Gravity = Gauge Theory

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What this talk is NOT about

~~Gravity = (Gauge Theory)^2~~

string-theory inspired
KLT relations

Even though many relations to that story as well

Main message:

General Relativity (in 4 dimensions) can be reformulated as an SU(2) gauge theory (of a certain type)

$\Lambda \neq 0$ **KK PRL106:251103,2011**

results on zero scalar curvature in early 90's Capovilla, Dell, Jacobson

Why should one be interested in any reformulations?

There are many:

- Tetrad (first order) formulation
- Plebanski (Ashtekar) self-dual formulation
- Mac Dowell-Mansouri SO(2,3) gauge theoretic formulation
- ...

Have not helped. The quantum gravity problem (non-renormalizability) is still open. And still best understood in the original metric formulation

Some exceptional things happen in the new formulation!

The gauge-theoretic formulation

- Simpler than the metric-based GR

perturbative calculations (scattering amplitudes) are easier in this formulation

conformal mode does not propagate even off-shell

- Suggests generalizations that are impossible to imagine in the usual formulation

GR is not the only theory of interacting massless spin 2 particles!

Suggests new (speculative at the moment) ideas as to what may be happening with gravity at very high energies

General Relativity

$g_{\mu\nu}$ - spacetime metric

$$S_{\mathrm{EH}}[g] = -\frac{1}{16\pi G}\int (R - 2\Lambda)$$

Beautiful geometric theory that physicists study for already about a century!

$$\Downarrow$$

$$R_{\mu\nu} \sim g_{\mu\nu}$$

Very “rigid” theory! Any modification messes it up

Several GR uniqueness theorems

GR is the unique theory of interacting massless spin 2 particles

spin two field - $h_{\mu\nu}$

But things also do get ugly...

Expansion around an arbitrary background $g_{\mu\nu}$

quadratic order (together with the gauge-fixing term)

$$L_{g.f.} = -\sqrt{-g}\left(h^{\mu\nu}{}_{;\nu} - \frac{1}{2}h_\nu{}^{\nu;\mu}\right)\left(h^\rho{}_{\mu;\rho} - \frac{1}{2}h^\rho{}_{\rho;\mu}\right)$$

$$L_2 = \sqrt{-g}\left\{-\frac{1}{2}h^{\alpha\beta}{}_{;\gamma}h_{\alpha\beta}{}^{;\gamma} + \frac{1}{4}h^\alpha{}_{\alpha;\gamma}h_\beta{}^{\beta;\gamma} + h_{\alpha\beta}h_{\gamma\delta}R^{\alpha\gamma\beta\delta} - h_{\alpha\beta}h^\beta{}_\gamma R^{\delta\alpha\gamma}{}_\delta \right.$$
$$\left. + h^\alpha{}_\alpha h_{\beta\gamma}R^{\beta\gamma} - \frac{1}{2}h_{\alpha\beta}h^{\alpha\beta}R + \frac{1}{4}h^\alpha{}_\alpha h^\beta{}_\beta R\right\}.$$

from Goroff-Sagnotti "2-loop" paper

cubic order

$$L_3 = \sqrt{-g}\left\{-\frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\gamma\delta;\beta} + 2h^{\alpha\beta}h^{\gamma\delta}{}_{;\alpha}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^\gamma{}_{\gamma;\alpha}h^\delta{}_{\beta;\delta} - \frac{1}{2}h^\alpha{}_\alpha h^{\beta\gamma;\delta}h_{\beta\delta;\gamma}\right.$$
$$+ \frac{1}{4}h^\alpha{}_\alpha h^{\beta\gamma;\delta}h_{\beta\gamma;\delta} - h^{\alpha\beta}h^\gamma{}_{\gamma;\delta}h^\delta{}_{\alpha;\beta} + \frac{1}{2}h^{\alpha\beta}h^\gamma{}_{\gamma;\alpha}h^\delta{}_{\delta;\beta} - h^{\alpha\beta}h_{\alpha\beta;\gamma}h^{\gamma\delta}{}_{;\delta}$$
$$+ \frac{1}{2}h^\alpha{}_\alpha h^\beta{}_{\beta;\gamma}h^{\gamma\delta}{}_{;\delta} + h^{\alpha\beta}h_{\alpha\beta;\gamma}h_\delta{}^{\delta;\gamma} + \frac{1}{4}h^\alpha{}_\alpha h^\beta{}_{\beta;\gamma}h_\delta{}^{\delta;\gamma} - h^{\alpha\beta}h^\gamma{}_{\alpha;\delta}h_{\beta\gamma}{}^{;\delta}$$
$$+ h^{\alpha\beta}h^\gamma{}_{\alpha;\delta}h^\delta{}_{\beta;\gamma} + R_{\alpha\beta}(2h^{\alpha\gamma}h_{\gamma\delta}h^{\beta\delta} - h^\gamma{}_\gamma h^{\alpha\delta}h^\beta{}_\delta - \frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta}$$
$$\left. + \frac{1}{4}h^{\alpha\beta}h^\gamma{}_\gamma h^\delta{}_\delta) + R(-\frac{1}{3}h^{\alpha\beta}h_{\beta\gamma}h^\gamma{}_\alpha + \frac{1}{4}h^\alpha{}_\alpha h^{\beta\gamma}h_{\beta\gamma} - \frac{1}{24}h^\alpha{}_\alpha h^\beta{}_\beta h^\gamma{}_\gamma)\right\}$$

quartic order

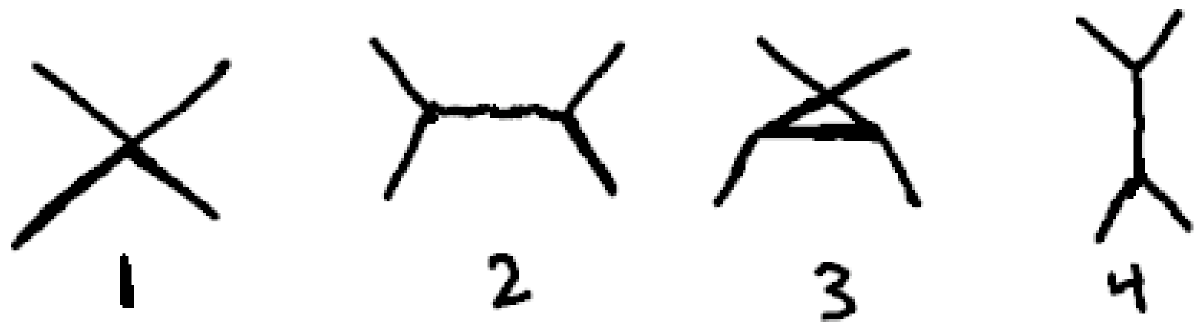
$$
\begin{aligned}
L_4 = \sqrt{-g}\,\Big\{&(h^\alpha{}_\alpha h^\beta{}_\beta - 2h^{\alpha\beta}h_{\alpha\beta})\Big(\frac{1}{16}h^{\gamma\delta;\sigma}h_{\gamma\delta;\sigma} - \frac{1}{8}h^{\gamma\delta;\sigma}h_{\gamma\sigma;\delta} + \frac{1}{8}h^\gamma{}_{\gamma;\delta}h^{\delta\sigma}{}_{;\sigma} \\
&- \frac{1}{16}h^\gamma{}_{\gamma;\delta}h_\sigma{}^{\sigma;\delta}\Big) + h^\alpha{}_\alpha h^{\beta\gamma}\Big(-\frac{1}{2}h_{\beta\gamma;\delta}h^{\delta\sigma}{}_{;\sigma} + \frac{1}{2}h_{\beta\gamma;\delta}h_\sigma{}^{\sigma;\delta} - \frac{1}{2}h^\delta{}_{\delta;\beta}h^\sigma{}_{\sigma;\gamma} \\
&+ \frac{1}{4}h^\delta{}_{\delta;\beta}h^\sigma{}_{\sigma;\gamma} + h^\delta{}_{\beta;\sigma}h^\sigma{}_{\delta;\gamma} - \frac{1}{4}h^{\delta\sigma}{}_{;\beta}h_{\delta\sigma;\gamma} - \frac{1}{2}h^\delta{}_{\beta;\sigma}h_{\delta\gamma}{}^{;\sigma} - \frac{1}{2}h^\delta{}_{\delta;\sigma}h^\sigma{}_{\beta;\gamma} \\
&+ \frac{1}{2}h_{\beta\delta;\sigma}h_\gamma{}^{\sigma;\delta}\Big) + h^\alpha{}_\beta h^{\beta\gamma}\Big(h^\delta{}_{\delta;\sigma}h^\sigma{}_{\alpha;\gamma} - h_{\alpha\gamma;\delta}h_\sigma{}^{\sigma;\delta} + \frac{1}{2}h^{\delta\sigma}{}_{;\alpha}h_{\delta\sigma;\gamma} \\
&- h^\delta{}_{\alpha;\sigma}h^\sigma{}_{\gamma;\delta} - 2h^\delta{}_{\alpha;\sigma}h^\sigma{}_{\delta;\gamma} + h_{\alpha\gamma;\delta}h^{\delta\sigma}{}_{;\sigma} + h^\delta{}_{\delta;\alpha}h^\sigma{}_{\gamma;\sigma} - \frac{1}{2}h^\delta{}_{\delta;\alpha}h^\sigma{}_{\sigma;\gamma} \\
&+ h^\delta{}_{\alpha;\sigma}h_{\gamma\delta}{}^{;\sigma}\Big) + h^{\alpha\gamma}h^{\beta\delta}\Big(h_{\alpha\gamma;\beta}h^\sigma{}_{\delta;\sigma} - h_{\alpha\gamma;\delta}h^\sigma{}_{\sigma;\beta} + \frac{1}{2}h_{\alpha\beta;\sigma}h_{\gamma\delta}{}^{;\sigma} \\
&- \frac{1}{2}h_{\alpha\gamma;\sigma}h_{\beta\delta}{}^{;\sigma} + h^\sigma{}_{\alpha;\beta}h_{\gamma\sigma;\delta} - h^\sigma{}_{\alpha;\beta}h_{\delta\sigma;\gamma} + h_{\alpha\beta;\delta}h^\sigma{}_{\sigma;\gamma} - 2h^\sigma{}_{\alpha;\beta}h_{\delta\gamma;\sigma} \\
&+ h_{\alpha\gamma;\sigma}h^\sigma{}_{\beta;\delta}\Big) + R_{\alpha\beta}\Big(-2h^{\alpha\gamma}h_{\gamma\delta}h^{\delta\sigma}h_\sigma{}^\beta + h^\gamma{}_\gamma h^{\alpha\delta}h_{\delta\sigma}h^{\sigma\beta} + \frac{1}{2}h^{\alpha\gamma}h_\gamma{}^\beta h^{\delta\sigma}h_{\delta\sigma} \\
&- \frac{1}{4}h^{\alpha\gamma}h_\gamma{}^\beta h^\delta{}_\delta h^\sigma{}_\sigma + \frac{1}{3}h^{\alpha\beta}h^{\gamma\delta}h_{\delta\sigma}h^\sigma{}_\gamma - \frac{1}{4}h^{\alpha\beta}h^\gamma{}_\gamma h^{\delta\sigma}h_{\delta\sigma} + \frac{1}{24}h^{\alpha\beta}h^\gamma{}_\gamma h^\delta{}_\delta h^\sigma{}_\sigma\Big) \\
&+ R\Big(-\frac{1}{192}h^\alpha{}_\alpha h^\beta{}_\beta h^\gamma{}_\gamma h^\delta{}_\delta + \frac{1}{16}h^\alpha{}_\alpha h^\beta{}_\beta h^{\gamma\delta}h_{\gamma\delta} + \frac{1}{4}h^{\alpha\beta}h_{\beta\gamma}h^{\gamma\delta}h_{\delta\alpha} \\
&- \frac{1}{16}h^{\alpha\beta}h_{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta} - \frac{1}{6}h^\alpha{}_\alpha h^{\beta\gamma}h_{\gamma\delta}h^\delta{}_\beta\Big)\Big\}
\end{aligned}
$$

Imagine having to do calculations with these interaction vertices!

Last 5-6 years: have learnt that computing Feynman diagrams is the wrong approach to the problem

BCFW recursion relation

In 1963 I gave [Walter G. Wesley] a student of mine the problem of computing the cross section for a graviton-graviton scattering in tree approximation, for his Ph.D. thesis. The relevant diagrams are these:



Given the fact that the vertex function in diagram 1 contains over 175 terms and that the vertex functions in the remaining diagrams each contain 11 terms, leading to over 500 terms in all, you can see that this was not a trivial calculation, in the days before computers with algebraic manipulation capacities were available. And yet the final results were ridiculously simple. The cross section for scattering in the center-of-mass frame, of gravitons having opposite helicities, is

$$d\sigma/d\Omega = 4G^2E^2\cos^{12}\tfrac{1}{2}\theta/\sin^4\tfrac{1}{2}\theta$$

where G is the gravity constant and E is the energy.

From: Bryce DeWitt
arXiv:0805.2935
Quantum Gravity, Yesterday and Today

Using BCFW technology, this calculation becomes a simple homework exercise

However, having an off-shell formulation that explains this magic would be very important - non-perturbative statements then possible

Diffeomorphism invariant gauge theories

$\mathfrak{g}$ - Lie algebra of G

Let f be a function on $\mathfrak{g} \otimes_S \mathfrak{g}$

$$f : X \rightarrow \mathbb{R}(\mathbb{C})$$

defining function

$$X \in \mathfrak{g} \otimes_S \mathfrak{g}$$

satisfying

1) $$f(\alpha X) = \alpha f(X)$$ homogeneous degree 1

2) $$f(gXg^T) = f(X), \quad \forall g \in G$$ gauge-invariant

Then $f(F \wedge F)$ is a well-defined 4-form (gauge-invariant)

Can define a gauge and diffeomorphism invariant action

$$F = dA + (1/2)[A, A]$$

$$\boxed{S[A] = \mathrm{i} \int_M f(F \wedge F)}$$

no dimensionful coupling constants!

Field equations: $d_A B = 0$

where $B = \frac{\partial f}{\partial X} F$ and $X = F \wedge F$

Second-order (non-linear) PDE's

compare Yang-Mills equations: $d_A B = 0$

where $B = {}^* F$

$*$ - encodes the metric

Dynamically non-trivial theory with 2n-4 propagating DOF

apart from the single point $f_{\rm top} = {\rm Tr}(F \wedge F)$

Gauge symmetries:

$\delta_\phi A = d_A \phi$ gauge rotations

$\delta_\xi A = \iota_\xi F$ diffeomorphisms

The simplest non-trivial theory:

G=SU(2) - gravity
(interacting massless spin 2 particles)

for any choice of f()

specific f() - GR

$$\boxed{S_{\mathrm{GR}}[A] = \frac{\mathrm{i}}{16\pi G\Lambda}\int_M \left(\mathrm{Tr}\sqrt{F\wedge F}\right)^2}$$

$\Lambda \neq 0$

related to Plebanski self-dual formulation of GR

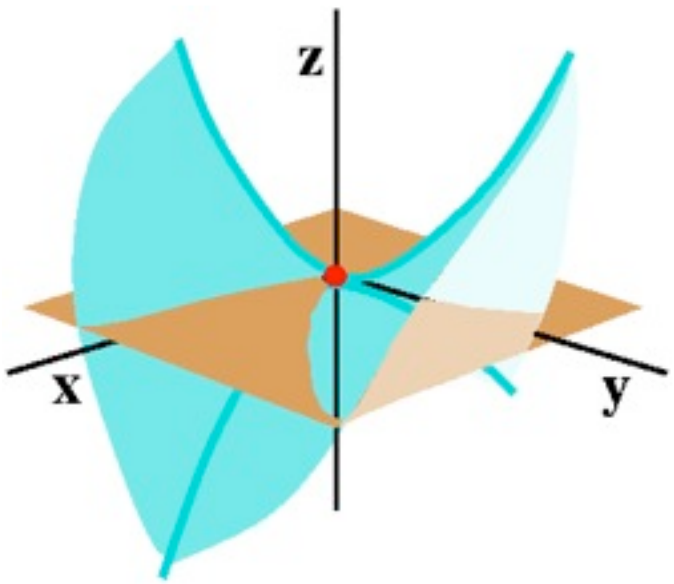
(only) on-shell equivalent description:

connection satisfying the resulting Euler-Lagrange equations $\Rightarrow$ Einstein metric (of non-zero scalar curvature)

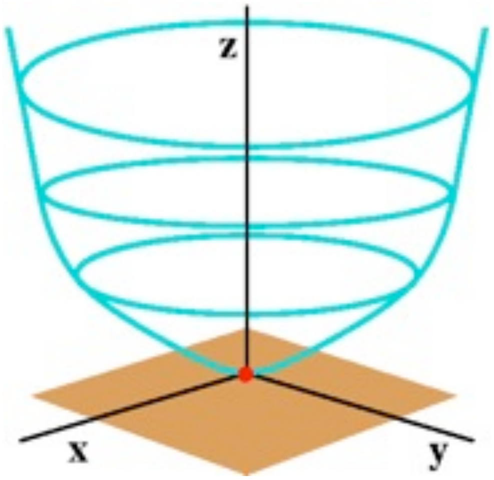
On-shell equivalent description of gravitons

Description of GR without the conformal mode problem!

Why only on-shell equivalent:



(Euclidean) EH functional is not convex (conformal mode problem)



The new action (its Euclidean version) is a convex functional

Have a different functional with the same critical points

The key to the simplicity of the new formulation is that the conformal mode is absent even off-shell

Perturbation theory

Background:

background connection $$A^i = \frac{a(\eta)}{\mathrm{i}} dx^i$$ η - time

$$F^i = dA^i + (1/2)\epsilon^{ijk} A^j \wedge A^k$$

$$F^i = -a^2 \left(\mathrm{i}\frac{a'}{a^2} d\eta \wedge dx^i + \frac{1}{2}\epsilon^{ijk} dx^j \wedge dx^k \right)$$

reparameterize time $$\frac{a'}{a^2} d\eta := dt \quad \Rightarrow \quad a(t) = \frac{1}{t_0 - t}$$

t_0 - integration constant

introduce scale M and a constant curvature metric

$$c(t) := \frac{1}{M(t_0 - t)}$$

$$ds^2 = c^2(t) \left(-dt^2 + \sum (dx^i)^2 \right)$$

de Sitter metric in flat slicing

‘t Hooft symbols

$$M^2 = \Lambda/3$$

$$\Sigma^i = c^2(t) \left(\mathrm{i} dt \wedge dx^i + \frac{1}{2}\epsilon^{ijk} dx^j \wedge dx^k \right)$$

self-dual two-forms

Background curvature

$$\boxed{F^i = -M^2 \Sigma^i}$$

Perturbative expansion:

$$X^{ij} \sim F^i \wedge F^j$$

$$\delta S = \int \frac{\partial f}{\partial X^{ij}} \delta X^{ij}$$

$$\delta^2 S = \int \frac{\partial^2 f}{\partial X^{ij} \partial X^{kl}} \delta X^{ij} \delta X^{kl} + \frac{\partial f}{\partial X^{ij}} \delta^2 X^{ij}$$

$$\delta^3 S = \int \frac{\partial^3 f}{\partial X^{ij} \partial X^{kl} \partial X^{mn}} \delta X^{ij} \delta X^{kl} \delta X^{mn} + 3 \frac{\partial^2 f}{\partial X^{ij} \partial X^{kl}} \delta^2 X^{ij} \delta X^{kl} + \frac{\partial f}{\partial X^{ij}} \delta^3 X^{ij}$$

Convenient to define $\hat{X}^{ij} = \frac{1}{8\mathrm{i}M^4} \epsilon^{\mu\nu\rho\sigma} F^i_{\mu\nu} F^j_{\rho\sigma}$

whose evaluation on the background $\hat{X}^{ij} \hat{=} \delta^{ij}$

The action takes the form

$$S[A] = -2M^4 \int \sqrt{-g}\, f(\hat{X})$$

Rescaled perturbations:

$$\delta \hat{X}^{ij} \doteq -\frac{1}{M^2}\Sigma^{(i\mu\nu}D_\mu \delta A_\nu^{j)},$$

$$\delta^2 \hat{X}^{ij} \doteq \frac{1}{\mathrm{i}M^4}\epsilon^{\mu\nu\rho\sigma}D_\mu\delta A_\nu^i D_\rho \delta A_\sigma^j - \frac{1}{M^2}\Sigma^{(i\mu\nu}\epsilon^{j)kl}\delta A_\mu^k \delta A_\nu^l,$$

$$\delta^3 \hat{X}^{ij} \doteq \frac{3}{\mathrm{i}M^4}\epsilon^{\mu\nu\rho\sigma}D_\mu \delta A_\nu^{(i}\epsilon^{j)kl}\delta A_\rho^k \delta A_\sigma^l.$$

$$\delta^4 \hat{X}^{ij} = 0$$

Can now compute the expansion of the action to any order

Remark: $\mathrm{Tr}\left(\delta^2 \hat{X}\right)$ - total derivative

Linearization:

Independent of the defining function

after a rescaling to give canonical normalization $a^i_\mu \sim \delta A^i_\mu$

simplest known Lagrangian for massless spin 2 in de Sitter space

$$\mathcal{L}^{(2)} = -\frac{1}{2} P^{ij|kl} \left(\Sigma^{i\,\mu\nu} D_\mu a^j_\nu\right) \left(\Sigma^{k\,\rho\sigma} D_\rho a^l_\sigma\right)$$

for any choice of f()

where $$P^{ij|kl} = \delta^{i(k}\delta^{l)j} - \frac{1}{3}\delta^{ij}\delta^{kl}$$

M-independent

projector on spin 2 in $V^1 \otimes V^1$

The flat limit is easy to take $D \to \partial$

Simple Hamiltonian analysis $\Rightarrow$ two propagating graviton polarizations

Spinorial description:

$$TM = S_+ \otimes S_-$$

$$\mu \to AA'$$

using GR notations for spinors

the Lie algebra index $i \to (AB)$

$$a^i_\mu \to a_{AA'}{}^{BC} \in S_+ \otimes S_- \otimes S^2_+ = S^3_+ \otimes S_- \oplus S_+ \otimes S_-$$

$a^{(ABC)}_{A'}$ $\quad$ $a_{A'E}{}^E{}_A$

pure gauge
(diffeomorphisms) part

$$\boxed{\mathcal{L}^{(2)} \sim \left(\partial^{(A}_{A'} a^{BCD)A'}\right)^2}$$

only depends on the
$S^3_+ \otimes S_-$ part of $a_{AA'}{}^{BC}$

$\dim(S^3_+ \otimes S_-) = 8$ (per point)

explicitly positive-definite (Euclidean signature) functional

Comparison with the metric description:

$$\mathcal{L}^{(2)} = -\frac{1}{2}(\partial_\mu h_{\rho\sigma})^2 + \frac{1}{2}(\partial_\mu h)^2 + (\partial^\mu h_{\mu\nu})^2 + h\partial^\mu\partial^\nu h_{\mu\nu}$$

"wrong" sign

conformal mode

$$h = h^\mu{}_\mu$$

$$h_{\mu\nu} \to h_{AA'BB'} \qquad \in S^2_+ \otimes S^2_- \oplus (\text{trivial})$$

$$\dim(h_{\mu\nu}) = 10$$ (per point) $$-\underbrace{4 - 4}_{\text{diffeomorphisms}} \to 2$$ propagating DOF

In the gauge theory description a smaller space of fields to start with!

$$8 - \underbrace{3 - 3}_{\text{SU(2) gauge rotations}} \to 2$$ propagating DOF

Comparison with Yang-Mills:

can rewrite the Lagrangian as

(gauge indices suppressed)

$$\mathcal{L}_{\rm YM} = -\frac{1}{4g^2}(F^+_{\mu\nu})^2$$

F^+- self-dual part of the curvature

Spinorial description:

$$TM = S_+ \otimes S_-$$
$$\mu \to AA'$$

$$A_{AA'} \in S_+ \otimes S_-$$

spin 1

quadratic order (not gauge-fixed)

$$\mathcal{L}^2_{\rm YM} \sim (\partial^{(A}_{A'} A^{B)A'})^2$$

our linearized graviton Lagrangian is just the generalization to the case

$$A_{ABCA'} \in S^3_+ \otimes S_-$$

spin 2

Gauge-fixing:

gauge-fixing condition invariant under shifts $\partial^\mu \left(P^{(3/2,1/2)} A^i_\mu \right) = 0$

in spinor terms $(\partial a)^{BC} \equiv \partial^A_{A'} a^{BCA'}_A = 0$ where $a^{ABC}_{A'} \in S^3_+ \otimes S_-$

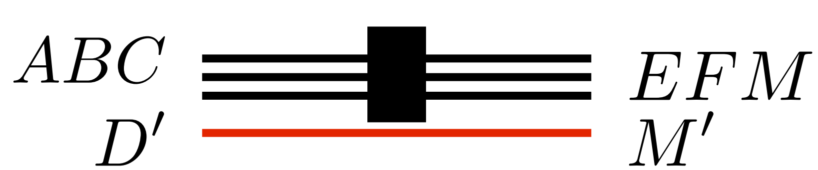
gauge-fixed Lagrangian - functional on $\mathcal{C}^\infty(S^3_+ \otimes S_-)$

Analog of Feynman gauge in YM

$$\mathcal{L}^{(2)} + \mathcal{L}^{(2)}_{gf} = \left(\partial^{(A}_{A'} a^{BCD)A'} \right)^2 + \frac{3}{4} \left((\partial a)^{AB} \right)^2 = -\frac{1}{2} a_{ABC}{}^{A'} \partial^2 a^{ABC}{}_{A'}$$

Thus, the propagator

$$\Delta(k)_{EFM}{}^{M'ABC}{}_{D'} = \frac{\epsilon_E{}^{(A} \epsilon_F{}^{B} \epsilon_M{}^{C)} \epsilon_{D'}{}^{M'}}{k^2}$$



$$\frac{1}{k^2}$$

only the (3/2,1/2) component propagates

Interactions: Case of GR

$$M_p^2 := \frac{1}{16\pi G}$$

$$S_{\mathrm{GR}}[A] = -\frac{2M_p^2 M^2}{3}\int_M \sqrt{-g}\left(\mathrm{Tr}\sqrt{\hat{X}}\right)^2$$

<u>complete off-shell</u> cubic vertex

significantly more complicated expression in the metric case

zero on-shell

$$\mathcal{L}^{(3)} = \frac{2}{MM_p}(\partial a)^{ABCD}(\partial a)^{M'N'}{}_{AB}(\partial a)_{M'N'CD} - \frac{1}{4MM_p}(\partial a)^{ABCD}(\partial a)_{AB}(\partial a)_{CD}$$
$$+\frac{4M}{M_p}(\partial a)^{M'N'AB}(aa)_{M'N'AB}.$$

the only part that is relevant for MHV

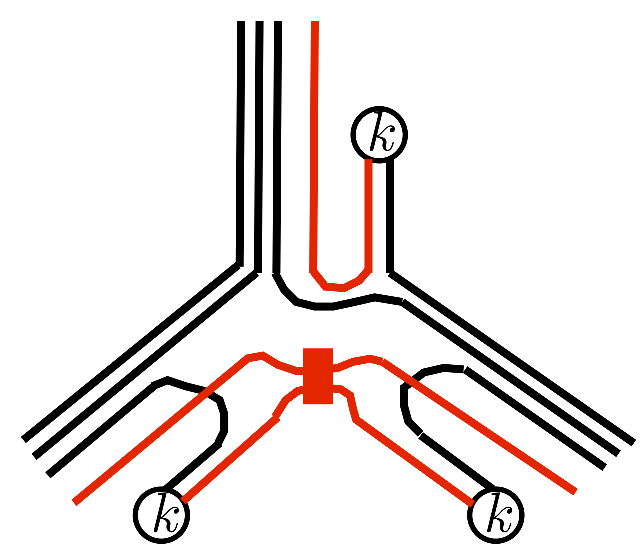
graphical notation for the 3-derivative vertex

where the spinor contraction notations are

$$(\partial a)^{ABCD} = \partial^{(A}{}_{M'}a^{B)CDM'},$$

$$(\partial a)^{M'N'AB} = \partial^{C(M'}a_C{}^{ABN')}$$

$$(aa)^{M'N'CD} = a^{CD(AM'}a_{CD}{}^{B)N'}$$



Spinor helicity states

$$\varepsilon^+(k)^{ABC}{}_{D'} = \frac{1}{M}\frac{k^A k^B k^C p_{D'}}{[kp]}, \qquad \varepsilon^-(k)^{ABC}{}_{D'} = M\frac{q^A q^B q^C k_{D'}}{(kq)^3}$$

here, as usual p^A, q^A are arbitrary spinors not aligned with k^A

and $[kp] := k_{A'}p^{A'}, \qquad (kp) := k^A p_A$ are spinor products

To take the $M \to 0$ limit

need to make the (positive helicity) <u>external</u> momenta slightly <u>massive</u>

$$k^{AA'} = k^A k^{A'} + \frac{M^2 q^A q^{A'}}{(kq)[kq]} \qquad \text{so that} \qquad k^{AA'}k_{AA'} = -2M^2$$

Relation to the metric description

both valid on-shell only

$$h_{ABA'B'} \sim \frac{1}{M} (\partial a)_{ABA'B'} \qquad a^{ABCA'} \sim \frac{1}{M} \partial^{(A}_{B'} h^{BC)A'B'}$$

both are true on $k^2 = 2M^2$

then our helicity states are just images of the usual metric states

3-vertex in the metric language

square of the YM vertex

$$\mathcal{L}^{(3)} \sim \frac{1}{M_p} \left(\partial^{(A}_{A'} \partial^{B}_{B'} h^{CD)A'B'} \right) h^{M'N'}{}_{AB} h_{M'N'CD}$$

calculations done with the usual metric helicity states and the above vertex give the same amplitudes

Compare with the Yang-Mills vertex in the form

gauge indices suppressed

$$(F_{\rm sd})^2 \sim \left(\partial^{(A}_{A'} a^{B)A'} \right) a^{M'}{}_{A} a_{M'B} + \ldots$$

Summary of perturbation theory:

- Using $S^3_+ \otimes S_-$ instead of $S^2_+ \otimes S^2_-$ to describe spin 2 gravitons

parity invariance non-manifest!

- Much easier way the diffeomorphisms are realized - the corresponding field components can be projected out from the outset

- Much simpler linearized action, much simpler interaction vertices!

e.g. off-shell 4-vertex contains only 7 terms, as compared to a page in the metric-based case

- Formulation in which the off-shell 3-vertex is (basically) (YM vertex)^2

became possible because the conformal mode does not propagate even off-shell

as close to the explanation Gravity=(YM)^2 as one can currently get

Deformations of GR

All other choices of f() lead to different (from GR) interacting theories of massless spin 2 particles

can be shown to correspond to the EH Lagrangian with an infinite set of counterterms added

seemingly impossible due to the GR uniqueness, but specific (sometimes innocuous) assumptions that go into each version of the uniqueness theorems are explicitly violated here

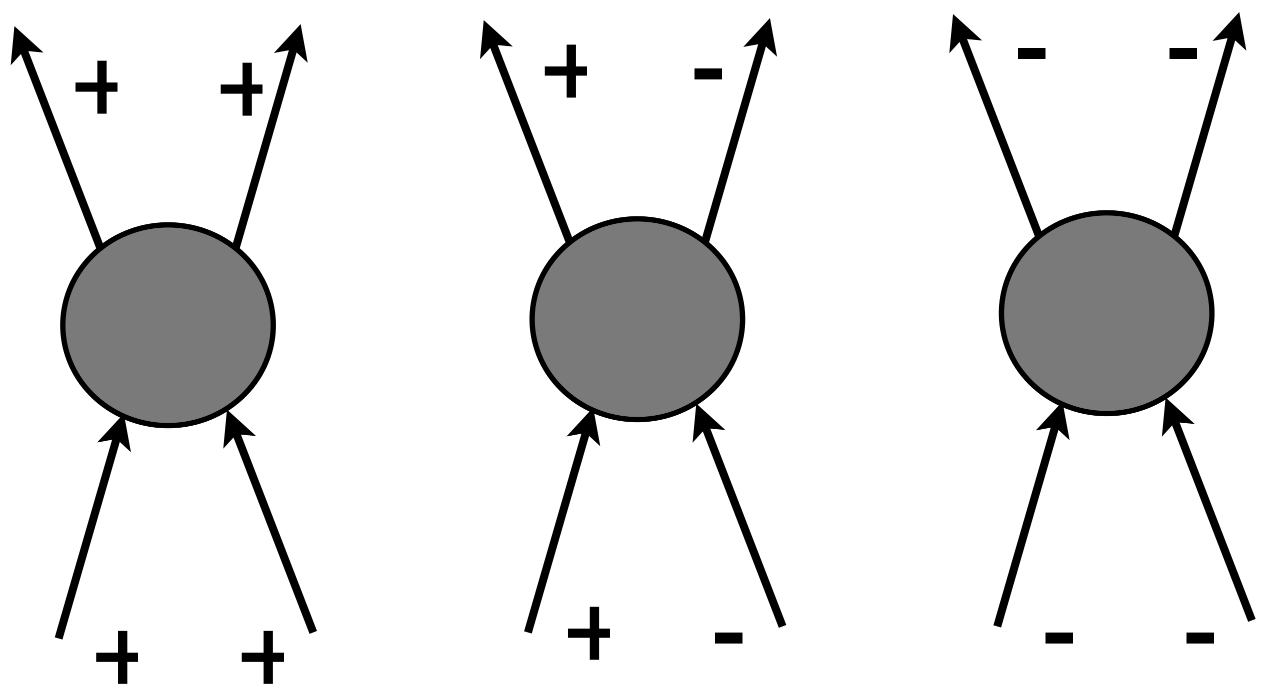
Not a dynamical theory of $g_{\mu\nu}$

(in its second-order formulation)

A generic theory is not parity invariant!

Modified gravity theories with 2 propagating DOF - a very interesting object of study

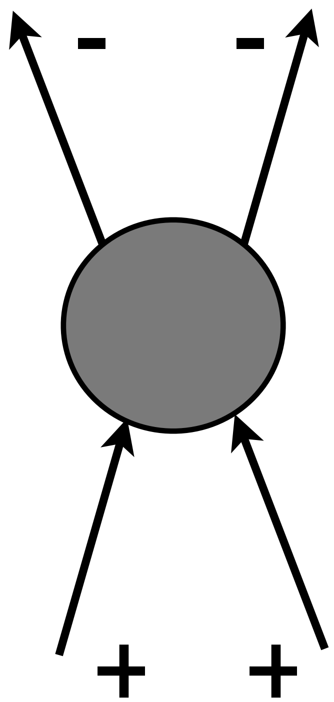
In GR only parity-preserving processes:



amplitude $$\mathcal{A} \sim \frac{1}{M_p^2}\frac{s^3}{tu} \sim \left(\frac{E}{M_p}\right)^2$$

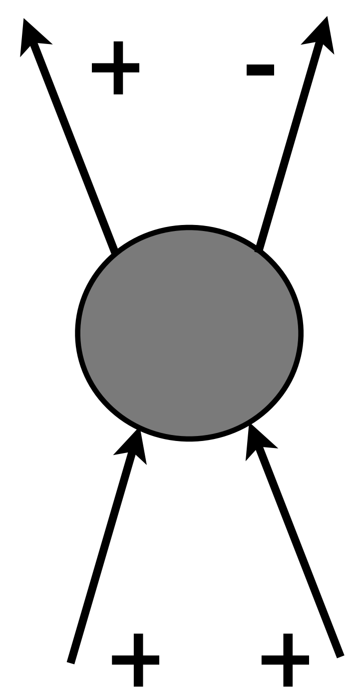
becomes larger than unity at Planck energies, cannot trust perturbation theory

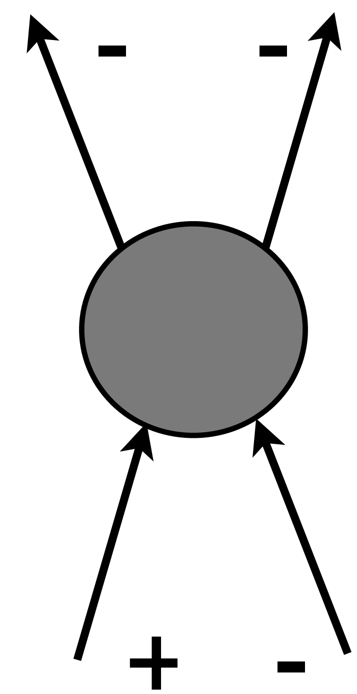
In a general theory from our family parity-violating processes become allowed:



$$\mathcal{A} \sim \frac{s^4 + t^4 + u^4}{M_p^8} \sim \left(\frac{E}{M_p} \right)^8$$

$$\mathcal{A} \sim \frac{stu}{M_p^6} \sim \left(\frac{E}{M_p} \right)^6$$





A general theory likes negative helicity gravitons!

Can speculate that at high energies these processes will dominate and all gravitons will get converted into negative helicity ones (strongly coupled by the parity-preserving processes)

Quantum Theory Hopes

Remark: no dimensionful coupling constants
in any of these gravitational theories

(negative) dimension coupling constant comes when expanded around a background

Non-renormalizable in the usual sense

Hope: the class of theories - all possible f() - is large enough to be closed under renormalization

$$\frac{\partial f(F \wedge F)}{\partial \log \mu} = \beta_f(F \wedge F)$$

I.e. physics at higher energies continues to be described by theories from the same family

= no new DOF appear at Planck scale, just the dynamics changes

The speculative RG flow

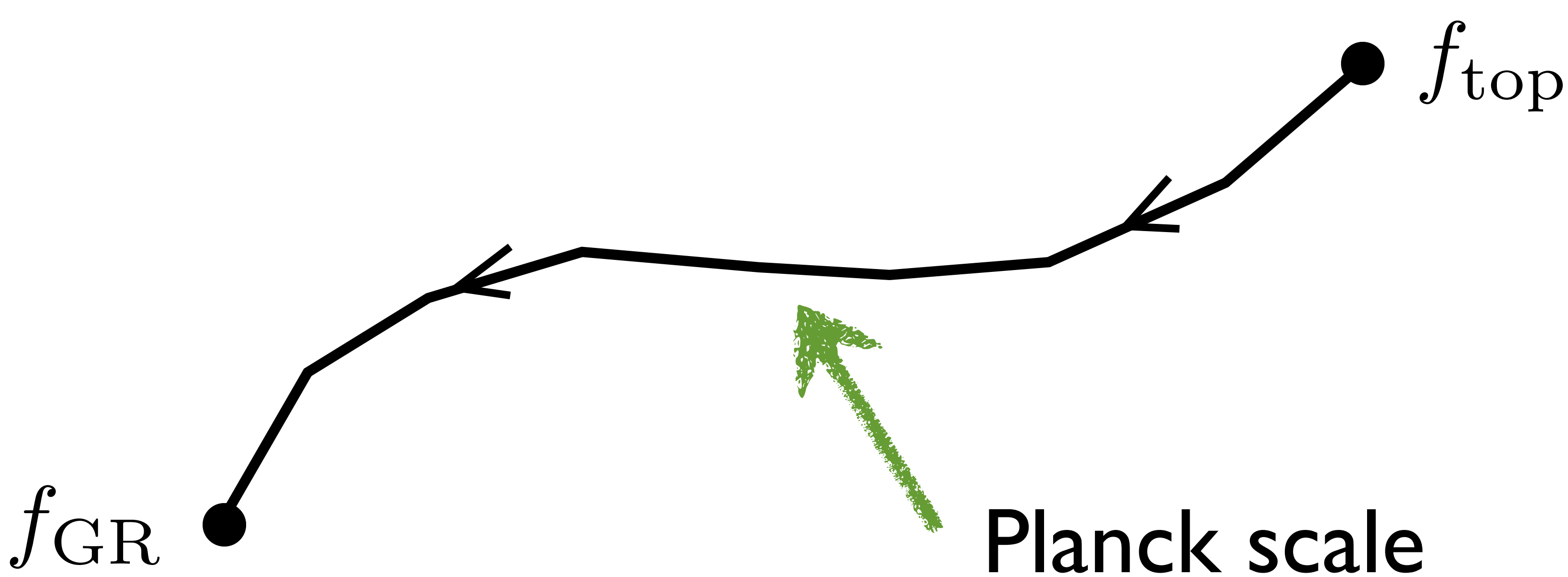
strongly coupled negative helicity gravitons at high energies

$\Rightarrow$ no propagating DOF ? $\Rightarrow$ topological theory ?

$$f_{\rm top}(F \wedge F) = {\rm Tr}(F \wedge F)$$

necessarily a fixed point of the RG flow

corresponds to a topological theory (no propagating DOF)



flow from very steep in IR towards very flat in UV potential

Summary:

- Dynamically non-trivial diffeomorphism invariant gauge theories
- The simplest non-trivial such theory G=SU(2) - gravity
- GR can be described in this language (on-shell equivalent only)

  $\Rightarrow$ possibly different quantum theory

- Computationally efficient alternative to the usual description (no propagating conformal mode even off-shell)
- Different from GR (parity-violating) theories of interacting massless spin 2 particles
- If this class of theories is closed under renormalization

  $\Rightarrow$ understanding of the gravitational RG flow description of the Planck scale physics

Open problems

- Chiral, thus complex description. Unitarity?

- Coupling to matter?

  Enlarging the gauge group - rather general types of matter coupled to gravity can be obtained. Fermions?

- Closedness under renormalization?

  Are these just some effective field theory models, or they are UV complete as Yang-Mills?