Naturally Aligned Two Higgs Doublet Model and its Collider Signatures

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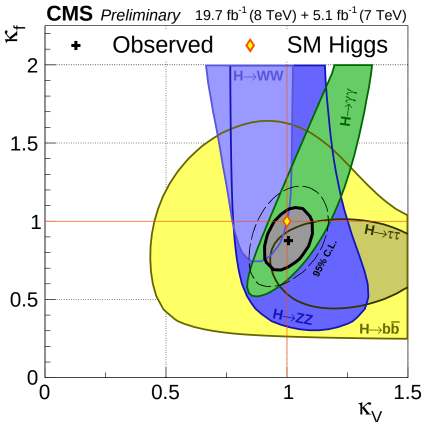
Outline

- Introduction
- Natural Alignment
- Symmetry Justification
- Maximally Symmetric 2HDM
- Higgs Spectrum
- Collider Phenomenology
- Conclusion

'A' Higgs or 'the' Higgs?



- Measured couplings and spin-parity are consistent with the SM predictions.
- Unique opportunity in search of/constraining BSM Higgs scenarios.
  - Precision Higgs Study (Higgcision).
  - Search for additional Higgses.

Two Higgs Doublets

- Several theoretical motivations to go for an extended Higgs sector (e.g. SUSY).
- Any scalar sector in a local $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge theory must be consistent with $\rho_{\mathrm{exp}} = 1.0004^{+0.0003}_{-0.0004}$. [PDG '14]
- With n Higgs multiplets Φ_i (with $i = 1, 2, ..., n$):

$$\rho_{\mathrm{tree}} = \frac{\sum_{i=1}^{n} \left[T_i(T_i + 1) - Y_i^2 \right] v_i}{2 \sum_{i=1}^{n} Y_i^2 v_i} .$$

- Simplest choice: Add multiplets with $T(T + 1) = 3Y^2$, where $n = 2T + 1$.
- SM: One $SU(2)_L$ doublet Φ with $Y = \frac{1}{2}$.
- A simple extension: two $SU(2)_L$ doublets $\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}$ (with $i = 1, 2$).

General 2HDM Potential

- Most general 2HDM potential in doublet field space $\Phi_{1,2}$:

$$
\begin{aligned}
V = & -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \left[m_{12}^2(\Phi_1^\dagger\Phi_2) + \mathrm{H.c.}\right] \\
& + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2) + \mathrm{H.c.}\right].
\end{aligned}
$$

- Four real mass parameters $\mu_{1,2}^2$, Re(m_{12}^2), Im(m_{12}^2), and 10 real quartic couplings $\lambda_{1,2,3,4}$, Re($\lambda_{5,6,7}$), Im($\lambda_{5,6,7}$).
- Rich vacuum structure. [Battye, Brawn, Pilaftsis; Branco *et al* '12]
- Consider normal vacua with real vevs $v_{1,2}$, where $\sqrt{v_1^2+v_2^2} = v_{\rm SM}$ and $\tan\beta = v_2/v_1$.
- Eight real scalar fields: $\phi_j = \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j+\rho_j+i\eta_j) \end{pmatrix}$ (with $j=1,2$).
- After EWSB, 3 Goldstone bosons ($G^\pm$, G^0), eaten by $W^\pm$ and Z, and five physical scalar fields: two CP-even (h, H), one CP-odd (a) and two charged ($h^\pm$).

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Higgs Spectrum in a General 2HDM

- In the charged sector, $\begin{pmatrix} G^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$.

$$M^2_{h^\pm} = \frac{1}{s_\beta c_\beta}\left[\mathrm{Re}(m^2_{12}) - \frac{1}{2}\left(\{\lambda_4 + \mathrm{Re}(\lambda_5)\}\, s_\beta c_\beta + \mathrm{Re}(\lambda_6) c^2_\beta + \mathrm{Re}(\lambda_7) s^2_\beta\right)\right].$$

- In the *CP*-odd sector, $\begin{pmatrix} G^0 \\ a \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$.

$$\begin{aligned} M^2_a &= \frac{1}{s_\beta c_\beta}\left[\mathrm{Re}(m^2_{12}) - v^2\left(\mathrm{Re}(\lambda_5) s_\beta c_\beta + \frac{1}{2}\left\{\mathrm{Re}(\lambda_6) c^2_\beta + \mathrm{Re}(\lambda_7) s^2_\beta\right\}\right)\right] \\ &= M^2_{h^\pm} + \frac{1}{2}\left[\lambda_4 - \mathrm{Re}(\lambda_5)\right] v^2. \end{aligned}$$

- In the *CP*-even sector, $\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$

$$\begin{aligned} M^2_S &= M^2_A \begin{pmatrix} s^2_\beta & -s_\beta c_\beta \\ -s_\beta c_\beta & c^2_\beta \end{pmatrix} \\ &+ v^2 \begin{pmatrix} 2\lambda_1 c^2_\beta + \mathrm{Re}(\lambda_5) s^2_\beta + 2\mathrm{Re}(\lambda_6) s_\beta c_\beta & \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c^2_\beta + \mathrm{Re}(\lambda_7) s^2_\beta \\ \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c^2_\beta + \mathrm{Re}(\lambda_7) s^2_\beta & 2\lambda_2 s^2_\beta + \mathrm{Re}(\lambda_5) c^2_\beta + 2\mathrm{Re}(\lambda_7) s_\beta c_\beta \end{pmatrix} \end{aligned}$$

with $\tan 2\alpha = 2C/(A-B)$ [*new* mixing angle].

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$$M_{h^\pm}^2 = \frac{1}{s_\beta c_\beta}\left[\mathrm{Re}(m_{12}^2) - \frac{1}{2}\left(\{\lambda_4 + \mathrm{Re}(\lambda_5)\}\, s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2\right)\right].$$

- In the *CP*-odd sector, $\begin{pmatrix} G^{0} \\ a \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$.

$$\begin{aligned} M_a^2 &= \frac{1}{s_\beta c_\beta}\left[\mathrm{Re}(m_{12}^2) - v^2\left(\mathrm{Re}(\lambda_5) s_\beta c_\beta + \frac{1}{2}\left\{\mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2\right\}\right)\right] \\ &= M_{h^\pm}^2 + \frac{1}{2}\left[\lambda_4 - \mathrm{Re}(\lambda_5)\right] v^2. \end{aligned}$$

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$$\begin{aligned} M_S^2 &= M_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} \\ &\quad + v^2 \begin{pmatrix} 2\lambda_1 c_\beta^2 + \mathrm{Re}(\lambda_5) s_\beta^2 + 2\mathrm{Re}(\lambda_6) s_\beta c_\beta & \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2 \\ \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2 & 2\lambda_2 s_\beta^2 + \mathrm{Re}(\lambda_5) c_\beta^2 + 2\mathrm{Re}(\lambda_7) s_\beta c_\beta \end{pmatrix} \end{aligned}$$

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Higgs Couplings in a General 2HDM

- The SM Higgs boson is given by

$$H_{\rm SM} \quad = \quad \rho_1 \cos\beta + \rho_2 \sin\beta = H\cos(\beta-\alpha) + h\sin(\beta-\alpha)\ .$$

- With respect to the SM Higgs couplings $H_{\rm SM}VV$ ($V = W^\pm, Z$),

$$g_{hVV} \ = \ \sin(\beta-\alpha)\ , \qquad g_{HVV} \ = \ \cos(\beta-\alpha)\ .$$

  Unitarity constraints uniquely fix other V-Higgs-Higgs couplings [Gunion, Haber, Kane, Dawson '90]

$$g_{haZ} \ = \ \frac{g}{2\cos\theta_w}\cos(\beta-\alpha)\ , \qquad g_{HaZ} \ = \ \frac{g}{2\cos\theta_w}\sin(\beta-\alpha)\ ,$$
$$g_{h^+hW^-} \ = \ \frac{g}{2}\cos(\beta-\alpha)\ , \qquad g_{h^+HW^-} \ = \ \frac{g}{2}\sin(\beta-\alpha)\ .$$

- Motivated by the LHC Higgs data, we scrutinize the SM alignment limit $\alpha \to \beta$ (or $\beta - \pi/2$).
- Usually attributed to either decoupling or accidental cancellations.

  [Gunion, Haber '03; Carena, Low, Shah, Wagner '13]
- **Explore symmetries of the 2HDM potential to *naturally* justify the alignment limit.**

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- **Explore symmetries of the 2HDM potential to *naturally* justify the alignment limit.**

Natural Alignment Condition

- Rewrite *CP*-even mass matrix as

$$M_S^2 = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \widehat{A}v^2 & \widehat{C}v^2 \\ \widehat{C}v^2 & M_a^2 + \widehat{B}v^2 \end{pmatrix} \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \equiv O\widehat{M}_S^2 O^{\mathsf{T}} .$$

$$\widehat{A} = 2\Big[c_\beta^4\lambda_1 + s_\beta^2 c_\beta^2 \lambda_{345} + s_\beta^4\lambda_2 \; + \; 2s_\beta c_\beta\Big(c_\beta^2\lambda_6 + s_\beta^2\lambda_7\Big)\Big] ,$$

$$\widehat{B} = \lambda_5 \; + \; 2\Big[s_\beta^2 c_\beta^2\Big(\lambda_1 + \lambda_2 - \lambda_{345}\Big) \; - \; s_\beta c_\beta\Big(c_\beta^2 - s_\beta^2\Big)\Big(\lambda_6 - \lambda_7\Big)\Big] ,$$

$$\widehat{C} = s_\beta^3 c_\beta\Big(2\lambda_2 - \lambda_{345}\Big) - c_\beta^3 s_\beta\Big(2\lambda_1 - \lambda_{345}\Big) + c_\beta^2\Big(1 - 4s_\beta^2\Big)\lambda_6 + s_\beta^2\Big(4c_\beta^2 - 1\Big)\lambda_7 .$$

- Exact alignment ($\alpha = \beta$) iff $\widehat{C} = 0$, i.e.

$$\lambda_7 t_\beta^4 - (2\lambda_2 - \lambda_{345})t_\beta^3 + 3(\lambda_6 - \lambda_7)t_\beta^2 + (2\lambda_1 - \lambda_{345})t_\beta - \lambda_6 \; = \; 0 .$$

- Natural alignment if happens for *any* value of $\tan\beta$, independent of non-SM Higgs spectra:

$$\boxed{\lambda_1 = \lambda_2 = \lambda_{345}/2 , \quad \lambda_6 = \lambda_7 = 0}$$

- CP-even Higgs masses are given by

$$M_H^2 = 2v^2(\lambda_1 c_\beta^4 + \lambda_{345} s_\beta^2 c_\beta^2 + \lambda_2 s_\beta^4) \equiv \lambda_{\mathrm{SM}} v^2 ,$$

$$M_h^2 = M_a^2 + \lambda_5 v^2 + 2v^2 s_\beta^2 c_\beta^2(\lambda_1 + \lambda_2 - \lambda_{345}) .$$

- **Role of symmetries of the 2HDM potential to realize this without fine-tuning.**

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An Alternative Formulation of the 2HDM Potential

- Gauge-invariant bilinear scalar-field formalism.
  [Nishi '06; Ivanov '06; Maniatis, von Manteuffel, Nachtmann, Nagel '06]
- Introduce an 8-dimensional complex multiplet: [Battye, Brawn, Pilaftsis '11; Nishi '11; Pilaftsis '12]

$$\mathbf{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ i\sigma^2\Phi_1^* \\ i\sigma^2\Phi_2^* \end{pmatrix} .$$

- $\mathbf{\Phi}$ satisfies the Majorana property: $\mathbf{\Phi} = C\mathbf{\Phi}^*$, where $C = \sigma^2 \otimes \sigma^0 \otimes \sigma^2 = C^{-1} = C^*$.
- Define a null 6-dimensional Lorentz vector bilinear in Φ:

$$R^A = \Phi^\dagger \Sigma^A \Phi ,$$

(with $A = 0, 1, 2, 3, 4, 5$), where

$$\Sigma^0 = \frac{1}{2}\sigma^0 \otimes \sigma^0 \otimes \sigma^0 \equiv \frac{1}{2}\mathbf{1}_8, \quad \Sigma^1 = \frac{1}{2}\sigma^0 \otimes \sigma^1 \otimes \sigma^0, \quad \Sigma^2 = \frac{1}{2}\sigma^3 \otimes \sigma^2 \otimes \sigma^0,$$
$$\Sigma^3 = \frac{1}{2}\sigma^0 \otimes \sigma^3 \otimes \sigma^0, \quad \Sigma^4 = -\frac{1}{2}\sigma^2 \otimes \sigma^2 \otimes \sigma^0, \quad \Sigma^5 = -\frac{1}{2}\sigma^1 \otimes \sigma^2 \otimes \sigma^0.$$

- Σ^A satisfy the Majorana condition: $C^{-1}\Sigma^A C = (\Sigma^A)^T$.

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$$\Sigma^3 = \frac{1}{2}\sigma^0 \otimes \sigma^3 \otimes \sigma^0, \quad \Sigma^4 = -\frac{1}{2}\sigma^2 \otimes \sigma^2 \otimes \sigma^0, \quad \Sigma^5 = -\frac{1}{2}\sigma^1 \otimes \sigma^2 \otimes \sigma^0.$$

- Σ^A satisfy the Majorana condition: $C^{-1}\Sigma^A C = (\Sigma^A)^\mathsf{T}$.

2HDM Potential in Bilinear Field Space

- The general 2HDM potential takes a simple form:

$$
\begin{aligned}
V &= -\frac{1}{2} M_A R^A + \frac{1}{4} L_{AB} R^A R^B \,, \qquad \text{where} \\
M &= \Big(\mu_1^2 + \mu_2^2, \quad 2\mathrm{Re}(m_{12}^2), \quad -2\mathrm{Im}(m_{12}^2), \quad \mu_1^2 - \mu_2^2, \quad 0, \quad 0 \Big), \\
R &= \begin{pmatrix} \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \\ \Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \\ -i(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1) \\ \Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2 \\ \Phi_1^{\mathsf{T}} i\sigma^2 \Phi_2 - \Phi_2^\dagger i\sigma^2 \Phi_1^* \\ -i(\Phi_1^{\mathsf{T}} i\sigma^2 \Phi_2 + \Phi_2^\dagger i\sigma^2 \Phi_1^*) \end{pmatrix}, \\
L &= \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & \mathrm{Re}(\lambda_6 + \lambda_7) & -\mathrm{Im}(\lambda_6 + \lambda_7) & \lambda_1 - \lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6 - \lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6 + \lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4 - \mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6 - \lambda_7) & 0 & 0 \\ \lambda_1 - \lambda_2 & \mathrm{Re}(\lambda_6 - \lambda_7) & -\mathrm{Im}(\lambda_6 - \lambda_7) & \lambda_1 + \lambda_2 - \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}
$$

- The bilinear field space spanned by the 6-vector R^A realizes an $SO(1,5)$ symmetry.

Symmetry Classifications of the 2HDM Potential

- Three classes of accidental symmetries of the 2HDM potential:
  - Higgs Family (HF) Symmetries involving transformations of $\Phi_{1,2}$ only (but not $\Phi^*_{1,2}$), e.g. Z_2 [Glashow, Weinberg '58], $\mathrm{U(1)_{PQ}}$ [Peccei, Quinn '77], $\mathrm{SO(3)_{HF}}$ [Deshpande, Ma '78; Ivanov '07; Ma, Maniatis '09; Ferreira, Haber, Maniatis, Nachtmann, Silva '10].
  - CP Symmetries relating $\Phi_{1,2}$ to $\Phi^*_{1,2}$, e.g. $\Phi_{1(2)} \to \Phi^*_{1(2)}$ (CP1) [Lee '73; Branco '80], $\Phi_{1(2)} \to (-)\Phi^*_{2(1)}$ (CP2) [Davidson, Haber '05], CP1 combined with $\mathrm{SO(2)_{HF}}/Z_2$ (CP3) [Ivanov '07; Ferreira, Haber, Silva '09; Ma, Maniatis '09; Ferreira, Haber, Maniatis, Nachtmann, Silva '10].
  - Additional mixed HF and CP symmetries that leave the gauge-kinetic terms of $\Phi_{1,2}$ invariant [Battye, Brawn, Pilaftsis '11].
- Includes *all* custodial symmetries of the 2HDM potential.
- Maximum of 13 *distinct* accidental symmetries of the general 2HDM potential.
- Each of them imposes specific relations among the scalar parameters.

Symmetry Classifications of the 2HDM Potential

- Three classes of accidental symmetries of the 2HDM potential:
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Symmetry Classifications of the 2HDM Potential

[Pilaftsis '12]

Table 1
Parameter relations for the 13 accidental symmetries [1] related to the $U(1)_Y$-invariant 2HDM potential in the diagonally reduced basis, where $\mathrm{Im}\,\lambda_5 = 0$ and $\lambda_6 = \lambda_7$. A dash signifies the absence of a constraint.

No.	Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	$\mathrm{Re}\,\lambda_5$	$\lambda_6 = \lambda_7$
1	$Z_2 \times O(2)$	–	–	Real	–	–	–	–	–	Real
2	$(Z_2)^2 \times SO(2)$	–	–	0	–	–	–	–	–	0
3	$(Z_2)^3 \times O(2)$	–	μ_1^2	0	–	λ_1	–	–	–	0
4	$O(2) \times O(2)$	–	–	0	–	–	–	–	0	0
5	$Z_2 \times [O(2)]^2$	–	μ_1^2	0	–	λ_1	–	–	$2\lambda_1 - \lambda_{34}$	0
6	$O(3) \times O(2)$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0
7	$SO(3)$	–	–	Real	–	–	–	–	λ_4	Real
8	$Z_2 \times O(3)$	–	μ_1^2	Real	–	λ_1	–	–	λ_4	Real
9	$(Z_2)^2 \times SO(3)$	–	μ_1^2	0	–	λ_1	–	–	$\pm\lambda_4$	0
10	$O(2) \times O(3)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	–	0	0
11	$SO(4)$	–	–	0	–	–	–	0	0	0
12	$Z_2 \times O(4)$	–	μ_1^2	0	–	λ_1	–	0	0	0
13	$SO(5)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	0	0	0

- *Maximal* symmetry group in the bilinear field space: $G^R_{\mathrm{2HDM}} = \mathrm{SO(5)}$.
- In the original Φ-field space, $\mathrm{G}^{\Phi}_{\mathrm{2HDM}} = (\mathrm{Sp}(4)/\mathrm{Z}_2) \otimes \mathrm{SU}(2)_L$ [due to $\mathrm{SO}(5) \sim \mathrm{Sp}(4)/\mathrm{Z}_2$].
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- For the SM (with $n = 1$), reproduces the well-known result $\mathrm{G}^{\Phi}_{\mathrm{SM}} = (\mathrm{SU}(2)_C/\mathrm{Z}_2) \otimes \mathrm{SU}(2)_L$ [Sikivie, Susskind, Voloshin, Zakharov '80], since $\mathrm{Sp}(2) \sim \mathrm{SU}(2)_C$.

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Maximally Symmetric 2HDM

- In the SO(5) limit:

$$\mu_1^2 = \mu_2^2 \,, \quad m_{12}^2 = 0 \,, \quad \lambda_2 = \lambda_1 \,, \quad \lambda_3 = 2\lambda_1 \,, \quad \lambda_4 = \mathrm{Re}(\lambda_5) = \lambda_6 = \lambda_7 = 0 \,.$$

- Satisfies the natural alignment condition: $\lambda_1 = \lambda_2 = \lambda_{345}/2$.
- MS-2HDM potential is parametrized by *single* mass parameter μ^2 and *single* quartic coupling λ:

$$V = -\mu^2 \left(|\Phi_1|^2 + |\Phi_2|^2\right) + \lambda \left(|\Phi_1|^2 + |\Phi_2|^2\right)^2 = -\frac{\mu^2}{2}\,\mathbf{\Phi}^\dagger\,\mathbf{\Phi} + \frac{\lambda}{4}\,(\mathbf{\Phi}^\dagger\,\mathbf{\Phi})^2 \,.$$

- More minimal than the MSSM scalar potential, which in the custodial limit $g' \to 0$, has a smaller symmetry: $\mathrm{O(2)} \otimes \mathrm{O(3)} \subset \mathrm{SO(5)}$.

- After EWSB in the MS-2HDM, one massive Higgs boson H with $M_H^2 = 2\lambda_2 v^2$, whilst remaining four (h, a and $h^\pm$) are massless [Goldstone theorem].
- **Natural SM alignment limit with $\alpha = \beta$.** [Recall $H_{\mathrm{SM}} = H\cos(\beta-\alpha) + h\sin(\beta-\alpha)$]
- (Pseudo)-Goldstones can naturally pick up mass due to g' and Yukawa coupling effects.
- In Type-II 2HDM, *only* two other symmetries satisfy the natural alignment condition: (i) $\mathrm{O(3)} \otimes \mathrm{O(2)}$ and (ii) $\mathrm{Z_2} \otimes [\mathrm{O(2)}]^2$.

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Custodial Symmetries in the MS-2HDM

- Quark-sector Yukawa Lagrangian

$$
\begin{aligned}
-\mathcal{L}_Y^q &= \bar{Q}_L(h_1^u \Phi_1 + h_2^u \Phi_2) u_R + \bar{Q}_L(h_1^d \widetilde{\Phi}_1 + h_2^d \widetilde{\Phi}_2) d_R \\
&= (\bar{u}_L\, , \bar{d}_L) \left(\Phi_1 \, , \Phi_2 \, , \widetilde{\Phi}_1 \, , \widetilde{\Phi}_2 \right) \underbrace{\begin{pmatrix} h_1^u & \mathbf{0} \\ h_2^u & \mathbf{0} \\ \mathbf{0} & h_1^d \\ \mathbf{0} & h_2^d \end{pmatrix}}_{\mathcal{H}} \begin{pmatrix} u_R \\ d_R \end{pmatrix} .
\end{aligned}
$$

- To find *all* custodial symmetries of this Lagrangian, consider all the Lie generators of Sp(4): $K^a = \kappa^a \otimes \sigma^0$, where [with normalization: $\mathrm{Tr}(\kappa^a \kappa^b) = \delta^{ab}$]

$$
\kappa^{0,1,3} = \frac{1}{2}\sigma^3 \otimes \sigma^{0,1,3} , \quad \kappa^2 = \frac{1}{2}\sigma^0 \otimes \sigma^2 , \quad \kappa^4 = \frac{1}{2}\sigma^1 \otimes \sigma^0 , \quad \kappa^5 = \frac{1}{2}\sigma^1 \otimes \sigma^3 ,
$$

$$
\kappa^6 = \frac{1}{2}\sigma^2 \otimes \sigma^0 , \quad \kappa^7 = \frac{1}{2}\sigma^2 \otimes \sigma^3 , \quad \kappa^8 = \frac{1}{2}\sigma^1 \otimes \sigma^1 , \quad \kappa^9 = \frac{1}{2}\sigma^2 \otimes \sigma^1 .
$$

- K^0 is the hypercharge generator associated with $\mathrm{U}(1)_Y$ rotations.
- Candidate Sp(4) generators of the custodial symmetry are those which do *not* commute with K^0, i.e. K^a with $a = 4, 5, 6, 7, 8, 9$.

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Custodial Symmetries in the MS-2HDM

- 3 inequivalent realizations of custodial symmetry: (i) $K^{0,4,6}$, (ii) $K^{0,5,7}$, (iii) $K^{0,8,9}$. [Pilaftsis '12]
- Satisfy the symmetry 'commutation' relation [PSBD, Pilaftsis '14]

$$\kappa^a \mathcal{H} - \mathcal{H}\, t^b = \mathbf{0}_{4\times 2} ,$$

where $t^b = \sigma^b/2$ (with $b = 1, 2, 3$).

- 3 different relations among the up- and down-sector Yukawa couplings:

$$\begin{array}{lccc}
\text{(i)} & h_1^u = e^{i\theta} h_1^d & \text{and} & h_2^u = e^{i\theta} h_2^d , \\
\text{(ii)} & h_1^u = e^{i\theta} h_1^d & \text{and} & h_2^u = -e^{i\theta} h_2^d , \\
\text{(iii)} & h_1^u = e^{i\theta} h_2^d & \text{and} & h_2^u = e^{-i\theta} h_1^d ,
\end{array}$$

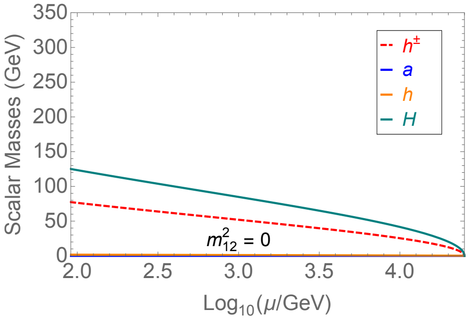
- Equivalent only in the SO(5) limit.

g' and Yukawa Coupling Effects

- Custodial symmetry broken by non-zero g' and Yukawa couplings.

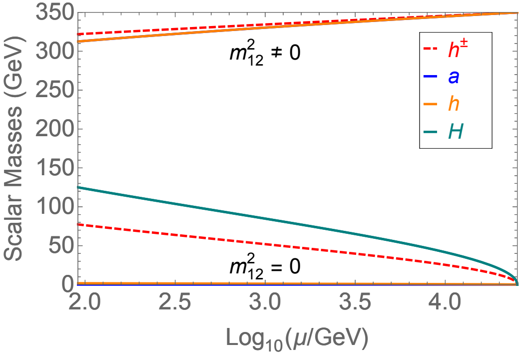
$$
\begin{aligned}
\mathrm{SO}(5)\otimes\mathrm{SU}(2)_L \quad & \xrightarrow{g'\neq 0} \quad \mathrm{O}(3)\otimes\mathrm{O}(2)\otimes\mathrm{SU}(2)_L \;\sim\; \mathrm{O}(3)\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(2)_L \\
& \xrightarrow{\text{Yukawa}} \quad \mathrm{O}(2)\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(2)_L \;\sim\; \mathrm{U}(1)_{\mathrm{PQ}}\otimes\mathrm{U}(1)_Y\otimes\mathrm{SU}(2)_L \\
& \xrightarrow{\langle\Phi_{1,2}\rangle\neq 0} \quad \mathrm{U}(1)_{\mathrm{em}}\ .
\end{aligned}
$$

- Assume SO(5)-symmetry scale $\mu_X \gg v$, and use RG running down to the weak scale.
- Does NOT yield a viable Higgs spectrum with only g' and Yukawa coupling effects.



Soft Breaking Effects

- Include soft SO(5)-breaking effects by $\mathrm{Re}(m_{12}^2) \neq 0$.
- Does yield a viable Higgs spectrum.

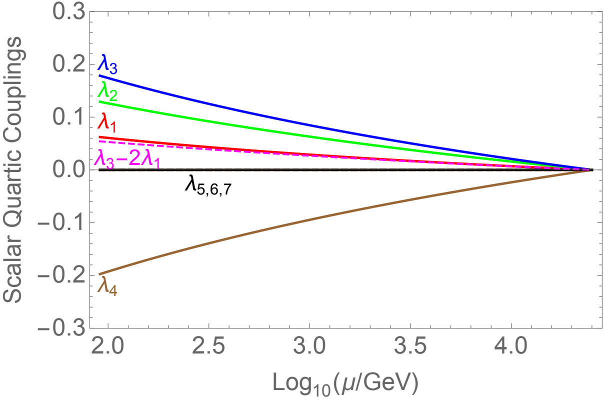


- In the $SO(5)$ limit for quartic couplings,

$$M_H^2 = 2\lambda_2 v^2 , \qquad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta} .$$

- Still preserves natural alignment, irrespective of other 2HDM parameters.

Quartic Coupling Unification



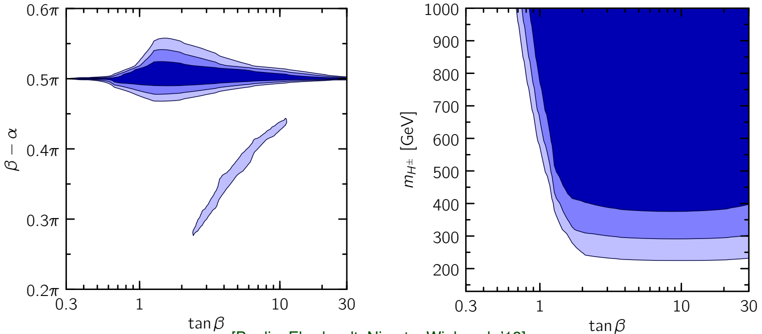
Global Fit

- Electroweak precision observables.
- LHC signal strengths of the light *CP*-even Higgs boson.
- Limits on heavy *CP*-even scalar from $h \to WW, ZZ, \tau\tau$ searches.
- Flavor observables such as B_s mixing and $B \to X_s\gamma$.

- Stability of the potential:

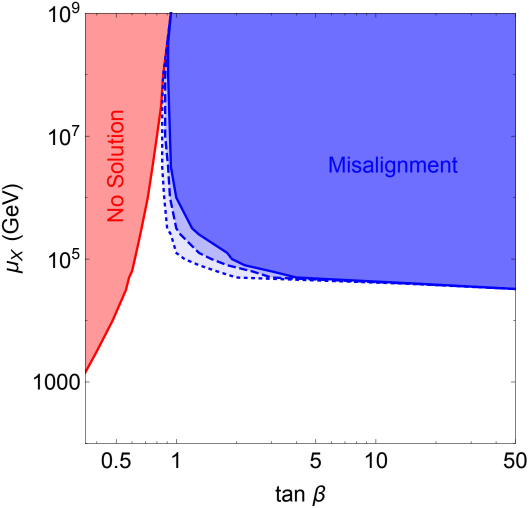
$$\lambda_{1,2} > 0, \quad \lambda_3 + \sqrt{\lambda_1\lambda_2} > 0, \quad \lambda_3 + \lambda_4 + \sqrt{\lambda_1\lambda_2} - \mathrm{Re}(\lambda_5) > 0.$$

- Perturbativity of the Higgs self-couplings: $\|S_{\Phi\Phi\to\Phi\Phi}\| < \frac{1}{8}$.



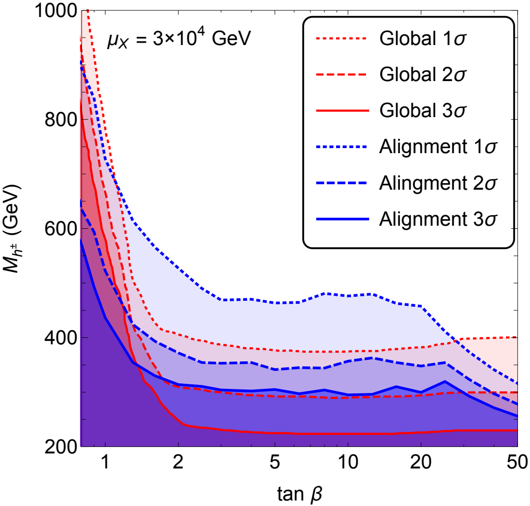
[Baglio, Eberhardt, Nierste, Wiebusch '13]

Misalignment Predictions



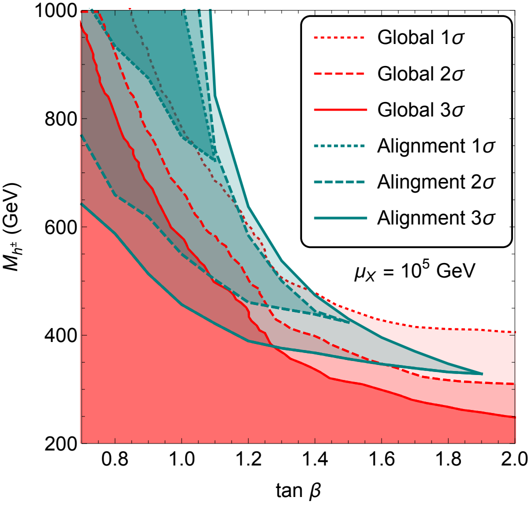
[PSBD, Pilaftsis '14]

Lower Limit on Charged Higgs Mass



[PSBD, Pilaftsis '14]

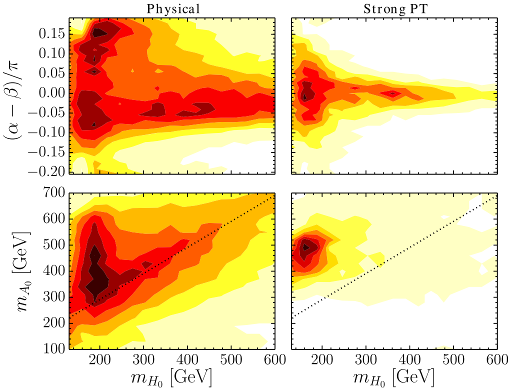
Lower *and* Upper Limits on Charged Higgs Mass



[PSBD, Pilaftsis '14]

Electroweak Phase Transition

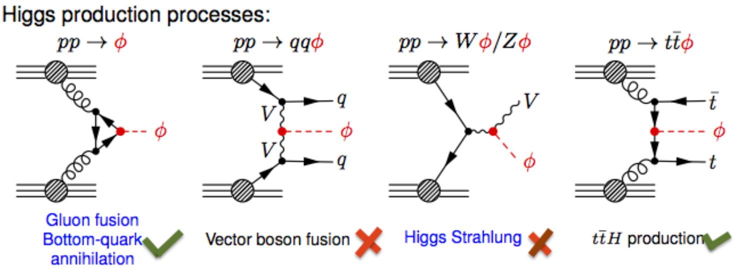
- Alignment limit is favorable to EWPT. [Dorsch, Huber, Mimasu, No '14]



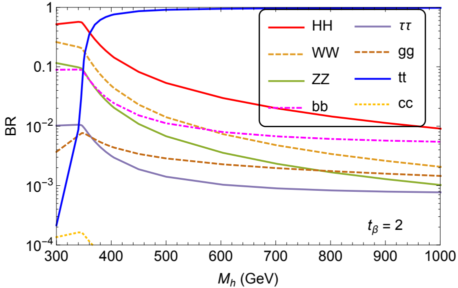
- In the SO(5) limit, the heavy Higgs sector is quasi-degenerate.
- Not many solutions for strongly first-order EWPT.
- Might be possible to have $M_a - M_h \gtrsim v$ in other naturally aligned scenarios with a lower symmetry group, i.e. $\mathrm{O}(3) \otimes \mathrm{O}(2)$ or $\mathrm{Z}_2 \otimes [\mathrm{O}(2)]^2$.

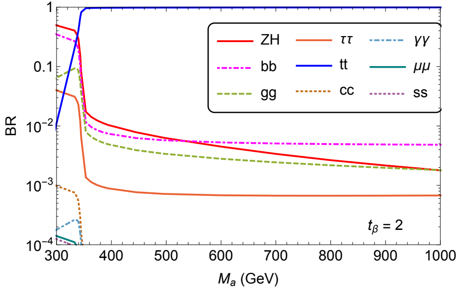
Implications of Alignment for the LHC Searches

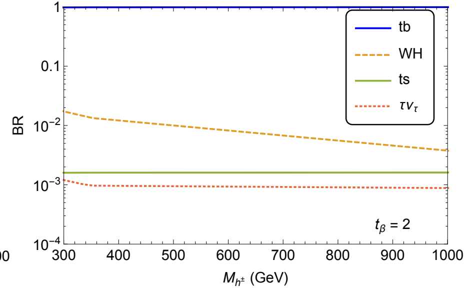
- Recall that $g_{hVV} = \sin(\beta - \alpha)$, $g_{HVV} = \cos(\beta - \alpha)$.
- In the alignment limit $\alpha \to \beta$, H is SM-like and the heavy Higgs h is gaugephobic.
- Dominant production modes at the LHC: ggF and associated production with $t\bar{t}$.



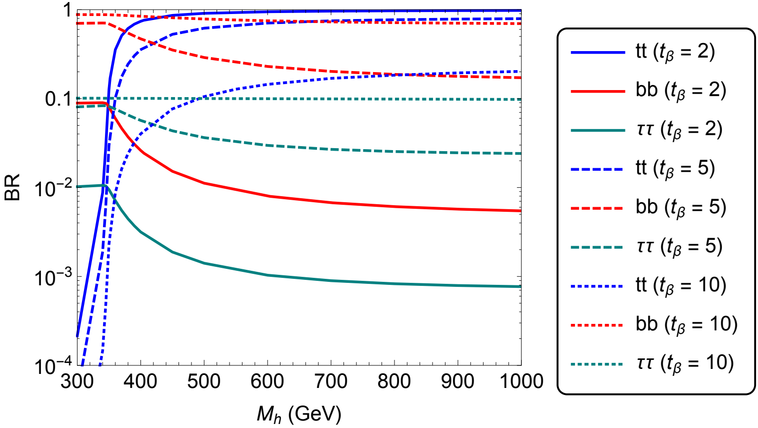
Branching Fractions







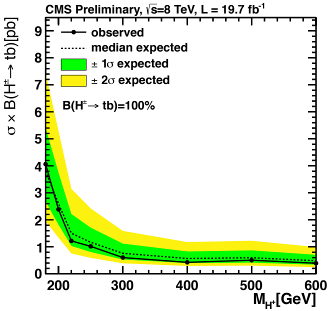
tan β Dependance



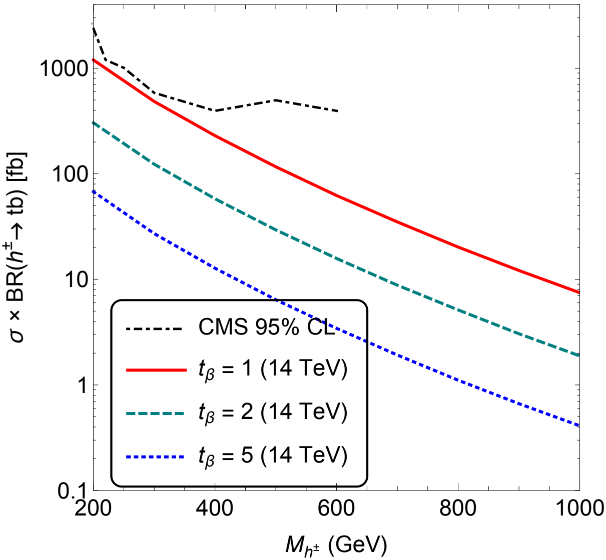
Existing LHC Searches

- Existing collider limits on the heavy Higgs sector derived from *WW* and *ZZ* modes are not applicable in the alignment limit.
- Limits from $gg \to h \to \tau^+\tau^-$ and $gg \to b\bar{b}h \to b\bar{b}\tau^+\tau^-$ are easily satisfied.
- Similarly for $h \to HH \to \gamma\gamma bb$.
- In the charged-Higgs sector, most of the searches focus on the low-mass regime ($M_{h^\pm} < M_t$): $pp \to tt \to Wbbh^+,\ h^+ \to cs$.
- Recently, the search was extended beyond the top-threshold: [CMS-PAS-HIG-13-026]

$$gg \to h^+tb \to (\ell\nu bb)(\ell'\nu b)b$$



Predictions in the MS-2HDM



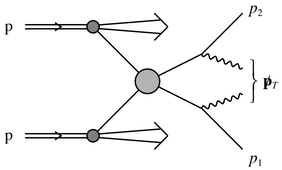
Simulations for $\sqrt{s} = 14$ TeV LHC

- Used `MadGraph5_aMC@NLO`.
- Event reconstruction using the CMS cuts:

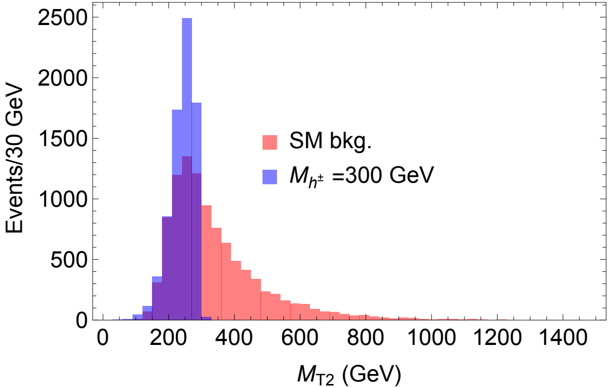
$$\begin{aligned} p_T^\ell &> 20\ \mathrm{GeV}, \quad |\eta^\ell| < 2.5, \quad \Delta R^{\ell\ell} > 0.4, \\ M_{\ell\ell} &> 12\ \mathrm{GeV}, \quad |M_{\ell\ell} - M_Z| > 10\ \mathrm{GeV}, \\ p_T^j &> 30\ \mathrm{GeV}, \quad |\eta^j| < 2.4, \quad \not{E}_T > 40\ \mathrm{GeV}. \end{aligned}$$

- Jet reconstruction using the anti-k_T clustering algorithm with a distance parameter of 0.5.
- At least two b-tagged jets are required in the signal events (each has a b-tagging efficiency of about 70%).
- For charged Higgs mass reconstruction, used 'stransverse mass' variable [Lester, Summers '99]

$$M_{T2} = \min_{\left\{\not{\mathbf{p}}_{T_1} + \not{\mathbf{p}}_{T_2} = \not{\mathbf{p}}_T\right\}} \left[\max\left\{m_{T_1}, m_{T_2}\right\}\right].$$

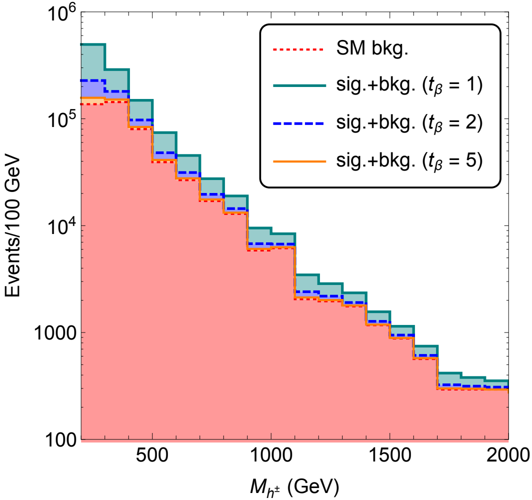


Mass Reconstruction using M_{T2}



[PSBD, Pilaftsis '14]

Reach at 14 TeV LHC

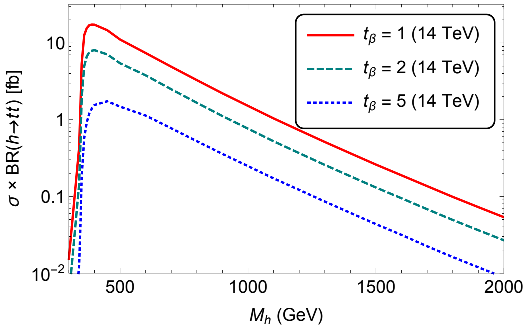


[PSBD, Pilaftsis '14]

New Signal in the Neutral Higgs Sector

$$gg \to t\bar{t}h \to t\bar{t}t\bar{t}$$

- Existing 95% CL experimental upper limit on $\sigma_{t\bar{t}t\bar{t}}$ is 32 fb (CMS).
- SM prediction for $\sigma(pp \to t\bar{t}t\bar{t} + X) \simeq$ 10–15 fb at NLO. [Bevilacqua, Worek '12]
- Still lot of room for BSM contribution.



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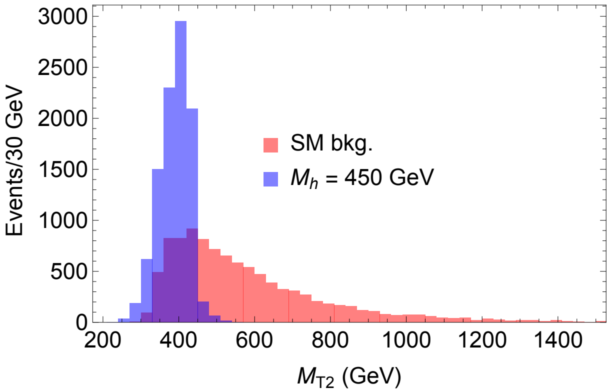

[PSBD, Pilaftsis '14]

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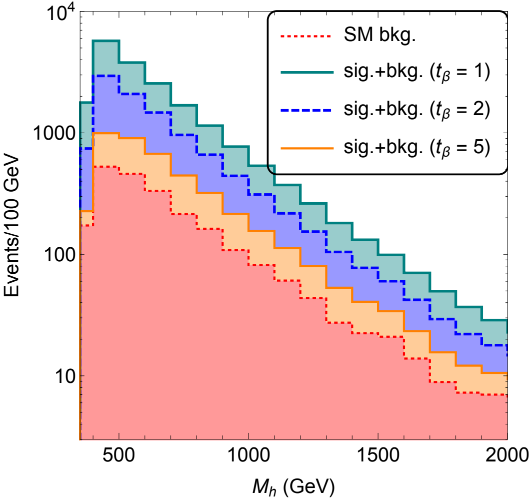

[PSBD, Pilaftsis '14]

Towards a Full Analysis of the $4t$ Signal

35 final states, grouped into five channels:

- Fully hadronic: 12 jets, with 4 b-jets.
- Mostly hadronic: 6 light jets, 4 b-jets, one charged lepton and $\not{E}_T$.
- Semi-leptonic/hadronic: 4 light jets, 4 b-jets, 2 charged leptons and $\not{E}_T$.
- Mostly leptonic: 2 light jets, 4 b-jets, 3 charged leptons and $\not{E}_T$.
- Fully leptonic: 4 b-jets, 4 charged leptons and $\not{E}_T$.

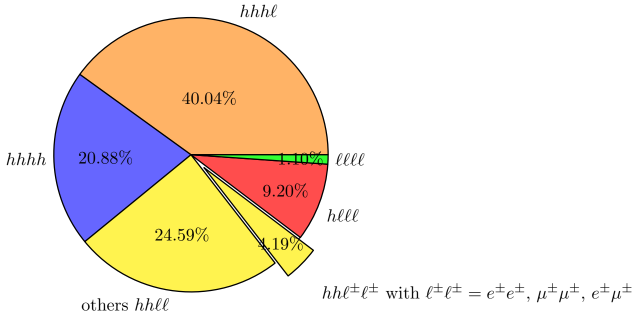

Figure 1.4: Branching fractions for the different decays of the four top quarks, depending on whether the W boson decays hadronically (h) or leptonically (ℓ). [Figure Courtesy: D. P. Hernández (ATLAS)]

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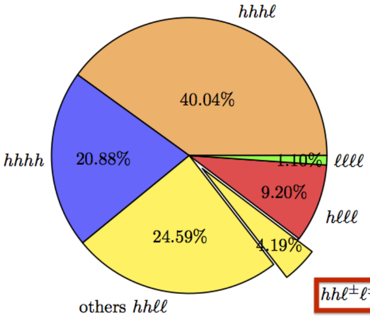

$hh\ell^{\pm}\ell^{\pm}$ with $\ell^{\pm}\ell^{\pm} = e^{\pm}e^{\pm},\ \mu^{\pm}\mu^{\pm},\ e^{\pm}\mu^{\pm}$

experimentally favored channel

Figure 1.4: Branching fractions for the different decays of the four top quarks, depending on whether the W boson decays hadronically (h) or leptonically (ℓ). [Figure Courtesy: D. P. Hernández (ATLAS)]

Conclusions

- Analyzed the symmetry classifications and custodial symmetries of the general 2HDM scalar potential.
- Maximal reparametrization group is SO(5).
- Maximally Symmetric 2HDM potential has a single quartic coupling.

- SM alignment limit is realized naturally, *independently* of the heavy Higgs spectrum and the value of tan β.
- Deviations from alignment limit can be naturally induced by RG effects due to g' and Yukawa couplings.
- In addition, non-zero soft SO(5)-breaking mass parameter is required to yield a viable Higgs spectrum.

- Using the current Higgs data, we derive important constraints on the MS-2HDM parameter space.
- Predict lower limits on the heavy Higgs spectrum, which prevail the present limits in a wide range of parameter space.
- Depending on the SO(5)-breaking scale, we also obtain an upper limit on the heavy Higgs masses, which could be completely probed during LHC run-II.
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Symmetry Generators

Table 2
Symmetry generators [cf. (10), (14)] and discrete group elements [cf. (17)] for the 13 accidental symmetries of the $U(1)_Y$-invariant 2HDM potential. For each symmetry, the maximally broken SO(5) generators compatible with a neutral vacuum are displayed, along with the pseudo-Goldstone bosons (given in parentheses) that result from the Goldstone theorem.

No.	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete group elements	Maximally broken SO(5) generators	Number of pseudo-Goldstone bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
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[Pilaftsis '12]

- T^a and K^a are the generators of $SO(5)$ and $Sp(4)$ respectively ($a = 0, \ldots, 9$).
- T^0 is the hypercharge generator in R-space, which is equivalent to the electromagnetic generator $Q_{\rm em} = \frac{1}{2}\sigma^0 \otimes \sigma^0 \otimes \sigma^3 + K^0$ in Φ-space.
- $Sp(4)$ contains the custodial symmetry group $SU(2)_C$.
- Three *independent* realizations of custodial symmetry induced by (i) $K^{0,4,6}$, (ii) $K^{0,5,7}$, (iii) $K^{0,8,9}$.

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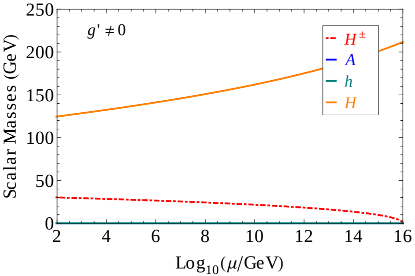
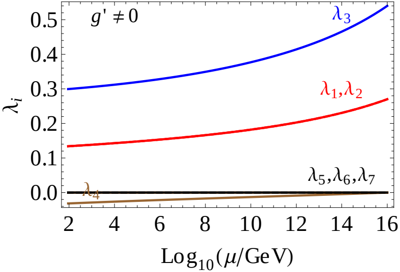
Quark Yukawa Couplings

- By convention, choose $h_1^u = 0$. For Type-I (Type-II) 2HDM, $h_1^d (h_2^d) = 0$.
- Quark yukawa couplings w.r.t. the SM are given by

Coupling	Type-I	Type-II
$g_{ht\bar{t}}$	$\cos\alpha / \sin\beta$	$\cos\alpha / \sin\beta$
$g_{hb\bar{b}}$	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$
$g_{Ht\bar{t}}$	$\sin\alpha / \sin\beta$	$\sin\alpha / \sin\beta$
$g_{Hb\bar{b}}$	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$
$g_{at\bar{t}}$	$\cot\beta$	$\cot\beta$
$g_{ab\bar{b}}$	$-\cot\beta$	$\tan\beta$

g' Effect





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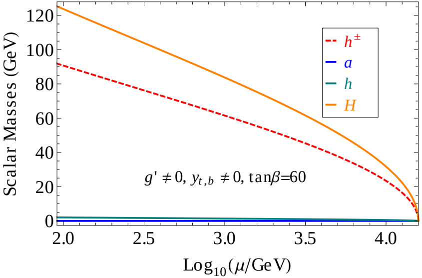
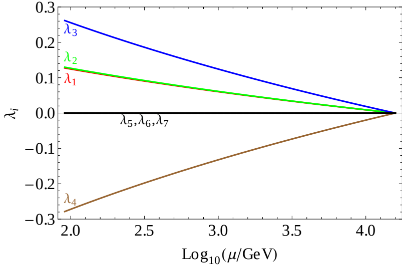
Yukawa Coupling Effects





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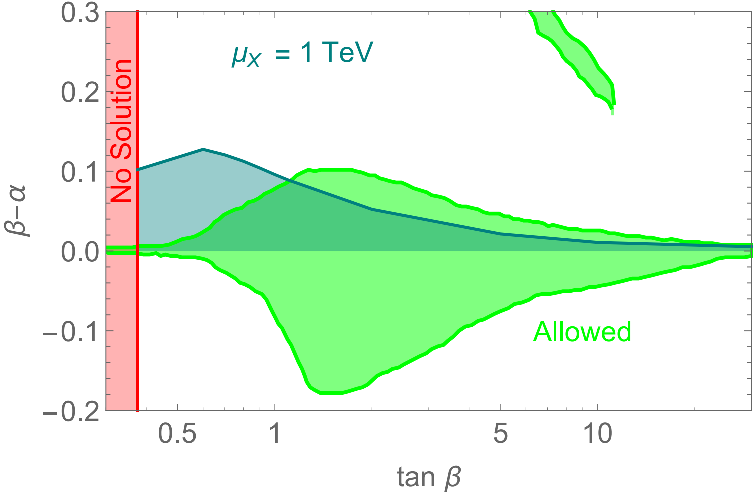
With $SO(5)$ Boundary Conditions at μ_X



[PSBD, Pilaftsis (preliminary)]

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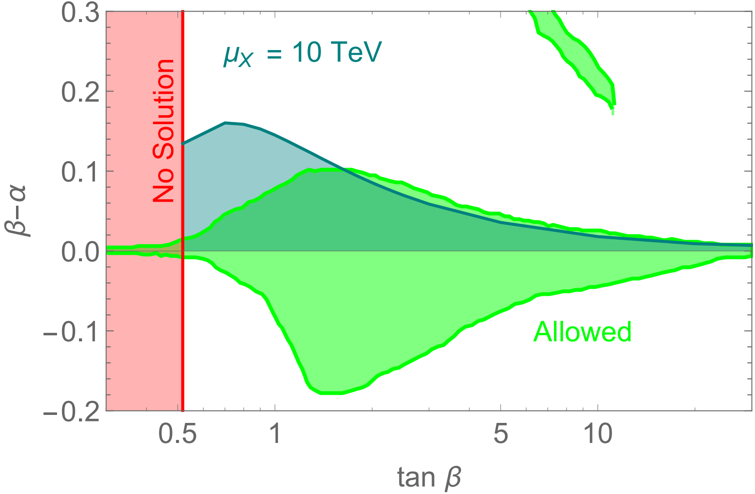

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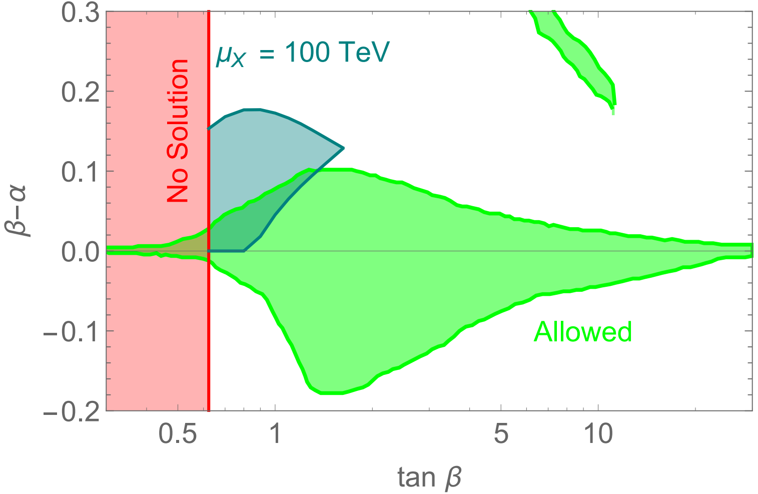

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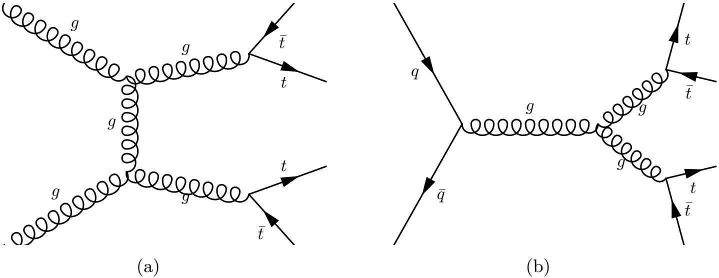
Production of 4 tops in the SM



Production of 4 tops in BSM