Phenomenological Quantum Gravity

Sabine Hossenfelder

Nordita

Freeman Dyson

"According to my hypothesis [...] the two theories are mathematically different and cannot be applied simultaneously. But no inconsistency can arise from using both theories, because any differences between their predictions are physically undetectable."

Freeman Dyson's Pessimism
The New York Review of Books Volume 51, Number 8 May 13, 2004
The World on a String By Freeman Dyson
Review of The Fabric of the Cosmos: Space, Time, and the Texture of Reality by Brian Greene

Simplify the Complicated

Top-down

Top-down inspired bottom-up approaches

…Extra Dimensions…Minimal Length …

…DSR …Holographic Principle …

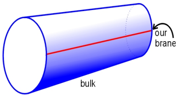
Bottom-up

Top-down inspired bottom-up approaches

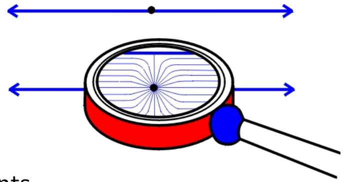
- **Extra Dimensions**: KK-excitations, graviton-production, black hole production
- **Deformed Special Relativity**: Shift of reaction-thresholds, energy dependent speed of light (Amelino-Camelia, Magueijo, Smolin, Ellis, SH...)
- **Generalized Uncertainty**: Stagnation of cross-section, modifications of loop contributions (Kempf, Niemeyer, Cavaglia, SH ...)
- **Violation of Lorentz invariance**: Preferred frame effects, higher oder operators (Jacobson, Kostolecky, Mattingly, Liberati ...)
- **Quantum Cosmology**: Imprints of QG fluctuations in the CMB/ν background, spectral index (Hofmann, Danielsson, Smolin, Sudarsky ...)
- **Space-time Foaminess**: CPT violation, stochastic deviations from lightcone, accelerated decoherence, noise, (Mavromatos, Farakos, Hogan, ...)
- **Emergent Gravity**: imprints in CMB through modified inflation/phase transition, non-local links, potentially violations of Lorenz-invariance (Konopka, Markopoulou, Prescod-Weinstein, Smolin, Visser ...)

Models with Extra Dimensions

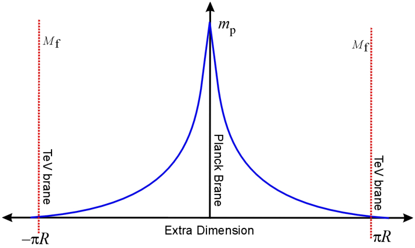
- ADD-model: large extra dimensions $R \gg 1/M_f$
  - $+$ Solves Hierarchy-problem, $m_p^2 = R^d M_f^{d+2}$

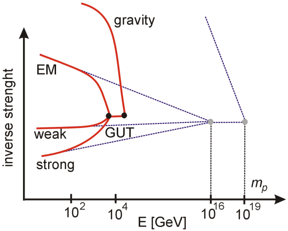


- RS-model (I and II), extra dimension is curved
  - $+$ AdS-CFT Correspondence
  - $+$ Allows non-compact extra dimension



- UXD, TeV-scale dimensions
  - $+$ Accelerated unification of coupling constants





Gravitation as Effective Theory (ADD)

Philosophy: use naively quantized gravity in perturbative limit

T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D **59** (1999) 105006
S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. D **62**, 055012 (2000)
T. G. Rizzo, Phys. Rev. D **64**, 095010 (2001)
J. Hewett and M. Spiropulu, Ann. Rev. Nucl. Part. Sci. **52**, 397 (2002)

- Perturbation of metric: $g_{AB} = \eta_{AB} + \Psi_{AB}$
- Decompose: spin-2 $h_{\mu\nu}$, vector $V_{\mu i}$, scalar ϕ_{ij} (trace $\phi^i{}_i = \phi$)
- Coupling to matter $\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_M$
- Energy momentum tensor on brane $T_{AB} = \eta^\mu{}_A \eta^\nu{}_B T_{\mu\nu}(x)\delta(y)$
- Yields coupling terms: $\mathcal{L}_{int} = -\frac{1}{2} T\phi - T^{\mu\nu} h_{\mu\nu}$

Massive Gravitons

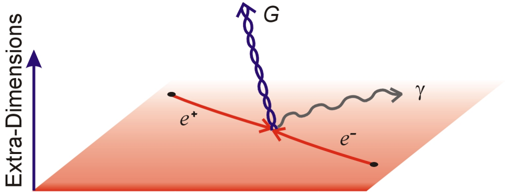
ADD: Yields tower of massive gravitons with tiny spacing

- Large phase space makes contributions important at $\sqrt{s} \sim M_f$
- # of excitations with energy E is $N(E) \sim (ER)^d$

$$\text{E.g.} \quad e^+e^- \to \gamma G: \quad \sigma \sim \frac{\alpha}{m_p^2} N(\sqrt{s}) \sim \frac{\alpha}{s}\left(\frac{\sqrt{s}}{M_f}\right)^{d+2}$$

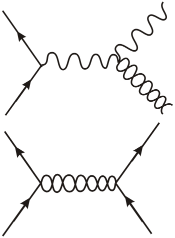
- Brane breaks Poincaré invariance and momentum conservation on brane

RS: Distinctly different signature! Discrete resonances at multiples of TeV.



Signatures of Gravitons

Collider physics (current bounds on M_f in ADD in TeV-range)



- Real gravitons lead to missing energy
- Virtual exchange modifies cross sections

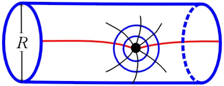
Astrophysics (ADD bounds weak for $d > 4$, strong for $d \leq 4$):

- Enhanced cooling of supernovae/red giants from graviton emmission
- Cooling in early universe and contributions to background from decay of bulk excitations
- Anomalous re-heating of neutron stars by decay of gravitationally trapped massive gravitons

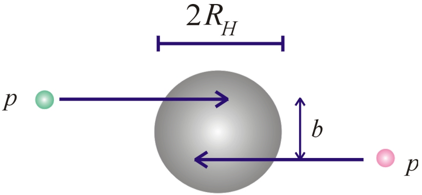
Black Holes in Extra Dimensions

In large extra dimensions (ADD)

- Gravity stronger at small distances $\Rightarrow$ horizon radius R_H larger
- For $M \sim 1$ TeV , R_H increases from $\sim 10^{-38}$ fm to 10^{-4} fm!
- For these black holes it is $R_H \ll R$ and they have approx higher dimensional spherical symmetry

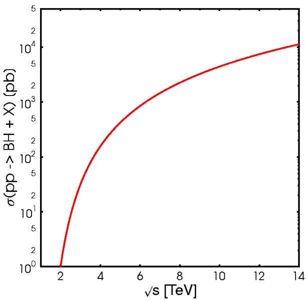


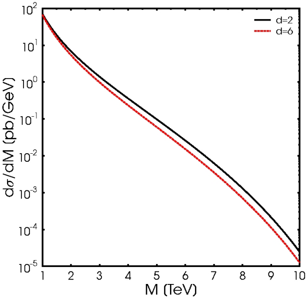
- At the LHC partons can come closer than their Schwarzschild horizon $\longrightarrow$ a black hole can be created!



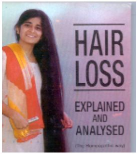
Production of Black Holes

- Semi-classical cross-section $\sigma \sim \pi R_H^2$
- Can be improved by modelling colliding wave packets
- Yields $\sim 10^8$ black holes per year for LHC pp-collisions
- Numerical tools available for event simulation





Evaporation of Black Holes



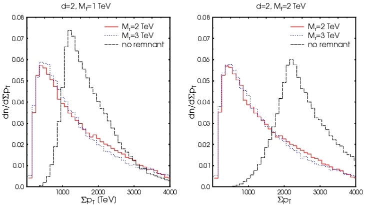
Evaporation proceeds in 3 stages:

1. Balding phase: hair loss – the black holes radiates off angular momentum and multipole moments
2. Hawking phase: thermal radiation into all particles of the standard model as well as gravitons
3. Final decay or remaining black hole relic

Black hole thermodynamics: $T = \kappa/2\pi$ and $dS/dM = 1/T$

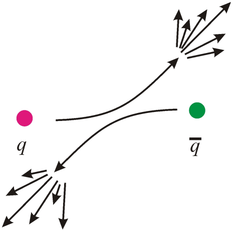
Numerical investigation:
black hole event generator CHARYBDIS

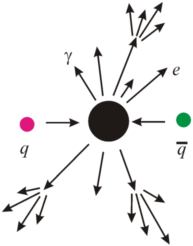
Tanaka *et al*, [arXiv:hep-ph/0411095]
Harris *et al*, [arXiv:hep-ph/0411022]



Observables of Black Holes

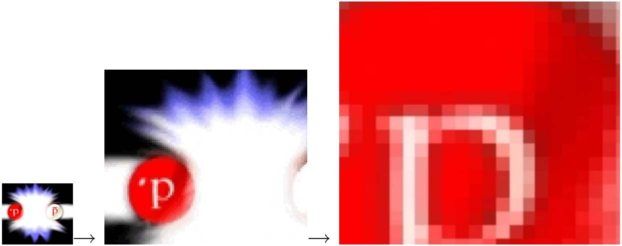
- Multi-jet like events, spherical, typical temperature $\sim$ 200 GeV
- Momentum cut-off at $\sim M_f$
- Thermal spectrum $\rightarrow$ (ideally) allows to reconstruct d and M_f
- Virtual black holes: baryon/flavor non-conservation





The Minimal Length Scale

- Very general expectation for quantum gravity: fluctuations of spacetime itself disable resolution of small distances
- Can be found e.g. in string theory, Loop Gravity, NCG, etc.
- Minimal length scales acts as UV cutoff
- Lowering the Planck mass means raising the Planck length

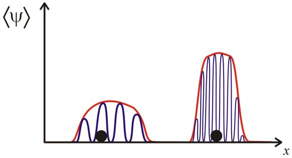


...is there a fundamental limit to the resolution of structures?

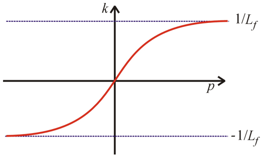
Discreteness $\Rightarrow$ finite resolution, but finite resolution $\not\Rightarrow$ discretenesss !

A Model for the Minimal Length

- For large momenta, p, Compton-wavelength $\lambda = 1/k$ can not get arbitrarily small $\lambda > L_{\rm f} = 1/M_{\rm f}$



- Model by modifing relation between wave-vector k and momentum p. Results in modified commutation relations $k = k(p) = \hbar p + a_1 p^3 + a_2 p^5 \ldots \Rightarrow [p_i, x_j] = i \partial p_i / \partial k_j$



SH *et al*, Phys. Lett. B598 (2004) 92-98; SH, Phys. Rev. D **73**, 105013 (2006)

Consequences of the Minimal Length

- Implies a generalized uncertainty principle, first correction

$$\Delta x \Delta p \geq \frac{1}{2}\hbar \left(1 + b_1 \frac{\Delta p^2}{M_{\mathrm{f}}^2}\right) \quad ,$$

- A squeezed phase space at high energies

$$\langle p|p'\rangle = \frac{\partial p}{\partial k}\delta(p-p') \Rightarrow dk \to \frac{dp}{\hbar}\frac{\partial k}{\partial p} = \frac{dk}{\hbar} e^{-p^2 L_{\mathrm{min}}^2} \quad ,$$

- And a modified dispersion relation

$$\omega^2 - k^2 - \mu^2 = \Pi(k,\omega)$$

- Can but need not have a energy dependent speed of light $d\omega/dk \neq 1$.

Quantisation with a Minimal Length

- Lagrangian for free fermions

$$\mathcal{L}_f = \mathrm{i}\overline{\Psi}(\not{p}(k) - m)\Psi \qquad \mathcal{L}_f = \mathrm{i}\overline{\Psi}(g^{\nu\kappa}(k)\gamma_\nu k_\nu - m)\Psi$$

- Coupling of the gauge field via $\partial_\nu \to D_\nu := \partial_\nu - \mathrm{i}eA_\nu$ yields the gauge- and Lorentz-invariant higher order derivative interaction

$$\mathcal{L} = \bar{\Psi}\not{p}(D)\Psi \qquad \mathcal{L} = \bar{\Psi}\gamma_\nu g^{\nu\kappa}(D)D_\kappa\Psi$$

- To first order one finds the usual $\mathcal{L} = \mathcal{L}_f - e\overline{\Psi}\eta^{\kappa\nu}\gamma_\kappa A_\nu\Psi + \mathcal{O}(eL_{\mathrm{min}}^2)$ and the dominant modification comes from the propagators

$$(\not{p}(k) - m)^{-1} \qquad (g^{\nu\kappa}(k)\gamma_\nu k_\kappa - m)^{-1}$$
$$(p^\nu(k)p_\nu(k) - m^2)^{-1} \qquad (g^{\nu\kappa}(k)k_\nu k_\kappa - m^2)^{-1}$$

- Recipe: replace p with $p(k)$ $\longrightarrow$ higher order derivative Lagrangian

The Locality Bound*

From the commutator

$$[a_p, a^\dagger_{p'}] = \delta(p - p')\left|\frac{\partial p}{\partial k}\right|$$

And the field expansion

$$\phi(x) = \int d^3p \left|\frac{\partial k}{\partial p}\right| \left[v_p(x)a_p + v^*_p(x)a^\dagger_p\right]$$

One finds the equal time commutator for $x = (\mathbf{x}, t), y = (\mathbf{y}, t)$.

$$[\phi(x), \pi(y)] = i\int \frac{d^3p}{(2\pi)^3}\left|\frac{\partial k}{\partial p}\right| e^{ik(x-y)} \rightarrow i\int \frac{d^3p}{(2\pi)^3} e^{ik(x-y)-\varepsilon p^2}$$

where $\varepsilon \sim L^2_{\text{min}}$. I.e.

$$[\phi(x), \pi(y)] \neq \delta(x - y)$$

*Giddings and Lippert, Phys. Rev. D **65**, 024006 (2002), Phys. Rev. D **69**, 124019 (2004).

The Propagator

$$\frac{1}{p^{\nu}(k)p_{\nu}(k) - m^2}$$

- Since $p(k)$ has exactly one zero, there are no additional poles on the real axis

$\longrightarrow$ This goes wrong in the first order approx (signs of coefficients are not fixed)

- For the same reason, the characteristic polynomial of the wave-equation has only one (real) zero.

Applications of the Model

The model is useful to examine effects of a minimal length scale

- Modified quantum mechanics:

$\longrightarrow$ Schrödinger's equation, levels in hydrogen atom, g-2, Casimir-effect

- Derivation of modified Feynman-rules:

$\longrightarrow$ General prescription for calculations

- Tree-level cross-sections (e.g. $e^+e^- \rightarrow f^+f^-$):

$\longrightarrow$ Show overall suppression relative to SM-result

- Loop-contributions (e.g. running coupling):

$\longrightarrow$ Finite, minimal length acts as UV-regulator

Deformed Special Relativity

- Minimal length L_{min} requires new Lorentz-transformations
- New transformations have 2 invariants: c and L_{min}
- Generalized Uncertainty $\Longleftrightarrow$ Deformed Special Relativity
  - * When relation $k(p)$ is known and p's (usual) transformation, then also the transformation of k is known.
  - * When the new transformation on k is known, then one gets $k(p)$ by boosting in and out of the restframe where $k = p$.

SH, Class. Quantum Grav. 23 (2006) 1815.

Deformed, Non-linear Action on Momentum Space

- Lorentz-algebra remains unmodified

$$[J^i, K^j] = \varepsilon^{ijk} K_k \ , \ [K^i, K^j] = \varepsilon^{ijk} K_k \ , \ [J^i, J^j] = \varepsilon^{ijk} J_k$$

- But it acts non-linearly on momentum space, e.g.*

$$e^{-iL_{ab}\omega^{ab}} \to U^{-1}(p_0) e^{-iL_{ab}\omega^{ab}} U(p_0) \quad \text{with} \quad U(p_0) = e^{L_{\min} p_0 p_a \partial p^a}$$

- Leads to Lorentz-boost (z-direction)

$$\begin{aligned} p_0' &= \frac{\gamma(p_0 - v p_z)}{1 + L_{\min}(\gamma - 1) p_0 - L_{\min} \gamma v p_z} \\ p_z' &= \frac{\gamma(p_z - v p_0)}{1 + L_{\min}(\gamma - 1) p_0 - L_{\min} \gamma v p_z} \end{aligned}$$

which transforms $(1/L_{\min}, 1/L_{\min}) \to (1/L_{\min}, 1/L_{\min})$

*Magueijo and Smolin, Phys. Rev. Lett. **88**, 190403 (2002).

Interpretation of an Invariant Minimal Length

Besides c there is a second invariant $L_{\rm min}$ for all observers

- DSR approach (from SR)
  - * Deformed transformation applies to free particles
  - * Physical momentum is subject to deformed transformation
  - ? If caused by quantum gravity effects what sets the scale?
- GUP approach (from particle physics)
  - * Two observers can not compare lengths without interaction
  - * The strength of gravitational effects sets the scale for the importance of quantum gravity
  - * Free particles do not experience any quantum gravity or DSR
  - * Effects apply for virtual particles in the interaction region only
  - * Physical momentum transforms under standard Lorentz transformation

$\longrightarrow$ Propagator of exchange particles is modified

Features and Observables of DSR

- Non-linear transformation of physical momenta results in unusual addition law

$$\begin{aligned} \widetilde{\Lambda}(p_1 + p_2) &\neq \widetilde{\Lambda}(p_1) + \widetilde{\Lambda}(p_2) \\ p_1 \oplus p_2 &= p(k_1 + k_2) \neq p(k_1) + p(k_2) \end{aligned}$$

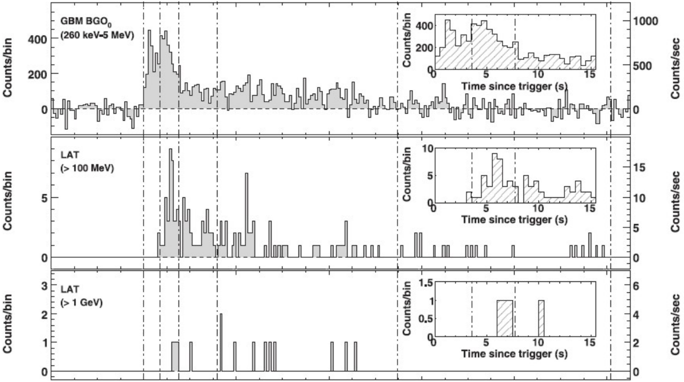
$\longrightarrow$ Modifications of interaction thresholds: GZK cutoff... (Aloisio, Grillo et al, Amelino-Camelia, Alfaro & Palma), No-no (SH, Phys. Rev. D 73:105013 (2006))

- Modified dispersion relation for free particles

$$m^2 \approx E^2 - \vec{p}^2 + \eta \left(\frac{E}{m_\mathrm{p}} \right)^n$$

$\longrightarrow$ Energy dependend speed of light: Modifications in the time of flight for γ-ray bursts (Amelino-Camelia, Magueijo & Smolin, Judes & Visser), No-no (SH, Phys. Rev. D 75:105005 (2007))

Time Delay In GRBs



- Fermi Observations of High-Energy Gamma-Ray Emission from GRB 080916C
- Science 27 March 2009, Vol. 323. no. 5922, pp. 1688 - 1693
- $z \approx 4.35$, highest energetic photon $\approx$ 13 GeV, arriving $\approx$ 16.5 seconds after the onset of the burst
- lower limit on the scale of quantum gravity $M_{QG} > 1.3 \times 10^{18}$ GeV.

Time Delay In GRBs

At 11:05:15 UT on 2 Sep 2009, the Fermi Large Area Telescope (LAT) detected gamma rays from the long GRB 090902B, which was triggered and located by the Fermi Gamma-ray Burst Monitor (GBM) (trigger 273582310/090902462, GCN9866). The angle of the GBM best position (RA, Dec= 264.5, 26.5) with respect to the LAT boresight was 51 degrees at the time of the trigger, which is close the edge of our field of view. [...]

More than 200 photons above 100 MeV and more than 30 photons above 1 GeV are observed within 100 seconds. The highest energy photon is a 33.4 GeV event which is observed 82 seconds after the GBM trigger [...]

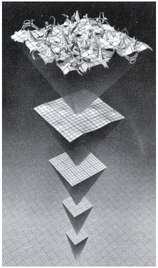
Further analysis is ongoing.

The point of contact for this burst is Francesco de Palma (francesco.depalma@ba.infn.it)

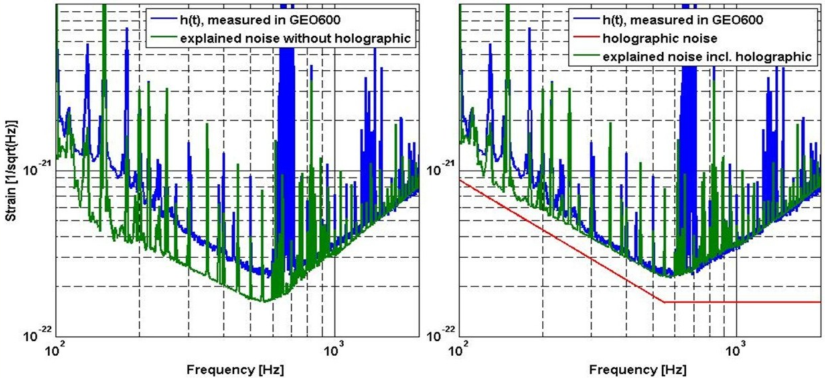
$\longrightarrow$ A puzzle!

Space-Time Foam

- Fluctuations of background metric
- Leading to deviations from lightcone (Ford, Phys. Rev. D **51**, 1692 (1995)), modified dispersion relations or CPT violation
- Halos in images of quasars, decoherence (eg in neutrino oscillations), novel CPT violating effects in entangled states of neutral kaons (Alexandre, Farakos, Mavromatos and Pasipoularides, Phys. Rev. D **77**, 105001 (2008))
- Noise in graviational wave interferometers...



GEO600 Mystery Noise



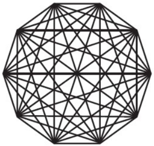
GEO600 noise with and without holographic noise as in Hogan, arXiv:0806.0665
Plot: Stefan Hild

⟶ A mystery? News last month: with different readout method most of the noise can be explained.

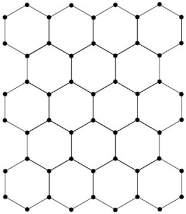
Geometrogenesis

- If space-time is not fundamental, then the early phases of the universe might not be describable by a geometry at all. Konopka, Markopoulou, Severini, Phys. Rev. D **77**, 104029 (2008)
- Phase transition can have consequences for CMB/structure formation. Magueijo, Smolin, Contaldi, Class. Quant. Grav. **24**, 3691 (2007)
- Remaining distortions of locality constitute deviations from QFT and be responsible for the cosmological constant Prescod-Weinstein, Smolin [arXiv:0903.5303]

High-T



Low-T



⟶ A challenge!

Summary

- Various effective models that incorporate quantum gravitational features, some of which make predictions that will be testable soon.
- The "conservative" ones are typically very hard to test. The not-so-conservative ones are frequently weak on the side of consistency.
- The connection between these models and a possibly underlying fundamental theory of quantum gravity is currently unsatisfactory.

$\longrightarrow$ Develop models that can be applied to various effects, and combine predictions to solve inverse problem.