Relativistic distortions in large-scale structure

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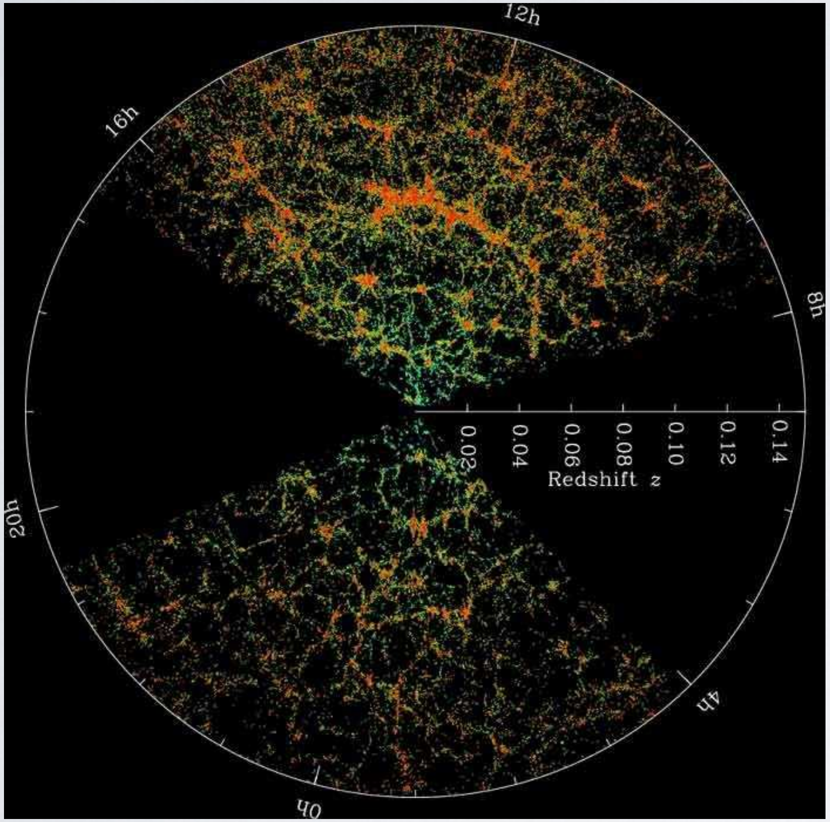
Galaxy survey

The **distribution** of galaxies is determined by:

Credit: M. Blanton, SDSS

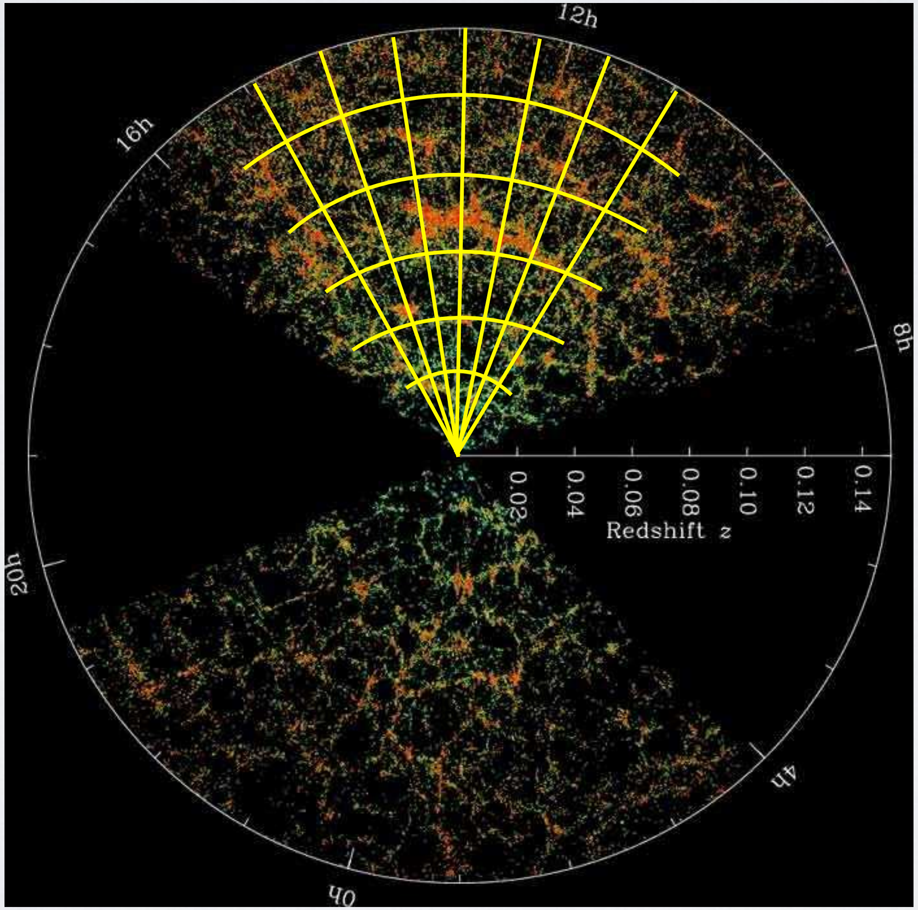


- The initial conditions
- The theory of gravity
- The content of the universe

To interpret properly the information from large-scale structure, we need to understand **what** we are **measuring**.

Galaxy survey

- We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N-\bar{N}}{\bar{N}}$
- How is Δ related to: the initial conditions, the theory of gravity and dark energy?

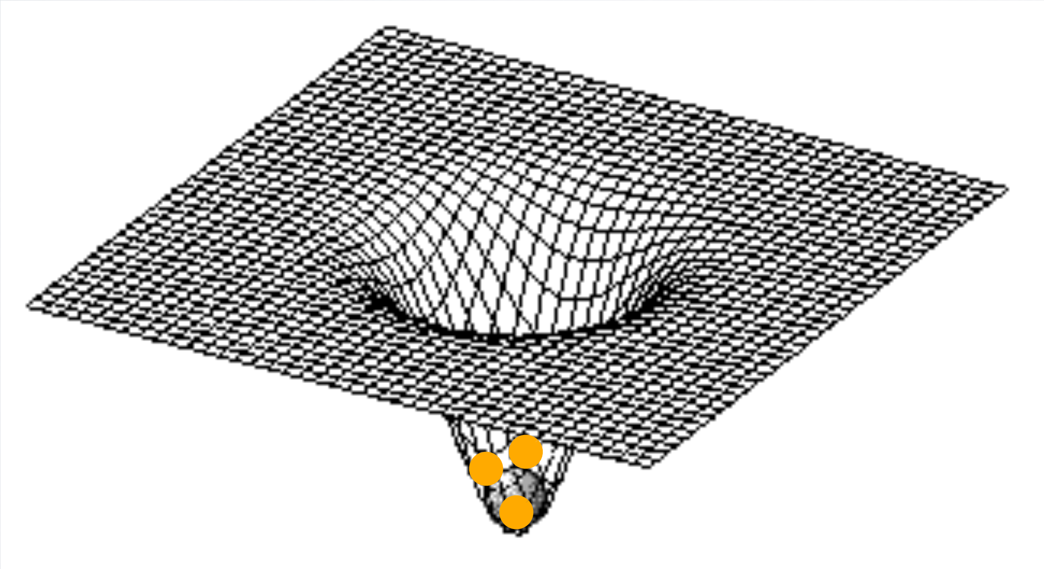


Credit: M. Blanton, SDSS

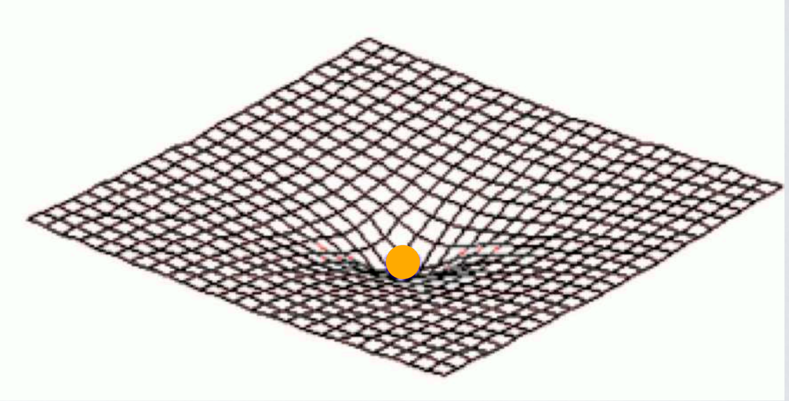
Galaxy distribution

Simple picture:
- ♦ **dark matter** is inhomogeneously distributed
- ♦ it creates **gravitational potential** wells
- ♦ **baryons** fall into them and form galaxies.

More dark matter



Less dark matter



$$\Delta = \frac{\delta\rho}{\rho} \equiv \delta$$

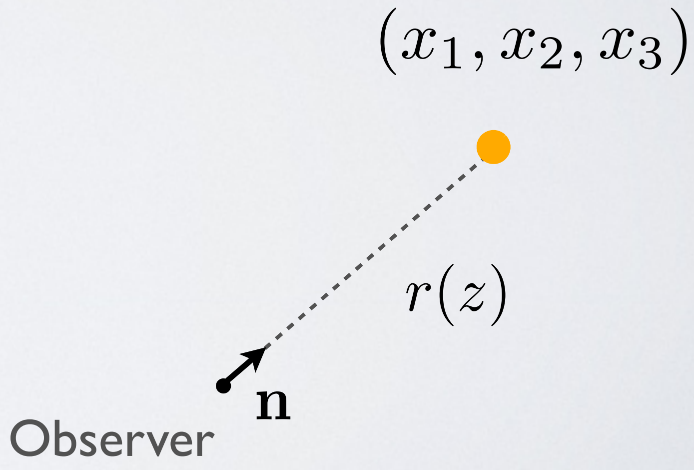
Complications

- **Bias**: the distribution of galaxies does not trace directly the distribution of dark matter $\Delta = b \cdot \delta$

- We never observe directly the **position** of galaxies, we observe the **redshift** and the **direction** of incoming photons.

In a **homogeneous** universe:

- we calculate $r(z)$
- light propagates on straight lines

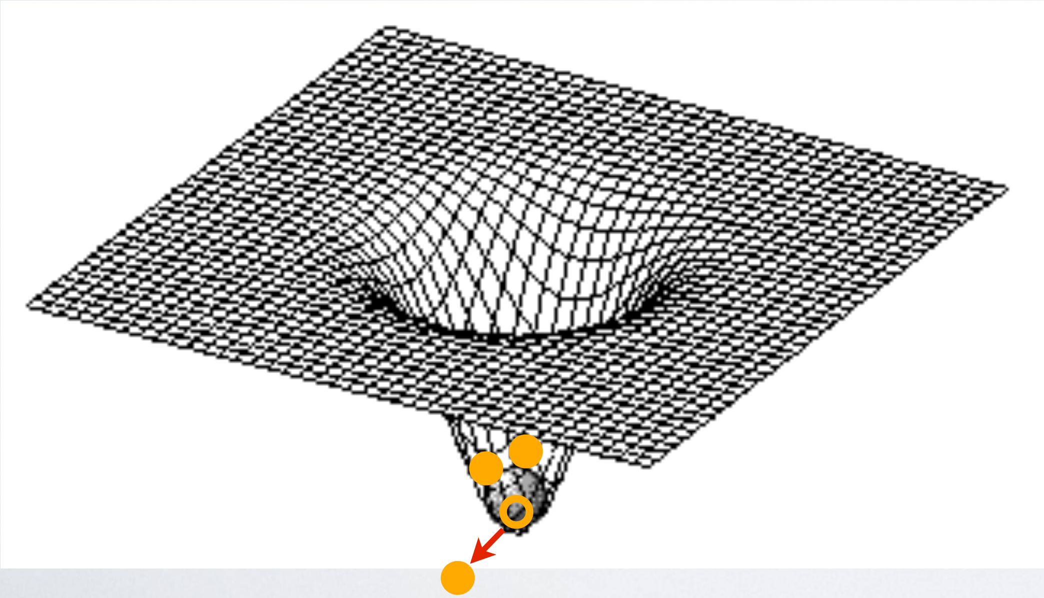


Redshift

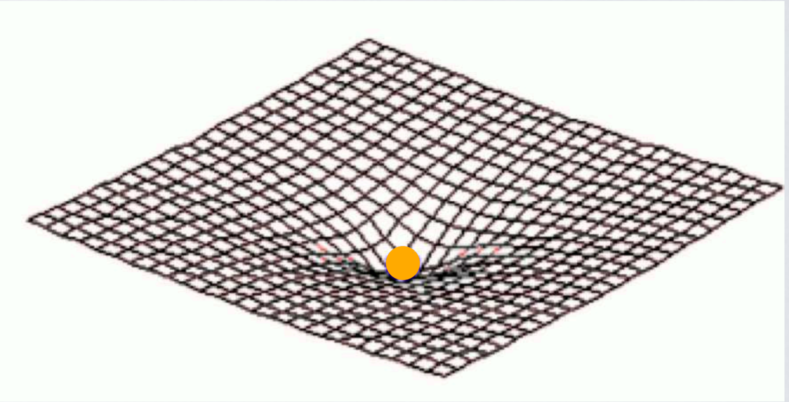
In an **inhomogeneous** universe: the redshift is affected by fluctuations, e.g. **Doppler** effect due to peculiar velocities.

→ **radial** shift in the galaxy position

More dark matter



Less dark matter



Redshift distortions

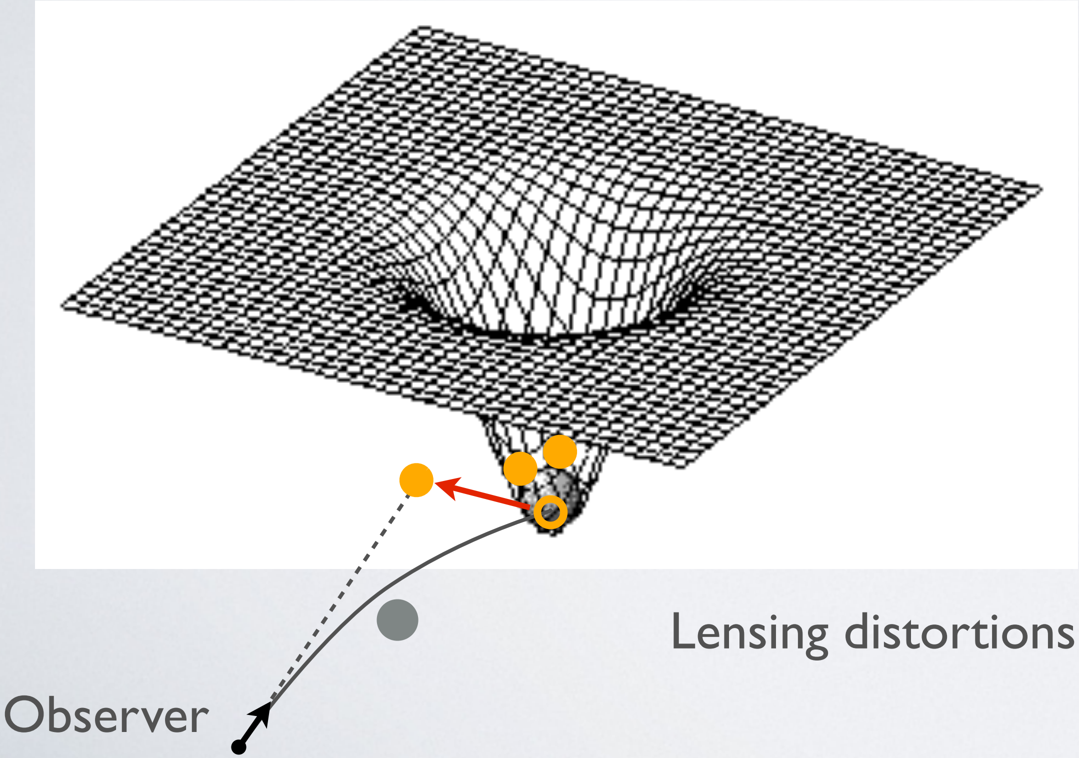
Observer

Lensing

In an **inhomogeneous** universe: light is **lensed** by matter between the galaxies and the observer

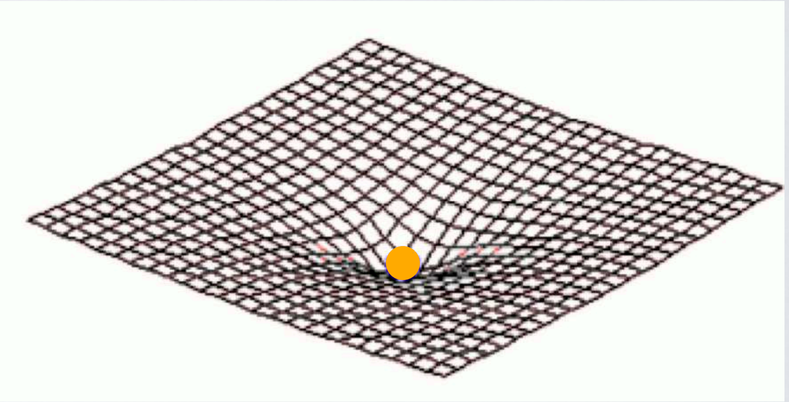
→ **transverse** shift in the galaxy position

More dark matter



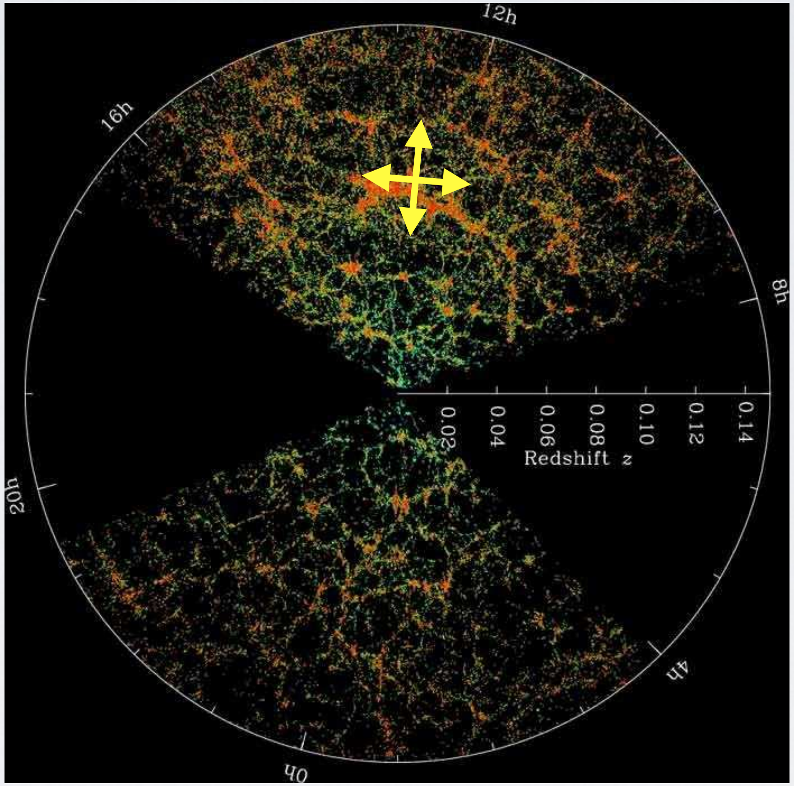
Lensing distortions

Less dark matter



Galaxy distribution

The **structures** seen on a galaxy map do **not reflect** directly the underlying dark matter structures. The observed **positions** of galaxies are **shifted** radially and transversally.



Credit: M. Blanton, SDSS

To extract **information** from a galaxy map, we need to understand exactly what the **distortions** are.

Outline

- Calculate the **distortions** that affect the large-scale structure: number density Δ and convergence κ (galaxy's size).

$$\boxed{\begin{aligned}\Delta = &\ \text{density} + \text{redshift distortions}\\ &+ \text{lensing} + \textbf{relativistic distortions}\\ \kappa = &\ \text{lensing} + \textbf{relativistic distortions}\end{aligned}}$$

- The relativistic distortions should not be considered as a noise but rather as a **new signal**.
- Impact of the different terms on the **correlation** function: the relativistic distortions change the properties of the two-point function → we can **isolate** them.
- We can combine the relativistic distortions with standard observables to **test** the theory of gravity.

The over-density of galaxies

- We count $N(z, \mathbf{n})$ **galaxies** in a pixel of **volume** $V(z, \mathbf{n})$

- We want to calculate the fluctuations in $N(z, \mathbf{n})$ with respect to the average number.

- At each redshift, we average over the direction: $\bar{N}(z)$

- The observed **over-density** is:

$$\Delta(z, \mathbf{n}) = \frac{N(z, \mathbf{n}) - \bar{N}(z)}{\bar{N}(z)}$$

- Relation with the dark matter density:

$$N(z, \mathbf{n}) = \rho(z, \mathbf{n}) \cdot V(z, \mathbf{n}) \quad \text{and} \quad \bar{N}(z) = \bar{\rho}(z) \cdot \bar{V}(z)$$

Derivation

$$\bar{\rho} + \delta\rho \qquad\qquad \bar{V} + \delta V$$

$$\Delta = \frac{\rho(z,\mathbf{n}) \cdot V(z,\mathbf{n}) - \bar{\rho}(z) \cdot \bar{V}(z)}{\bar{\rho}(z) \cdot \bar{V}(z)}$$

We keep only linear terms:

$$\Delta = \frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} + \frac{\delta V(z,\mathbf{n})}{\bar{V}(z)}$$

the background redshift is different
from the observed redshift

$$\delta(z,\mathbf{n}) \equiv \frac{\rho(z,\mathbf{n}) - \bar{\rho}(\bar{z})}{\bar{\rho}(\bar{z})} \neq \frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \frac{\rho(z,\mathbf{n}) - \bar{\rho}(z)}{\bar{\rho}(z)}$$

Derivation

$$\Delta = \frac{\overset{\bar{\rho}+\delta\rho}{\nwarrow}\!\!\!\rho(z,\mathbf{n}) \cdot \overset{\bar{V}+\delta V}{\nearrow}\!\!\!V(z,\mathbf{n}) - \bar{\rho}(z)\cdot\bar{V}(z)}{\bar{\rho}(z)\cdot\bar{V}(z)}$$

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$$\delta(z,\mathbf{n}) \equiv \frac{\rho(z,\mathbf{n}) - \bar{\rho}(\bar{z})}{\bar{\rho}(\bar{z})} \quad \overset{\uparrow}{\neq} \quad \frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \frac{\rho(z,\mathbf{n}) - \bar{\rho}(z)}{\bar{\rho}(z)}$$

Derivation

$z = \bar{z} + \delta z$ Taylor: $\bar{\rho}(z) = \bar{\rho}(\bar{z} + \delta z) \simeq \bar{\rho}(\bar{z}) + \partial_z \bar{\rho} \cdot \delta z$

$$\frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \frac{\rho(z,\mathbf{n}) - \bar{\rho}(z)}{\bar{\rho}(z)} \simeq \frac{\rho(z,\mathbf{n}) - \bar{\rho}(\bar{z}) - \partial_z\bar{\rho}\cdot\delta z}{\bar{\rho}(\bar{z})}$$

$$\frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \delta(z,\mathbf{n}) - 3\frac{\delta z}{1+z}$$

$$\boxed{\Delta(z,\mathbf{n}) = b\cdot\delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}}$$

describes what we observe in the linear regime

Derivation

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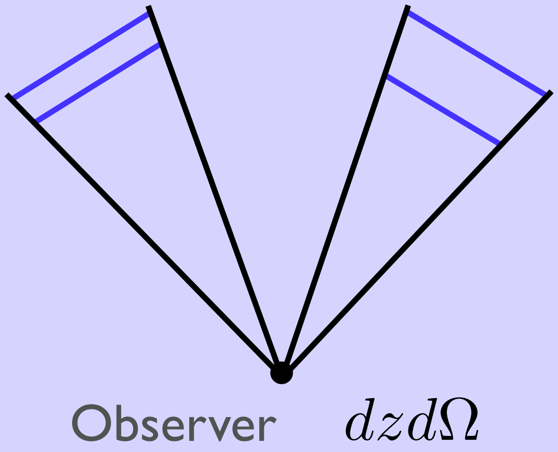
same redshift bin
different physical volume



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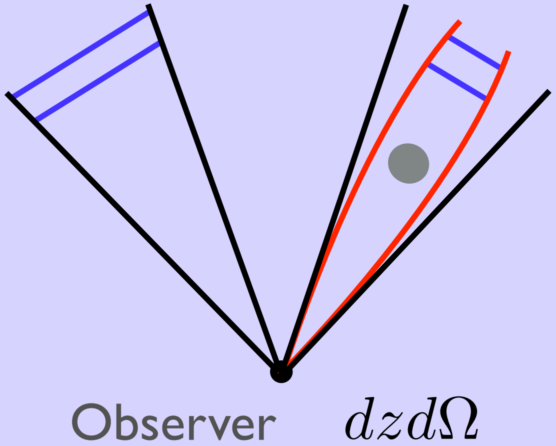
same solid angle
different physical volume



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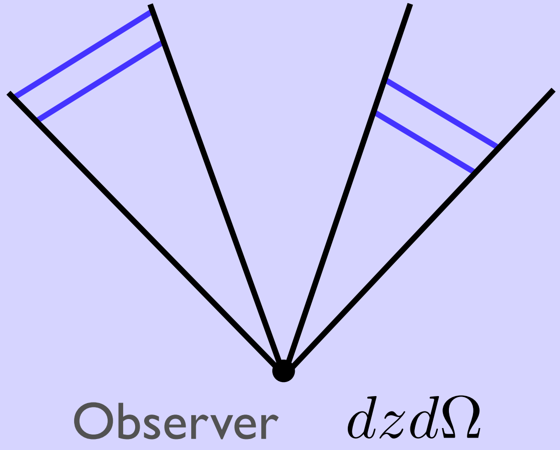
same radial bin
different distance



$$\frac{\delta\rho(z,\mathbf{n})}{\bar{\rho}(z)} = \delta(z,\mathbf{n}) - 3\frac{\delta z}{1+z}$$

$$\boxed{\Delta(z,\mathbf{n}) = b \cdot \delta(z,\mathbf{n}) + \frac{\delta V(z,\mathbf{n})}{V} - 3\frac{\delta z}{1+z}}$$

describes what we observe
in the linear regime

Redshift

CB and Durrer (2011)

$$ds^2 = -a^2\Big[\big(1+2\Psi\big)d\eta^2 + \big(1-2\Phi\big)\delta_{ij}dx^i dx^j\Big]$$

Effect of inhomogeneities on the redshift: $1+z=\dfrac{\nu_S}{\nu_O}=\dfrac{E_S}{E_O}$

Photons travel on **null geodesics**.

$$1+z=\frac{a_O}{a_S}\Big[1+\underbrace{\mathbf{V}_S\cdot\mathbf{n}-\mathbf{V}_O\cdot\mathbf{n}}_{\text{Doppler}}+\underbrace{\Psi_O-\Psi_S}_{\text{Gravitational redshift}}-\int_0^{r_S}\underbrace{dr(\dot\Phi+\dot\Psi)}_{\text{Integrated Sachs-Wolfe}}\Big]$$

Gravitational redshift:

Redshift

CB and Durrer (2011)

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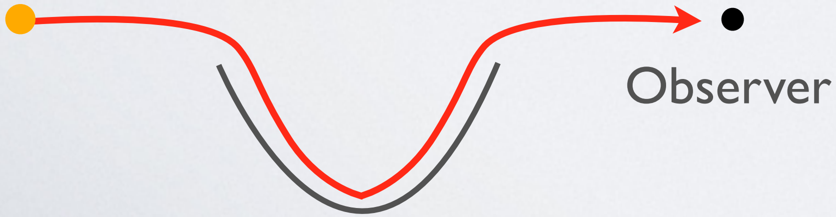
$$1+z=\frac{a_O}{a_S}\left[1+\mathbf{V}_S\cdot\mathbf{n}-\mathbf{V}_O\cdot\mathbf{n}+\Psi_O-\Psi_S-\int_0^{r_S}dr(\dot{\Phi}+\dot{\Psi})\right]$$

Doppler — Gravitational redshift — Integrated Sachs-Wolfe

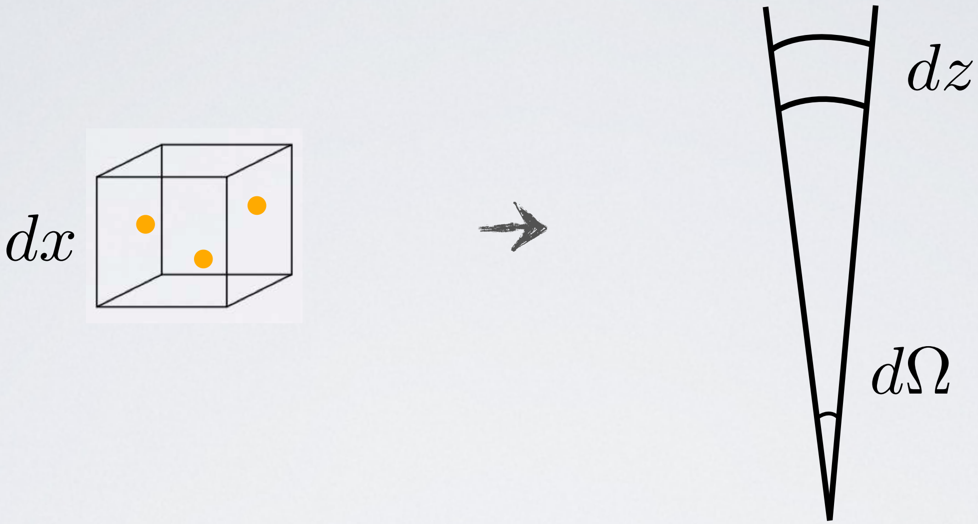
ISW: integrated along the trajectory, sensitive to dark energy.

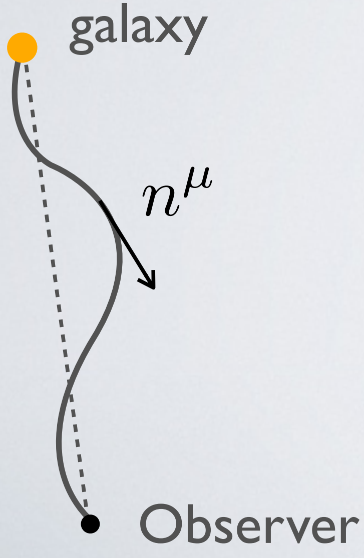


Volume fluctuation

CB and Durrer (2011)

We want to calculate the relation between a **physical volume** element at the position of the galaxies and an **observed pixel**.





The **null geodesic** equation tells us how directions and distances are **perturbed**.

$$\theta_S = \theta_O + \delta\theta \qquad \varphi_S = \varphi_O + \delta\varphi \qquad r = \bar{r} + \delta r$$

Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

density

redshift space distortion

lensing

gravitational redshift

Doppler

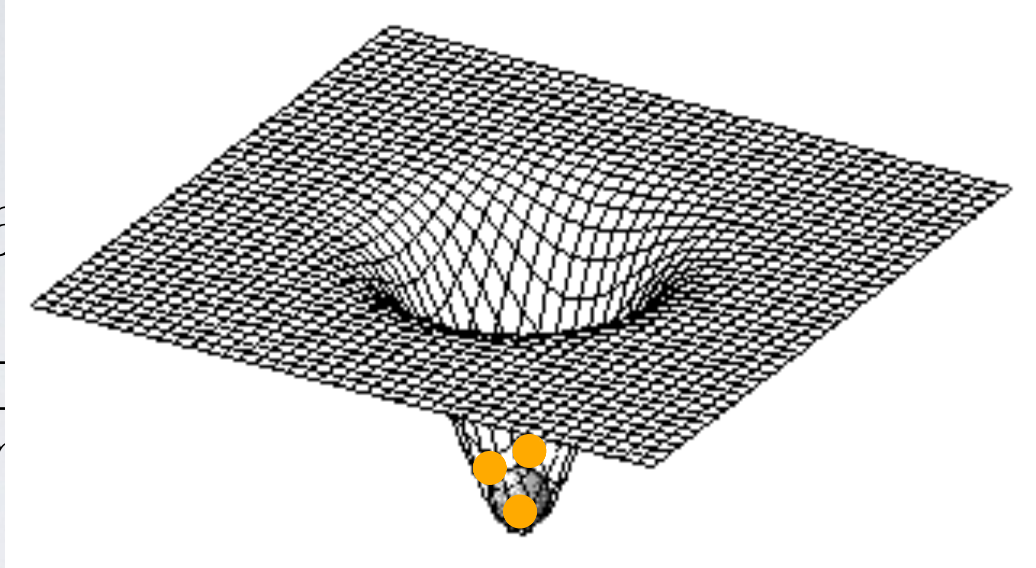
$$\begin{aligned}\Delta(z,\mathbf{n}) &= b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})\\ &-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)\\ &+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi\\ &\left.\begin{aligned}&+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)\\ &+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]\end{aligned}\right]\end{aligned}$$

→ potential

Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

density



$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial$$

$$-\int_0^r dr'\frac{r-}{rr}$$

$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

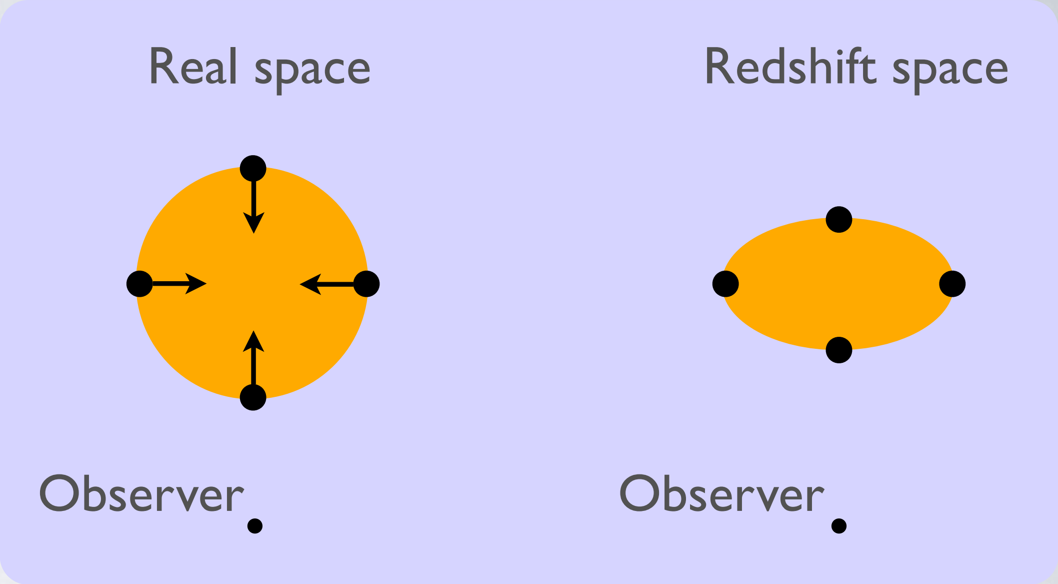
$$+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$

$$+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]$$

Result

Kaiser 1987, Hamilton 1992

redshift space distortion



$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

$$-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

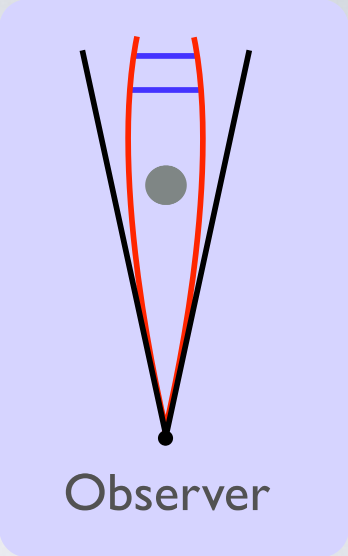
$$+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$

$$+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]$$

Result

$$\begin{aligned}\Delta(z,\mathbf{n}) = {} & b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\ & - \int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi) \\ & + \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\ & + \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\ & + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]\end{aligned}$$

↗ lensing



Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

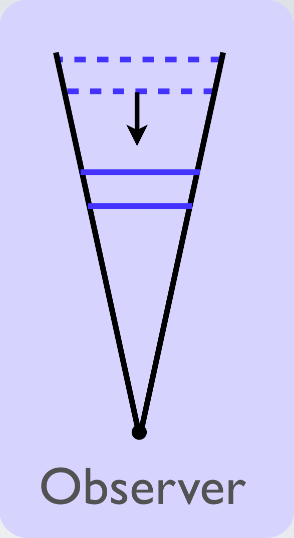
$$-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

Doppler $\nwarrow$

$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+$$

$$+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$

$$+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]$$



Result

Yoo et al (2010)
CB and Durrer (2011)
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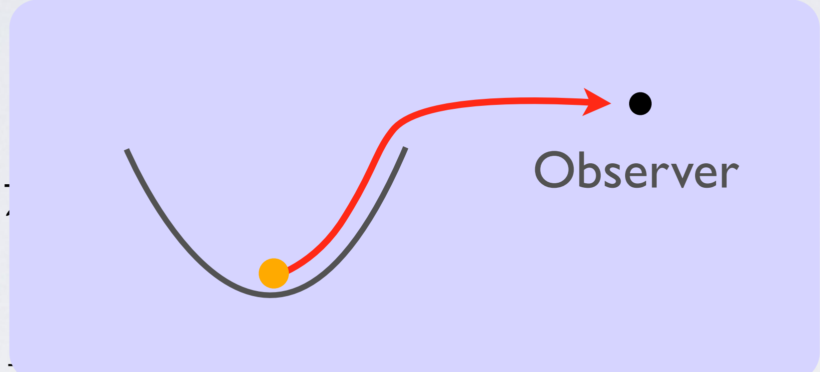
$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

$$-\int_0^r dr'\,\frac{r-r'}{\quad}\,\Delta_\Omega(\Phi+\Psi)$$

$$+\Bigg(1-\quad\qquad+\frac{1}{\mathcal{H}}\partial_r\Psi$$



gravitational redshift

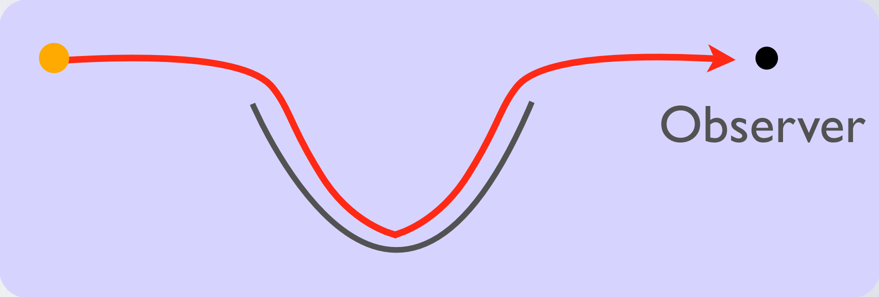
$$+\Psi-2\Phi+\frac{\quad}{\mathcal{H}}\Psi-3\frac{\quad}{k}V+\frac{\quad}{r}\int_0 dr'(\Phi+\Psi)$$

$$+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot\Phi+\dot\Psi)\right]$$

Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$\begin{aligned}
\Delta(z,\mathbf{n}) &= b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
&- \int_0^r dr'\,\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi) \\
&+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \\
&\left.\begin{aligned}
&+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\
&+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]
\end{aligned}\right] \rightarrow \text{potential}
\end{aligned}$$



Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

density

redshift space distortion

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

lensing

$$-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

gravitational redshift

Doppler

$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

$$\left.\begin{array}{l} +\Psi-2\Phi+\dfrac{1}{\mathcal{H}}\dot{\Phi}-3\dfrac{\mathcal{H}}{k}V+\dfrac{2}{r}\displaystyle\int_0^r dr'(\Phi+\Psi) \\ +\left(\dfrac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\dfrac{2}{r\mathcal{H}}\right)\left[\Psi+\displaystyle\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right] \end{array}\right] \rightarrow \text{potential}$$

Result

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

standard expression

$$\Delta(z,\mathbf{n}) = \boxed{b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})}$$

lensing: important at high z

$$\boxed{-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)}$$

relativistic distortions: important at large scale

$$\boxed{\begin{aligned} &+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi \\ &+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\ &+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right] \end{aligned}}$$

$$\frac{\mathcal{H}}{k}\delta$$

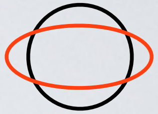
$$\left(\frac{\mathcal{H}}{k}\right)^2\delta$$

Convergence

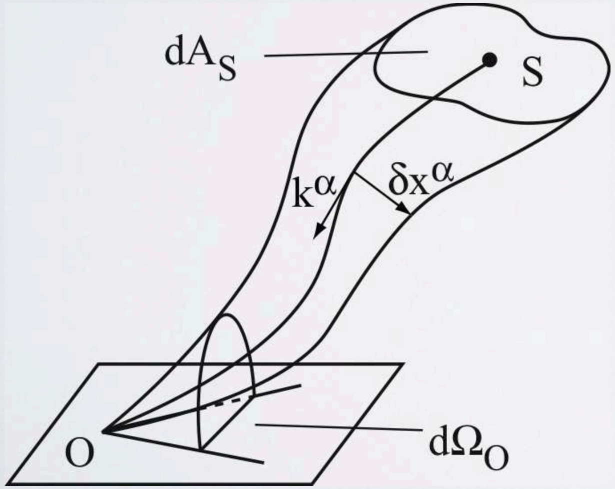
- Galaxy surveys observe also the shape and the luminosity of galaxies → measure of the **convergence**. Schmidt et al (2012) Alsing et al (2014)

- The convergence κ measures distortions in the **size**.

- The shear γ measures distortions in the **shape**.



- **Relativistic** distortions affect the convergence at **linear** order.



We solve the geodesic deviation equation

$$\frac{D^2\delta x^\alpha(\lambda)}{D\lambda^2} = R^\alpha_{\ \beta\mu\nu}k^\beta k^\mu \delta x^\nu$$

Convergence

CB (2008)
Bolejko et al (2013)
Bacon et al (2014)

Gravitational lensing

Doppler lensing

$$\begin{aligned}\kappa =&\frac{1}{2r}\int_0^r dr'\frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi)+\left(\frac{1}{r\mathcal{H}}-1\right)\mathbf{V}\cdot\mathbf{n}\\ &-\frac{1}{r}\int_0^r dr'(\Phi+\Psi)+\left(1-\frac{1}{r\mathcal{H}}\right)\int_0^r dr'(\dot\Phi+\dot\Psi)\end{aligned}$$

Integrated terms

$$+\left(1-\frac{1}{r\mathcal{H}}\right)\Psi+\Phi \rightarrow$$ Sachs Wolfe

Convergence

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Bolejko et al (2013)
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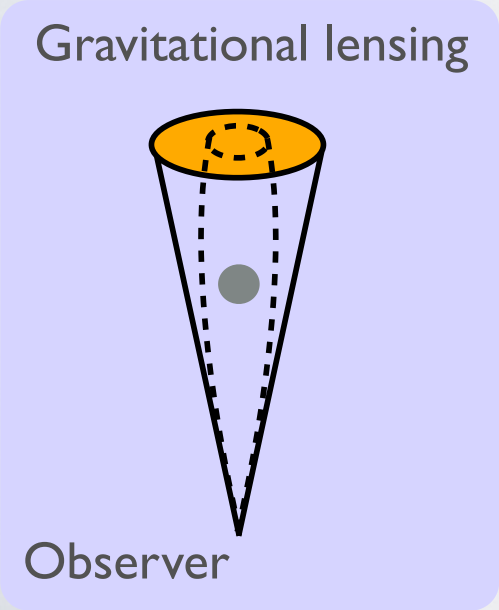
$$-\frac{1}{r}\int_0^r dr'(\Phi+\Psi) + \left(1-\frac{1}{r\mathcal{H}}\right)\int_0^r dr'(\dot\Phi+\dot\Psi)$$

Integrated terms

$$+\left(1-\frac{1}{r\mathcal{H}}\right)\Psi+\Phi \rightarrow \text{Sachs Wolfe}$$



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Integrated terms

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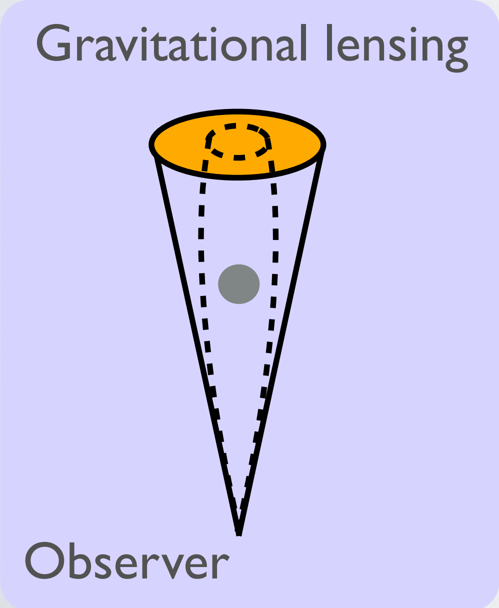
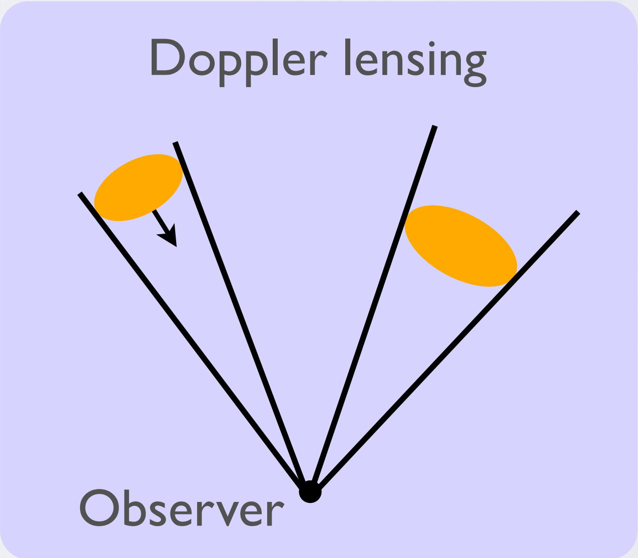




The moving galaxy is further away → it looks smaller, i.e. demagnified

Observations

- Due to relativistic effects, Δ and κ contain additional **information**.

$$\delta, V, \Phi, \Psi \qquad\qquad (\Phi + \Psi), V, \Phi, \Psi$$

- This can help **testing gravity** by probing the **relations** between density, velocity and gravitational potentials.

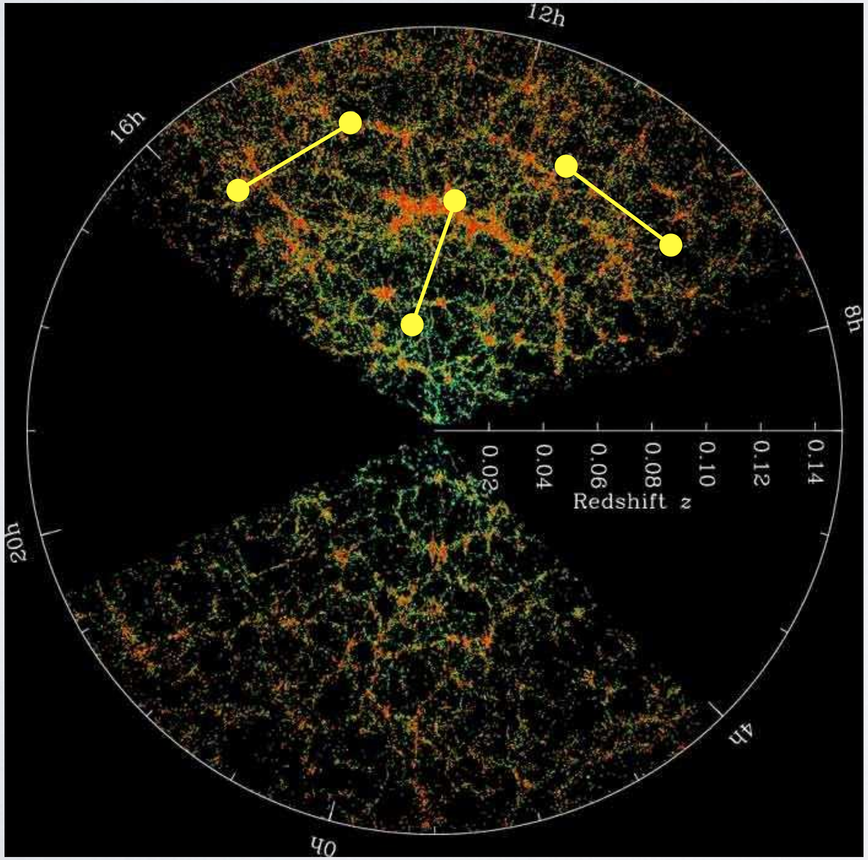
- Two **difficulties**:
  - The relativistic effects are small: we need to go to large scales.
  - We always measure the sum of all the effects.

- We need a way of **isolating** the relativistic effects.

  Method: look for **anti-symmetries** in the correlation function.

Correlation function

We do not **compare** $\Delta(\mathbf{n}, z)$ with observations, because we do not know the values of the density, velocity and gravitational potentials at $(\mathbf{n}, z)$

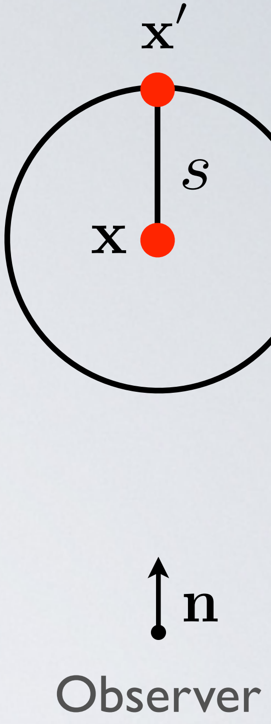


Credit: M. Blanton, SDSS

We compare **ensemble averages**

$$\xi = \langle \Delta(\mathbf{x}) \Delta(\mathbf{x}') \rangle$$

Density



The **density** contribution $\Delta = b \cdot \delta$, generates an **isotropic** correlation function.

$\xi(s) = \langle \Delta(\mathbf{x}) \Delta(\mathbf{x}') \rangle$ depends only on the **separation** $s = |\mathbf{x} - \mathbf{x}'|$

$$\xi(s) = \frac{Ab^2 D_1^2}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0} \right)^{n_s - 1} T_\delta^2(k)\, j_0(k \cdot s)$$

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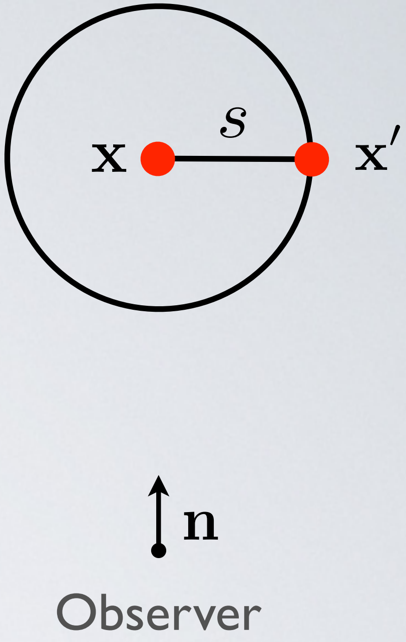

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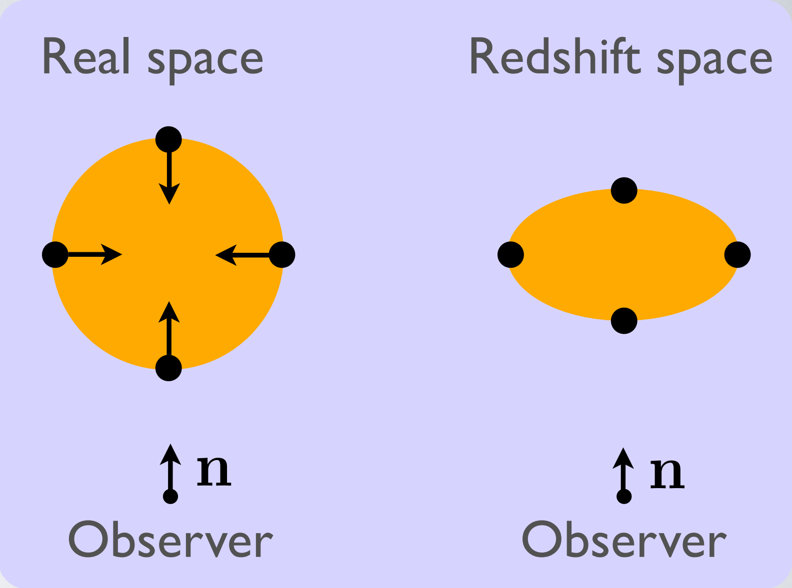
Redshift distortions

Redshift distortions **break** the **isotropy** of the correlation function.

$$\Delta = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$



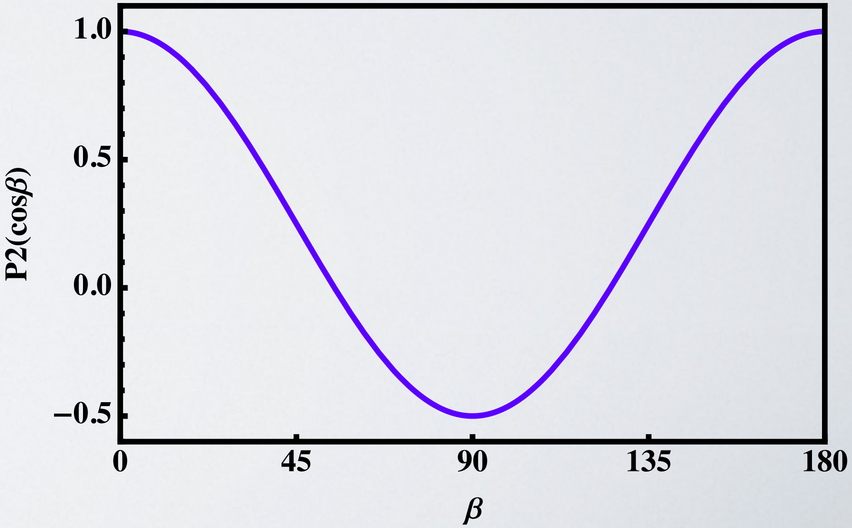
Quadrupole

Hamilton (1992)

$$\xi_2 = -D_1^2\left(\frac{4f}{3}+\frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta)$$

$$\mu_\ell(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1}T_\delta^2(k)\,j_\ell(k\cdot s)$$

$$P_2(\cos\beta) = \frac{3}{2}\cos^2\beta - \frac{1}{2}$$



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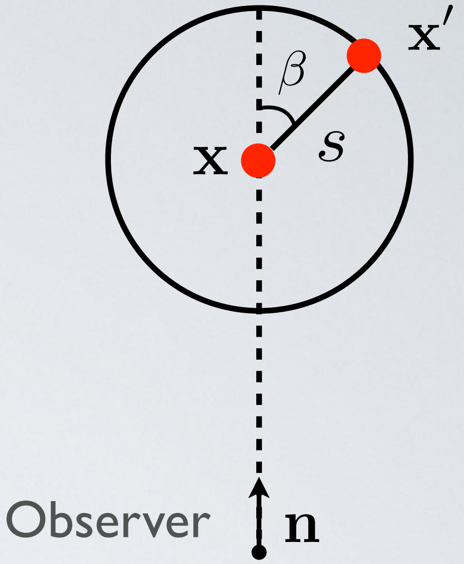

Quadrupole Hamilton (1992)

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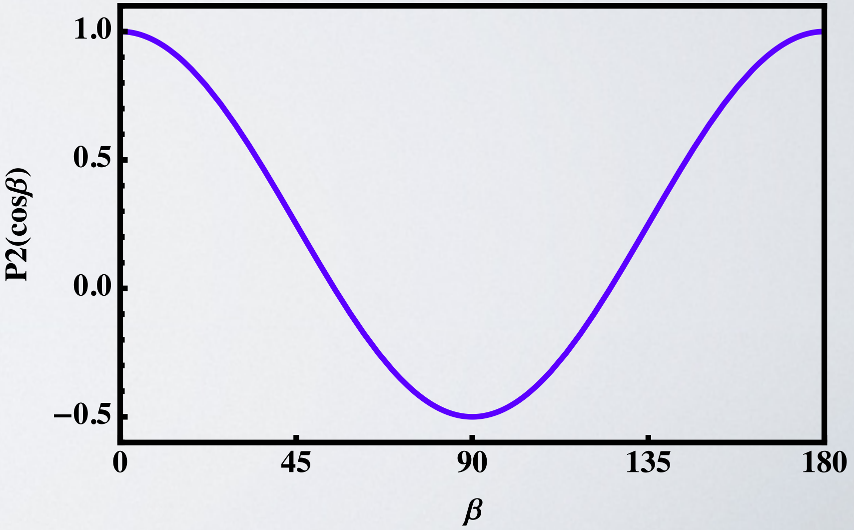

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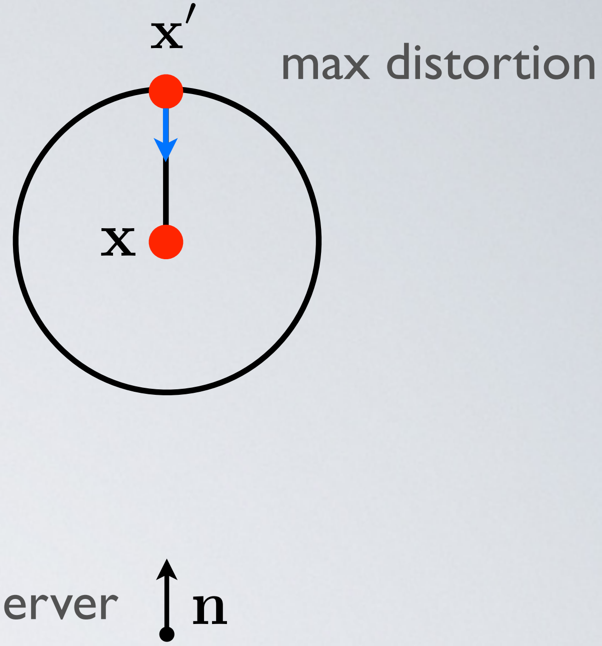

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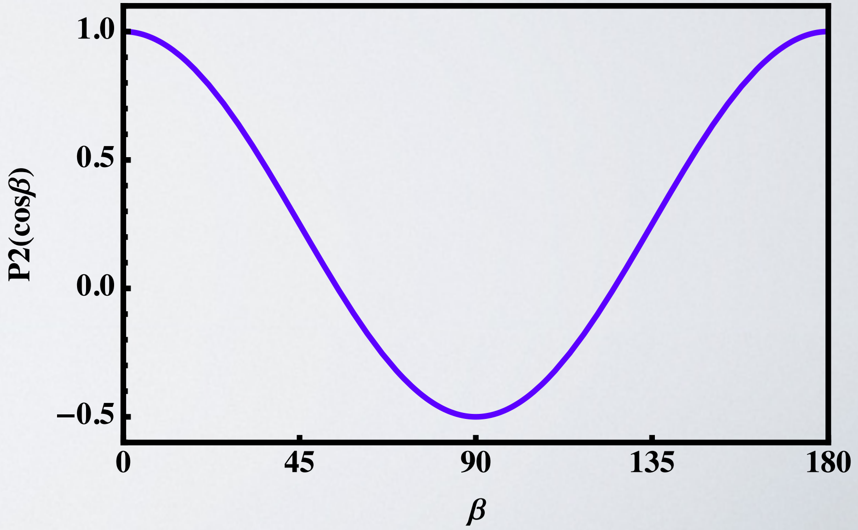

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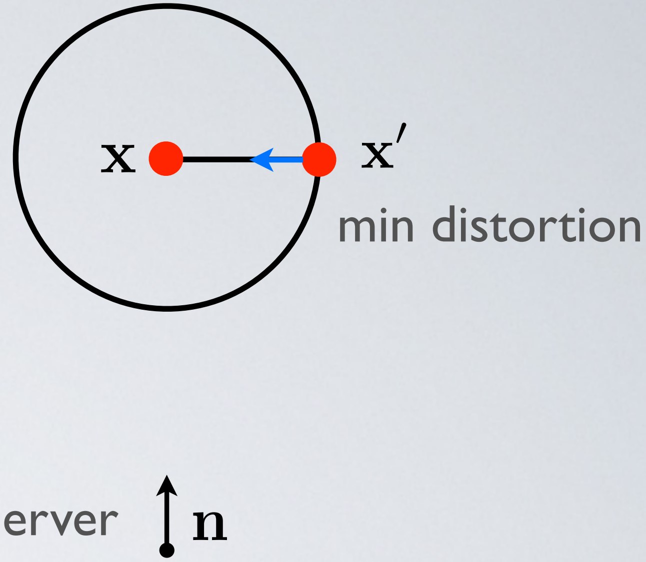

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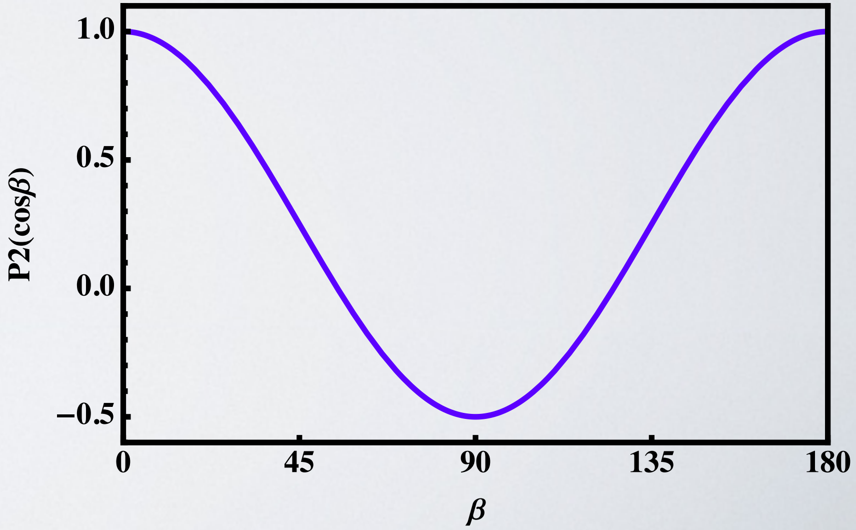

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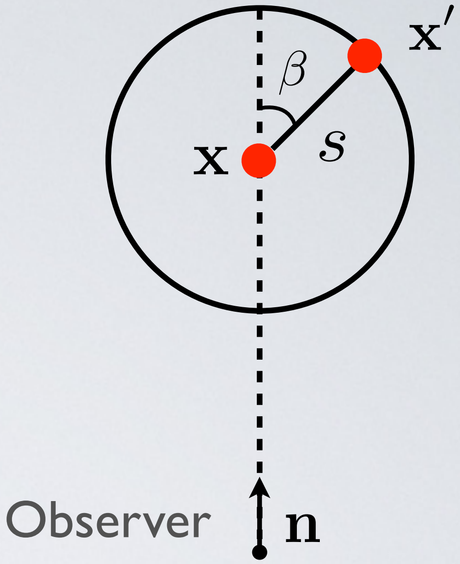

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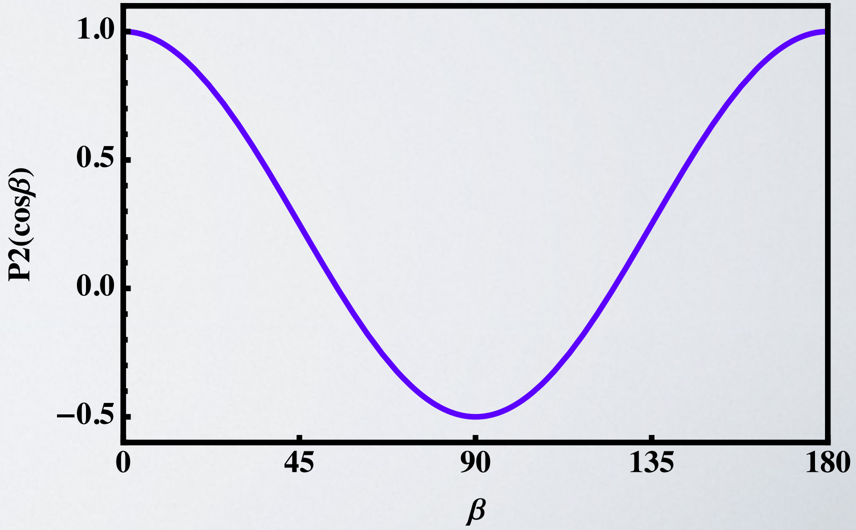

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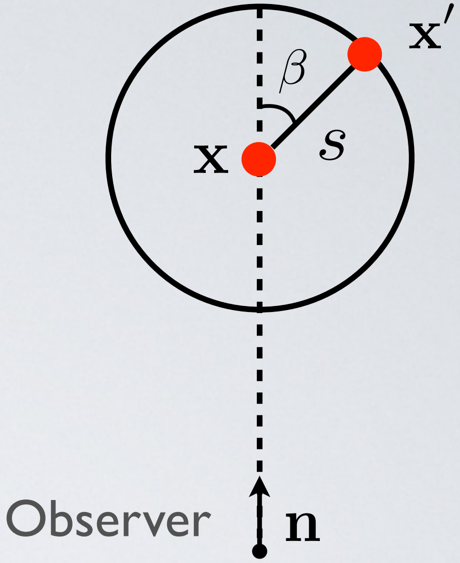
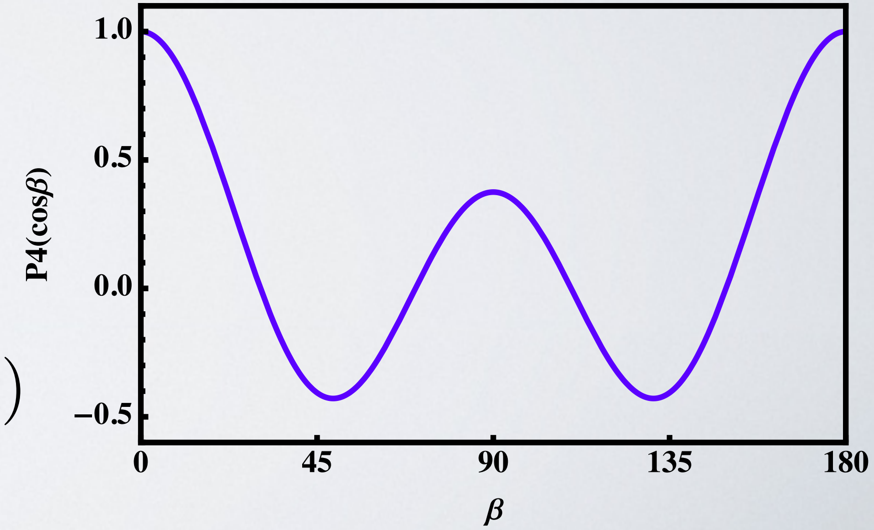


Hexadecapole Hamilton (1992)

$$P_4(\cos\beta) = \frac{1}{8}\left[35\cos^4\beta - 30\cos^2\beta + 3\right]$$

$$\xi_4 = D_1^2 \frac{8f^2}{35} \mu_4(s) P_4(\cos\beta)$$

$$\mu_4(s) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0}\right)^{n_s - 1} T_\delta^2(k)\, j_4(k \cdot s)$$



Redshift distortions

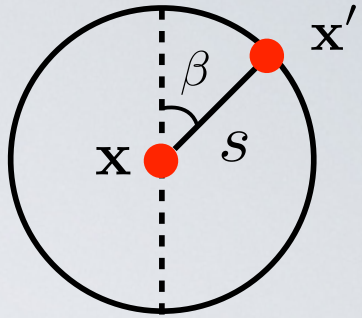
Samushia et al (2014)

$$\xi_0(s) = \frac{1}{2}\int_{-1}^{1} d\mu\ \xi(s,\mu)$$

$$\xi_2(s) = \frac{5}{2}\int_{-1}^{1} d\mu\ \xi(s,\mu) P_2(\mu)$$

$$\xi_4(s) = \frac{9}{2}\int_{-1}^{1} d\mu\ \xi(s,\mu) P_4(\mu)$$

$$\mu = \cos\beta$$



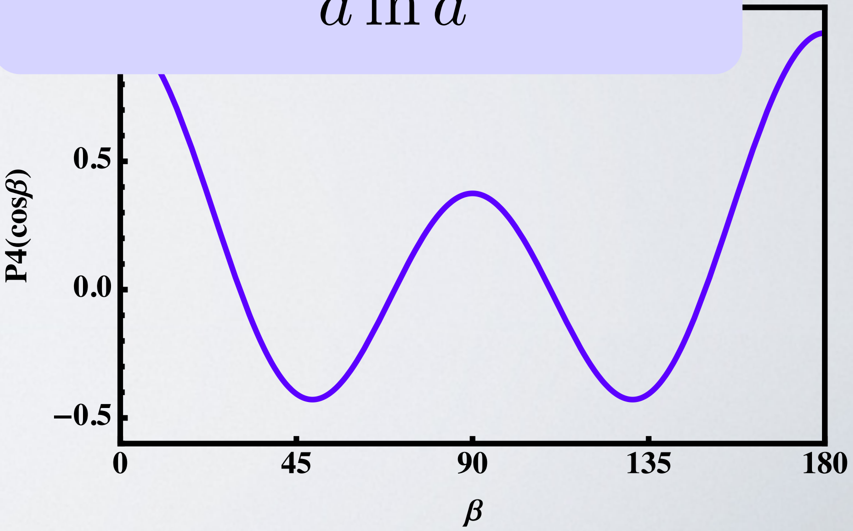
→

Measure separately:

$b\cdot\sigma_8$ and $f\cdot\sigma_8$

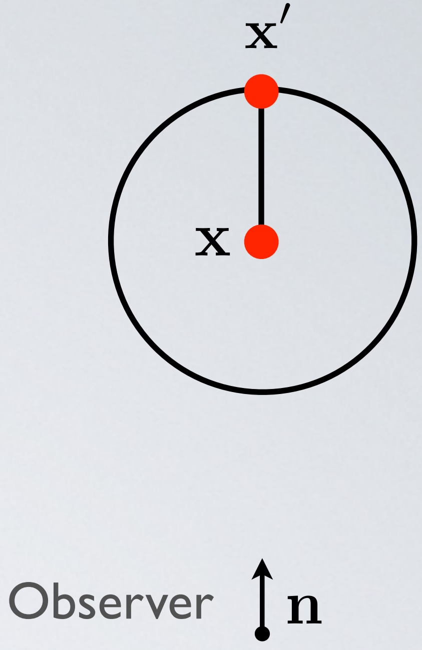
$$f = \frac{d\ln D_1}{d\ln a}$$

$$\mu_4(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_4(k\cdot s)$$



CB, Hui and Gaztanaga (2013)
McDonald (2009)

Relativistic distortions



The relativistic distortions break the **symmetry** of the correlation function.

The correlation function differs for galaxies **behind** or in **front** of the central one.

This differs from the breaking of **isotropy**, which is symmetric: redshift distortions have **even** powers of $\cos\beta$.

→ The amplitude is the same for $\beta = 0$ and $\beta = \pi$

To measure the asymmetry, we need **two populations** of galaxies: faint and bright.

CB, Hui and Gaztanaga (2013)
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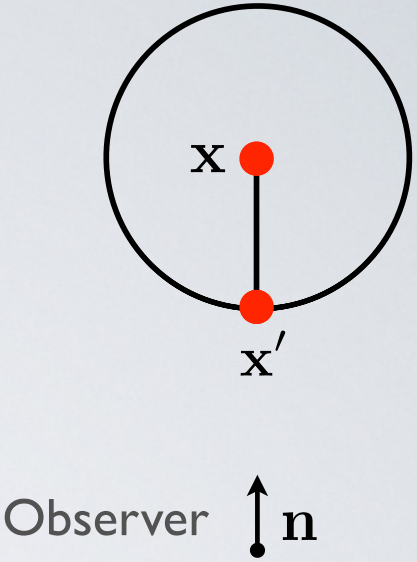

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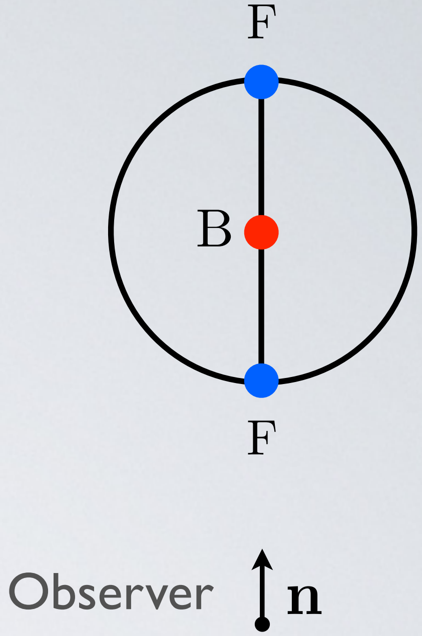

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Anti-symmetries

density

redshift space distortion

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

lensing

$$- \int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

gravitational redshift

Doppler

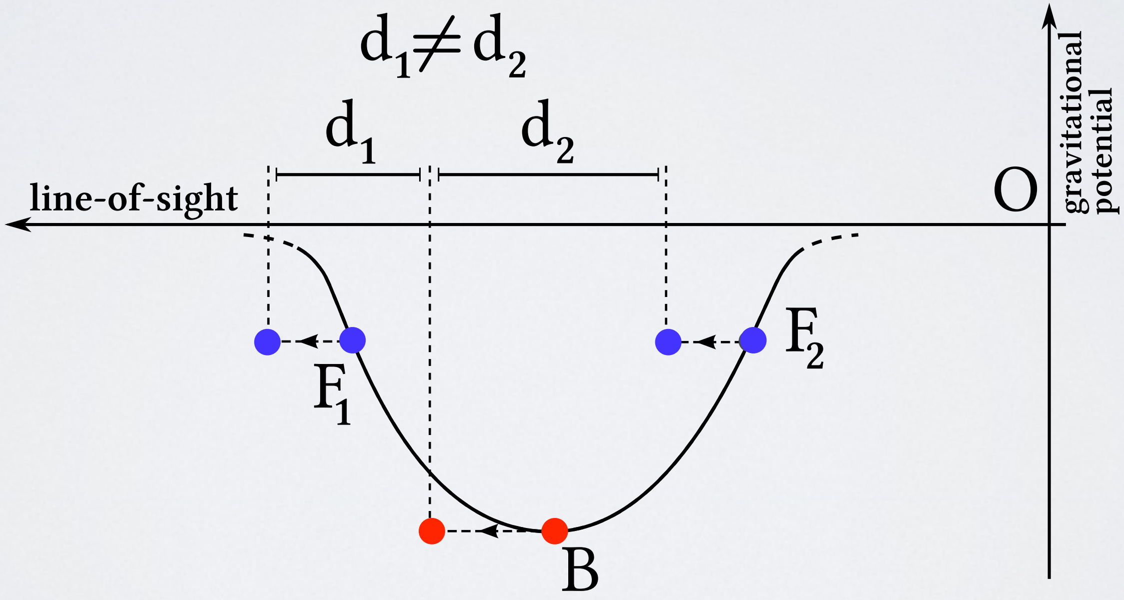
$$+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$

$$\left.\begin{aligned}&+\Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi)\\ &+\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]\end{aligned}\right] \rightarrow \text{potential}$$

Cross-correlation

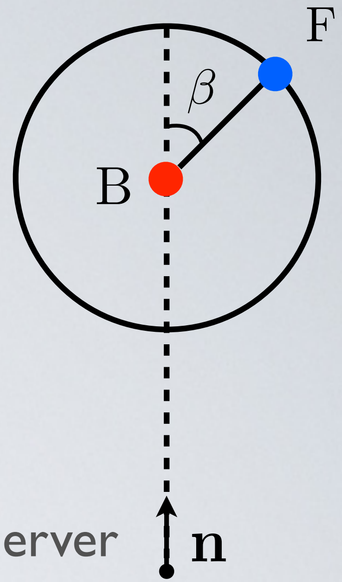
The following terms **break** the **symmetry**:

$$\Delta_{\rm rel} = \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi$$



Dipole in the correlation function

CB, Hui and Gaztanaga (2013)



$$\xi(s,\beta) = D_1^2\, f\, \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\rm B} - b_{\rm F})\nu_1(s)\cdot \cos(\beta)$$

$$\nu_1(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k) T_\Psi(k)\, j_1(k\cdot s)$$

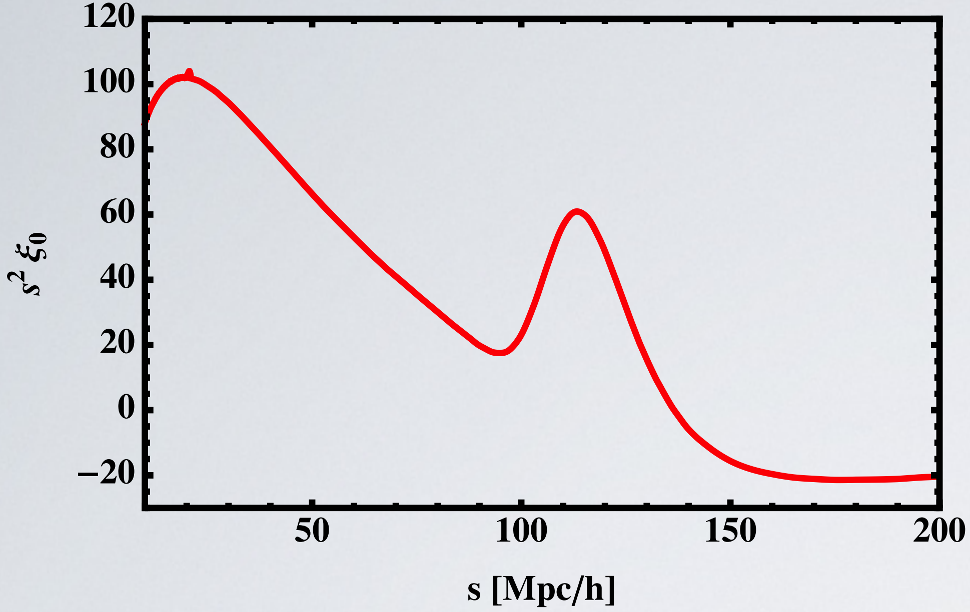
By fitting for a **dipole** in the correlation function, we can measure **relativistic distortions**, and separate them from the density and redshift space distortions.

$$\boxed{\xi_1(s) = \frac{3}{2}\int_{-1}^{1} d\mu\; \xi(s,\mu)\cdot\mu \qquad \mu = \cos\beta}$$

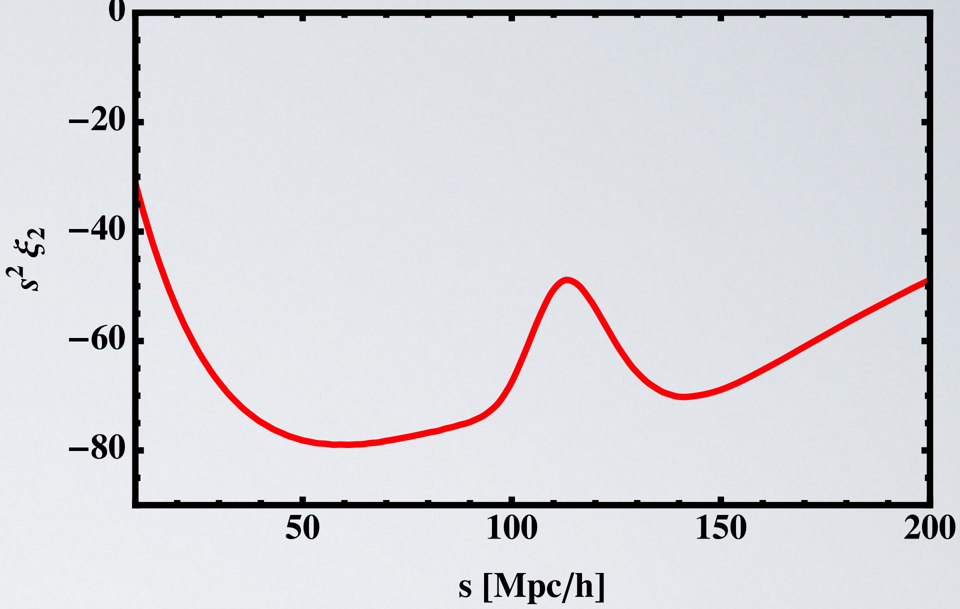
$z = 0.25$

Multipoles

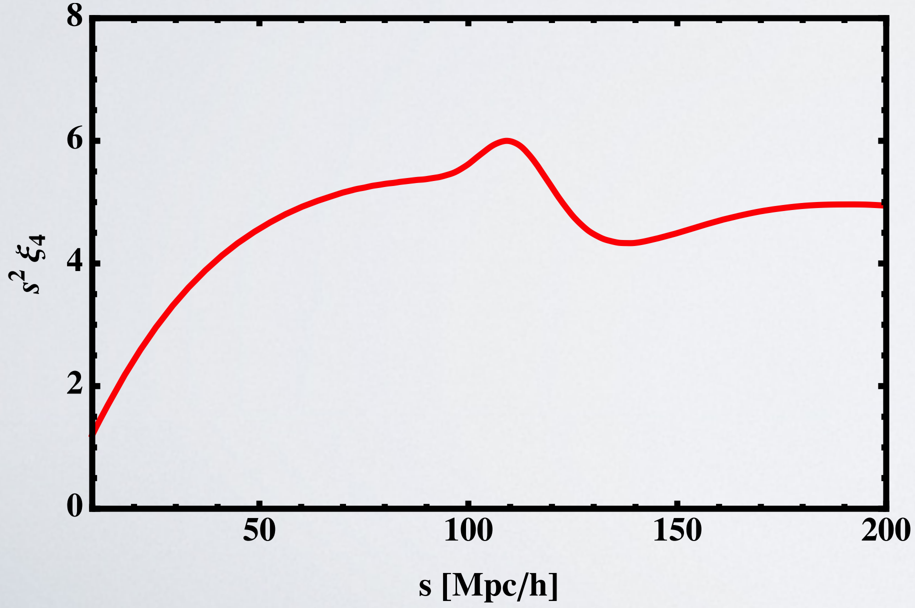
Monopole



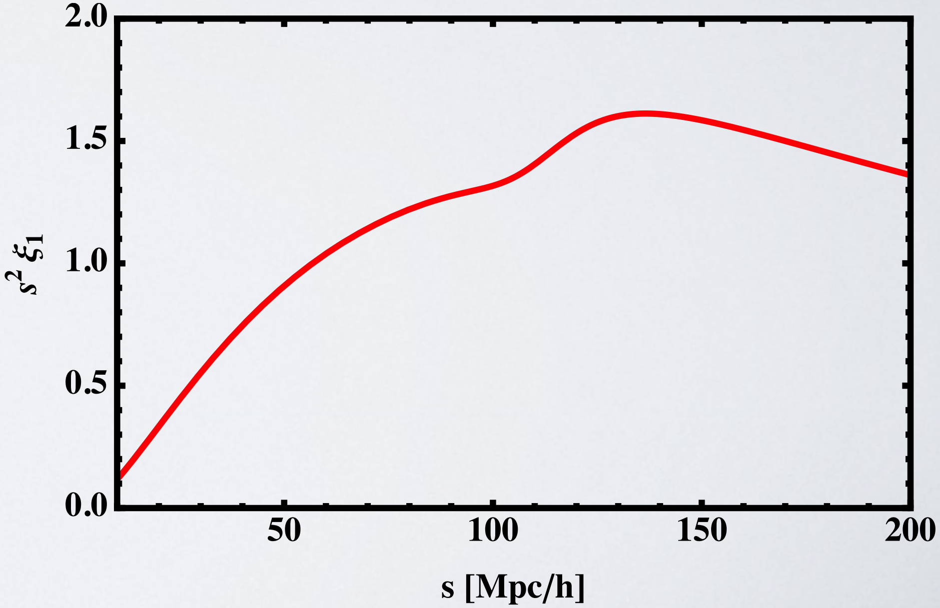
Quadrupole



Hexadecapole



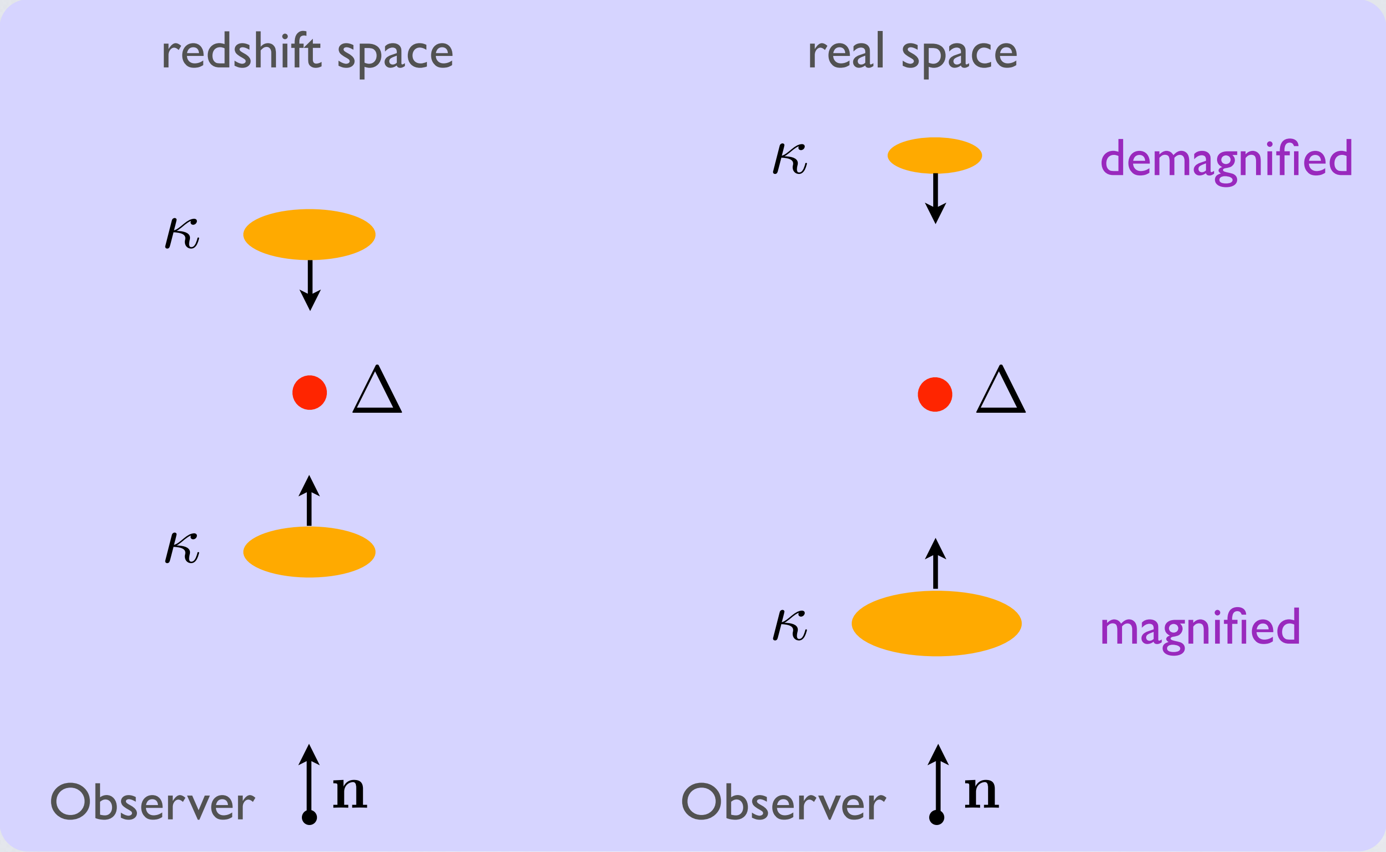
Dipole



Convergence

$$\kappa_{\rm g} = \frac{1}{2r}\int_0^r dr' \frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi) \qquad\qquad \kappa_{\rm v} = \left(\frac{1}{r\mathcal{H}}-1\right)\mathbf{V}\cdot\mathbf{n}$$

We can isolate the Doppler lensing by looking for **anti-symmetries** in $\langle\Delta\kappa\rangle$



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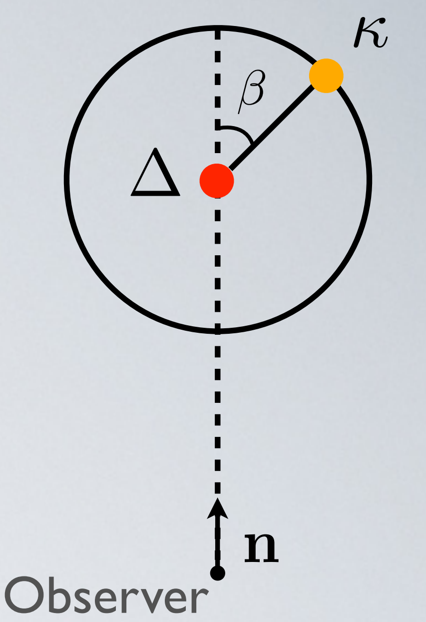
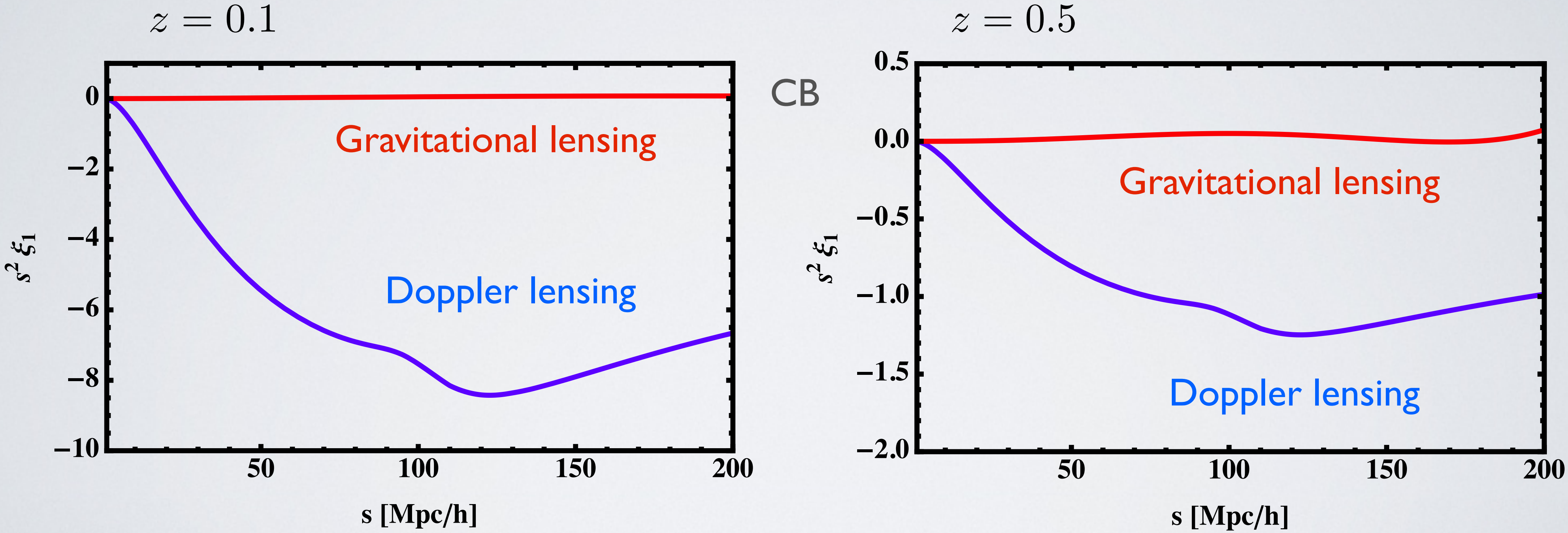


CB, Bacon, Andrianomena, Clarkson and Maartens (in preparation)

$$\xi(s,\beta) = \frac{2A}{9\pi^2\Omega_m^2} D_1^2 \frac{\mathcal{H}}{\mathcal{H}_0} f \left(1 - \frac{1}{\mathcal{H}r}\right)\left(b + \frac{3f}{5}\right) \nu_1(s) \cos(\beta)$$



The dipole due to gravitational lensing is completely subdominant.

Testing Euler equation

- The monopole and quadrupole in Δ allow to measure V
- The **dipole** allows to measure:

$$\Delta_{\rm rel} = \frac{1}{\mathcal{H}}\partial_r\Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$

- If **Euler** equation is valid: $\dot{\mathbf{V}}\cdot\mathbf{n} + \mathcal{H}\mathbf{V}\cdot\mathbf{n} + \partial_r\Psi = 0$

$$\Delta_{\rm rel} = -\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}$$

→ With the dipole, we can **test** the Euler equation.

Measuring the anisotropic stress

- The **dipole** in the convergence is sensitive to:

$$\kappa_{\rm v} = \left(\frac{1}{r\mathcal{H}} - 1\right)\mathbf{V}\cdot\mathbf{n}$$

- The standard part $\kappa_{\rm g} = \frac{1}{2r}\int_0^r dr'\frac{r-r'}{r'}\Delta_\Omega(\Phi+\Psi)$

  can be measure through $\langle\kappa\kappa\rangle$ and $\langle\gamma\gamma\rangle$

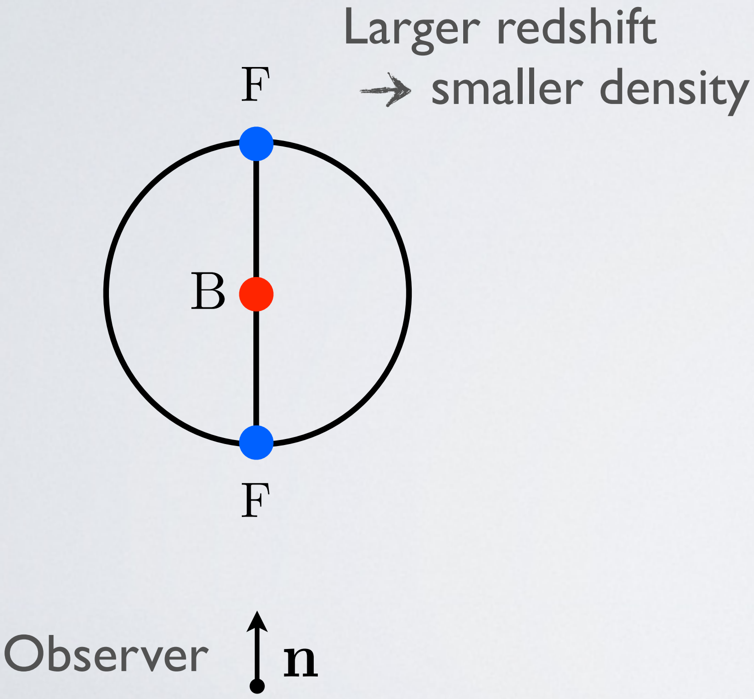
- Assuming Euler equation, we can test the **relation** between the two metric **potentials** Φ and Ψ.

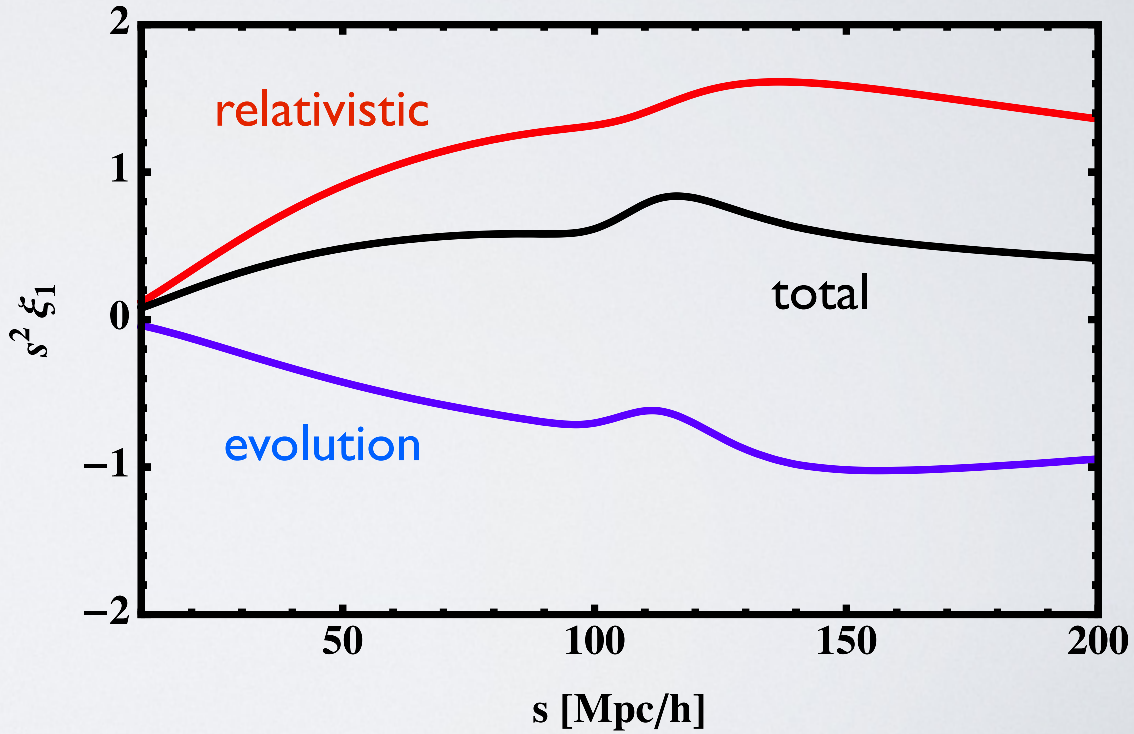
Conclusion

- Our **observables** are affected by relativistic effects.
- These effects have a different **signature** in the **correlation** function: they induce anti-symmetries.
- By measuring these anti-symmetries we can isolate the relativistic effects and use them to **test** the **relations** between the density, velocity and gravitational potentials.

Contamination

The density and velocity **evolve** with time: the density of the faint galaxies in front of the bright is larger than the density behind. This also induces a **dipole** in the correlation function.

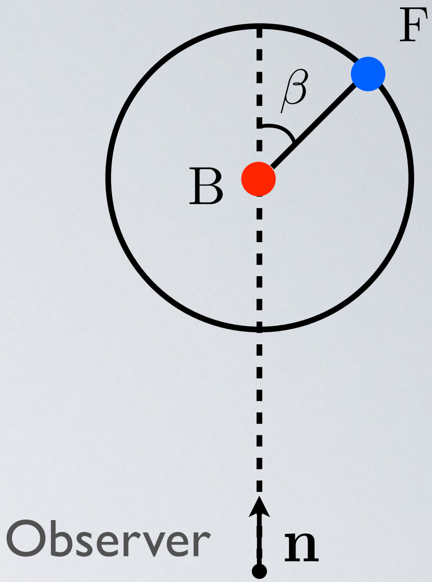


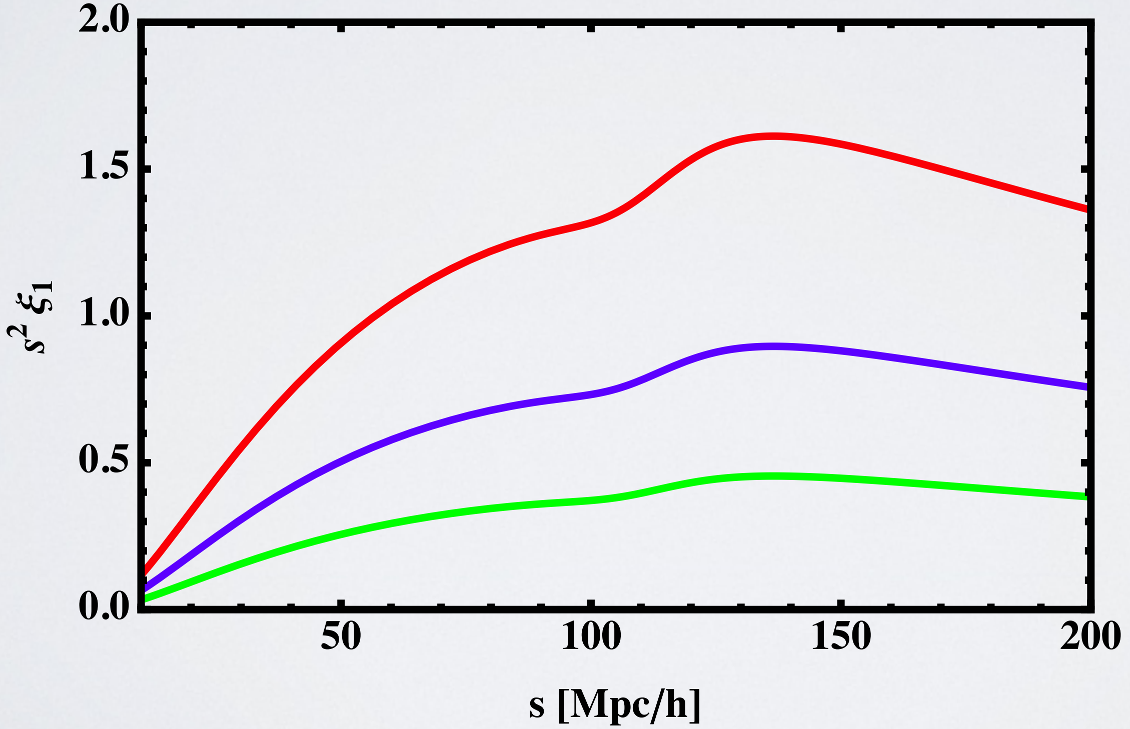


Dipole in the correlation function

$$\xi(s,\beta) = D_1^2\, f \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\rm B} - b_{\rm F})\nu_1(s)\cdot \cos(\beta)$$

$$\nu_1(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k)T_\Psi(k)\, j_1(k\cdot s)$$





$b_{\rm B} - b_{\rm F} \simeq 0.5$