On the critical endpoint in the QCD phase diagram

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Formal Seminars

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Outline

- **QCD Phase Diagram**
- **Chiral QCD-like models with 'statistical' confinement**
- **Importance of fluctuations:**
  - ▷ Matter back-coupling to Yang-Mills sector
- **Anatomy of the critical region around the CEP**
- **Higher moments**
- **QCD for $N_c = 2$**

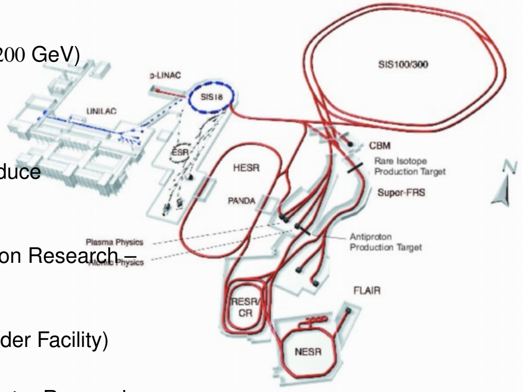
Heavy-Ion Collision Experiments

aim: create hot and dense QCD matter $\rightarrow$ elucidate its properties

QCD under extrem conditions: very active field (April 2012)

6 large experiments

1. RHIC @ BNL (Au-Au collisions $\sqrt{s_{NN}} \sim 200$ GeV)
2. LHC @ CERN (higher energies)
3. LeRHIC @ BNL (low-energy scan to produce $n_B >> n_0 \sim 0.17$ fm^{-3})
4. FAIR @ GSI (Facility for Antiproton and Ion Research – hopefully SIS-300)
5. NICA @ JINR (Nuclotron-based Ion Collider Facility)
6. J-PARC @ JAERI (Japan Proton Accelerator Research Complex)



QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \longrightarrow SU_L(N_f) \times SU_R(N_f)$

   order parameter:

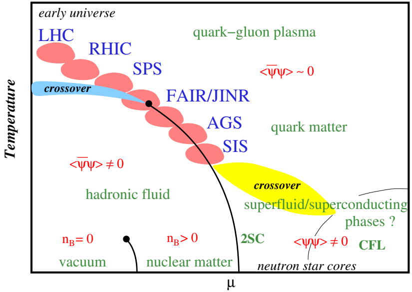
   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

2. de/confinement

   order parameter: Polyakov loop variable

   $$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase, } T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase, } T > T_c \end{cases}$$

   $$\Phi = \left\langle \mathrm{tr}_c \mathcal{P} \exp \left(i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle / N_c$$



At densities/temperatures of interest only model calculations available

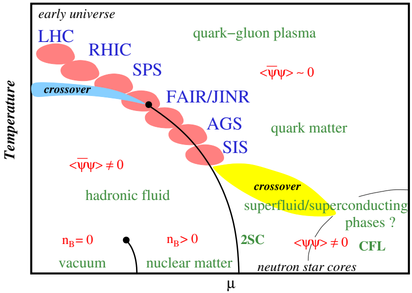
alternative: ➜ dressed Polyakov loop (dual condensate)

relates chiral and deconfinement transition $\rightarrow$ spectral properties of Dirac operator

effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

The conjectured QCD Phase Diagram



At densities/temperatures of interest
only model calculations available

➜ can one improve the model calculations?

➜ remove model parameter dependency?

non-perturbative functional methods (FunMethods)

$\rightarrow$ complementary to lattice

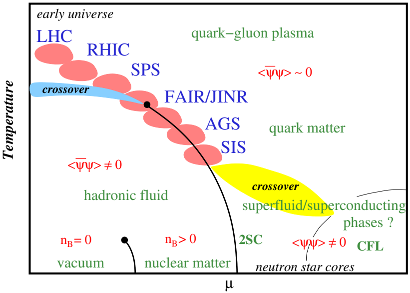
• no sign problem $\mu > 0$ • chiral symmetry/fermions (small masses/chiral limit) etc...

Open issues:

related to chiral & deconfinement transition

- ▷ existence/location of CEP?
  How many? Additional CEPs?
- ▷ coincidence of both transitions at $\mu = 0$ and $\mu > 0$ (quarkyonic phase)?
- ▷ relation between chiral and deconfinement?
  chiral CEP/deconfinement CEP?
- ▷ are there finite volume effects?
  $\rightarrow$ lattice comparison
- ▷ mostly only MFA results
  effects of fluctuations?
  $\rightarrow$ **size of crit. region**
- ▷ What are good exp. signatures?
  $\rightarrow$ higher moments more sensitive

The conjectured QCD Phase Diagram



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non-perturbative functional methods (FunMethods)

Method of choice: Funtional Renormalization Group Method (FRG)
one needs a truncation: e.g. (Polyakov)-quark-meson model

• good for chiral sector • what about deconfinement sector • baryonic degrees of freedom ?

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- Anatomy of the critical region around the CEP
- Higher moments
- QCD for $N_c = 2$

Chiral Sector:

- Three flavor Quark-Meson (QM) Model Lagrangian:

$$\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$$

Quark part with Yukawa coupling $\mathbf{h}$:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar $\sigma_{\mathbf{a}}$ and pseudoscalar $\pi_{\mathbf{a}}$ nonet

meson fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2 \text{tr}[M^\dagger M] - \lambda_1 (\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)]$$
$$+ \text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

Mean-Field Approximation

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling $\mathbf{h}$:

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Meson part: scalar σ_a and pseudoscalar π_a nonet

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- Mean-field approximation (MFA):

  Integrate over quarks and neglect mesonic fluctuations

Grand potential:

$$\Omega(T,\mu) = \Omega_{\text{vac}} + \Omega_{q\bar{q}} + U_{\text{class}}$$

no-sea MFA: neglect Ω_{vac}

Beyond Mean-Field Approximation

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$
- Renormalizable model:

regularization **(Λ regularization scale)**

$$\Omega_{\text{vac}}(\Lambda) = -\frac{N_c}{8\pi^2} \sum_{f=u,d,s} m_f^4 \log\left[\frac{m_f}{\Lambda}\right]$$

renormalized grand potential

$$\Omega(T,\mu) = \Omega_{\text{vac}}^{\text{renorm}} + \Omega_{q\bar{q}} + U_{\text{class}}$$

- all physical observables independent of choice Λ

  (remaining model parameters cancel the scale dependence)

- later - full FRG treatment

$N_f = 2 + 1$ Phase Diagram $\quad (\mu \equiv \mu_q = \mu_s)$

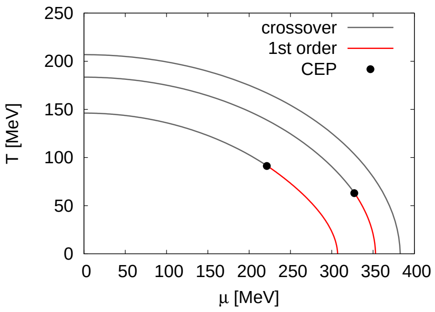
- model parameters fitted to (pseudo)scalar meson spectrum:
- one parameter precarious: $f_0(600)$ 'particle' (sigma)$\rightarrow$ broad resonance

PDG: mass = $(400 \ldots 1200)$ MeV

➜ existence of CEP depends on m_σ!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here no-sea MFA)

with $U(1)_A$ **[BJS, M. Wagner '09]**



- m_σ smaller $\rightarrow$ CEP towards T axis
- including vacuum term Ω_{vac} (beyond no-sea approximation)

  $\rightarrow$ CEP moves down
- renormalized model has NO CEP for $m_\sigma > 500$ MeV

  here: use smaller m_σ to study influence of fluctuations
- including mesonic fluctuations (full FRG calculation):

  $\rightarrow$ CEP moves up again

... including 'statistical' confinement aspects

via Polyakov-loop (not yet dynamical)

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

- **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

beyond MFA:

improve this model by back-coupling of quarks (QCD matter sector) to the Yang-Mills sector

this yields to a N_f and μ-modifications in presence of dynamical quarks:

$T_0 = T_0(N_f, \mu, m_q)$ **BJS, Pawlowski, Wambach; 2007**

N_f	0	1	2	$2+1$	3
T_0 [MeV]	270	240	208	187	178

This becomes clear by considering the FRG

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- ○ Higher moments
- ○ QCD for $N_c = 2$

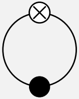
Functional RG Approach

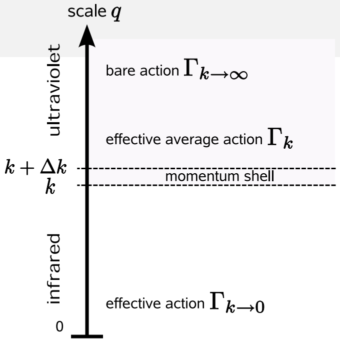
$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) Wetterich 1993

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



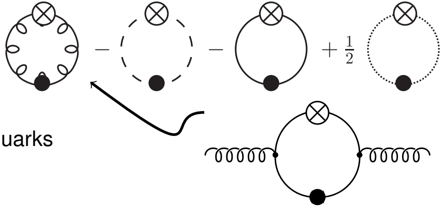


$T_0(N_f, \mu)$ modification

full dynamical QCD FRG flow: fluctuations of
gluon, ghost, quark and meson (via hadronization) fluctuations

Braun, Haas, Marhauser, Pawlowski; 2009

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ ... $-$ ... $-$ ... $+ \frac{1}{2}$ ...

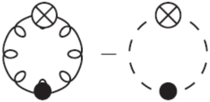


in presence of dynamical quarks
gluon propagator modified:

$\rightarrow$ pure Yang Mills flow + these modifications

pure Yang Mills flow
replaced by effective Polyakov loop potential:
(fit to YM thermodynamics)

$\partial_t \Gamma_k[\phi] = \frac{1}{2}$ ... $-$ ...



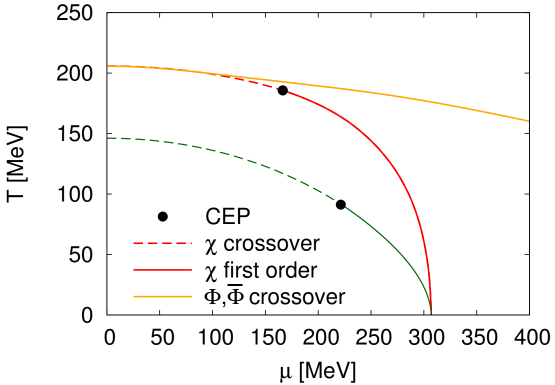
$$T_0 \leftrightarrow \Lambda_{QCD} \qquad : \qquad T_0 \rightarrow T_0(N_f, \mu, m_q)$$

BJS, Pawlowski, Wambach; 2007

$N_f = 2 + 1$ (P)QM phase diagrams

Summary of QM and PQM models in no-sea MFA

chiral transition and 'deconfinement' coincide at small densities



- for PQM model

  (upper lines)

- for QM model

  (lower lines)

BJS, M. Wagner; arXiv:1111.6871

$N_f = 2$: **BJS, Pawlowski, Wambach; 2007**

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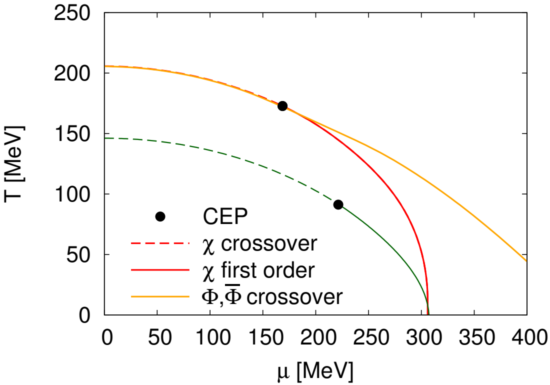


- for PQM model

  (upper lines)

  with
  matter back reaction in Polyakov loop potential
  i.e. $T_0(\mu)$

  → shrinking of possible quarkyonic phase

- for QM model

  (lower lines)

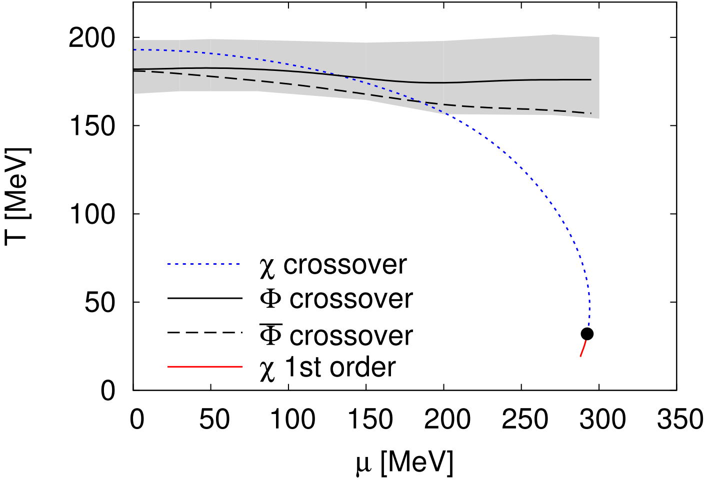
BJS, M. Wagner; arXiv:1111.6871

$N_f = 2$: **BJS, Pawlowski, Wambach; 2007**

Functional Renormalization Group

Polyakov-quark-meson model
no matter back-reaction to YM system (T_0 const.)

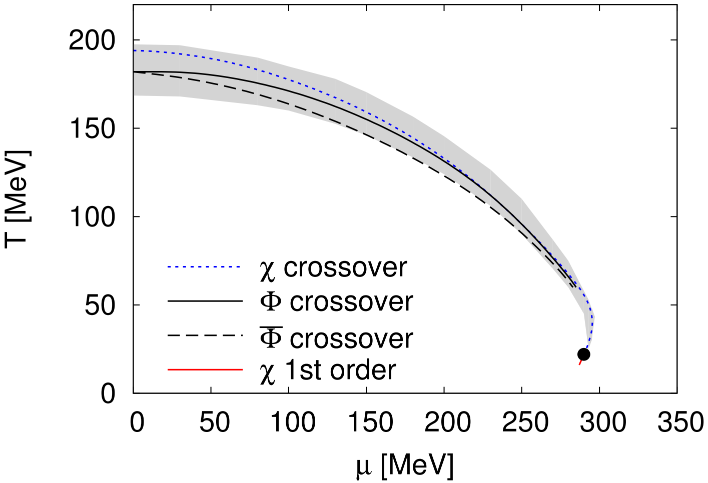
Herbst, Pawlowski, BJS; 2011



Functional Renormalization Group

Polyakov-quark-meson model
with matter back-reaction to YM system ($T_0(\mu)$)

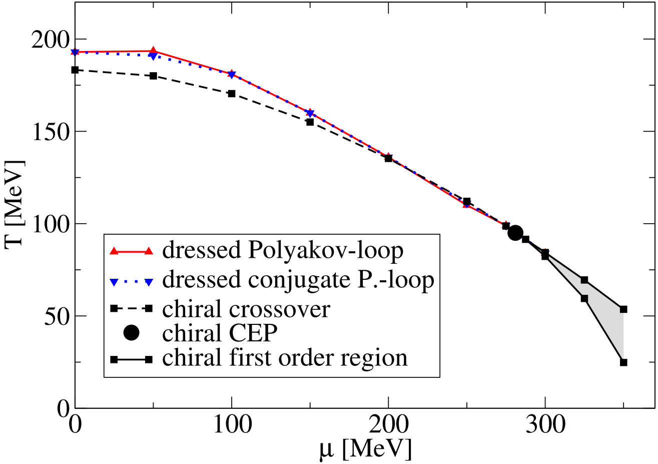
Herbst, Pawlowski, BJS; 2011



Functional Renormalization Group

Dyson-Schwinger calculation
matter back-reaction to YM system estimated via HTL

Ch. S. Fischer, J. Luecker, J. A. Mueller; 2011



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- ◦ **Chiral QCD-like models with 'statistical' confinement**
- ◦ **Importance of fluctuations:**
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- ◦ **Anatomy of the critical region around the CEP**
- ◦ **Higher moments**
- ◦ **QCD for $N_c = 2$**

Critical region

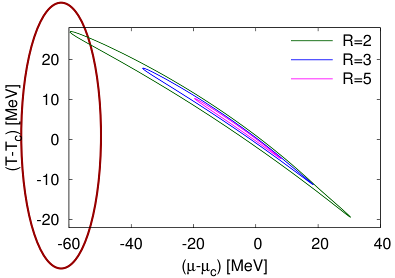
contour plot of **size of the critical region** around CEP

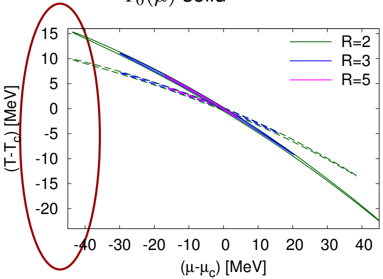
defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop

QM model (2+1) no-sea MFA

PQM model (2+1) no-sea MFA: T_0 dashed $T_0(\mu)$ solid





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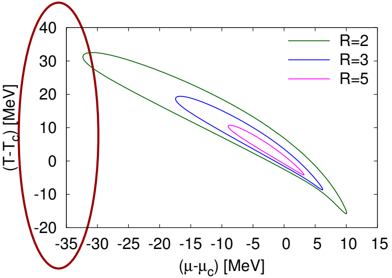
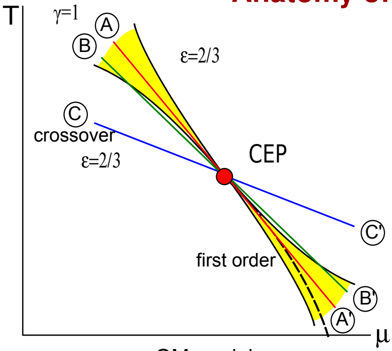
QM model (2+1) renormalized

PQM model (2+1) renormalized: T_0 dashed $T_0(\mu)$ solid



BJS, M. Wagner; arXiv:1111.6871

Anatomy of the critical region

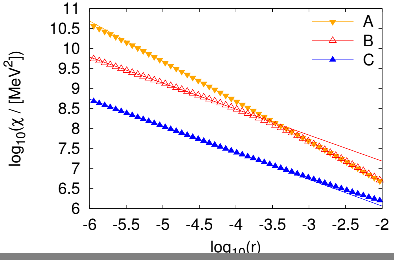


- crit. exponents depend on path (cf. different slopes)

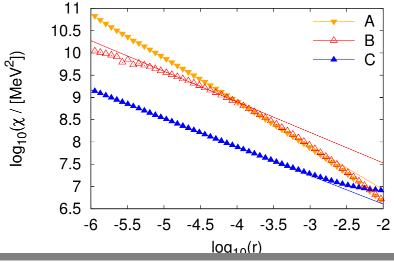
  → reason for the elongation

BJS, M. Wagner; 2012

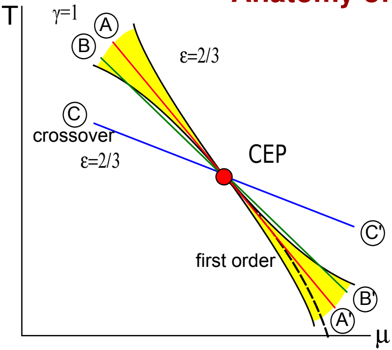
QM model



PQM model (T_0 const)

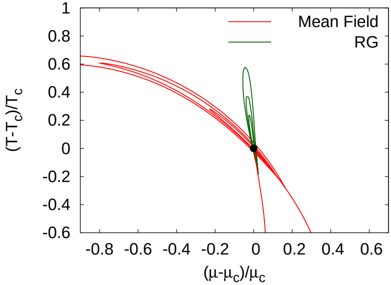


Anatomy of the critical region



- crit. exponents depend on path (cf. different slopes)

  → reason for the elongation
- another influence: fluctuations
- (P)QM models with/without vacuum (zero modes) quark contribution
- Mean-field approximations vs. RG calculation



BJS, J. Wambach; 2006

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- ◦ **Higher moments**
- ◦ QCD for $N_c = 2$

Higher moments

- limited theoretical guidance for experimental search of CEP
  (e.g. STAR @ BNL/RHIC)
  unfortunately model predictions vary a lot

- possible signatures are based on singular behavior of thermodynamic functions

- BUT: realistic heavy-ion collision correlation length ξ is always finite!
  (finite volume and critical slowing down)

  example estimates: $\xi \sim 2 - 3$ fm near a critical point (only factor 3 larger)

- hope: non-monotonic behavior in the fluctuations of particle numbers (near CEP)
  might serve as a probe *Volker Koch: The Details are in the tails*

- more sensible quantities (on ξ) are needed
  $\rightarrow$ higher (cumulants) moments $\equiv$ generalized susceptibilities

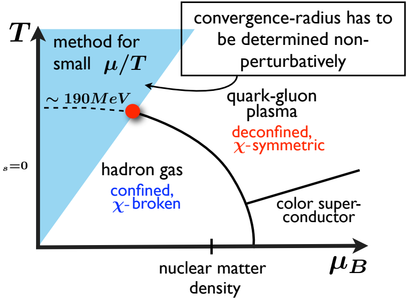
  higher order moments depend on higher powers of ξ; example: $\chi_4 \sim \xi^7$ near CEP

  generalized susceptibities are related to Taylor expansion coefficients

Taylor expansion coefficients

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial (\mu/T)^n}\Bigg|_{\mu=0}$$



C. Schmidt 2009

convergence radii:

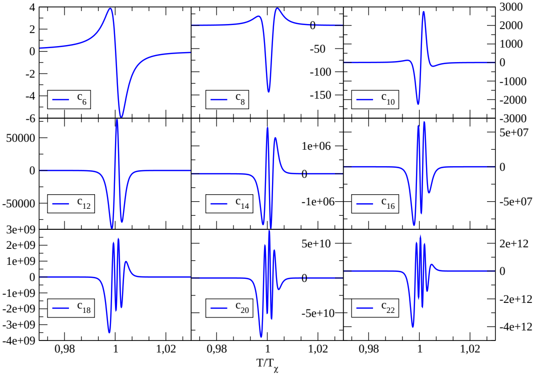
limited by first-order line?

$$\rho_{2n} = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

Taylor coefficients for $N_f = 2 + 1$ PQM model

New method: based on algorithmic differentiation — M. Wagner, A. Walther, BJS; 2010

Taylor coefficients c_n numerically known to high order, e.g. $n = 22$



- ▷ technique applied to PQM model
- ▷ investigation of convergence properties of Taylor series

properties of c_n

- oscillating
- increasing amplitude
- no numerical noise
- small outside transition region
- number of roots increasing
- 26th order

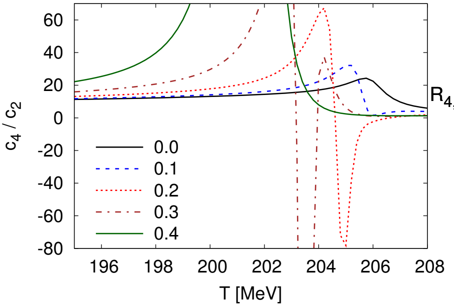
Can we use these coefficients to locate the CEP experimentally ?

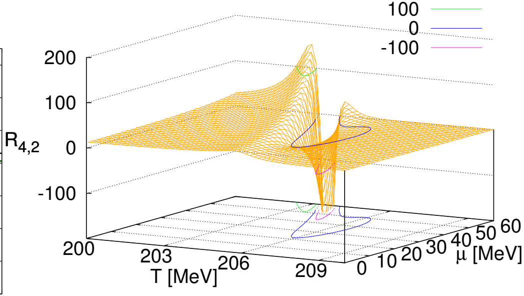
F. Karsch, BJS, M. Wagner, J. Wambach; 2010

Generalized Susceptibilities ≡ higher moments

- Higher moments are increasingly sensitive to critical behavior even at $\mu = 0$
- Example: Kurtosis $R^B_{4,2} = \frac{1}{9}\frac{c_4}{c_2}$ $\rightarrow$ probe of deconfinement?

  It measures quark content of particles carrying baryon number B
- in Hadron Resonance Gas (HRG) model $R_{4,2} = 1$ (always positive)
- PQM three flavor calculation (no-sea MFA)



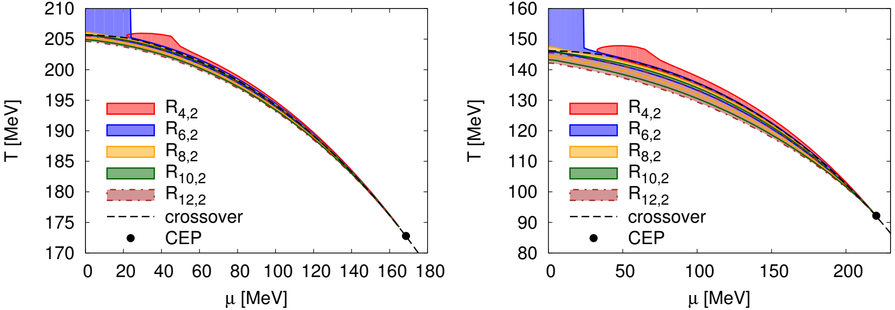


Generalized Susceptibilities ≡ higher moments

Fluctuations of higher moments exhibit **strong variation from HRG model**

- → turn negative — **Karsch, Redlich, Friman et al.; 2011**
- higher moments: $R^q_{n,m} = c_n/c_m$
- regions where $R_{n,2}$ are negative along crossover line in the phase diagram

PQM with $T_0(\mu)$ no-sea MFA — QM model no-sea MFA



BJS, M.Wagner; 2012

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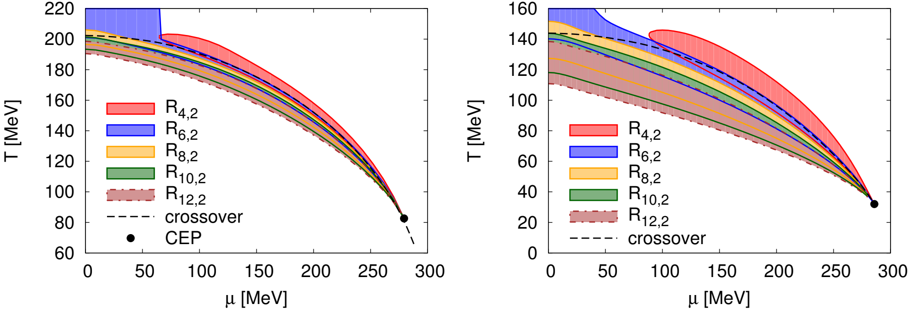
renormalized PQM with $T_0(\mu)$ renormalized QM model



BJS, M.Wagner; 2012

Generalized Susceptibilities $\equiv$ higher moments

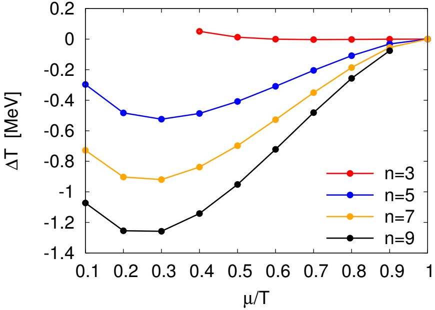
Fluctuations of higher moments exhibit **strong variation from HRG model**

- can we exclude the existence of the CEP ?

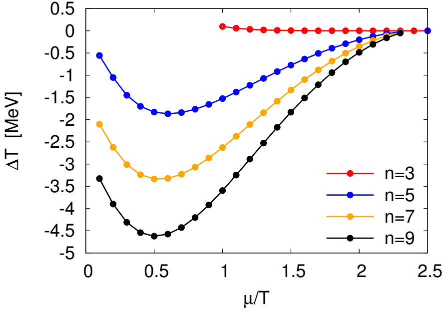
  $\rightarrow$ linear fit PQM yes - QM no

- distance $\Delta T \equiv T_n - T_\chi$ of the first root in $R_{n,2}$ to the chiral temperature T_χ

PQM with $T_0(\mu)$



QM model



BJS, M.Wagner; 2012

Generalized Susceptibilities ≡ higher moments

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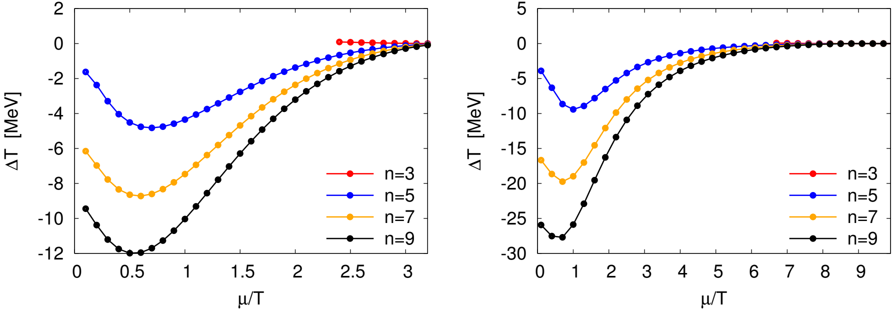

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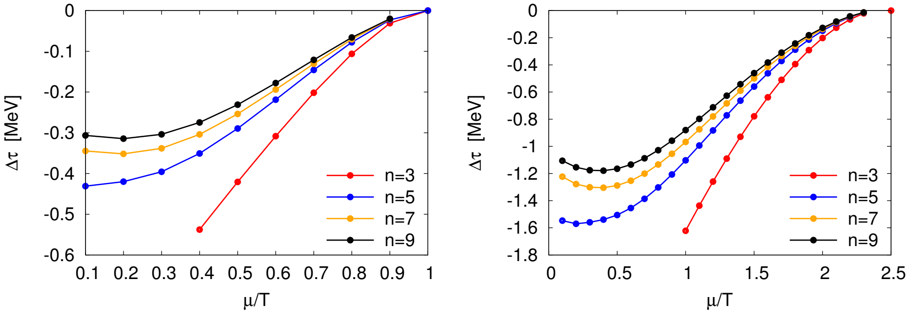

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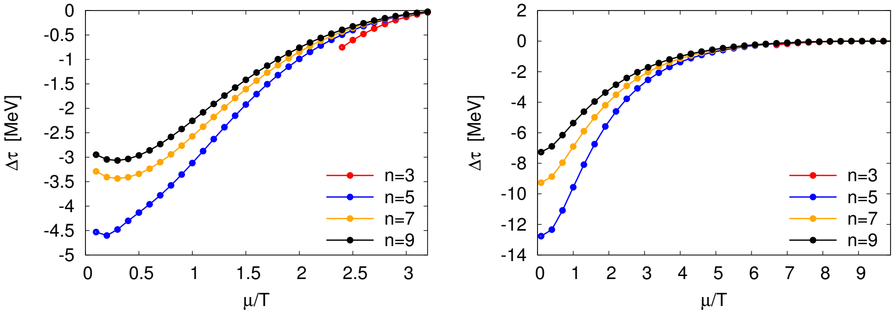


BJS, M.Wagner; 2012

Outline

- QCD Phase Diagram
- Chiral QCD-like models with ’statistical’ confinement
- Importance of fluctuations:
  - ▷ Matter back-coupling to Yang-Mills sector
- Anatomy of the critical region around the CEP
- Higher moments
- **QCD for $N_c = 2$**

Role of baryonic degrees of freedom

baryonic dof important for larger densities!

$N_c = 2$: scalar diquarks $\longleftrightarrow$ bosonic baryons

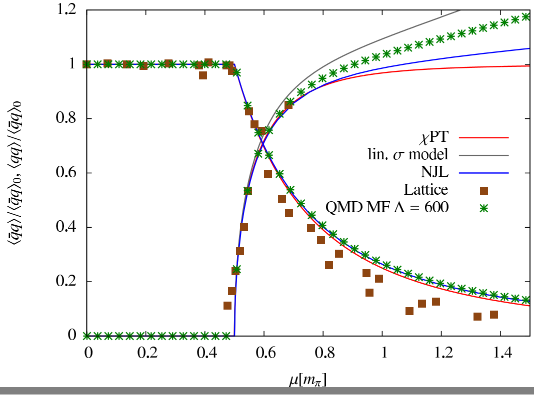
Quark-Meson-Diquark (QMD) model

$\langle qq \rangle$& $\langle \bar{q}q \rangle$ condensates

MFA with vacuum term

lattice data: Hands 2000

LO χPT



- $\langle qq \rangle$ depends on m_σ
- variation in diquark due to finite m_σ
- finite m_σ: beyond LO χPT

  LO χPT corresponds $m_\sigma \to \infty$ (non-linear σ model)
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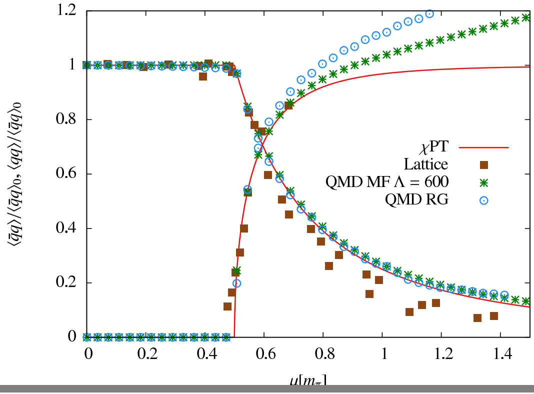

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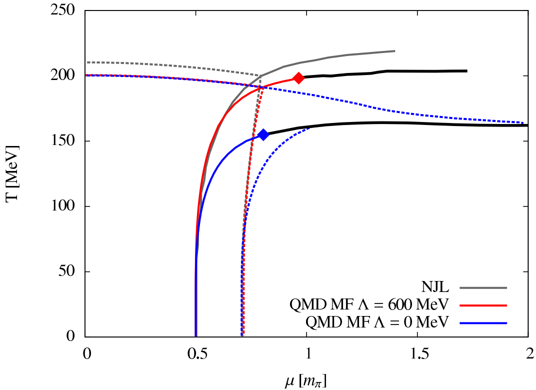
<u>NJL and QMD phase diagrams</u>

MFA with & without vacuum term



NJL: continuous $\langle qq \rangle$ condensation

QMD: MFA 2nd order & TCP

➜ NLO χ PT predicts also TCP

■ BUT this is a MFA artifact!

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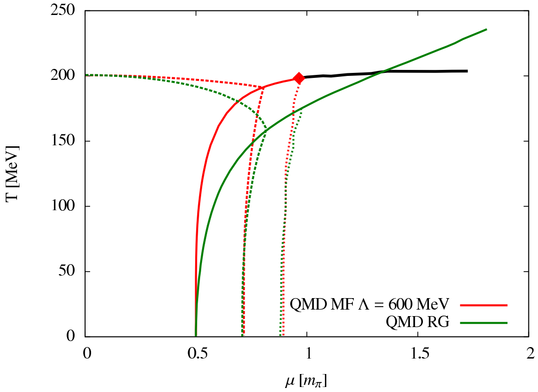
RG and MFA



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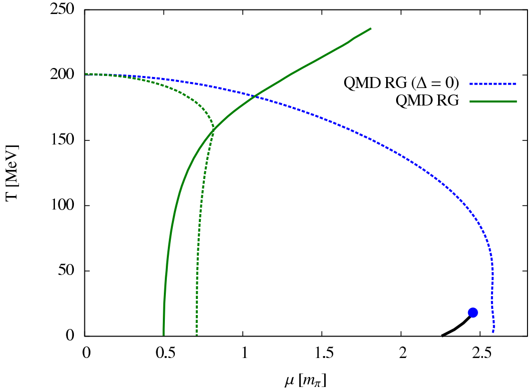
QMD phase diagrams

RG with/without baryonic fluctuations ($\Delta = 0$)



QMD: RG no endpoint

- No 1st order $T = 0$!

➜ more tests are needed

- first step towards covariant quark-diquark description with a quark-meson-baryon model for QCD

Summary and Outlook

- chiral (Polyakov)-quark-meson model study (three flavor)

➜ **role of fluctuations: important**

(location, existence of the CEP, critical region)

➜ experimental signatures: higher moments

➜ role of baryonic degrees of freedom

PRD 85 (2012) 034027 `arXiv:1111.6871`

and

PRD 85 (2012) 074007 `arXiv:1112.5401`

functional approaches (such as the FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

➜ **FunMethods guide the way towards full QCD**