}$$

- Then, since $d\mathcal{L}_D = 0$, the lowest dimension nonvanishing cochain (or "generator") $\mathcal{L}_{D-q,q}$ must satisfy $t_0 \mathcal{L}_{D-q,q} = 0$, so $\mathcal{L}_{D-q,q}$ belongs to the t_0 cohomology group $H_t^{D-q,q}$.

- Starting with the t_0 cohomology groups $H_t^{p,q}$, one then defines a spinorial exterior derivative $d_s : H_t^{p,q} \to H_t^{p,q+1}$ by $d_s[\omega] = [d_1 \omega]$, where the [ ] brackets denote H_t classes.

Cederwall, Gran, Nilsson & Tsimpis
Howe & Tsimpis

- One finds that d_s is nilpotent, $d_s^2 = 0$, and so one can define spinorial cohomology groups $H_s^{p,q} = H_{d_s}(H_t^{p,q})$.

The groups $H_s^{0,q}$ give multi *pure spinors*.

- This formalism gives a way to reformulate the algebraic renormalization cohomology in terms of spinorial cohomology. The lowest dimension cochain, or *generator*, of a counterterm's superform will be d_s closed, *i.e.* it must be an element of $H_s^{D-q,q}$.

- Solving $d_s[\mathcal{L}_{D-q,q}] = 0$ then allows one to solve for all the higher components of $\mathcal{L}_D$ in terms of $\mathcal{L}_{D-q,q}$.

- To see how this formalism works, consider N=1 supersymmetry in D=10. Corresponding to the κ symmetries of strings and 5-branes, we have the D=10 Gamma matrix identities $t_0\Gamma_{1,2}=0 \qquad t_0\Gamma_{5,2}=0$.

- The second of these is relevant to the construction of d-closed forms in D=10. One may have a generator

$$L_{5,5}=\Gamma_{5,2}M_{0,3}$$

where $d_s[M_{0,3}]=0$. The simplest example of such a form corresponds to a full superspace integral over S:

$$M_{\alpha\beta\gamma}=T_{\alpha\beta\gamma,\delta_1\cdots\delta_5}(D^{11})^{\delta_1\cdots\delta_5}S$$

where $T_{\alpha\beta\gamma,\delta_1\cdots\delta_5}$ is constructed from the D=10 Gamma matrices; it is totally symmetric in $\alpha\beta\gamma$ and totally antisymmetric in $\delta_1\cdots\delta_5$.

- One finds that the lowest dimension cochain in the D=10 SYM Lagrangian cocycle also has structure $\mathcal{L}_{5,5} = \Gamma_{5,2} Q_{0,3}$, *i.e.* it is of the *same* structure as that for the full superspace integral counterterm.

- Consequently, full superspace integral cocycles have the same structure as that of the SYM Lagrangian cocycle and thus are *not* subject to a nonrenormalization theorem.

- Examples of operators that *are* ruled out by the ectoplasm/algebraic renormalization analysis include half-BPS counterterms such as the $\mathrm{tr}(F^4)$ or $(\mathrm{tr}(F^2))^2$ SYM counterterms. In D dimensions, the generator component of such a 1/2 BPS cocycle is an (0,D) form of dimension 8-D/2. Since the structure of this cocycle is different (i.e. it is longer) from than that of the SYM Lagrangian, the corresponding 1/2 BPS counterterm is *illegal*.

Double-trace SYM non-renormalization

- Similar analysis of the D=7 $\mathrm{tr}(\partial F^2)\mathrm{tr}(\partial F^2)$ L=2 double-trace candidate shows that its lowest cocycle components may be removed by the addition of exact terms, consistent with the D=7 SU(2) R-symmetry, thus leaving a (2,5) lowest dimension form like that of the classical Lagrangian. Thus, this structure is *not* protected.
  Bossard, Howe & K.S.S
- In D=6, however, the situation is different. The R-symmetry is now SU(2)xSU(2) and one finds that there is *no* trivial term that can be added to shorten the D=6 double-trace cocycle so as to agree with the D=6 Lagrangian cocycle structure. Thus, the double-trace L=3 counterterm is *ruled out* in D=6.

Current outlook

- Algebraic renormalization / ectoplasm analysis explains all of the current calculational results in max. SYM theory. Thus, there is so far no evidence for "miraculous" SYM cancellations which are not understandable purely within ordinary field theory.
- The supergravity cases remain to be clarified. Providing the D=6, L=3 vs D=5, L=4 max. supergravity cases work similarly to SYM, the current SG calculational results may also be understood purely within field theory.