Holographic Slow-Roll Inflation

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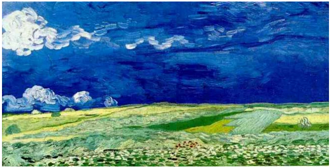
Work in collaboration with **Kostas Skenderis** and **Paul McFadden**.



University of Sussex, 15.10.2012.

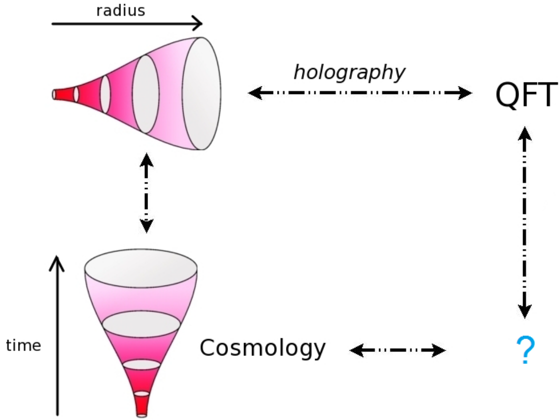
Rotate the Universe

Euclidean AdS

$$ds^2 = dr^2 + e^{2\alpha r} d\vec{x}^2.$$

de Sitter solution

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2.$$

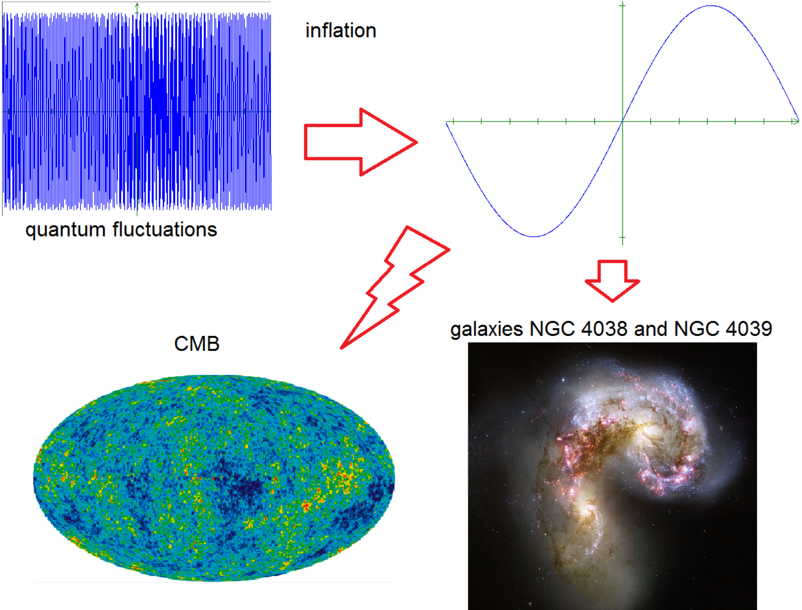


Cosmological observables ↔ correlation functions in CFT

Overview

1. Standard inflation and cosmology.
2. AdS/CFT and the holographic cosmology.
3. Two models:
    1. Slow-roll inflation as a deformation of a dual CFT.
    2. Holographic cosmology based on the super-renormalizable dual QFT.
4. Conclusions.

Cosmic Microwave Background



Primordial perturbations

Perturbations $\delta\phi$ of the *inflaton* (matter) $\Rightarrow$
perturbations in the *stress-energy tensor* $\delta T_{\mu\nu}$ $\Rightarrow$
perturbations of the *metric* $\delta g_{\mu\nu}$ via Einstein equations.

In the gauge where $\delta\phi = 0$,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\delta_{ij} + e^{2\zeta}[e^{\gamma}]_{ij}\right),$$

where

- ζ is the *scalar perturbation*,
- γ_{ij} is the *transverse-traceless perturbation*.

Observables

We use a notation

$$\gamma^{(s)} = \bar{\epsilon}^{(s)}_{ij} \gamma^{ij},$$

where $\epsilon^{(s)}_{ij}$ is a *helicity projector*, $s = \pm 1$, i.e. it spans a two-dimensional space of *transverse-traceless, symmetric tensors*

$$\epsilon^{(s)}_{ij} = \epsilon^{(s)}_{ji}, \qquad p^i \epsilon^{(s)}_{ij} = 0, \qquad \epsilon^{(s)}_{ii} = 0.$$

Standard observables are:

- *scalar and tensor power spectra*

$$\langle\langle \zeta(p)\zeta(-p) \rangle\rangle, \qquad \langle\langle \gamma^{(s)}(p)\gamma^{(s)}(-p) \rangle\rangle,$$

- bispectra or *non-gaussianities*

$$\langle\langle \zeta(p_1)\zeta(p_2)\zeta(p_3) \rangle\rangle, \qquad \langle\langle \gamma^{(s_1)}(p_1)\zeta(p_2)\zeta(p_3) \rangle\rangle$$
$$\langle\langle \gamma^{(s_1)}(p_1)\gamma^{(s_2)}(p_2)\zeta(p_3) \rangle\rangle, \qquad \langle\langle \gamma^{(s_1)}(p_1)\gamma^{(s_2)}(p_2)\gamma^{(s_3)}(p_3) \rangle\rangle,$$

- higher-point correlations functions.

Power spectra

One usually defines

$$\Delta_S^2(p) = \frac{p^3}{(2\pi)^2}\langle\langle\zeta(p)\zeta(-p)\rangle\rangle, \qquad \Delta_T^2(p) = \frac{p^3}{(2\pi)^2}\langle\langle\gamma^{(s)}(p)\gamma^{(s)}(-p)\rangle\rangle.$$

- If $\Delta_{S,T}$ does not depend on any dimensionful parameters different than the momentum – the spectrum is *scale-invariant*.
- If correlation functions are those of free fields – the spectrum is *gaussian*.
- Non-gaussianities are related to interactions in the underlying *fundamental theory*.

What can be measured?

Experimental parametrisation:

$$\Delta_S^2(p) = \Delta_S^2(p_0) \cdot \left(\frac{p}{p_0}\right)^{n_S - 1}, \qquad \Delta_T^2(p) = \Delta_T^2(p_0) \cdot \left(\frac{p}{p_0}\right)^{n_T - 3}.$$

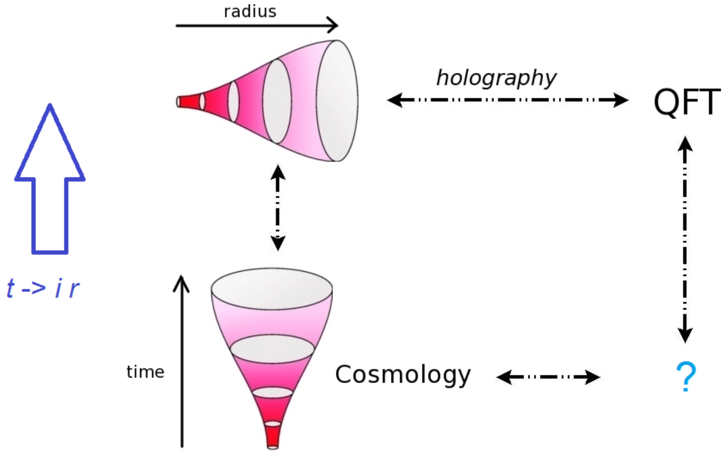
Current data:

$$\Delta_S^2(p_0) = (2.445 \pm 0.096) \cdot 10^{-9}, \qquad n_s - 1 = 0.040 \pm 0.013,$$

at $p_0 = 0.002\text{Mpc}^{-1}$.

1. Scalar power spectrum has *small amplitude* and is *almost scale invariant*.
2. In principle *non-gaussianities* were measured, but uncertainties usually exceed the measured value.
3. Not sufficient data for the remaining characteristics.

Cosmology/domain-wall duality



More explicitly

More formally, cosmology/domain-wall duality changes the following signs

$$\mathrm{d}s^2 = \mp\mathrm{d}z^2 + a^2(z)\mathrm{d}\vec{x}^2, \qquad \Phi = \Phi(z),$$

as well as

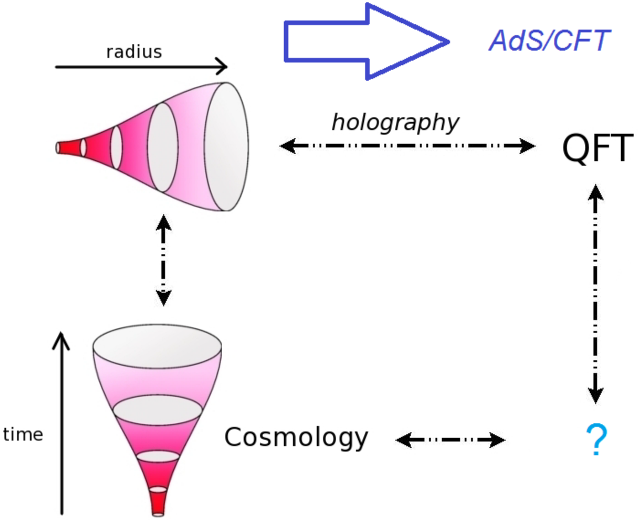
$$S = \frac{\mp 1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{|g|}\left[-R + (\partial\Phi)^2 + 2\kappa^2 V(\Phi)\right].$$

Therefore, cosmology/domain-wall duality leads to

$$\bar{\kappa}^2 = -\kappa^2, \qquad \bar{p} = -\mathrm{i}p,$$

[Maldacena (2005)], [Skenderis, Townsend (2006, 2007)], [McFadden, Skenderis (2010)]

Gauge/Gravity duality



To be precise

For every field Φ in AdS_{d+1} there exists an operator $\mathcal{O}_\Phi$ in CFT such that

$$Z_{\mathrm{SUGRA}}[\phi_{(0)}] = \left\langle \exp\left(- \int_{\partial AdS} \phi_{(0)} \mathcal{O}_\Phi \right) \right\rangle_{\mathrm{CFT}} .$$

For example

- scalar field Φ in AdS of mass m $\Leftrightarrow$ conformal operator $\mathcal{O}_\Phi$ of dimension Δ given by $m^2 = \Delta(d - \Delta)$.
- metric $g^{\mu\nu}$ in AdS $\Leftrightarrow$ energy-momentum tensor $T^{\mu\nu}$.

Similarly, there exists *a gauge/gravity duality* between some gauge theory without gravity and a gravity theory with the metric asymptotically *power law*

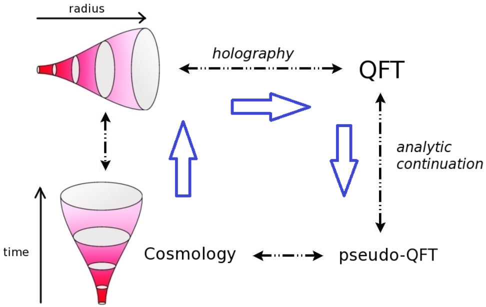
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + t^{2\alpha}\mathrm{d}\vec{x}^2,$$

[Kanitscheider, Taylor, Skenderis (2008)]

Analytic continuation

Euclidean AdS

de Sitter



AdS side

$$
\begin{aligned}
\bar{\kappa}^2 &= -\kappa^2, \\
\bar{p} &= -\mathrm{i}p.
\end{aligned}
$$

CFT side

$$
\begin{aligned}
\bar{N} &= -\mathrm{i}N, \\
\bar{p} &= -\mathrm{i}p.
\end{aligned}
$$

Cosmology/CFT correspondence

Two-point functions

$$\langle\!\langle \zeta(p)\zeta(-p)\rangle\!\rangle = \frac{-1}{8\,\mathrm{Im}\,B(\bar{p})}, \qquad \langle\!\langle \gamma^{(s)}(p)\gamma^{(s')}(-p)\rangle\!\rangle = \frac{-\delta^{ss'}}{\mathrm{Im}\,A(\bar{p})},$$

where

$$\langle\!\langle T^{ij}(\bar{p})T^{kl}(\bar{p})\rangle\!\rangle = A(\bar{p})\Pi^{ijkl} + B(\bar{p})\pi^{ij}\pi^{kl}.$$

In [McFadden, Skenderis (2011)] all three-point function has been worked out, for example

$$\langle\!\langle \zeta(p_1)\zeta(p_2)\zeta(p_3)\rangle\!\rangle = -\frac{1}{256} \times \frac{1}{\prod_{j=1}^{3}\mathrm{Im}\,B(\bar{q}_j)}\,\mathrm{Im}\Big[\langle\!\langle T(\bar{p}_1)T(\bar{p}_2)T(\bar{p}_3)\rangle\!\rangle$$
$$+3\sum_{j=1}^{3}\langle\!\langle T(\bar{p}_j)T(-\bar{p}_j)\rangle\!\rangle - 2\left(\delta^{ij}\langle\!\langle \frac{\delta T}{\delta g^{ij}}(\bar{p}_1,\bar{p}_2)T(\bar{p}_3)\rangle\!\rangle_{g=\delta} + 2\text{ perm.}\right)\Big].$$

The models

The basic questions that can be asked are:

1. Given a *cosmology*, what is the *dual QFT*?
2. Given a *dual QFT*, what is the *corresponding cosmology*?

In our papers [AB, McFadden, Skenderis (2011)], [AB, McFadden, Skenderis (2012) to appear] we have been considering two models:

1. The cosmology is given by an inflaton with a *fake superpotential*. What is the dual QFT?
2. A dual QFT is *super-renormalizable*, defined by perturbations around *free theories*. What is the cosmology?

Holographic slow-roll inflation

The inflaton potential is given by a *fake superpotential* [Townsend (1984)], [Bond, Salopek (1991)], ...

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[R - (\partial\Phi)^2 - 2\kappa^2 V(\Phi) \right]$$

with

$$-2\kappa^2 V = W_{,\Phi}^2 - \frac{3}{2} W^2, \qquad W = -2 - \frac{\lambda}{2}\Phi^2 - \frac{c}{3}\Phi^3,$$

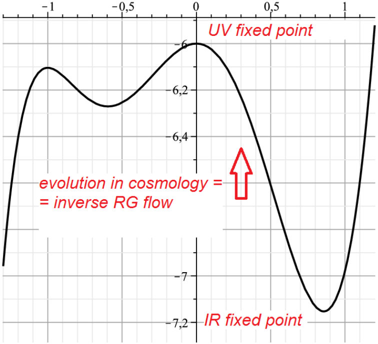
where $\lambda \ll 1$ and $c \sim 1$.

Equations of motion can be *solved exactly* and the background geometry turns out to be *asymptotically de Sitter*.

$$\Phi(t) = \frac{3\lambda}{c} \cdot \frac{1}{1+e^{\lambda t}},$$

$$a(t) = \left(1+e^{\lambda t}\right)^{-\frac{\lambda^2}{3c^2}} \exp\left[t\left(1 + \frac{\lambda^3}{3c^2}\right) + \frac{\lambda^2 e^{\lambda t}}{3c^2(1+e^{\lambda t})^2} \right].$$

Cosmology to follow



For this model

$$\epsilon_* = \frac{2\lambda^4}{c^2}\frac{p^{2\lambda}}{(1+p^\lambda)^4} + O(\lambda^7)$$

$$\eta_* = -\lambda + \frac{2\lambda}{1+p^\lambda} + O(\lambda^4)$$

- This is a *'hilltop' inflationary model*.
- In this model $\epsilon_* \sim \eta_*^4$, which differs from a textbook slow-roll inflation, where $\epsilon_* \sim \eta_*$.

In particular,

$$\langle\langle \zeta(p)\zeta(-p)\rangle\rangle = \frac{H_*^2}{4\epsilon_*}(1 + 2b\eta_* + O(\lambda^2)).$$

The dual description

With the potential

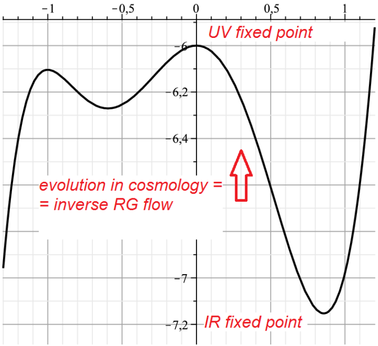
$$-2\kappa^2 V = W_{,\Phi}^2 - \frac{3}{2}W^2, \qquad W = -2 - \frac{\lambda}{2}\Phi^2 - \frac{c}{3}\Phi^3,$$

the mass of the inflaton Φ is $m^2 = \lambda(3-\lambda)$.

In the dual description it corresponds to the operator $\mathcal{O}_{\mathrm{UV}}$ of dimension $\Delta_{\mathrm{UV}} = 3 - \lambda$ turned on in the UV CFT,

$$S = S_{\mathrm{UV\ CFT}} + \frac{2\lambda}{c}\int \mathrm{d}^3x\, \mathcal{O}_{\mathrm{UV}}.$$

The dual description in the IR



- The inflationary phase is described by the *inverse RG flow* from the IR to the UV CFT.
- In the IR the deformation is given by an irrelevant conformal primary operator $\mathcal{O}_{\rm IR}$ of dimension $\Delta_{\rm IR} = 3 + \lambda + O(\lambda^4)$ and

$$S = S_{\rm IR\ CFT} - \frac{2\lambda}{c} \int \mathrm{d}^3x\, \mathcal{O}_{\rm IR}.$$

What we want to achieve

- We want to show that the cosmological results can be obtained via the cosmology/CFT duality.
- In other words, the *slow-roll inflation* can be understood as a simple *deformation of the dual CFT*.

- Since $\lambda \ll 1$, we can use the *perturbation theory* around the CFTs.
- *CFT does not have to be weakly coupled!*
- Since the gravity side is weakly coupled, the agreement on the both sides of the correspondence leads to a non-trivial check on AdS/CFT.

Sketch of the calculations in the CFT

We need to calculate the correlation functions

$$\langle\!\langle TT \rangle\!\rangle, \qquad \langle\!\langle T^{(s)} T^{(s')} \rangle\!\rangle, \qquad \langle\!\langle TTT \rangle\!\rangle$$
$$\langle\!\langle T^{(s_1)} TT \rangle\!\rangle, \qquad \langle\!\langle T^{(s_1)} T^{(s_2)} T \rangle\!\rangle, \qquad \langle\!\langle T^{(s_1)} T^{(s_2)} T^{(s_3)} \rangle\!\rangle$$

in the *deformed CFT*, i.e. in the QFT with the action

$$S = S_{\text{CFT}} + \frac{2\lambda}{c} \int \mathrm{d}^3x \, \mathcal{O},$$

where $\mathcal{O}$ is a scalar conformal primary of dimension $\Delta = 3 - \lambda$.

Since

$$\langle T(x) \rangle_{\text{src}} = (\Delta - d)\phi_0(x) \langle \mathcal{O}(x) \rangle_{\text{src}},$$

all correlation functions involving trace of T are related to the correlation functions involving $\mathcal{O}$.

Sketch of the calculations in the CFT, continued

$$\langle\langle \zeta(p)\zeta(-p)\rangle\rangle = -\frac{c^2}{8\lambda^4}\frac{1}{\mathrm{Im}\,\langle\langle \mathcal{O}(\bar{p})\mathcal{O}(-\bar{p})\rangle\rangle},$$

$$\langle\langle \zeta\gamma^{(s_2)}\gamma^{(s_3)}\rangle\rangle = \frac{c\left[\langle\langle \mathcal{O}(\bar{p}_1)T^{(s_2)}(\bar{p}_2)T^{(s_3)}(\bar{p}_3)\rangle\rangle + \text{semi-local terms}\right]}{2\lambda^2\,\mathrm{Im}\langle\langle \mathcal{O}(\bar{p}_1)\mathcal{O}(-\bar{p}_1)\rangle\rangle\cdot\mathrm{Im}\,A(\bar{p}_2)\,\mathrm{Im}\,A(\bar{p}_3)}$$

and then the perturbative expansion, e.g.

$$\begin{aligned}\langle \mathcal{O}(x)\mathcal{O}(y)\rangle &= \langle \mathcal{O}(x)\mathcal{O}(y)e^{-\frac{2\lambda}{c}\int\mathcal{O}}\rangle_{\mathrm{CFT}}\\ &= \langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\mathrm{CFT}} - \frac{2\lambda}{c}\int \mathrm{d}^3u\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(u)\rangle_{\mathrm{CFT}} + \dots\end{aligned}$$

Two challenges:

1. To find out the general expressions for three-point functions such as $\langle\langle \mathcal{O}(p_1)T^{(s_2)}(p_2)T^{(s_3)}(p_3)\rangle\rangle$ in *momentum space*.
2. Use the perturbative expansion in the *leading order* in λ (more complex than it looks).

Three-point functions in CFT

- The form of all *three-point functions in CFTs* involving a scalar conformal primary and the stress-energy tensor are given *in position space* by Osborn and Petkos.
- We have worked out a *complete set of three-point functions* of a scalar primary operator, conserved spin-1 current and stress-energy tensor. We chose a different approach via the *direct solution of conformal Ward identities*. This is a topic for another presentation...
- We use the method to evaluate all necessary three-point functions of stress-energy tensor and the operator of dimension $3 - \lambda$ to *leading order* in λ.

Now we want to evaluate

$$\begin{aligned}
\langle \mathcal{O}(x)\mathcal{O}(y) \rangle &= \langle \mathcal{O}(x)\mathcal{O}(y) e^{-\frac{2\lambda}{c}\int \mathcal{O}} \rangle_{\text{CFT}} \\
&= \langle \mathcal{O}(x)\mathcal{O}(y) \rangle_{\text{CFT}} - \frac{2\lambda}{c} \int \mathrm{d}^3 u \langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(u) \rangle_{\text{CFT}} + \ldots
\end{aligned}$$

in the leading order in λ.

Limits of n-point functions

- Problem:

$$\langle\langle \mathcal{O}(p_1)\mathcal{O}(p_2)\mathcal{O}(p_3)\rangle\rangle = \frac{|p_1|^3 + |p_2|^3 + |p_3|^3}{\lambda} + O(\lambda^0).$$

- One can show that the leading λ behaviour of

$$I_n = \int \mathrm{d}^3 u_1 \ldots \mathrm{d}^3 u_n \langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(u_1)\ldots\mathcal{O}(u_n)\rangle$$

in the *momentum space* is

$$I_n(p) \sim \frac{1}{\lambda^n} p^{3-(n+2)\lambda}.$$

- In the perturbative expansion

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\mathrm{CFT}} - \frac{2\lambda}{c}\int \mathrm{d}^3 u \langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(u)\rangle_{\mathrm{CFT}} + \ldots$$

all terms contribute!

Summation of the series

- One can show that the leading λ behaviour of the n-point function is

$$\lim_{p_1,p_2,\ldots,p_n\to 0}\langle\langle\mathcal{O}(p)\mathcal{O}(-p)\mathcal{O}(p_1)\ldots\mathcal{O}(p_n)\rangle\rangle$$
$$=\frac{(-1)^n(n+3)!}{2\cdot 3^{n+1}}\cdot\frac{p^{3-(n+2)\lambda}}{\lambda^n}+O\left(\frac{1}{\lambda^{n-1}}\right).$$

- The behaviour is *universal* assuming λ has 'nothing to do' with other operators in the theory.
- The summation is possible and we find the perfect *agreement with cosmological two and three-point functions*.

The model with non-geometric inflation

This model is analysed in [AB, McFadden, Skenderis (2011)]. We start with the dual QFT

$$S = \frac{1}{g_{\rm YM}^2}\int \mathrm{d}^3x \ \mathrm{Tr}\left[\frac{1}{2}(F_{ij}^I)^2 + \frac{1}{2}(D\phi^J)^2 + \frac{1}{2}(D\chi^K)^2 + \bar{\psi}^L \not{D}\psi^L \right.$$
$$\left. + \text{interactions}\right],$$

where we have

- $\mathcal{N}_A$ $SU(N)$-gauge fields A_i^{aI},
- $\mathcal{N}_\phi$ minimal real scalars ϕ^J,
- $\mathcal{N}_\chi$ conformal real scalars χ^K,
- $\mathcal{N}_\psi$ fermions ψ_α^L,

with all fields transforming in the adjoint representation of $SU(N)$.

The QFT is assumed to be *weakly coupled*, therefore

- The *perturbative analysis* on the QFT side is possible.
- We should expect a *strongly coupled gravity* on the cosmology side.

Predictions

Predictions *differ* from the conventional inflation but are *compatible with the current data*. The scalar power spectrum is

$$\langle\!\langle \zeta\zeta \rangle\!\rangle \sim \frac{1}{\langle\!\langle TT \rangle\!\rangle} \sim \frac{1}{N^2(\mathcal{N}_\phi + \mathcal{N}_A)p^3 \left[1 + C g_{\text{eff}}^2 \ln(p/p_0) + O(g_{\text{eff}}^4)\right]},$$

where the dimensionless effective coupling is $g_{\text{eff}}^2 = g_{\text{YM}}^2 \bar{N}/\bar{p}$.

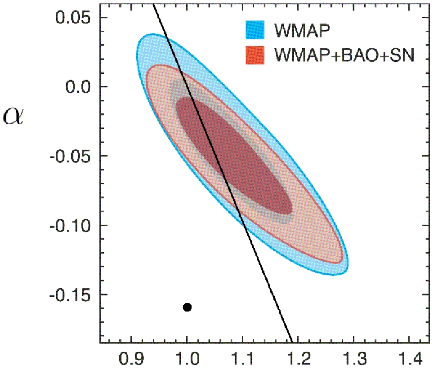
1. The amplitude is small $\Rightarrow N \sim 10^4$ – *large N expansion* is valid.
2. Almost scale invariance $\Rightarrow g_{\text{eff}}^2 \sim 10^{-2}$ at $p_0 = 0.002\text{Mpc}^{-1}$ – *QFT is weakly coupled*.
3. The dual *cosmology is strongly coupled* and stringy corrections are important. The inflation phase is *non-geometric*.
4. The geometry is asymptotically the *power-law inflation* $\mathrm{d}s^2 \sim -\mathrm{d}t^2 + t^{2n}\mathrm{d}\vec{x}^2$ for $t \to \infty$ and some n.

Running of the spectral index

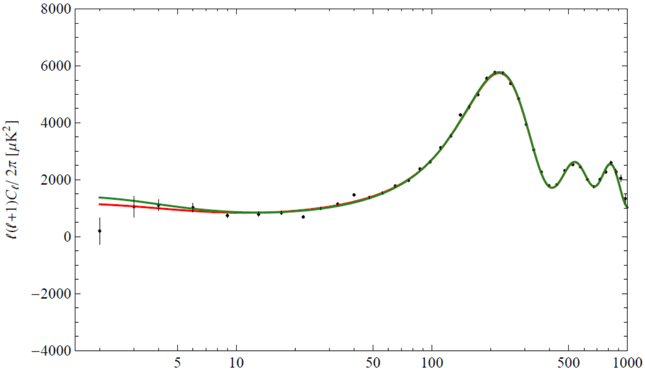
Holographic models predict [McFadden, Skenderis (2010)], [Dias (2011)]

$$\frac{\mathrm{d}n_s}{\mathrm{d}\log p} = -(n_s - 1) + O(g_{\mathrm{eff}}^4),$$

while conventional slow-roll inflation predicts that $\alpha_s/(n_s - 1)$ is very small (of first order in slow-roll).



ΛCDM vs. our model



ΛCDM model, our holographic model.

- The dedicated analysis was carried out in [Easther, Flauger, McFadden, Skenderis (2011)]. Related work: [Dias (2011)].
- Other characteristics such as non-gaussianities were calculated as well.

Conclusions

1. *Inflation is holographic*: standard observables such as power spectra and non-gaussianities can expressed and calculated by means of the correlation functions of the dual QFT.
2. The QFT dual to *slow-roll inflation* is a deformation of a CFT.
3. Slow-roll results are essentially fixed by *conformal invariance*.
4. There are *new holographic models* based on perturbative QFT that describe the inflation starting in a non-geometric, strongly coupled phase.
5. A class of models based on super-renormalizable QFT was fit to data and shown to provide a *competitive model to ΛCDM*. Data from the Planck satellite should permit a *definitive test* of this scenario.

Thank you for your attention.

arXiv:
[hep-th/0907.5542]
[hep-th/1001.2007]
[hep-th/1010.0244]
[hep-th/1011.0452]
[hep-th/1104.2040]
[hep-th/1104.3894]
[hep-th/1112.1967]