Time dependence and the effective potential in Higgs inflation

Damien P. George

DAMTP & The Cavendish,
The Herchel Smith Fund,
Cambridge, UK

University of Sussex — 25th February 2013

Overview

Higgs inflation paradigm.

Quantum corrections and the calculation of the effective potential:

- expanding the action,
- split 2-point interactions,
- in-in formalism,
- computing the tadpole diagrams,
- putting it all together.

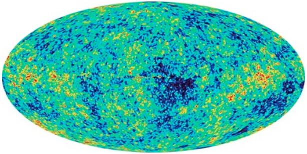
Work in progress:

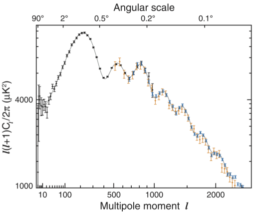
- full SM,
- running of the RGEs.

Higgs inflation paradigm

Inflation:

- solves flatness and horizon (and monopole) problems,
- provides seeds for structure formation,
- is driven by an inflaton field rolling slowly down its potential.





The standard model Higgs field can be play the role of the inflaton
Need a single additional term: $\xi|H|^2 R$.
Bezrukov & Shaposhnikov PLB **659** 703 (2008).

Such a term is expected to be generated, current bounds: $\xi \lesssim 2.6 \times 10^{15}$
Atkins & Calmet PRL **110** 051301 (2013).

Higgs inflation — main results

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\ldots + \xi H^\dagger H R + \ldots - \lambda\left(H^\dagger H - v^2/2\right)^2 + \ldots\right].$$

Go to Einstein frame and canonical kinetic term. In large field regime expand the potential in $\delta \equiv 1/(\xi\phi^2) \ll 1$. Slow roll parameters:

$$\epsilon = 4\delta^2/3, \quad \eta = -4(\delta - \delta^2)/3,$$

Inflation ends for $\epsilon \approx 1$, giving number of e-folds $N \approx \frac{6}{8\delta_*}$.
With $N \approx 60$, we find $\delta_* \sim 6/8N \approx 1/80$.

Cobe normalisation gives

$$(V/\epsilon)_* = (0.027)^4 \quad \Rightarrow \quad \lambda/\xi^2 = 4\times 10^{-10} \quad \Rightarrow \quad \xi = 5\times 10^4\sqrt{\lambda}$$

Spectral index and tensor-to-scalar ratio are compatible with data:

$$n_s = 1 + 2\eta - 6\epsilon \approx 1 - \frac{8}{3}\delta_* - \frac{16}{3}\delta_*^2 \approx 0.967, \qquad r = 16\epsilon = 8\delta_*^2 = 0.0033.$$

Running of λ and stability of Higgs vacuum with $m_h = 126$GeV??

Quantum corrections

To test BSM physics using cosmological data must have a precise understanding of the scalar field's dynamics.

$\rightarrow$ need to go beyond classical and include dominant quantum effects

One-loop effective action for a scalar field in an expanding universe.
Birrell & Davies, 1982; Candelas & Raine, 1975; Ringwald, 1987a, 1987b.

Describes the backreaction of the quantum fluctuations of the scalar field on the time-dependent background field, calculated systematically in a loop expansion.

Here: extend these results by including a coupling to a gauge field. The toy model is Abelian Higgs with a U(1) gauge field in FLRW background. Calculation found in DG, Mooij & Postma JCAP **11** 043 (2012).

Generalises the Coleman-Weinberg potential to *time-dependent* background fields in a curved space-time; Coleman & Weinberg 1973.

The toy model

A U(1) gauge field A_μ and charge scalar Φ in a curved background.

$$\mathrm{d}s^2 = a^2(\tau)\left(\mathrm{d}\tau^2 - \mathrm{d}\vec{x}^2\right), \qquad \Phi(x^\mu) = \frac{1}{\sqrt{2}}\big(\phi(\tau) + h + i\theta\big).$$

The action in R_ξ gauge:

$$S_{\mathrm{tot}} = \int \mathrm{d}^4x\sqrt{-g}(\mathcal{L} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}),$$

with

$$\mathcal{L} = -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}D_\mu\Phi(D_\nu\Phi)^\dagger - V(\Phi),$$

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi}G^2, \qquad G = g^{\mu\nu}\nabla_\mu A_\nu - \xi g(\phi + h)\theta,$$

$$\mathcal{L}_{\mathrm{FP}} = \bar{\eta}g\frac{\delta G}{\delta\alpha}\eta. \qquad (\alpha \text{ is a U(1) gauge transformation})$$

Quantum fields h, θ, A_μ, η in a time-dependent background $a(\tau)$, $\phi(\tau)$.
Background metric is fixed, backreaction assumed negligible.

Expanding the action

Expand the action up to 3rd order in the quantum fields.
The one-point vertex is

$$S^{(1)} = \int \mathrm{d}^4 x \left(-\hat{\lambda}_h \hat{h} \right) ,$$

where

$$\hat{\lambda}_h = \left(\partial_\tau^2 - (\mathcal{H}' + \mathcal{H}^2) \right) \hat{\phi} + \hat{V}_{\hat{\phi}} = a^3 \left[\ddot{\phi} + 3H\dot{\phi} + V_\phi \right] .$$

Working in conformal frame with $\mathcal{H} = a'/a$, $\hat{\phi} = a\phi$, $\hat{V} = a^4 V(\hat{\phi})$, etc.
Note A does not need a hat.

Easier to work in conformal time, as the resulting action has a form similar to the Minkowski action, and all the machinery developed for this can be used.

Mooij & Postma 2011; Heitmann, 1996.

Quadratic expansion

$$S^{(2)} = \int \mathrm{d}^4x \Bigg\{ -\frac{1}{2}A_\mu \left[-(\partial^2 + \hat{m}^2_{(\mu)})\eta^{\mu\nu} + \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu \right] A_\nu$$
$$- A_0(\hat{m}^2)^{i0}A_i - \hat{m}^2_{A\theta}A_0\hat{\theta}$$
$$- \frac{1}{2}\hat{h}(\partial^2 + \hat{m}^2_h)\hat{h} - \frac{1}{2}\hat{\theta}(\partial^2 + \hat{m}^2_\theta)\hat{\theta} - \hat{\bar{\eta}}(\partial^2 + \hat{m}^2_\eta)\hat{\eta} \Bigg\}.$$

2-point *interactions* are:

$$\hat{m}^2_{(\mu)} = g^2\hat{\phi}^2 + \frac{2}{\xi}\left(\mathcal{H}' - 2\mathcal{H}^2\right)\delta_{\mu 0},$$
$$\hat{m}^2_h = \hat{V}_{hh} - (\mathcal{H}' + \mathcal{H}^2),$$
$$\hat{m}^2_\theta = \hat{V}_{\theta\theta} + \xi g^2\hat{\phi}^2 - (\mathcal{H}' + \mathcal{H}^2),$$
$$\hat{m}^2_\eta = \xi g^2\hat{\phi}^2 - (\mathcal{H}' + \mathcal{H}^2).$$

Off-diagonal 2-point terms are:

$$\hat{m}^2_{A\theta} = 2g(\partial_\tau - \mathcal{H})\hat{\phi}, \qquad (\hat{m}^2)^{i0} = \frac{2}{\xi}\mathcal{H}\partial^i.$$

The 3-point interaction vertices are

$$S^{(3)} = \int \mathrm{d}^4x \Big[-\frac{1}{2}\hat{\lambda}_{hhh}\hat{h}^3 - \frac{1}{2}\hat{\lambda}_{h\theta\theta}\hat{h}\hat{\theta}^2 - \frac{1}{2}\hat{\lambda}_{hAA}\hat{h}\hat{A}^2 - \hat{\lambda}_{h\eta\eta}\hat{h}\hat{\bar{\eta}}\hat{\eta} - \hat{h}\hat{\lambda}_{hA\theta}A_0\hat{\theta}\Big],$$

where

$$\begin{aligned}
\hat{\lambda}_{hhh} &= \partial_{\hat{\phi}}\hat{m}_h^2 = \hat{V}_{\phi hh}, \\
\hat{\lambda}_{h\theta\theta} &= \partial_{\hat{\phi}}\hat{m}_\theta^2 = \hat{V}_{\phi\theta\theta} + 2\xi g^2\hat{\phi}, \\
\hat{\lambda}_{hAA} &= \partial_{\hat{\phi}}\hat{m}_A^2 = -2g^2\hat{\phi}, \\
\hat{\lambda}_{h\eta\eta} &= \partial_{\hat{\phi}}\hat{m}_\eta^2 = 2\xi g^2\hat{\phi}, \\
\hat{\lambda}_{hA\theta} &= 2g(-\partial_\tau - \mathcal{H}).
\end{aligned}$$

Expressed in terms of 2-point interactions; formally, $\partial_{\hat{\phi}}\mathcal{H} = 0$.

Integration by parts used to get $\lambda_{hA\theta}$ acting on $A_0\hat{\theta}$. Also discarded a ∂_i term.

Time dependence of interactions

The background is time-dependent: $a(\tau)$, $\phi(\tau)$.
$\rightarrow$ the "masses" (really 2-point interactions) are also time-dependent.

Split these interaction into time-independent and time-dependent parts:

$$\hat{m}^2(\tau) = \bar{\hat{m}}^2 + \delta\hat{m}^2(\tau),$$

with the split defined by

$$\delta\hat{m}^2(0) = 0.$$

- $\bar{\hat{m}}^2$ contributes to the free Lagrangian. We call it the mass and it determines the propagator.
- $\delta\hat{m}^2(\tau)$ is treated as a proper 2-point interaction in Feynman diagrams.

The loop expansion is independent of the split of the 2-point terms into a free and interacting part.

Also demand off-diagonal 2-point interactions vanish at $\tau = 0$.

In-in formalism

Since we are interested in expectation values of the background field and their evolution with time (not scattering amplitudes) we use the in-in formalism (closed-time-path (CTP) or Schwinger-Keldysh formalism).

Schwinger, 1961; Keldysh, 1964; Jordan, 1986; Bakshi & Mahanthappa 1963a, 1963b; Calzetta & Hu, 1987; Weinberg, 2005.

- Expectation values are computed using an action

$$S = S[\phi_i^+] - S[\phi_i^-]$$

  with boundary condition $\phi_i^+(t) = \phi_i^-(t)$.
- All fields, propagators and vertices are labelled by $\pm$ superscripts.
- The propagator $D^{\pm\pm}(x - x')$ connects vertices $\lambda^{\pm}(x)$ and $\lambda^{\pm}(x')$.
- The action of the minus-fields is defined with an overall minus sign, so

$$\left[m^2_{\alpha\beta}\right]^- = -\left[m^2_{\alpha\beta}\right]^+, \qquad \left[\lambda_{h\alpha\beta}\right]^- = -\left[\lambda_{h\alpha\beta}\right]^+.$$

In-in formalism — propagators

We construct the propagators from the free, time-independent part of the quadratic action. For example, for h the propagators are defined as the solutions of

$$\begin{pmatrix} \partial_x^2 + \hat{\bar{m}}_h^2 & 0 \\ 0 & -(\partial_x^2 + \hat{\bar{m}}_h^2) \end{pmatrix} \begin{pmatrix} D_h^{++}(x-y) & D_h^{+-}(x-y) \\ D_h^{-+}(x-y) & D_h^{--}(x-y) \end{pmatrix} = -i\delta(x-y)\mathbf{I}_2.$$

This defines D^{++} as the usual Feynman propagator, D^{--} as the anti-Feynman propagator, and D^{-+} and D^{+-} as Wightman functions.

$$\begin{aligned} D_h^{-+}(x_a - x_b) &= D_h^{+-}(x_b - x_a) = D_{h,ab}, \\ D_h^{++}(x_a - x_b) &= D_{h,ab}\Theta_{ab} + D_{h,ba}\Theta_{ba}, \\ D_h^{--}(x_a - x_b) &= D_{h,ab}\Theta_{ba} + D_{h,ba}\Theta_{ab}, \end{aligned}$$

Vanishing of the tadpole → equation of motion for $\phi(t)$.
Include 1-loop tadpoles → quantum corrected equation of motion.

$$\langle h^+(\tau, \vec{x})\rangle = 0.$$

Vanishing of the h^- component gives the same result.

From first principles, expand out the path integral:

$$\begin{aligned}
0 &= \langle h^+(x)\rangle \\
&= \int D\psi_\alpha^+ \mathcal{D}\psi_\beta^- h^+(x) \mathrm{e}^{i(S_0[\psi_\alpha^+]+S_{\mathsf{int}}[\psi_\alpha^+])-i(S_0[\psi_\beta^-]+S_{\mathsf{int}}[\psi_\beta^-])} \\
&= \int \mathcal{D}\psi_\alpha^+ \mathcal{D}\psi_\beta^- h^+(x) \mathrm{e}^{iS_0[\psi_\alpha^+]-iS_0[\psi_\beta^-]} \left[1 + i\int \mathrm{d}^4 y (\mathcal{L}_{\mathsf{int}}^+ - \mathcal{L}_{\mathsf{int}}^-) + \dots\right] \\
&= -i\int \mathrm{d}^4 y \left[D_h^{++}(x-y) - D_h^{+-}(x-y)\right] A(y).
\end{aligned}$$

Equation of motion: $A(y) = 0$.

The one-loop equation of motion

For divergent terms, need to go to 3^{rd} order in $\mathcal{L}_{\text{int}}$.

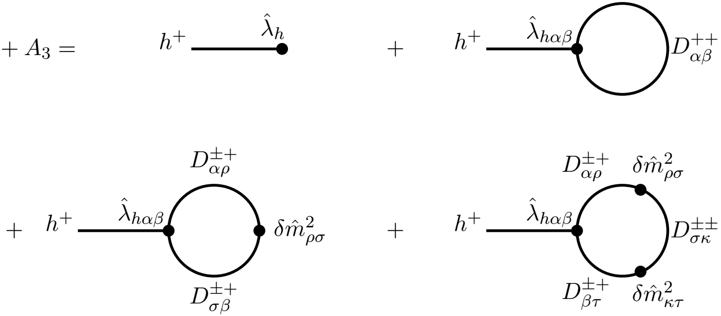
$$\begin{aligned}
0 &= A_{\text{cl}} + A_1 + A_2 + A_3 + \text{finite} \\
&= \hat{\lambda}_h(x) + S_{\alpha\beta}\hat{\lambda}_{h\alpha\beta}(x) D^{++}_{\alpha\beta}(0) \\
&\quad - iS_{\alpha\beta\gamma\delta}\hat{\lambda}_{h\alpha\beta}(x)\int \mathrm{d}^4x' D^{+\pm}_{\alpha\gamma}(x-x')\left[\delta\hat{m}^2_{\gamma\delta}(x')\right]^{\pm} D^{\pm+}_{\delta\beta}(x'-x) \\
&\quad - S_{\alpha\beta\gamma\delta\rho\sigma}\hat{\lambda}_{h\alpha\beta}(x)\int\mathrm{d}^4x'\int\mathrm{d}^4x''\Big\{D^{+\pm}_{\alpha\gamma}(x-x')\left[\delta\hat{m}^2_{\gamma\delta}(x')\right]^{\pm} D^{\pm\pm}_{\delta\rho}(x'-x'') \\
&\qquad\qquad \times \left[\delta\hat{m}^2_{\rho\sigma}(x'')\right]^{\pm} D^{\pm+}_{\sigma\beta}(x''-x)\Big\} \\
&\quad + \text{finite.}
\end{aligned}$$

- Sum over: field-type, Lorentz indices, possibilities for $\pm$.
- $S_{\alpha\beta\ldots}$ are appropriate symmetry factors.
- Masses $\hat{\bar{m}}^2_\alpha$ in propagators D_α.
- $\delta\hat{m}^2_{\alpha\beta}(\tau)$ and $\hat{\lambda}_{h\alpha\beta}(\tau)$: 2- and 3-point interaction vertices.
- Concerned only with the *UV divergent contributions* $\rightarrow$ consider only up to three vertices.

Tadpole diagrams

In terms of diagrams: all 1PI tadpole graphs with one external h^+ leg.

$A_{\text{cl}} + A_1 + A_2 + A_3 =$



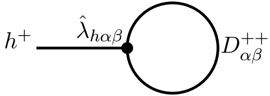
Computing these diagrams gives the quantum corrected equation of motion at the 1-loop level.

Classical part: $A_{\text{cl}} = \hat{\lambda}_h(x)$.

First order

First order diagrams as in Minkowski [Mooij & Postma, JCAP **1109**, 006 (2011)].

Four diagrams contribute:
h, θ, η, A^μ in the loop.



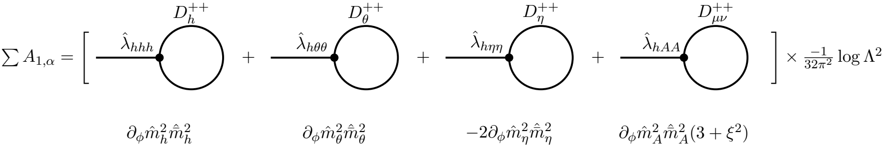
Each diagram has the same structure:

$$\begin{aligned} A_{1,\alpha} &= \frac{1}{2}\partial_\phi \hat{m}_\alpha^2 D_\alpha^{++}(0) \\ &= \frac{1}{2}\partial_\phi \hat{m}_\alpha^2 \frac{1}{4\pi^2}\int_0^{\hat{\Lambda}} k^2 \mathrm{d}k \left[\frac{1}{k} - \frac{1}{2}\frac{\hat{\bar{m}}_\alpha^2}{k^3} + \ldots\right] \\ &= \frac{1}{16\pi^2}\partial_\phi \hat{m}_\alpha^2 \left[\hat{\Lambda}^2 - \frac{1}{2}\hat{\bar{m}}_\alpha^2 \ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite}\right]. \end{aligned}$$

The variable k is the comoving momentum with $k < \hat{\Lambda}$ a comoving cutoff.
Gauge loop is expressed as scalar propagators $-\eta^{\mu\nu} D^{++}_{\mu\nu}(0) = 3D^{++}_A(0) + \xi D^{++}_\xi(0)$.

First order diagrams

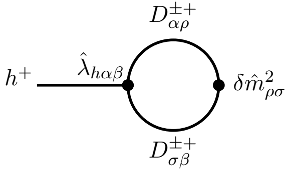
For the log-divergent part, sum of all first-order diagrams is



Note that $\partial_\phi \hat{m}_\alpha^2$ is time-dependent and evaluated at τ

For example, $\partial_\phi \hat{m}_A^2 = -2g^2 \hat{\phi}(\tau)$.

Thus, A_1 is a function of τ.



At second order in $\mathcal{L}_{\text{int}}$ the loop diagrams with one 2-point insertion contribute. We split them into three parts:

- A_2^{Mink} contains all scalar loops, and the gauge boson loop where only the diagonal part of $\hat{m}^2_{(\mu)}$ is inserted. Also a mixed θA^0-loop. This part is analogous to the equivalent Minkowski calculation.
- A_2^{mass} contains the gauge boson loop with a $\delta\hat{m}_0^2$.
- A_2^{mix} contains the gauge boson loop with a $(\delta\hat{m}^2)^{0i}$.

These last two diagrams are both absent in Minkowski.

Calculating $A_2^{\rm Mink}$

The scalar Higgs loop with one 2-point insertion gives

$$A_{2,h}^{\rm Mink} = -\frac{i}{2}\partial_\phi \hat{m}_h^2(\tau)\int \mathrm{d}^4x_b \delta\hat{m}_h^2(\tau_b)\Big[D_h^{++}(x_a-x_b)D_h^{++}(x_b-x_a) - D_h^{+-}(x_a-x_b)D_h^{-+}(x_b-x_a)\Big]$$

$$= -\frac{i}{2}\partial_\phi \hat{m}_h^2(\tau)\int \mathrm{d}^4x_b \delta\hat{m}_h^2(\tau_b)\Theta_{ab}\Big[D_{h,ab}^2 - D_{h,ba}^2\Big].$$

Everything expressed in terms of Wightman functions. Fourier transform, perform the d^3x_b integral, integrate by parts to extract the UV divergent piece:

$$A_{2,h}^{\rm Mink} = -\partial_\phi \hat{m}_h^2(\tau)\delta\hat{m}_h^2(\tau)\int \frac{\mathrm{d}^3k}{64\pi^3\vec{k}^3}$$

$$= -\partial_\phi \hat{m}_h^2(\tau)\delta\hat{m}_h^2(\tau)\frac{1}{32\pi^2}\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite.}$$

Calculating A_2^{Mink}

The Goldstone boson loop $A_{2,\theta}^{\mathrm{Mink}}$ and ghost loop $A_{2,\eta}^{\mathrm{Mink}}$ are similar to $A_{2,h}^{\mathrm{Mink}}$. Ghost has overall factor (-2).

Gauge boson loop follows the same steps but with a non-trivial Lorentz structure:

$$
\begin{aligned}
A_{2,A}^{\mathrm{Mink}} &= -\frac{i}{2}\partial_\phi \hat{m}_A^2 \int \mathrm{d}^4 x_b \delta\hat{m}_A^2(\tau_b)\eta^{\mu\nu}\eta^{\rho\sigma}\Big[D_{\mu\rho}^{++}(x_a - x_b)D_{\sigma\nu}^{++}(x_b - x_a) \\
&\qquad\qquad - D_{\mu\rho}^{+-}(x_a - x_b)D_{\sigma\nu}^{-+}(x_b - x_a)\Big] \\
&= -\partial_\phi \hat{m}_A^2 \int_0^\tau \mathrm{d}\tau_b \delta\hat{m}_A^2(\tau_b) \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{C_{IJ}\sin\left[(\bar{\omega}_I + \bar{\omega}_J)(\tau - \tau_b)\right]}{(2\bar{\omega}_I)(2\bar{\omega}_J)} \\
&= -\partial_\phi \hat{m}_A^2 \delta\hat{m}_A^2(\tau)\frac{(3+\xi^2)}{32\pi^2}\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite.}
\end{aligned}
$$

C_{IJ} encodes the structure arising from Wightman functions in Fourier space.

Finally, the mixed θA^0-loop gives

$$A^{\rm Mink}_{2,A\theta} = -i\hat{\lambda}_{h\theta A}(\tau)\int {\rm d}^4x_b\,\delta\hat{m}^2_{A\theta}(\tau_b)\left[D^{++}_{00,ab}D^{++}_{\theta,ba} - D^{+-}_{00,ab}D^{-+}_{\theta,ba}\right]$$
$$= -2\hat{\lambda}_{h\theta A}(\tau)\int_0^\tau {\rm d}\tau_b\,\delta\hat{m}^2_{A\theta}(\tau_b)\int\frac{{\rm d}^3k}{(2\pi)^3}\frac{C_I\sin[(\bar{\omega}_I+\bar{\omega}_\theta)(\tau-\tau_b)]}{(2\bar{\omega}_I)(2\bar{\omega}_\theta)}$$
$$= 2\hat{\lambda}_{h\theta A}(\tau)\delta\hat{m}^2_{A\theta}(\tau)\frac{(3+\xi)}{128\pi^2}\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite.}$$

The 3-point vertex is $\hat{\lambda}_{h\theta A}(\tau) = 2g(-\partial_\tau - \mathcal{H}(\tau))$.

Adding all the Minkowski pieces together gives

$$A_2^{\rm Mink} = \frac{-1}{32\pi^2}\sum_\alpha S_\alpha\partial_\phi\hat{m}^2_\alpha\delta\hat{m}^2_\alpha\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \frac{(3+\xi)}{64\pi^2}\hat{\lambda}_{hA\theta}\delta\hat{m}^2_{A\theta}\ln(\hat{\Lambda}/\hat{\bar{m}})^2.$$

Symmetry factors $S_\alpha = \{1, 1, -2, 3, 1\}$ for $\{h, \theta, \eta, A, \xi\}$; $\hat{m}^2_\xi = \xi\hat{m}^2_A$.

Calculating A_2^{mass}

Next, the Lorentz violating mass m_0^2 gives

$$
\begin{aligned}
A_2^{\mathrm{mass}} &= -\frac{i}{2}\partial_\phi \hat{m}_A^2(\tau)\int \mathrm{d}^4x_b\ \delta\hat{m}_0^2(\tau_b)\eta^{\mu\nu}\left[D^{++}_{\mu 0,ab}D^{++}_{0\nu,ba} - D^{+-}_{\mu 0,ab}D^{-+}_{0\nu,ba}\right] \\
&= -\partial_\phi \hat{m}_A^2(\tau)\int_0^\tau \mathrm{d}\tau_b\ \delta\hat{m}_0^2(\tau_b)\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{C_{IJ}\sin\left[(\bar{\omega}_I+\bar{\omega}_J)(\tau-\tau_b)\right]}{(2\bar{\omega}_I)(2\bar{\omega}_J)} \\
&= -\partial_\phi \hat{m}_A^2(\tau)\delta\hat{m}_0^2(\tau)\frac{(3+\xi^2)}{4\times 32\pi^2}\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite.}
\end{aligned}
$$

This diagram is not present in Minkowski.

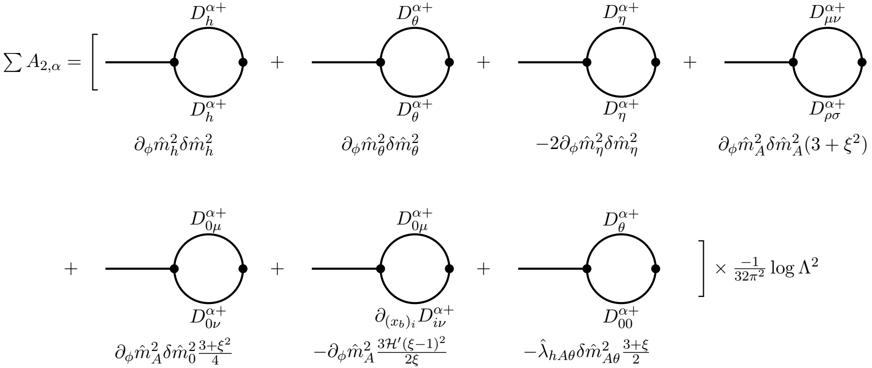
Calculating $A_2^{\rm mix}$

The off-diagonal interaction $(\delta\hat{m}^2)^{0i}$ contains a spatial derivative and brings down a factor of the momentum.

$$\begin{aligned} A_2^{\rm mix} &= i\partial_\phi \hat{m}_A^2(\tau)\int \mathrm{d}^4x_b (\delta\hat{m}^2)^{0i}(\tau_b)\eta^{\mu\nu}\left[D^{++}_{\mu0,ab}D^{++}_{i\nu,ba} - D^{+-}_{\mu0,ab}D^{-+}_{i\nu,ba}\right] \\ &= -\frac{2}{\xi}\partial_\phi \hat{m}_A^2(\tau)\int_0^\tau \mathrm{d}\tau_b \mathcal{H}(\tau_b)\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{2C_{IJ}\cos\left[(\bar{\omega}_I+\bar{\omega}_J)(\tau-\tau_b)\right]}{(2\bar{\omega}_I)(2\bar{\omega}_J)} \\ &= -\frac{2}{\xi}\partial_\phi \hat{m}_A^2(\tau)\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{2C_{IJ}\mathcal{H}'(\tau)}{(2\bar{\omega}_I)(2\bar{\omega}_J)(\bar{\omega}_I+\bar{\omega}_J)^2} + \text{finite} \\ &= \partial_\phi \hat{m}_A^2(\tau)\frac{3\mathcal{H}'(\tau)(1-\xi)^2}{64\pi^2\xi}\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite.} \end{aligned}$$

We have a cosine instead of a sine $\rightarrow$ integrate by parts twice to isolate the leading term in the UV limit. Obtain a result proportional to $\mathcal{H}'$.

Summary of second order tadpoles



These Feynman diagrams are in (conformal) coordinate space.

All time-dependent quantities ($\hat{\lambda}_{A\theta}$, $\hat{m}_\alpha^2$, $\delta \hat{m}_\alpha^2$ and $\mathcal{H}$) are evaluated at τ.

Third order

Not done yet!

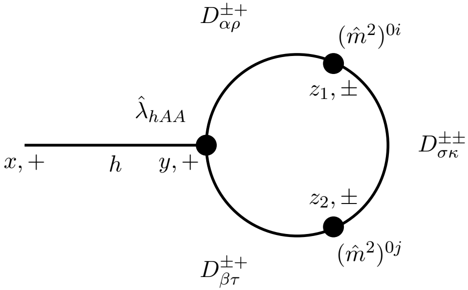
All 3rd order diagrams with 2-point insertions are UV finite, *except for one*.

Power counting:

- d^4k in 4d,
- $1/k^2$ per propagator,
- k for a derivative from $(\hat{m}^2)^{0i} = \frac{2}{\xi}\mathcal{H}\partial^i$.

In 4d, 3 propagators and 2 derivatives $\rightarrow$ logarithmically divergent.

The third order diagram



$$A_3 = \frac{1}{2}\partial_\phi \hat{m}_A^2(\tau)\int \mathrm{d}^4x_b \int \mathrm{d}^4x_c (\delta\hat{m}^2)^{0i}(\tau_b)(\delta\hat{m}^2)^{0j}(\tau_c)\eta^{\mu\nu}D^{ab}_{\mu\rho}D^{bc}_{\sigma\kappa}D^{ca}_{\tau\nu}.$$

Sum over $\{\rho, \sigma, \kappa, \tau\}$ from $\{0, i, j\}$. Sum over $\{a, b, c\}$ from $\pm$.

End result is

$$A_3 = \partial_\phi \hat{m}_A^2(\tau)\mathcal{H}^2(\tau)\frac{-6(1+\xi)}{64\pi^2\xi}\ln(\hat{\Lambda}/\hat{\bar{m}})^2 + \text{finite}.$$

Adding it all up

We have computed all *quadratically* and *logarithmically divergent* contributions to the one-loop equation of motion.

The first and second order combined $\hat{A}^{\rm Mink} = \hat{A}_1^{\rm Mink} + \hat{A}_2^{\rm Mink}$ is

$$\hat{A}^{\mathsf{Mink}} = \frac{1}{16\pi^2}\sum_\alpha S_\alpha \partial_{\hat{\phi}} \hat{m}_\alpha^2 \left[\hat{\Lambda}^2 - \frac{1}{2}\hat{m}_\alpha^2 \ln(\hat{\Lambda}/\hat{\bar{m}})^2\right] + \frac{(3+\xi)}{64\pi^2}\hat{\lambda}_{hA\theta}\hat{m}_{A\theta}^2 \ln(\hat{\Lambda}/\hat{\bar{m}})^2.$$

This is independent of how the 2-point interaction is split, since the 1st and 2nd order pieces combine to give $\hat{m}_\alpha^2 = \hat{\bar{m}}_\alpha^2 + \delta\hat{m}_\alpha^2$.

For A_0 mass insertions we have the 2nd order piece

$$\hat{A}^{\mathsf{mass}} = -\partial_{\hat{\phi}}\hat{m}_A^2 \delta\hat{m}_0^2 \frac{3+\xi^2}{128\pi^2}\ln(\hat{\Lambda}/\hat{\bar{m}})^2.$$

For the mixed piece we have contributions from 2nd and 3rd order

$$\hat{A}^{\mathsf{mix}} = \partial_{\hat{\phi}}\hat{m}_A^2 \left(\frac{3\mathcal{H}'(1-\xi)^2}{\xi} - \frac{6\mathcal{H}^2(1+\xi)}{\xi}\right)\frac{1}{64\pi^2}\ln(\hat{\Lambda}/\hat{\bar{m}})^2.$$

Effective action

Have the 1-loop equation of motion $\hat{A}^{\text{1-loop}}$. Corresponding Lagrangian is

$$\hat{A}^{\text{1-loop}} = \left(\frac{\delta\hat{\mathcal{L}}^{\text{1-loop}}}{\delta\hat{\phi}'}\right)' - \frac{\delta\hat{\mathcal{L}}^{\text{1-loop}}}{\delta\hat{\phi}}.$$

The action is then simply

$$\Gamma^{\text{1-loop}} = \int \mathrm{d}^3x\,\mathrm{d}\tau\;\hat{\mathcal{L}}^{\text{1-loop}}.$$

All terms polynomial in $\hat{\phi}$ are easily integrated to find the Lagrangian.

Only one is not polynomial; in $\hat{A}^{\text{Mink}}$ there is:

$$\hat{\lambda}_{hA\theta}\hat{m}^2_{A\theta} = 4g^2\left(-\hat{\phi}'' + \mathcal{H}'\hat{\phi} + \mathcal{H}^2\hat{\phi}\right).$$

This comes from a Lagrangian

$$-\frac{1}{2}\hat{m}^4_{A\theta} = -2g^2\left(\hat{\phi}'^2 - 2\mathcal{H}\hat{\phi}\hat{\phi}' + \mathcal{H}^2\hat{\phi}^2\right).$$

Gauge invariance

Going to coordinate time and taking off the hats, the total 1-loop effective action is

$$\Gamma^{\text{1-loop}} = \frac{-1}{16\pi^2} \int \mathrm{d}^3x \mathrm{d}t \sqrt{-g} \Bigg\{ \left(V_{hh} + V_{\theta\theta} + 3m_A^2\right) \Lambda^2 - \Bigg[\left(V_{hh} - \dot{H} - 2H^2\right)^2 + \left(V_{\theta\theta} - \dot{H} - 2H^2\right)^2 + 3m_A^4 + 2\xi V_{\theta\theta} m_A^2 - (6+2\xi) g^2 \dot{\phi}^2 + 6m_A^2 \left(\dot{H} + 2H^2\right) \Bigg] \frac{\ln(\Lambda/\bar{m})^2}{4} \Bigg\}.$$

Result is still gauge variant. Use the Nielsen identities

$$\frac{\partial V_{\text{eff}}}{\partial \xi} + \frac{\partial \phi}{\partial \xi} \frac{\partial V_{\text{eff}}}{\partial \phi} = 0.$$

V_{eff} is only gauge invariant when the background field is in a minimum of the potential, i.e. the background field satisfies its equation of motion.

Going on-shell enables us to rewrite the $\dot{\phi}^2$ term and eliminate ξ.

Self-contained summary of the results

FLRW metric background $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\vec{x}^2$.

Abelian Higgs model with U(1) gauge symmetry

$$S_{\mathrm{tot}} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}D_\mu\Phi(D_\nu\Phi)^\dagger - V(\Phi)\right].$$

Expand the Higgs around a time-dependent background

$$\Phi(x^\mu) = \frac{1}{\sqrt{2}}\big(\phi(t) + h(t,\vec{x}) + i\theta(t,\vec{x})\big).$$

UV divergent contributions at one loop are

$$\Gamma^{1-\mathrm{loop}} = \frac{-1}{16\pi^2}\int \mathrm{d}^3x\mathrm{d}t\sqrt{-g}\Bigg[(\tilde{V}_{hh} + \tilde{V}_{\theta\theta} + 3m_A^2)\Lambda^2 - \left(\tilde{V}_{hh}^2 + \tilde{V}_{\theta\theta}^2 + 3m_A^4 - 6\tilde{V}_{\theta\theta}m_A^2\right)\frac{\ln(\Lambda/\bar{m})^2}{4}\Bigg],$$

where the time-dependent "shifted scalar mass" is

$$\tilde{V}_{\alpha\alpha} \equiv V_{\alpha\alpha} - \dot{H} - 2H^2.$$

Additional scalars and fermions in the loops

If the Higgs field couples to additional scalars χ_α and/or fermion fields ψ_β, we get an additional contribution

$$\delta\Gamma^{1-\text{loop}} = -\frac{1}{16\pi^2}\int \mathrm{d}^3x\mathrm{d}t\sqrt{-g}\Bigg[\sum_{\chi_\alpha}\left(\tilde{V}_{\alpha\alpha}\Lambda^2 - \tilde{V}^2_{\alpha\alpha}\frac{\ln(\Lambda/\bar{m})^2}{4}\right) - \sum_{\psi_\beta}\left(m_\beta^2\Lambda^2 - \left(m_\beta^4 - \tilde{V}_{\theta\theta}m_\beta^2\right)\frac{\ln(\Lambda/\bar{m})^2}{4}\right)\Bigg].$$

The sum is over all bosonic and fermion real degrees of freedom, where a Weyl (Dirac) fermion counts as 2 (4) degrees of freedom.

The shifted scalar mass is as before $\tilde{V}_{\alpha\alpha} \equiv V_{\alpha\alpha} - \dot{H} - 2H^2$.

For fermions, assume a Yukawa interaction $m_\psi \propto \phi$.

Comparison with literature

The results agree with the expressions in the literature in the appropriate limits:

- Minkowski case ($H = \dot{H} = 0$, and thus $\tilde{V}_{\alpha\alpha} = V_{\alpha\alpha}$) it matches Mooij & Postma 2011.
- In the de Sitter limit $\dot{H} = 0$, and for a time-independent Higgs field ($V_{\theta\theta} = 0$ by Goldstone's theorem), it agrees with Garbrecht 2007.
- Taking both the Minkowski limit and a static background field we retrieve the classic CW potential, Coleman & Weinberg 1973.

Next steps

We only calculate the UV divergent terms, as these will generically give the dominant contribution. Using a renormalisation prescription, these terms (and wavefunction renormalisation) suffice to derive the RGEs and find the RG improved action.

To apply our results to Higgs inflation we need to extend them:

1. Include back reaction from gravity.
2. Go from U(1) toy model to SM gauge group.
3. Consider non-minimal coupling to gravity.
4. To relate parameters to low energy observables need RG flow.

Non-minimal coupling to gravity

Include non-minimal coupling to gravity, $\xi|\Phi|^2 R$.

Transform to the Einstein frame, then our 1-loop results can be applied.

For the SM, not so straight forward: Higgs H has 4 degrees of freedom, kinetic term is non-minimal in Einstein frame:

$$\frac{\mathcal{L}_e}{\sqrt{-g}} \supset \frac{1}{2}\gamma_{ij}\partial\phi_i\partial\phi_j = \frac{1}{2}\left[\frac{\delta_{ij}}{\Omega^2} + \frac{6\xi^2}{\Omega^4}\phi_i\phi_j\right]\partial\phi_i\partial\phi_j .$$

Field-space metric γ_{ij} cannot be diagonalised everywhere. Instead, diagonalise it at each point in field-space, giving a spectrum of 4 scalars with masses a function of ϕ_{BG}.

Can then apply our 1-loop results.

Running of the RGEs

To connect low energy observables (at LHC) with high energy ones (inflation and CMB) need to run the couplings from M_Z to M_{infl}.

Jordan versus Einstein frame:

- Jordan has gravity fluctuations which should be important (can we ignore them?). RGEs:

$$\beta_\lambda = \frac{9\lambda^2}{8\pi^2}, \quad \beta_{m^2} = \frac{3\lambda m^2}{8\pi^2}, \quad \beta_\xi = \frac{3\lambda(\xi + 1/6)}{8\pi^2},$$
$$\beta_\kappa = \frac{m^2(\xi + 1/6)}{8\pi^2}, \quad \beta_\Lambda = \frac{m^4}{32\pi^2}$$

- Einstein has non-minimal kinetic structure (diagonalise at each point in field-space?) and non-renormalisable terms.

  ξ will run, so is reintroduced in Einstein frame. Not such a problem.

Conclusions

Higgs inflation is simple and promising, although slight tension with $m_h = 126$GeV.

To constrain BSM models using cosmological data need quantum corrections to scalar potential, and running of the parameters.

We gave the effective potential with time-dependent FLRW background and time-dependent Higgs vev, with a gauge field.

Work in progress: go to full SM with non-minimal coupling, and determine RGEs.

Selected references

- S. R. Coleman, E. J. Weinberg, "Radiative Corrections as the Origin of Spontaneous Symmetry Breaking", Phys. Rev. **D7** (1973) 1888-1910.
- Bezrukov & Shaposhnikov, "The Standard Model Higgs boson as the inflaton", PLB **659** 703 (2008).
- K. Heitmann, " Thermalisierung bei kosmologischen Phasenübergängen: Eichfelder", Master's Thesis, 1996.
- K. Heitmann, "Non-equilibirum dynamics of symmetry breaking and gauge fields in quantum field theory", PhD Thesis, 2000.
- S. Weinberg, "Quantum contributions to cosmological correlations", Phys. Rev. **D72** (2005) 043514. [hep-th/0506236].
- S. Mooij and M. Postma, JCAP **1109**, 006 (2011) [arXiv:1104.4897].
- D. P. George, S. Mooij and M. Postma, "Effective action for the Abelian Higgs model in FLRW", JCAP **1211** (2012) 043 [arXiv:1207.6963].