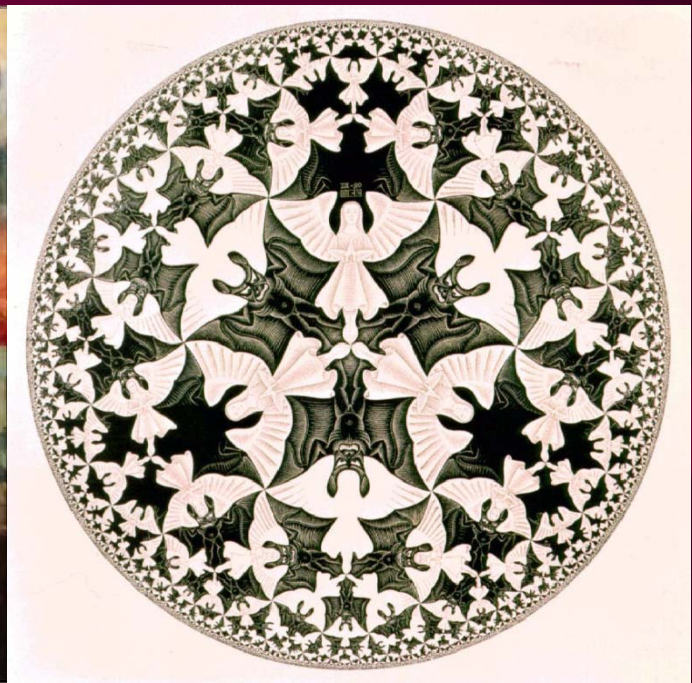
The holography of chiral symmetry breaking

Nick Evans   University of Southampton

+ Keun-Young Kim



Sussex January 2012

Introduction

Chiral symmetry breaking in QCD

AdS/CFT and quarks

Top down models, B field induced, a phenomenological model

Leaving the conformal window

T-μ phase diagrams

Out of equilibrium physics

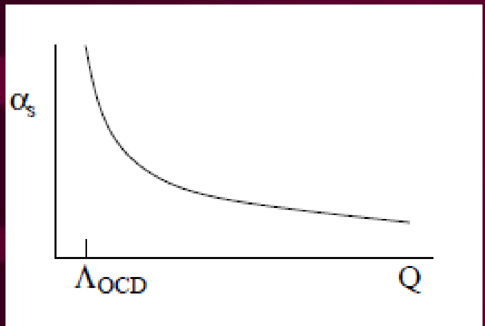
(Links to inflation, condensed matter, technicolour…)

Chiral Symmetry Breaking

Quarks are "massless" so where does the proton mass come from?

QCD

Log running creates a hierarchy naturally



Strong IR generates a symmetry breaking condensate

$$\langle \bar{u}_L u_R + \bar{d}_L d_R + h.c. \rangle \neq 0$$

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

There are three light Goldstone bosons – the pions…

…. except we can't understand off a supercomputer

cf superfluidity; superconductivity

Introduction

String theory has provided a new tool to study strongly coupled gauge theories. It can provide a caricature of QCD that describes:

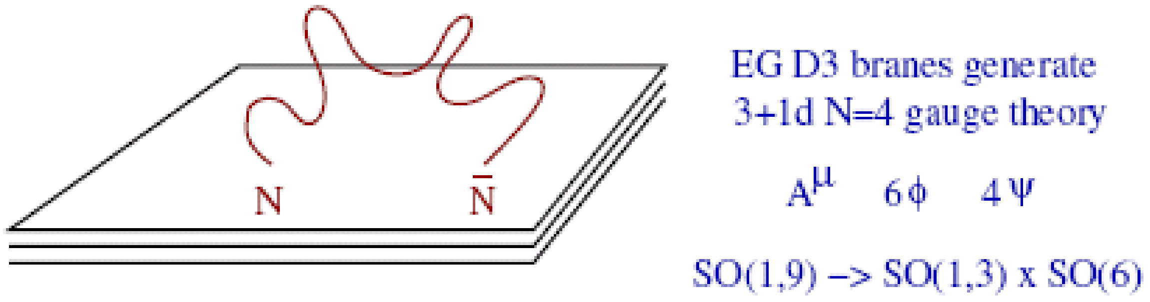
- Dynamical mass generation
- Pions as Goldstone bosons
- ρ meson spectra

Numerical matches are more challenging because we live near

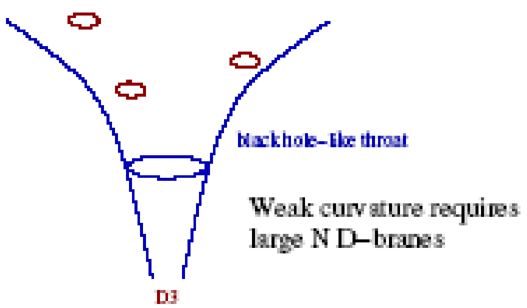
- Large N_c
- Conformal Symmetry
- Supersymmetry
- $N_f \ll N_c$

Strings, Branes & Fields

Open strings described gauge fields in 10d... their ends can be restricted to D-branes though



Alternative description of branes via the supergravity geometry they create (closed strings in 10d)



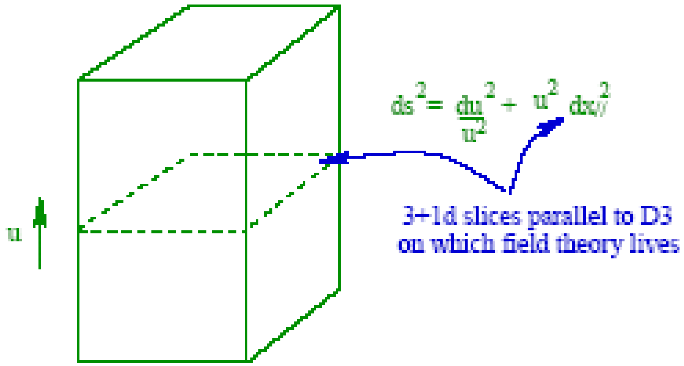
Now believe these are two dual descriptions

AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled $\mathcal{N}$=4 SYM = IIB strings on $AdS_5 \times S^5$

Pretty well established by this point!



u corresponds to energy (RG) scale in field theory

The SUGRA fields act as sources

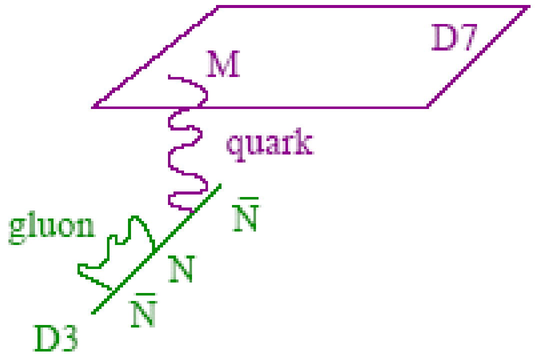
$$\int d^4x \; \Phi_{SUGRA}(u_0)\lambda\lambda$$

eg asymptotic solution ($u \to \infty$) of scalar

$$\varphi \simeq \frac{m}{u} + \frac{\langle\lambda\lambda\rangle}{u^3}$$

Adding Quarks

Bertolini, DiVecchia...; Polchinski, Grana; Karch, Katz...



Quarks can be introduced via D7 branes in AdS

The brane set up is

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	•	•	•	•	•	•
D7	—	—	—	—	—	—	—	—	•	•

Directions 4–7: fills radial dirn of AdS, S^3 embedded in S^5. Directions 8–9: $U(1)_A$

We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

Quarks In AdS

Myers et al

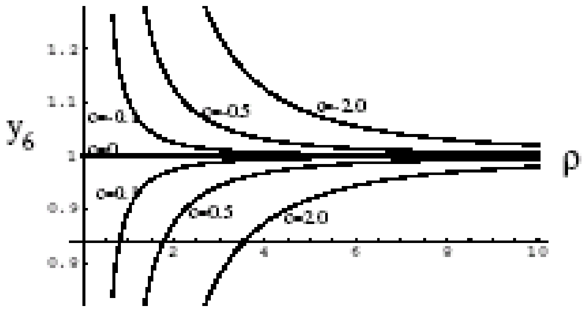
$$S_{D7} = -T_7 \int d^8\xi \; \epsilon_3 \; \rho^3 \sqrt{1 + \frac{g^{ab}}{\rho^2 + u_5^2 + u_6^2}(\partial_a u_5 \partial_b u_5 + \partial_a u_6 \partial_b u_6)}$$

EoM is: $$\frac{d}{d\rho}\left[\frac{\rho^3}{\sqrt{1+\left(\frac{du_6}{d\rho}\right)^2}}\frac{du_6}{d\rho}\right] = 0$$

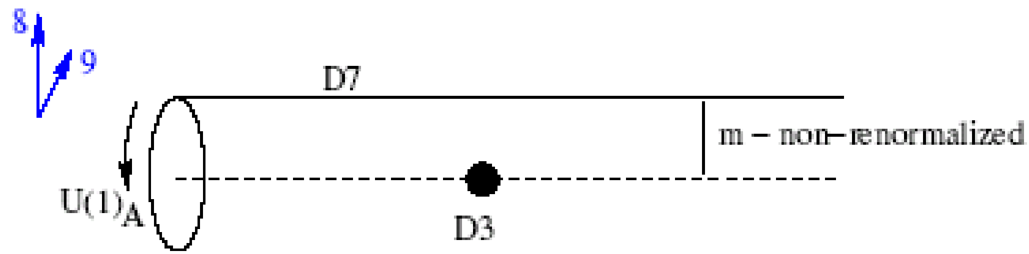
UV asymptotic solution is

$$u_6 = m + \frac{c}{\rho^2} + \ldots$$

m is the quark mass, c the $\langle \bar{q}q \rangle$ condensate



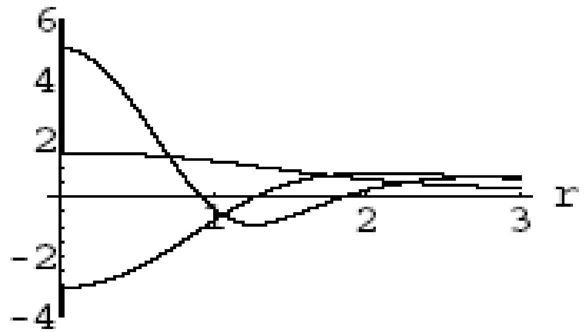
In AdS regular D7 solution is flat brane



Mesons in AdS$_5$

Kruczenski,Myers, Mateos, Winters

The D7 lie flat in AdS. We can consider fluctuations that describe R-chargeless mesons



$$w_6 + i w_5 = d + \delta(\rho) e^{ik.x}$$

δ satisfies a linearized EoM

$$\partial_\rho^2 \delta + \frac{3}{\rho}\partial_\rho \delta + \frac{M^2}{(\rho^2+1)^2}\delta = 0$$

and the mass spectrum is

$$M = \frac{2d}{R^2}\sqrt{(n+1)(n+2)} \sim \frac{2m}{\sqrt{\lambda_{YM}}}$$

Tightly bound - meson masses suppressed relative to quark mass

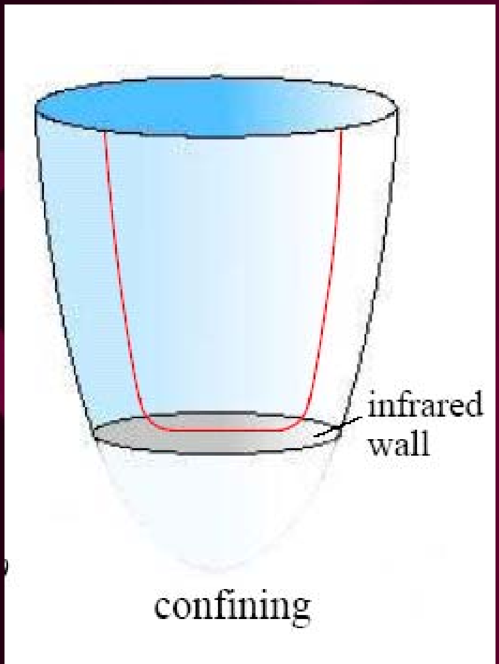
Add Confinement and Chiral Symmetry Breaking

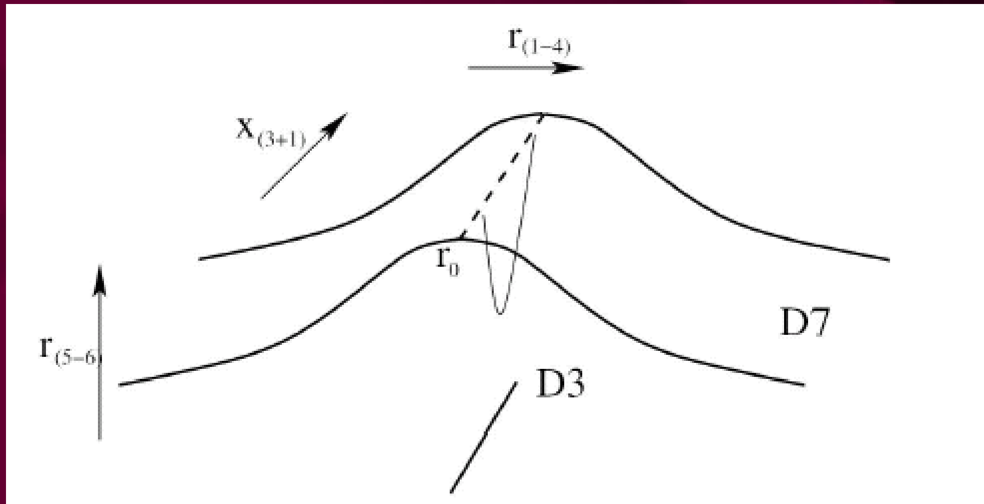
$$ds^2 = \frac{r^2}{R^2} A^2(r) dx_{3+1}^2 + \frac{R^2}{r^2} dr_6^2,$$

$$A(r) = \left(1 - (\frac{r_w}{r})^8\right)^{1/4}, \qquad e^{\phi} = \left(\frac{1 + (r_w/r)^4}{1 - (r_w/r)^4}\right)^{\sqrt{3/2}}$$

Dilaton Flow Geometry: Gubser, Sfetsos

Here, this is just a simple, back reacted, repulsive, hard wall….

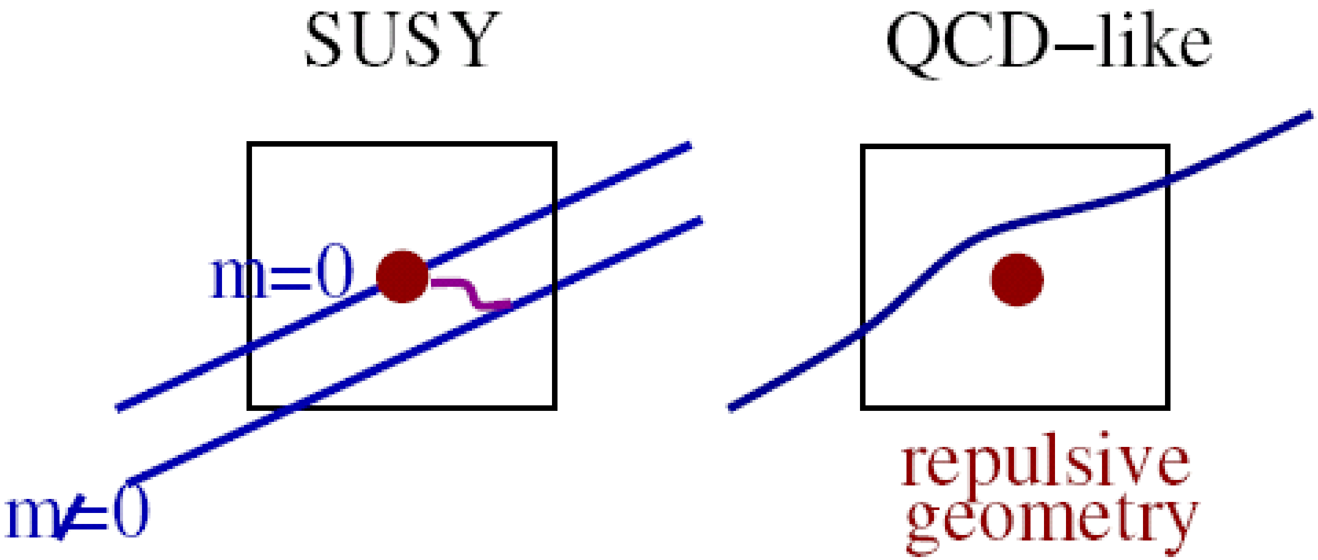




B,E, Erdmenger,G,Kirsch,

Geometrical Pions

D7s lie in $x_8 - x_9$ plane with explicit $U(1)_A$



Fluctuations about ring = massless pions!

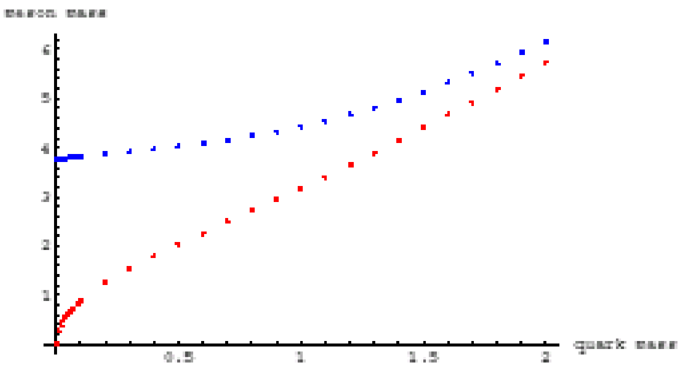
Pion Physics

Seek pion solutions of the form

$$\pi(x, r) = f(\rho)e^{ikx}, \qquad k^2 = -M^2$$

$f(\rho)$ must be smooth - normalizable - at all ρ

The pion and sigma masses can thus be computed as a function of quark mass



There is a Goldstone in the massless limit.

Expected $\sqrt{m}$ behaviour

Magnetic Field Induced Chiral Symmetry Breaking

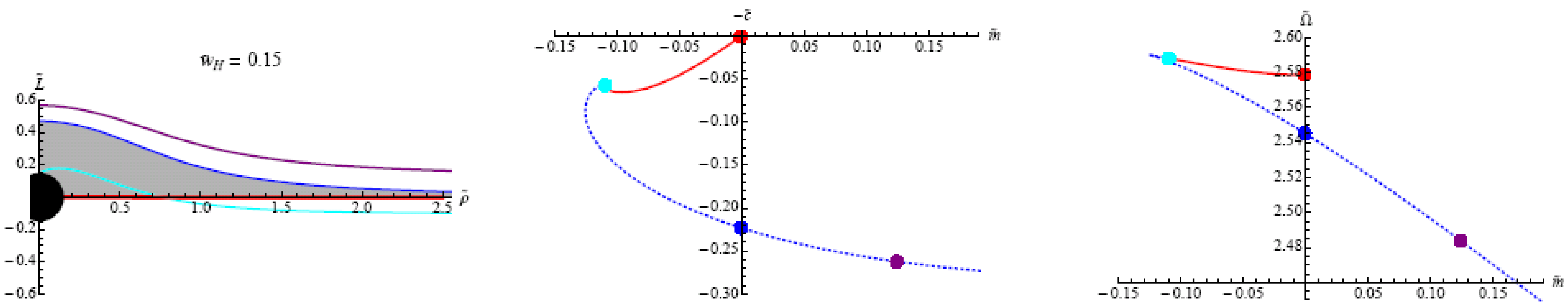
Johnson, Filev, Kundu….

$$P[G]+F = \begin{pmatrix} -\frac{r^2}{R^2} & & & & & & & \\ & \frac{r^2}{R^2} & B^z & & & & & \\ & -B^z & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho w_6)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & & & \frac{R^2}{r^2}.. \end{pmatrix}$$

$$\mathcal{L} = \rho^3\sqrt{1+(\partial_\rho w_6)^2}\sqrt{1+\frac{B^z R^4}{r^4}}$$

Put in B through susy partner of mesons..

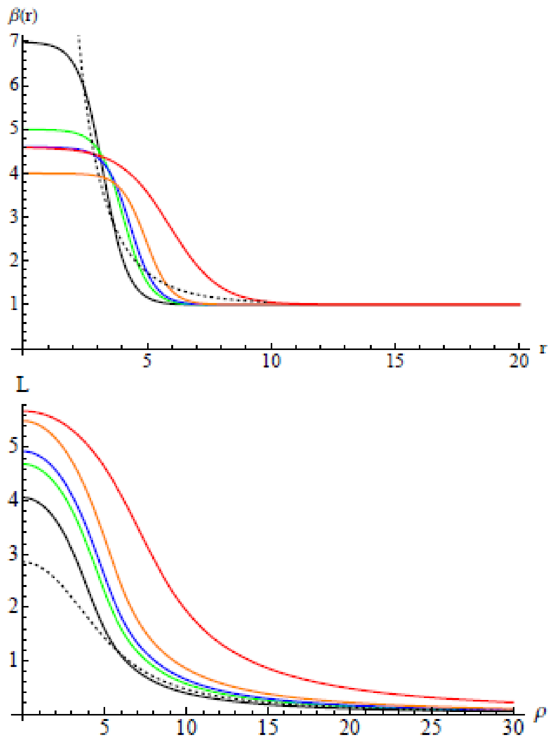
B "just" breaks conformal symmetry…



(a)Low temperature - $\tilde{w}_H = 0.15$. Here we see chiral symmetry breaking with the blue embedding thermodynamically preferred over the red at $\tilde{m} = 0$.

A Phenomenological Dilaton arXiv:1109.2633 [hep-th]

$$e^{\Phi} = g^2_{\rm YM}(r^2) = g^2_{\rm UV}\Big(A + 1 - A\tanh\left[\Gamma(r-\lambda)\right]\Big)$$



	Γ	A	λ
Black	1	3	3.240
Green	1	2	4.045
Blue	1	1.8	4.325
Orange	1	1.5	4.940
Red	0.5	1.8	5.882
Black(Dotted)	B = 35.6	-	-

Move to a phenomenological variant of the D3/D7 system

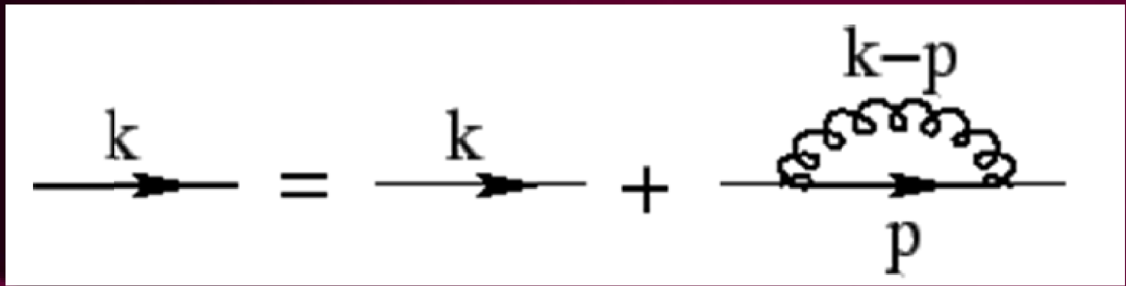
The dilaton interpolates between QCD like case and “walking” dynamics (black is B field induced chiral symmetry breaking)

But we’re not back reacted – this is AdS/QCD.

λ is the scale of the problem..
A is height
Γ is width

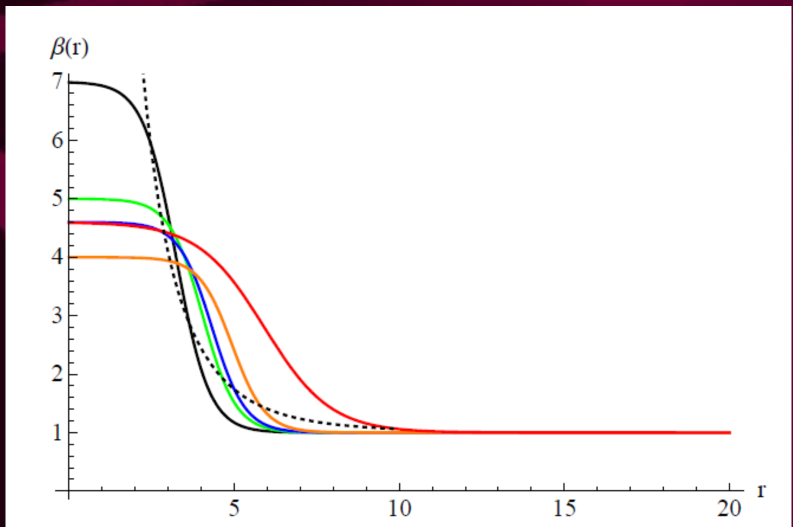
Critical Couplings

Schwinger-Dyson analysis tells an old story… coming from weak coupling there is a critical coupling for chiral symmetry breaking



$$\alpha_c \equiv \frac{\pi}{3\,C_2(R)} = \frac{2\pi\,N}{3\,(N^2-1)}$$

In walking theories though chiral symmetry breaking is triggered by movement in coupling from a strongly coupled conformal sector



We find there must be a critical departure from conformality

Eg fixed $\Gamma=1$ $A_c = 2.1$

The QCD Conformal Window

The Conformal Window

AF is gained at Nf = 11/2 Nc

At large Nc, Nf   Nf/Nc = x
is continuous

Banks Zak fixed point in 2 loop beta function

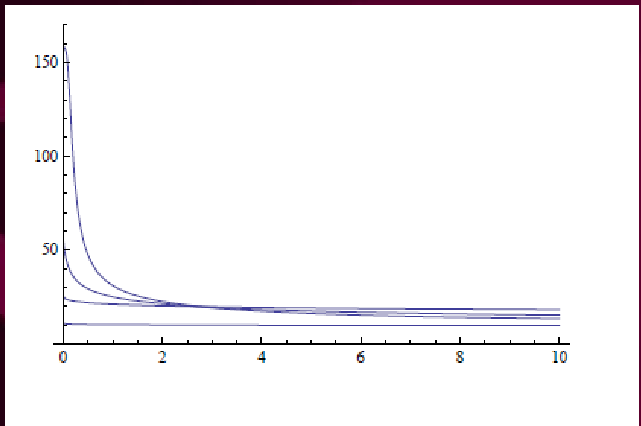
$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$



x = 3.. 4.5

$$x = 11/2 - \epsilon$$

$$\lambda_* = \frac{(8\pi)^2}{75}\epsilon$$

Perturbative IR CFT…. Then strongly coupled IR CFT…. Then chiral symmetry breaking cf QCD…

The Conformal Window

Flavour dependence of QCD

No AF

Nf=11/2 Nc

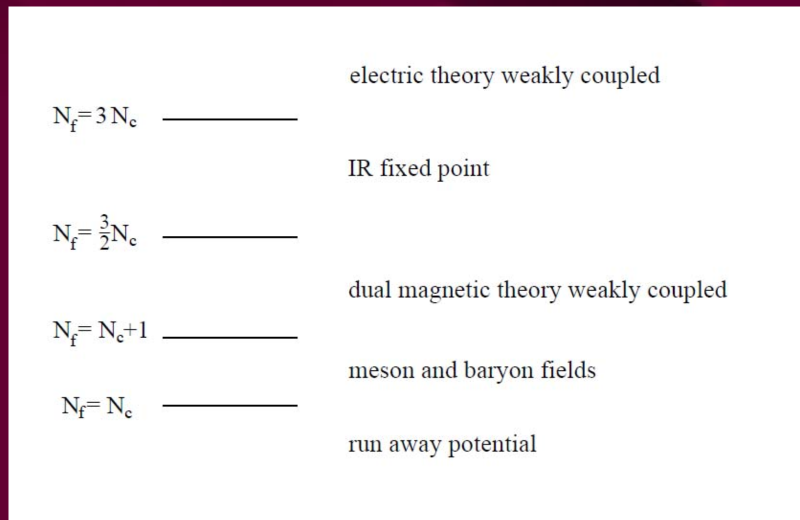
CFT

Nf = ? Nc

χ SB

Estimates of ? by Appelquist, Sannino… gap equations, “exact” beta functions..

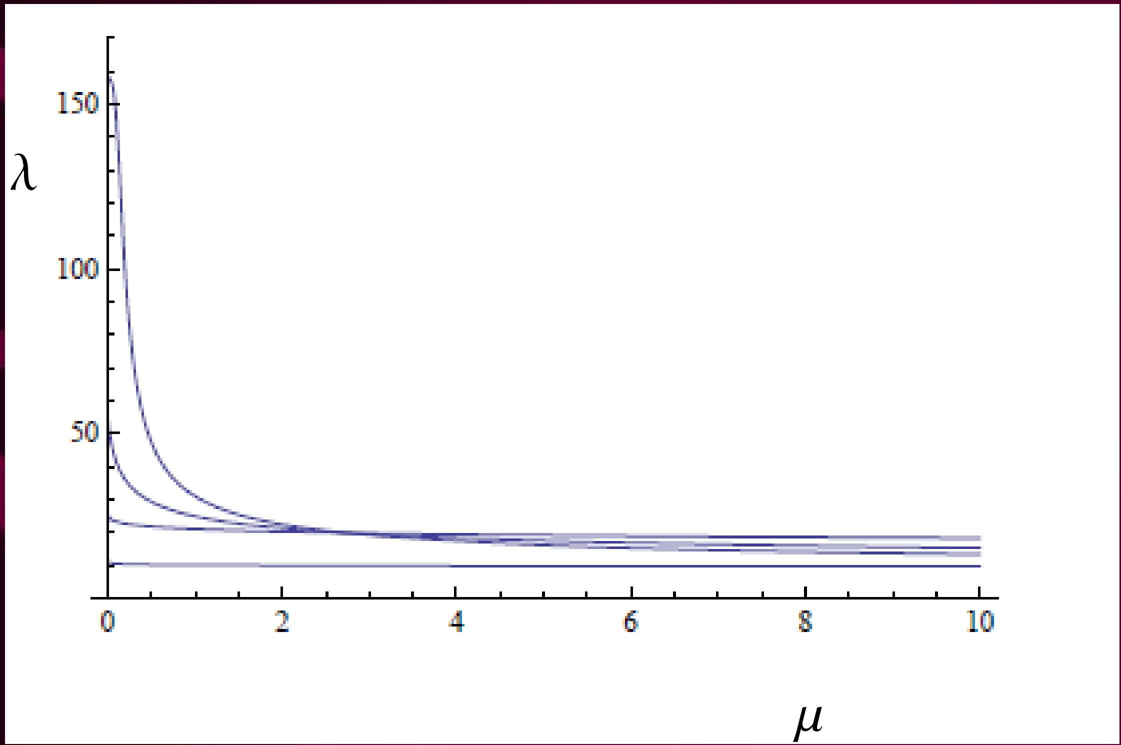
Flavour dependence of SQCD



The Conformal Window

Jarvinen, Kiritsis have used an elaborate holographic dual to estimate 4.2 < x? < 3.7 at large Nc

It’s pretty obvious in our simpler model though!



x? ~ 3.5

Of course there’s little reason to trust these runnings at these coupling values!!

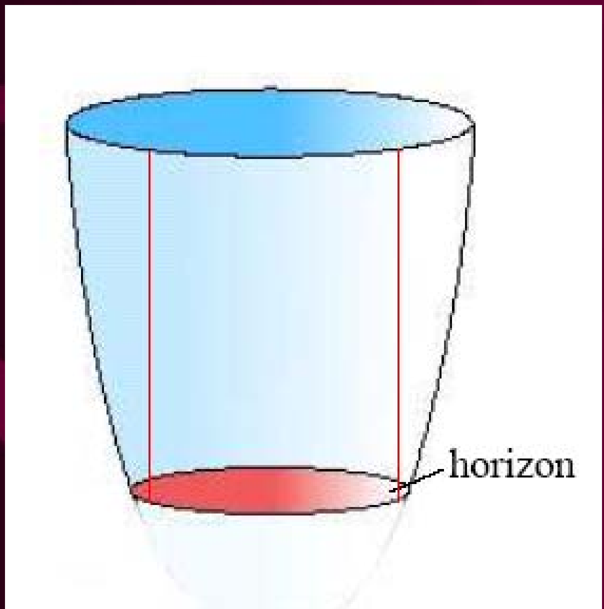
Phase Diagrams

Finite T - AdS-Schwarzschild

$$ds^2 = \frac{r^2}{R^2}(-f\,dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f}dr^2 + R^2 d\Omega_5^2$$

where $R^4 = 4\pi g_s N \alpha'^2$ and

$$f := 1 - \frac{r_H^4}{r^4}\,, \qquad r_H := \pi R^2 T\,.$$

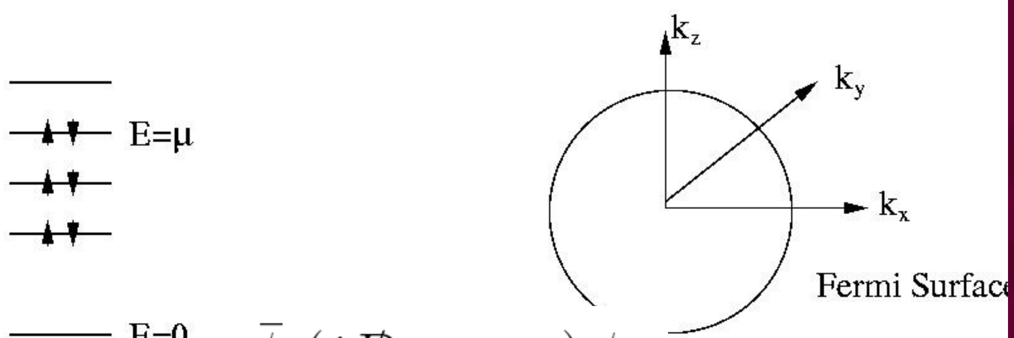


Quarks are screened by plasma

Asymptotically AdS, SO(6) invariant at all scales… horizon swallows information at rH …. Witten interpreted as finite temperature… black hole has right thermodynamic properties…

Chemical Potential

At finite density the Fermi-sea of quarks fills up to an energy called the chemical potential



$$\bar{\psi}_i(i\not{D}+\mu\gamma_0)\psi_i$$

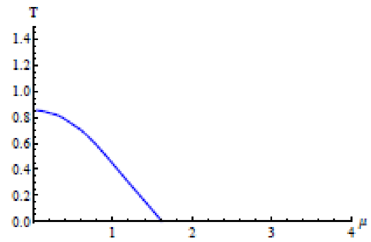
$$\bar{\psi}i(-iA^t\gamma_0)\psi \quad \rightarrow \quad \bar{\psi}\mu\gamma_0\psi$$

We can think of μ as a background vev for the temporal component of the photon…

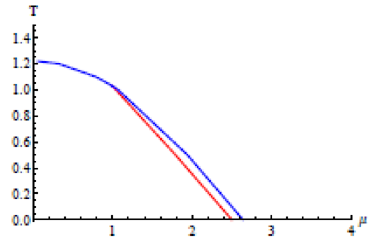
$$P[G]+F=\begin{pmatrix} -\frac{r^2}{R^2} & & & & \partial_\rho A_0 & & & \\ & \frac{r^2}{R^2} & & & & & & \\ & & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ -\partial_\rho A_0 & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho w_6)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & & & \frac{R^2}{r^2}.. \end{pmatrix}$$

Phase Diagrams

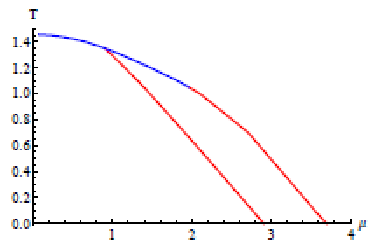
NE, K-Y K, Gebauer, Magou



(a) $A = 3$



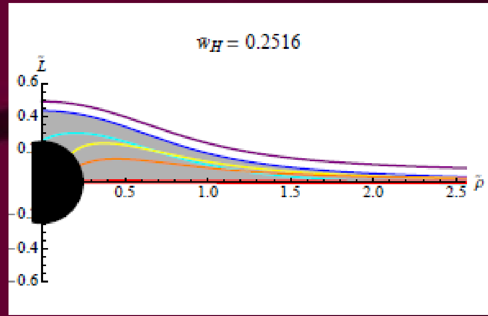
(b) $A = 5$



(c) $A = 8$

FIG. 6: Plots for three possible phase diagrams for the choices $A = 3, 5, 8$. Large (small) A gives second (first) order transition at low T. $\Gamma = 1, \lambda = 1.7$.

Walking encourages first order transition



Breaking the ρ-L symmetry

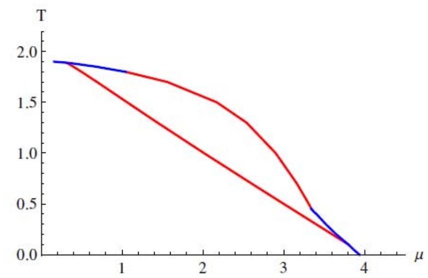
	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	•	•	•	•	•	•
D7	—	—	—	—	—	—	—	—	•	•

fills radial dirn of AdS / S^3 embedded in S^5 (4–7); $U(1)_A$ (8–9)

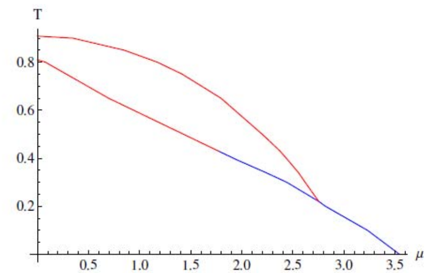
$$g_t = \frac{(w^4 - w_H^4)^2}{w^4(w^4 + w_H^4)}, \qquad g_x = \frac{w^4 + w_H^4}{w^4}$$

$$w^4 \to \rho^2 + \frac{1}{\tilde{\alpha}} L^2$$

QCD-like phase diagrams…



(a) $\tilde{\alpha} = 2, A = 3, \Gamma = 0.9, \lambda = 3.2$



(b) $\tilde{\alpha} = 3, A = 3 \Gamma = 1, \lambda = 1.715$

Quasi-normal modes & meson melting

BEEGK… Sonnenschein, Peeters,Zamaklar… Hoyos…. Myers, Mateos…

Linearized fluctuations in eg the scalars on the D7 brane must now enter the black hole horizon…

Quasi-normal modes are those modes that near the horizon have only in-falling pieces…

The mass of the bound states become complex – they decay into the thermal bath…

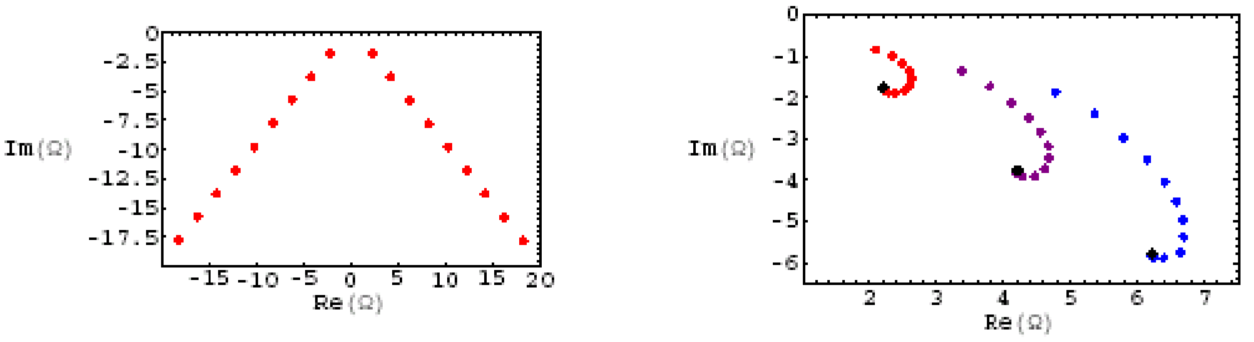


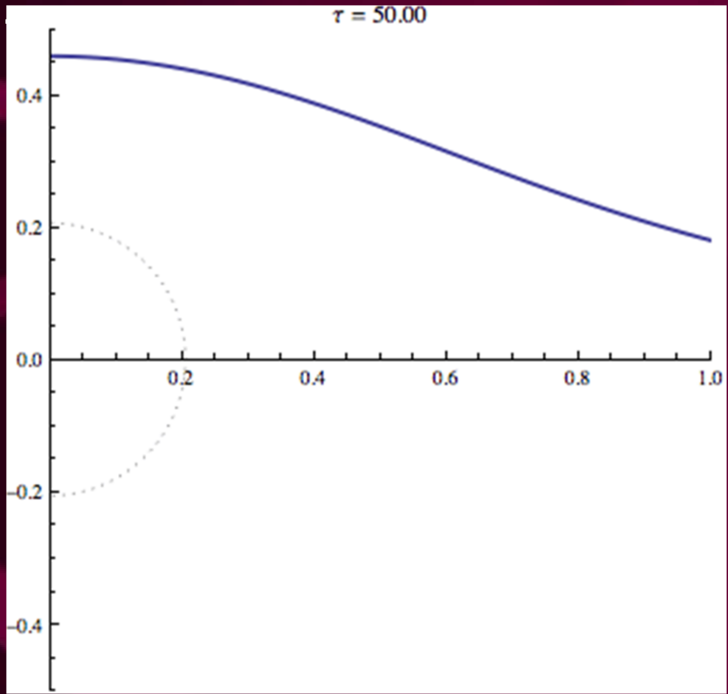
Figure 7.4: The lowest quasi-normal modes for $m_q = 0$ on the left and the three lowest quasi-normal modes for increasing m_q on the right. The black points on the right show the limiting values for $m_q = 0$.

Non-equilibrium Physics

Non-equilibrium Dynamics At The Transitions

AdS/CFT replaces strongly coupled quantum field theory with weakly coupled classical field theory – we can now do time dependent problems that the lattice can't…

…



EG This a D7 brane embedding in a heating geometry (Janik's) moving through a first order chiral phase transition

With Ingo Kirsch, Tigran Kalaydzhyan (DESY)

Chiral Transition in Janik's Cooling/Heating Geometry

The black hole grows/shrinks changing the effective potential…

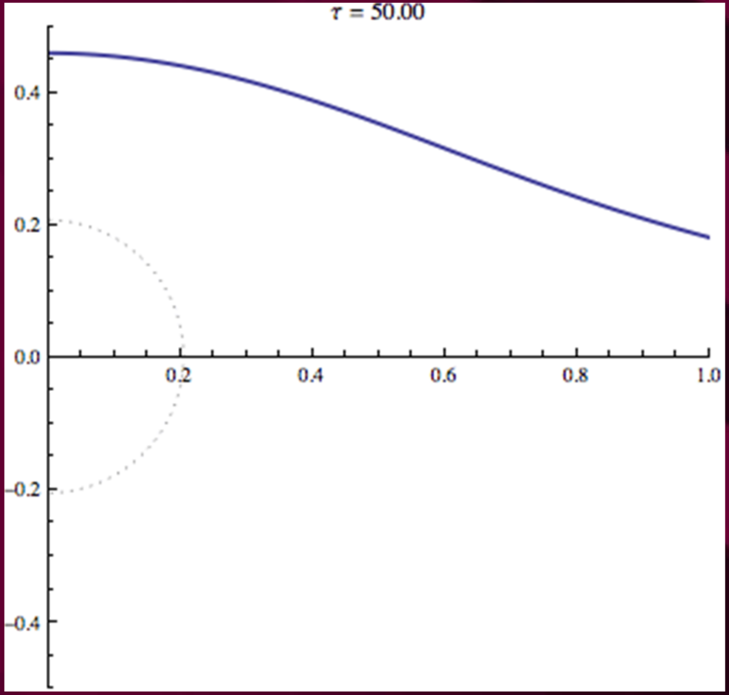
With Ingo Kirsch, Tigran Kalaydzhyan (DESY)

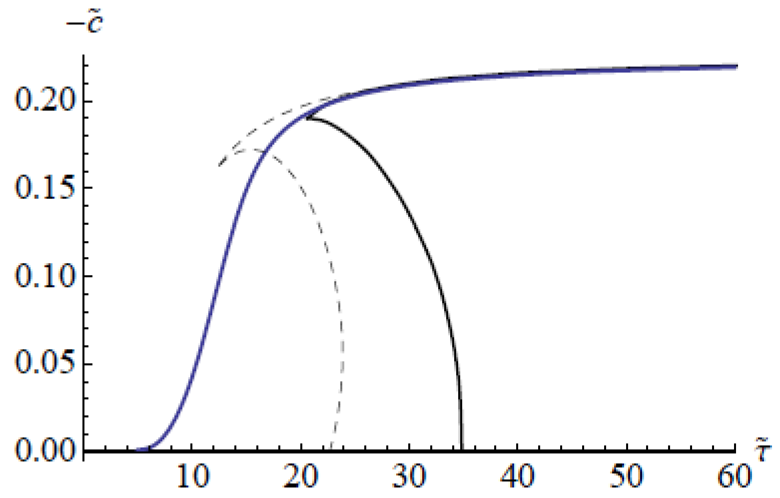
$$ds^2 = \frac{r^2}{R^2}(-e^{a(\tau,r)}d\tau^2 + e^{b(\tau,r)}\tau^2 dy^2 + e^{c(\tau,r)}dx_\perp^2) + \frac{R^2}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2)$$

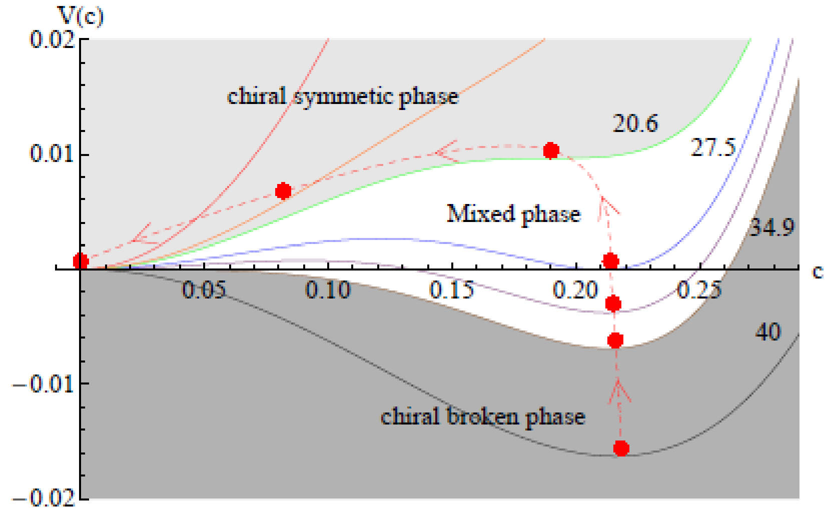
$$a(\tau,z) = \ln\left(\frac{(1-v^4/3)^2}{1+v^4/3}\right) + 2\eta_0 \frac{(9+v^4)v^4}{9-v^8}\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

$$b(\tau,z) = \ln(1+v^4/3) + \left(-2\eta_0 \frac{v^4}{3+v^4} + 2\eta_0 \ln\frac{3-v^4}{3+v^4}\right)\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

$$b(\tau,z) = \ln(1+v^4/3) + \left(-2\eta_0 \frac{v^4}{3+v^4} - \eta_0 \ln\frac{3-v^4}{3+v^4}\right)\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

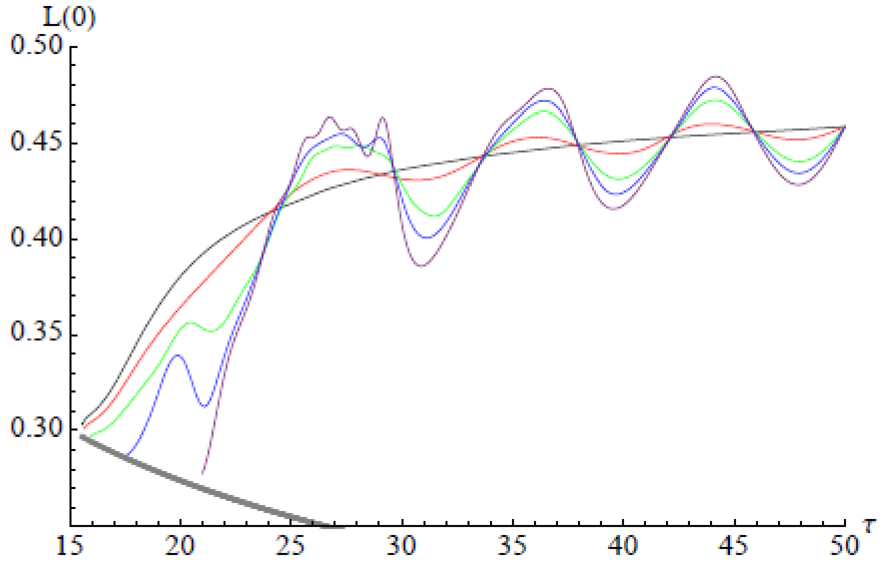






Equilibrium vs PDE solutions…

Bubble formation…



Conclusion

We have a wonderfully simple description of chiral symmetry breaking using holography…

We can induce CSB with top down running dilaton models, B-field induced, and with arbitrary running coupling ansatz…

Using the two loop beta function can estimate critcical Nf to leave conformal window (~ 4) … note role of critical conformal symmetry breaking…

We can compute the T-μ phase diagrams and phenomenologically construct models for many situations…

We can compute out of equilibrium behaviour in strongly coupled phase transitions…

Other applications: hadronization, technicolour, condensed matter systems (high Tc superconductors??) , composite inflatons…