A ΛCDM Bounce Scenario

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University of Sussex Seminar

Quantum Gravity Effects in the Early Universe

One of the main difficulties of any theory of quantum gravity is to obtain predictions (that are realistically testable) and confront them to experiment or observations.

The best hope in this direction appears to lie in the very early universe, where quantum gravity effects are expected to be strong and may have left some imprints on the cosmic microwave background (CMB). What form could these imprints have?

In this talk, I will focus on loop quantum cosmology (LQC) and some potential predictions concerning the CMB.

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In this talk, I will focus on loop quantum cosmology (LQC) and some potential predictions concerning the CMB.

Caveat: the dynamics of LQC (just like general relativity) depend on the matter content. Therefore the predictions of LQC will strongly depend on what the dominant matter field (radiation, inflaton, ...) is during the bounce.

The CMB and the Matter Bounce

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An alternative to inflation is the matter bounce scenario: Fourier modes that are initially in the quantum vacuum state that exit the Hubble radius in a contracting matter-dominated Friedmann universe become scale-invariant. [Wands]

Then, if this contracting branch can be connected to our currently expanding universe via some sort of a bounce, these scale-invariant perturbations can provide suitable initial conditions for the expanding branch. [Finelli, Brandenberger]

Cosmological Perturbation Theory

A commonly used gauge-invariant variable for scalar perturbations is the comoving curvature perturbation $\mathcal{R}$. In particular, the main quantity of interest is the power spectrum

$$\Delta^2_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \sim A \cdot \left(\frac{k}{k_\star}\right)^{n_s-1}.$$

However, for calculations the Mukhanov-Sasaki variable

$$v = z\mathcal{R}, \qquad z = \frac{a\sqrt{\rho + P}}{c_s H},$$

is commonly used since the differential equation that governs its dynamics is particularly simple:

$$v_k'' + c_s^2 k^2 v_k - \frac{z''}{z} v_k = 0.$$

Review of the Matter Bounce

For a contracting matter-dominated ($P = 0$) FLRW universe,

$$a(\eta) = \eta^2, \qquad -\infty < \eta < 0.$$

Then, the Mukhanov-Sasaki equation becomes

$$v_k'' + c_s^2 k^2 v_k - \frac{2}{\eta^2} v_k = 0,$$

as $\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\eta^2}$, and the solution to this differential equation is

$$v_k = A_1 \sqrt{-\eta}\, H^{(1)}_{\frac{3}{2}}(-c_s k \eta) + A_2 \sqrt{-\eta}\, H^{(2)}_{\frac{3}{2}}(-c_s k \eta).$$

At early times, $|\eta| \gg 1$ and the Fourier modes are inside the horizon. If one imposes quantum vacuum fluctuations as the initial conditions, then $A_1 \sim \sqrt{\hbar}, A_2 = 0$ and when the modes exit the (sound) horizon they become scale-invariant.

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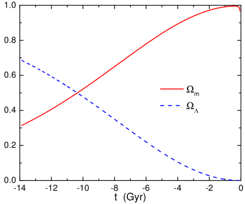
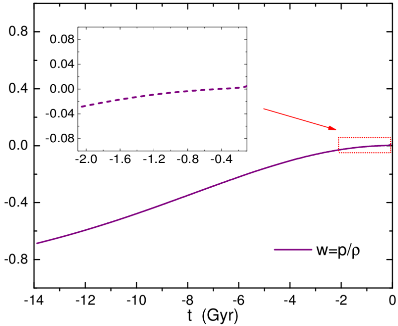
We assume that loop quantum cosmology (LQC) captures the relevant high-curvature dynamics, in which case a bounce occurs near the Planck scale and we can calculate the evolution of the perturbations through the bounce using the LQC Mukhanov-Sasaki equations.

Outline

Homogeneous Background I: ΛCDM Era

In the contracting branch, the dynamics of the space-time will initially be dominated by the cosmological constant Λ and afterwards by cold dark matter.





During the transition between these two epochs, there will be a period of time where the effective equation of state will be slightly negative.

The Fourier modes that exit the sound horizon at this time will be nearly scale-invariant with a slight red tilt.

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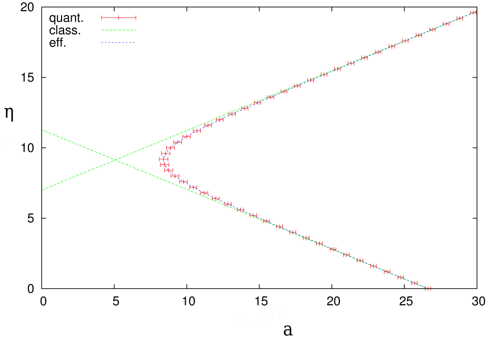
3. Assume that the area of the loop is given by the minimal non-zero eigenvalue of the area operator of LQG.

The result is a Hamiltonian (constraint) operator that can be solved.

Homogeneous Background II: Radiation Era

The Hamiltonian constraint operator can be solved numerically, and for a state that is semi-classical at late times —i.e., sharply-peaked around a classical solution— the result is the following:



[Pawłowski, Pierini, WE]

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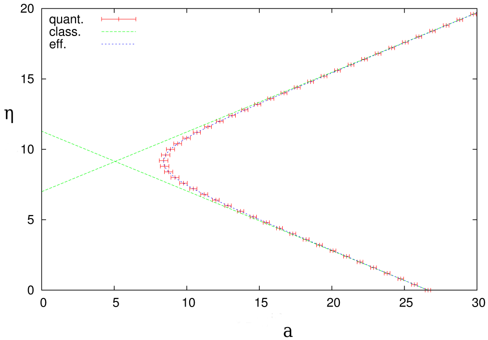

[Pawłowski, Pierini, WE]

An important result is that the effective equation [Taveras]

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

gives the dynamics of the expectation value of the scale factor at all times, including at the bounce point.

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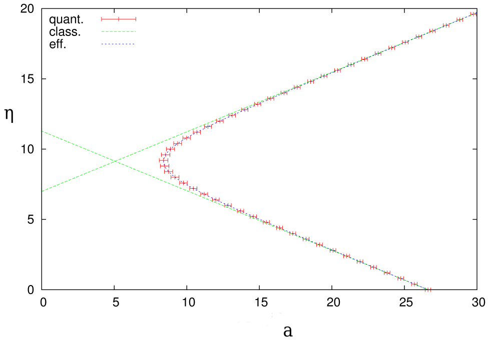

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Note that the bounce is generic and occurs also for states that are not sharply-peaked.

Some Approximations

In order to be able to analytically solve for the background and perturbations, some approximations are necessary.

1. Split the evolution of the universe into two parts: ΛCDM and radiation,
2. Assume constant equation of state during each of these eras:
   a) $\omega_{\rm eff} = -\delta$ during the ΛCDM era, with $\dot{\delta} = 0$ and $0 < \delta \ll 1$,
   b) $\omega_{\rm eff} = \frac{1}{3}$ during the radiation-dominated era.
3. Connect the two parts at the matter-raidiation time t_e by imposing continuity in the scale factor and in the Hubble rate.

This is a reasonable approximation for the Fourier modes that exit the sound horizon at a time when the effective equation of state is $\omega_{\rm eff} = -\delta$: this is a mode by mode calculation.

The Scale Factor

Choosing the bounce time to be $t = 0$ and setting the overall normalization of the scale factor so that $a(t = 0) = 1$, the approximations on the previous slide give

$$a(t) = \left(\frac{32\pi G \rho_c}{3} t^2 + 1 \right)^{1/4}$$

during radiation domination, and during the ΛCDM era

$$a(\eta) = a_e \left(\frac{\eta - \eta_o}{\eta_e - \eta_o} \right)^{2/(1-3\delta)}, \quad \text{where} \quad \eta_o = \eta_e - \frac{2}{(1-3\delta)\mathcal{H}_e}.$$

Here the subscript 'e' denotes matter-radiation equality.

The Key Ingredients

The key ingredients in the ΛCDM bounce scenario that the predictions will depend upon are the following:

- The effective equation of state when the Fourier modes of interest exit the sound horizon, $\omega_{\rm eff} = -\delta$ (recall $0 < \delta \ll 1$),
- The sound speed of CDM, $c_s = \epsilon \ll 1$,
- The (proper) Hubble rate at the time of matter-radiation equality H_e,
- The energy density of the radiation field at the time of the LQC bounce, $\rho_c \sim \rho_{\rm Pl}$.

Scalar Perturbations I: ΛCDM Background

The Mukhanov-Sasaki equation in the ΛCDM background is

$$v_k'' + \epsilon^2 k^2 v_k - \frac{2(1+3\delta)}{(1-3\delta)^2(\eta-\eta_o)^2} v_k = 0.$$

The solution (as usual for a constant equation of state) is a Hankel function. Assuming the quantum vacuum as the initial conditions,

$$v_k = \sqrt{\frac{-\pi\hbar(\eta-\eta_o)}{4}}\, H_n^{(1)}[-\epsilon k(\eta-\eta_o)],$$

where

$$n \approx \frac{3}{2} + 6\,\delta + O(\delta^2).$$

Note that to reach the long-wavelength limit, the scalar perturbations only need to exit the sound horizon, which is smaller than the Hubble radius by a factor of ϵ, the sound speed.

Scalar Perturbations II: Radiation Background

The Mukhanov-Sasaki equation for a radiation-dominated space-time (in the absence of quantum gravity effects) is

$$v_k'' + \frac{k^2}{3} v_k = 0,$$

and the solutions are simply plane waves. Requiring that v_k and v_k' be continuous during the transition between the ΛCDM era and the radiation-dominated period gives

$$v_k \sim \left(k^{-n} \cos \frac{k\eta_e}{\sqrt{3}} + k^{-n-1} \sin \frac{k\eta_e}{\sqrt{3}} \right) \cos \frac{k\eta}{\sqrt{3}} + \sin \frac{k\eta}{\sqrt{3}}.$$

Note that we have not imposed the condition that $k|\eta_e|/\sqrt{3} \ll 1$. However, as we can see here already, this condition will be necessary for scale-invariance.

Scalar Perturbations in Loop Quantum Cosmology

In loop quantum cosmology, scalar perturbations can be studied using the ‘separate universe’ approach. [Salopek, Bond; Wands, Malik, Lyth, Liddle]

a(1) φ(1)	a(2) φ(2)	a(3) φ(3)
a(4) φ(4)	a(5) φ(5)	a(6) φ(6)
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Take a cubic lattice and assume that each cell is homogeneous and isotropic. (The small variations between the parameters in each cell correspond to the perturbations.) Then the usual LQC quantization of a flat FLRW space-time can be done in each cell. [WE]

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The resulting LQC effective equations for scalar perturbations are expected to be valid for Fourier modes whose wavelength remains much larger than $\ell_{\rm Pl}$. [Rovelli, WE] For these modes, the long-wavelength Mukhanov-Sasaki equation in LQC is

$$v_k'' - \frac{z''}{z} v_k = 0, \qquad z = \frac{a\sqrt{\rho + P}}{c_s H}.$$

Scalar Perturbations III: Bounce

Knowing the background evolution and the form of the perturbations at the onset of the radiation-dominated era, the effective LQC Mukhanov-Sasaki equation can be solved, giving a hypergeometric function.

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Then, by taking the limit of $t \gg t_{\rm Pl}$, we obtain the form of the scalar perturbations after the bounce:

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi}|\mathcal{R}_k|^2 \sim \sqrt{\frac{\rho_c}{\rho_{\rm Pl}}} \cdot \frac{|H_e|\ell_{\rm Pl}}{\epsilon^3} \cdot \left(\frac{k}{k_o}\right)^{-12\delta},$$

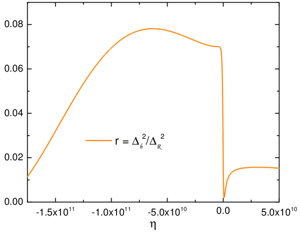
where the co-moving curvature perturbation is $\mathcal{R}_k \sim v_k/a$.

Here we have assumed that $k|\eta_e|/\sqrt{3} \ll 1$ in order to ensure that the resulting spectrum is nearly scale-invariant.

Tensor Perturbations

A similar calculation can be done for tensor modes, and the result is that their resulting amplitude is predicted to be significantly smaller than the amplitude of the scalar perturbations for two reasons:

1. The amplitude of the scalar perturbations are boosted by a factor of ϵ^{-3},
2. The amplitude of the tensor perturbations is damped by a factor of $1/4$ during the bounce due to LQC effects.



The predicted tensor-to-scalar ratio is

$$r = \frac{\Delta_h^2(k)}{\Delta_{\mathcal{R}}^2(k)} = 24\epsilon^3,$$

which satisfies the current observational bound of $r < 0.12$ [Planck+BICEP2/Keck] for $\epsilon \sim 0.1$.

An Asymmetric Bounce

To have scale-invariance, we required that

$$\frac{k|\eta_e|}{\sqrt{3}} \ll 1.$$

It is easy to check that, defining η_e^+ to be the time of matter-radiation equality in the expanding branch, for k observed in the CMB today $k\eta_e^+ \sim 1$.

Therefore, in order for the first condition to hold, $|\eta_e| \ll \eta_e^+$. This requires an asymmetric bounce where the radiation-dominated era lasts longer in the expanding branch than in the contracting branch.

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An asymmetric bounce has already been suggested to arise in LQC due to the effect of particle production during the bounce. [Mithani, Vilenkin] Whether this effect is strong enough to generate sufficient asymmetry is not yet clear.

The Effective Equation of State is not Constant

In the contracting branch during the ΛCDM epoch, the effective equation of state is not constant. Rather,

$$\frac{\mathrm{d}\omega_{\mathrm{eff}}}{\mathrm{d}\eta} > 0, \qquad \Rightarrow \qquad \frac{\mathrm{d}\omega_{\mathrm{eff}}}{\mathrm{d}k_h} > 0,$$

where the implication follows due to the fact that small k exit the sound horizon first.

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where the implication follows due to the fact that small k exit the sound horizon first.

Recall that the k-dependence of the scalar power spectrum depends directly on the effective equation of state when the Fourier mode exits the sound horizon. The same relation holds for tensor modes.

Since the sound speed for tensor modes is larger $(1 \gg \epsilon)$, the tensor modes exit their sound horizon later than their corresponding scalar modes. Therefore, by the above inequalities, the tensor index n_t must satisfy the relation

$$n_t > n_s - 1.$$

The Running of the Scalar Index

Furthermore,

$$\frac{\mathrm{d}n_s}{\mathrm{d}k} = \frac{\mathrm{d}n_s}{\mathrm{d}\omega_{\rm eff}} \cdot \frac{\mathrm{d}\omega_{\rm eff}}{\mathrm{d}k} = \frac{\mathrm{d}(1-12\delta)}{\mathrm{d}(-\delta)} \cdot \frac{\mathrm{d}\omega_{\rm eff}}{\mathrm{d}k} > 0.$$

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Observations currently weakly favour a negative running: [Planck]

$$\frac{\mathrm{d}n_s}{\mathrm{d}(\ln k)} = -0.008 \pm 0.016,$$

but a positive running is not ruled out.

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Most of the predictions are very robust. As we saw, the majority of the features of the spectrum are determined by the physics of the contracting branch far before the bounce and therefore the bounce will not affect these predictions.

However, there is one exception: in LQC, we found that the amplitude of the tensor modes is suppressed by a factor of $1/4$ during the bounce. Therefore, in other bouncing models (unless there is a similar suppression of the tensor modes), we would expect a larger value of r (for a given CDM sound speed ϵ).

This effect is a potential test for LQC versus other bouncing cosmology models.

Open Questions

There remain four main open questions in the ΛCDM bounce scenario:

- **Amplitude of the running of $\mathbf{n_s}$**: We showed that $\mathrm{d}n_s/\mathrm{d}(\ln k) > 0$, but to calculate the amplitude, more must be known about the details of the pre-bounce era.
- **Particle production during the bounce**: It has been pointed out that particle production may be important during the LQC bounce, but can it provide sufficient asymmetry?
- **Non-Gaussianities**: A small sound speed typically causes large non-Gaussianities. How large? What about the bounce?
- **Anisotropies during the bounce**: In the absence of an ekpyrotic phase, we expect anisotropies to dominate the dynamics during the bounce. How might the presence of anisotropies change the predictions?

Conclusions

In the ΛCDM bounce scenario, quantum vacuum fluctuations become nearly scale-invariant with a slight red tilt for the modes that exit the sound horizon when the effective equation of state is slightly negative.

This model requires an asymmetric bounce, and makes three important predictions beyond scale-invariance:

- A small tensor-to-scalar ratio,
- A positive running of n_s, and
- $n_t > n_s - 1$.

There is an LQC-specific effect of an extra damping of the tensor-to-scalar ratio by a factor of $1/4$ which is a potential observational test for the theory.

Thank you for your attention!