Towards an Asymptotically Safe SM

Steven Abel (Durham IPPP)

w/ Sannino, ArXiV:1704.00700 Phys.Rev. D96 (2017) no.10, 106013
w/ Sannino, ArXiV: 1707.06638 Phys.Rev. D96 (2017) no.5, 055021
w/ Molgaard,Sannino to appear
w/ D.Lewis to appear

Outline

- Motivation; the hierarchy versus triviality problem
- RG flows and the asymptotic safety idea
- Asymptotically safe 4D QFTs
- Adding relevant operators
- An ASSM ?
- AS in String theory?

Motivation: two problems to do with scalars

The hierarchy problem:

The hierarchy problem:

Candidate symmetries:

- Higgs is a Goldstone mode of ***some broken global symmetry*** (like the pions in chiral symmetry breaking) with breaking scale of a few TeV

The hierarchy problem:

Candidate symmetries:

- Higgs is a Goldstone mode of ***some broken global symmetry*** (like the pions in chiral symmetry breaking) with breaking scale of a few TeV
- ***Supersymmetry*** - relates boson to fermions. Divergences cancel level by level. Phenomenology requires soft (a.k.a. dimensionful) breaking.

The hierarchy problem:

Candidate symmetries:

- Higgs is a Goldstone mode of ***some broken global symmetry*** (like the pions in chiral symmetry breaking) with breaking scale of a few TeV
- ***Supersymmetry*** - relates boson to fermions. Divergences cancel level by level. Phenomenology requires soft (a.k.a. dimensionful) breaking.
- ***Scaling symmetry*** **-** Higgs is the Goldstone mode of a broken scale invariance (a.k.a. dilaton) (a trivial perturbative example of this is the Standard Model with vanishing higgs mass, but it can occur in nonperturbative models based on AdS/CFT).

The hierarchy problem:

Candidate symmetries:

- Higgs is a Goldstone mode of ***some broken global symmetry*** (like the pions in chiral symmetry breaking) with breaking scale of a few TeV
- ***Supersymmetry*** - relates boson to fermions. Divergences cancel level by level. Phenomenology requires soft (a.k.a. dimensionful) breaking.
- ***Scaling symmetry*** **-** Higgs is the Goldstone mode of a broken scale invariance (a.k.a. dilaton) (a trivial perturbative example of this is the Standard Model with vanishing higgs mass, but it can occur in nonperturbative models based on AdS/CFT).
- ***Misaligned Supersymmetry*** - even non-supersymmetric non-tachyonic strings are finite. (***Alternative route to naturalness***) (Dienes, Moshe, Myers (90's), SAA+Dienes+Mavroudi)

The triviality problem:

The triviality problem:

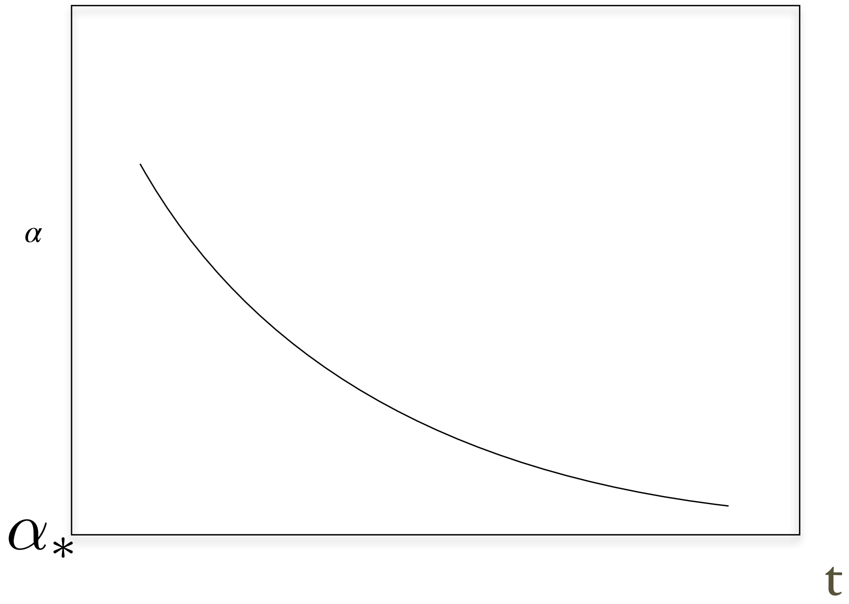
Scalars lead to Landau poles:

=> the theory is UV incomplete

But trying to UV complete it results in the hierarchy problem again! (see previous comments)

Hints from QCD

$$\partial_t \alpha = -B\alpha^2$$



$$\alpha_* = 0$$

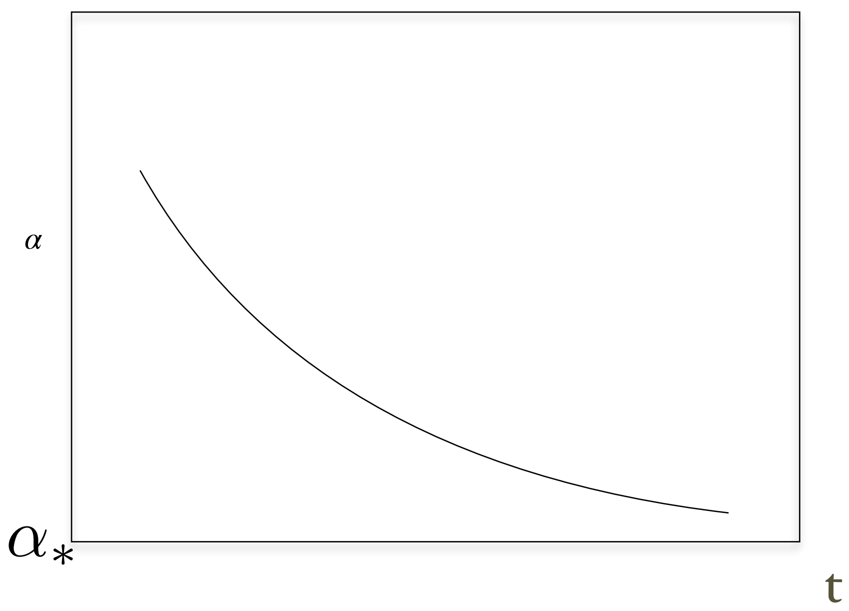
Hints from QCD

QCD is (unlike SUSY) a UV complete theory. Why?

1. ***There is no hierarchy problem:*** quark masses are protected by chiral symmetry

2. ***There is no triviality problem:*** QCD is **asymptotically free**

$$\partial_t \alpha = -B\alpha^2$$



$$\alpha_* = 0$$

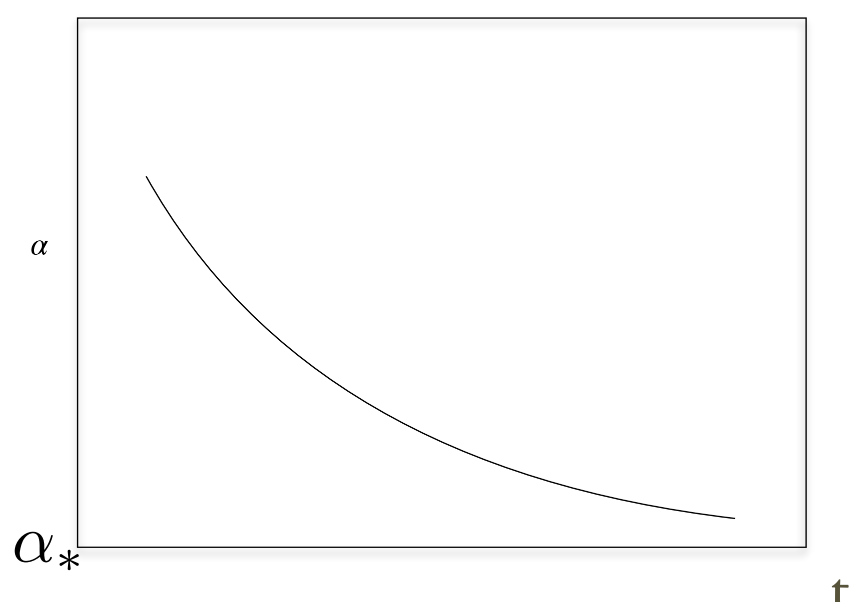
Hints from QCD

QCD is (unlike SUSY) a UV complete theory. Why?

1. ***There is no hierarchy problem:*** quark masses are protected by chiral symmetry

2. ***There is no triviality problem:*** QCD is **asymptotically free**

$$\partial_t \alpha = -B\alpha^2$$



$$\alpha_* = 0$$

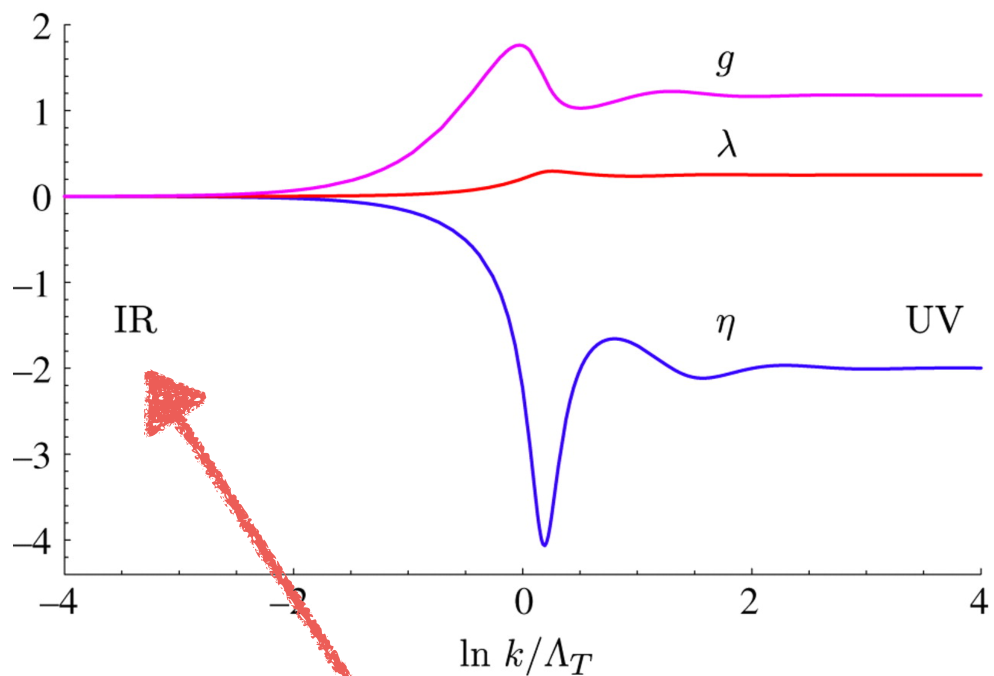
Note the philosophy of QCD: we do not mind running masses because they do not upset the Gaussian UV fixed point. We simply measure them and let them run. Or to put it another way: they are “relevant” operators that are effectively zero in the UV. They do not need to run to zero in the UV! (We also don’t care too much about couplings blowing up in the IR.)

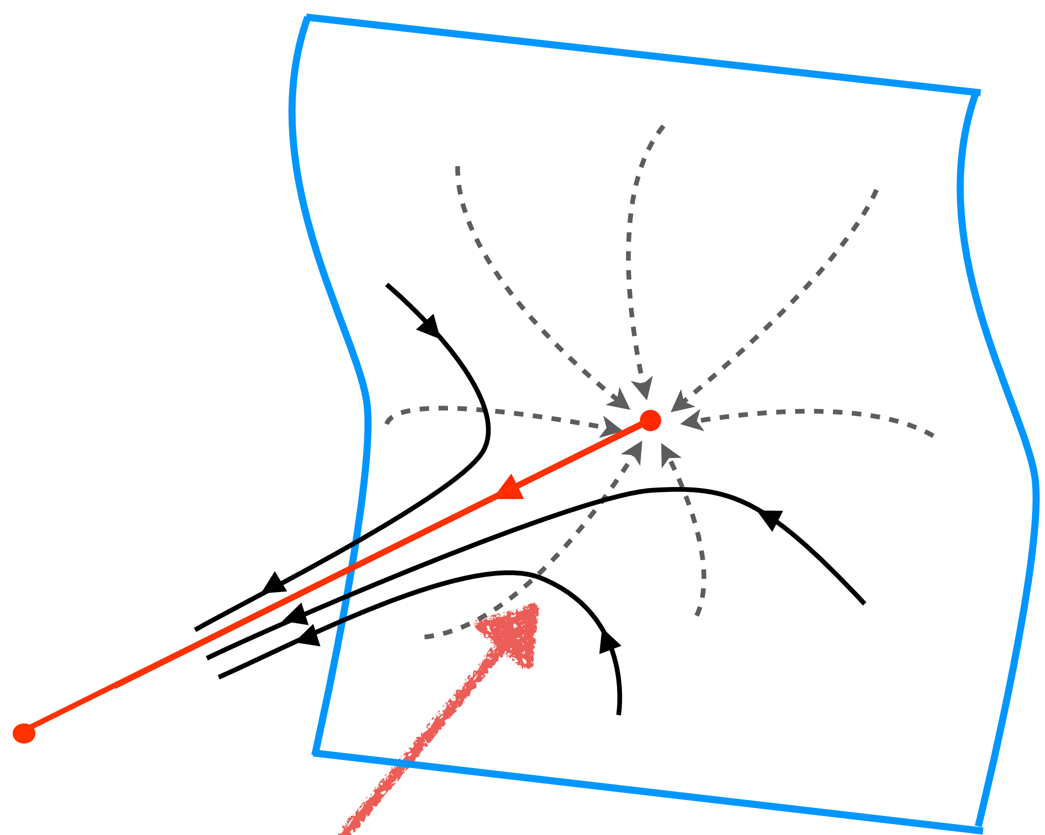
RG flows and the asymptotic safety idea

Gastmans et al '78
Weinberg '79
Peskin '80
Gawedski, Kupiainen '85
Kawai et al '90
de Calan et al '91
Litim '03
Morris '04

The Basic idea

Weinberg used this as a basis for his proposal of UV complete theories





Gaussian IR fixed point => perturbative

Interacting UV fixed point => finite anomalous dimensions

In a field theory replace $1/\epsilon$ with $1/\gamma$ => divergences of marginal operators (which affect the fixed point) cured

Categorise the possible content of a theory as follows:

Irrelevant operators: would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are assumed zero (exactly renormalizable trajectory)

Marginal operators: can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free) or irrelevant.

Relevant operators: become "irrelevant" in the UV but may determine the IR fixed point.

Dangerously irrelevant operators: grow in both the UV and IR (common in e.g. SUSY)

Harmless relevant operators: shrink in the UV and IR (not common)

Categorise the possible content of a theory as follows:

Irrelevant operators: would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are assumed zero (exactly renormalizable trajectory)

Marginal operators: can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free) or irrelevant.

Relevant operators: become "irrelevant" in the UV but may determine the IR fixed point.

Dangerously irrelevant operators: grow in both the UV and IR (common in e.g. SUSY)

Harmless relevant operators: shrink in the UV and IR (not common)

Note relevant or marginally relevant operators still have "infinities" at the FP - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (And by definition they become less important the higher you go in energy.)

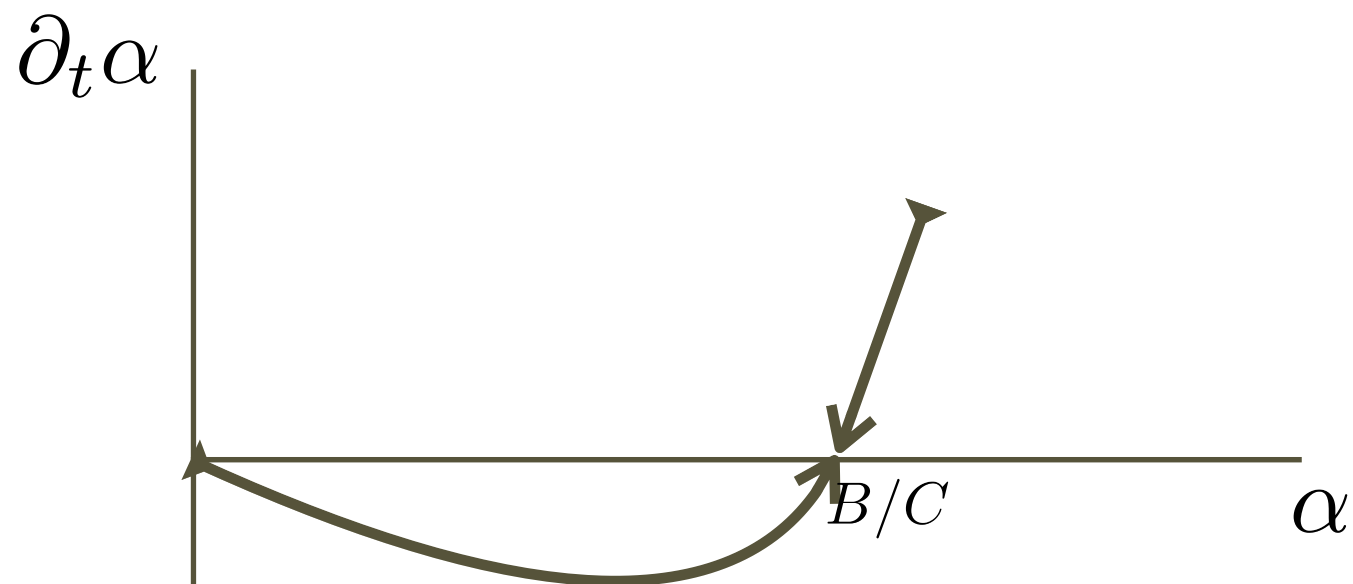
U.V. v. I.R. F.P.

Caswell-Banks-Zaks fixed point:

Take QCD with $SU(N_C)$ and N_F fermions but very large numbers of colours+flavours

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3 \qquad B \propto \epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

Turns out C>0, B>0: theory has *stable* IR fixed point at α = B/C and *unstable* one in UV α = 0



Note perturbativity: $\implies B \ll C$

requires many fields (Veneziano limit) with $N_F \approx 11 N_C/2$

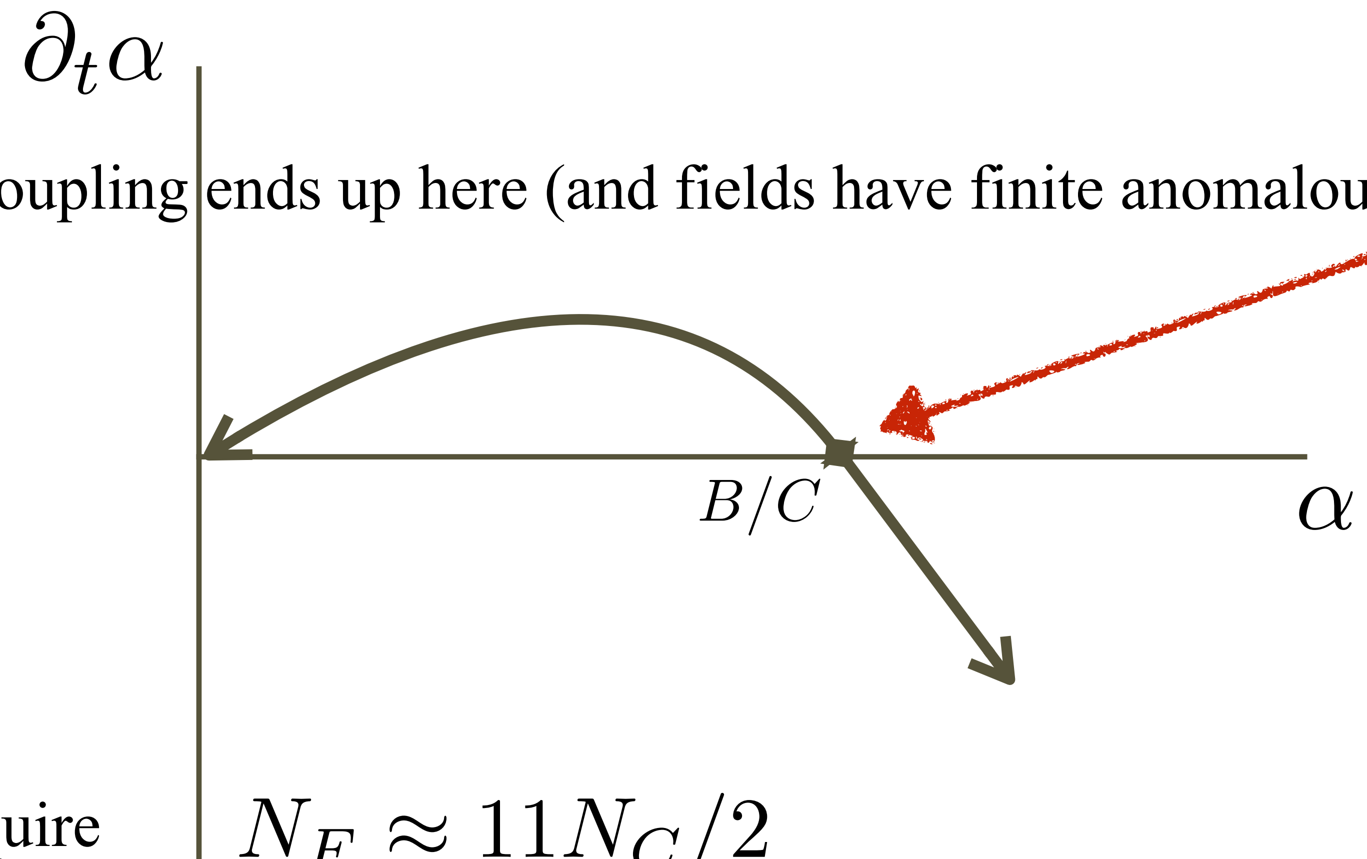
Familiar from Seiberg duality and weakly coupled $N_F \lesssim 3N_C$ $\mathcal{N}=1$ supersymmetry

Cartoon of a would-be Interacting UV FP:

Again would have …

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$

But requires C<0, B<0, this theory has *stable* IR fixed point at $\alpha = 0$ and *unstable* UV one at $\alpha = B/C$



At t -> infinity the coupling ends up here (and fields have finite anomalous dimensions)

Again perturbativity would require $N_F \approx 11 N_C/2$

Asymptotic safety in 4D QFT

Real situation requires several couplings to realise

Litim & Sannino '14

Need to add **scalars** and **Yukawa couplings:**

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu} F_{\mu\nu} + \mathrm{Tr}\left(\overline{Q}\, i\not{D}\, Q\right) + y\,\mathrm{Tr}\left(\overline{Q}\, H\, Q\right) + \mathrm{Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$$
$$-u\,\mathrm{Tr}\,[(H^\dagger H)^2] - v\,(\mathrm{Tr}\,[H^\dagger H])^2\,,$$

H is an $N_F \times N_F$ scalar

Initially have $U(N_F)_L \times U(N_F)_R$ flavour symmetry

Effect of Yukawa ….

$$\left(\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2} \right)$$

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right]$$

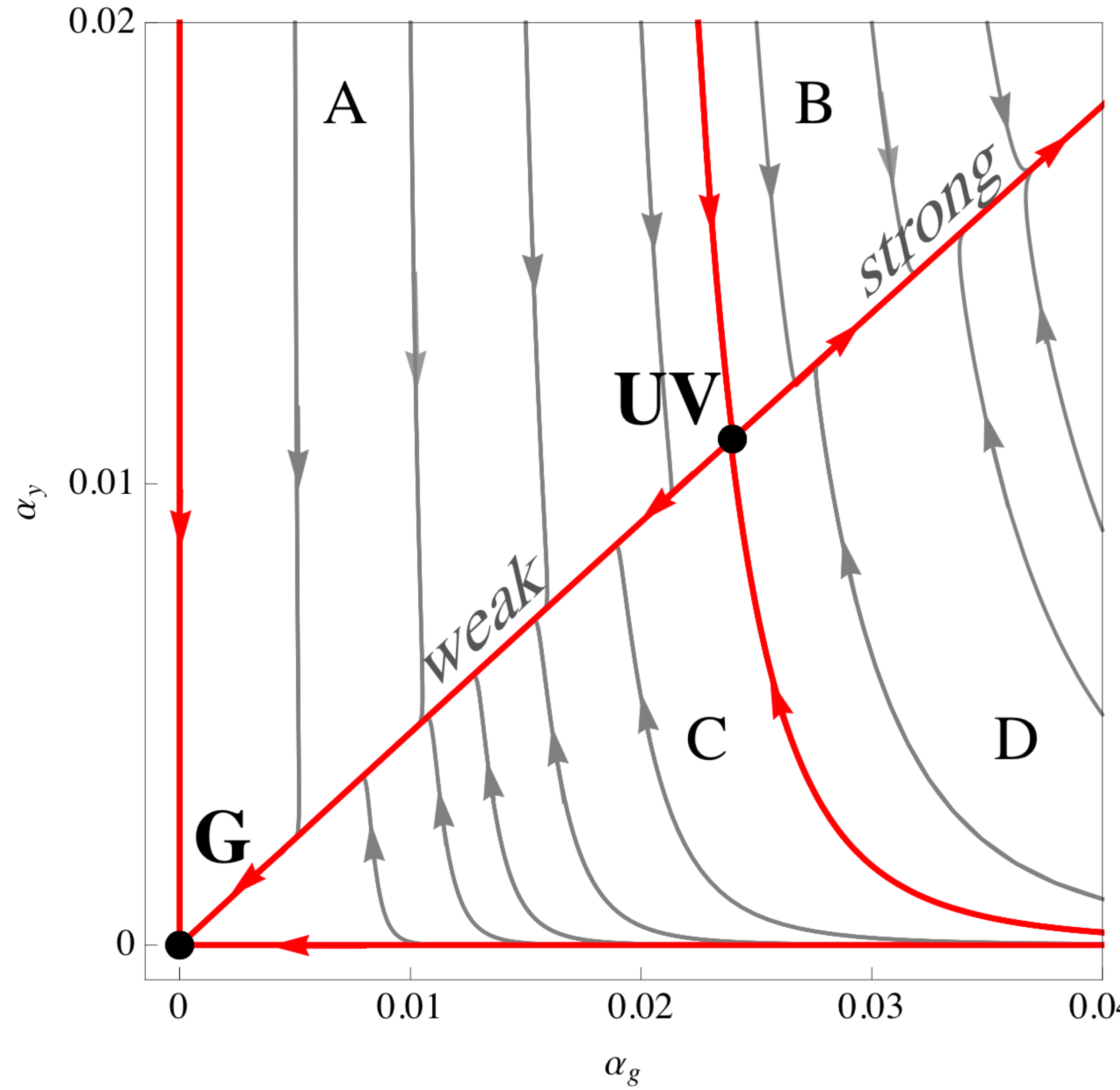
$$\beta_y = \alpha_y \left[(13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right]$$

Effect of Yukawa ….

$$\left(\alpha_g = \frac{g^2 N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}\right)$$

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y\right]$$

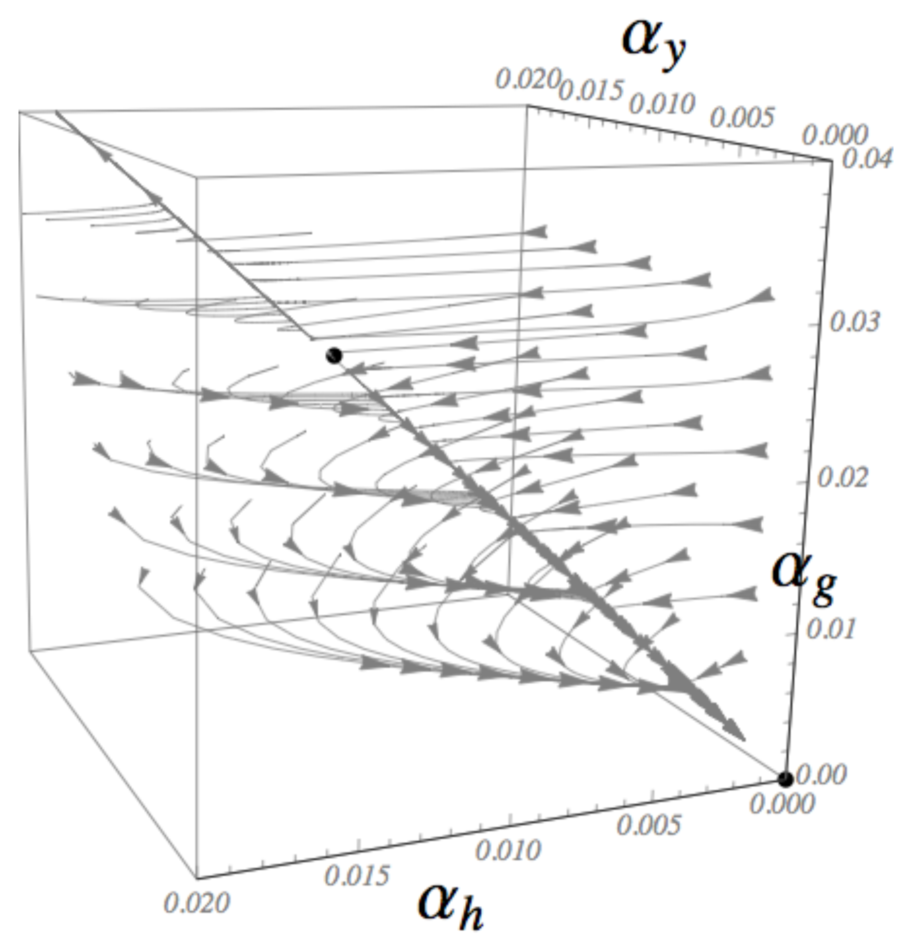
$$\beta_y = \alpha_y \left[(13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g\right]$$



Four ’t Hooft-like couplings - flow could in principle be four dimensional

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}\,, \quad \alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}$$

but …



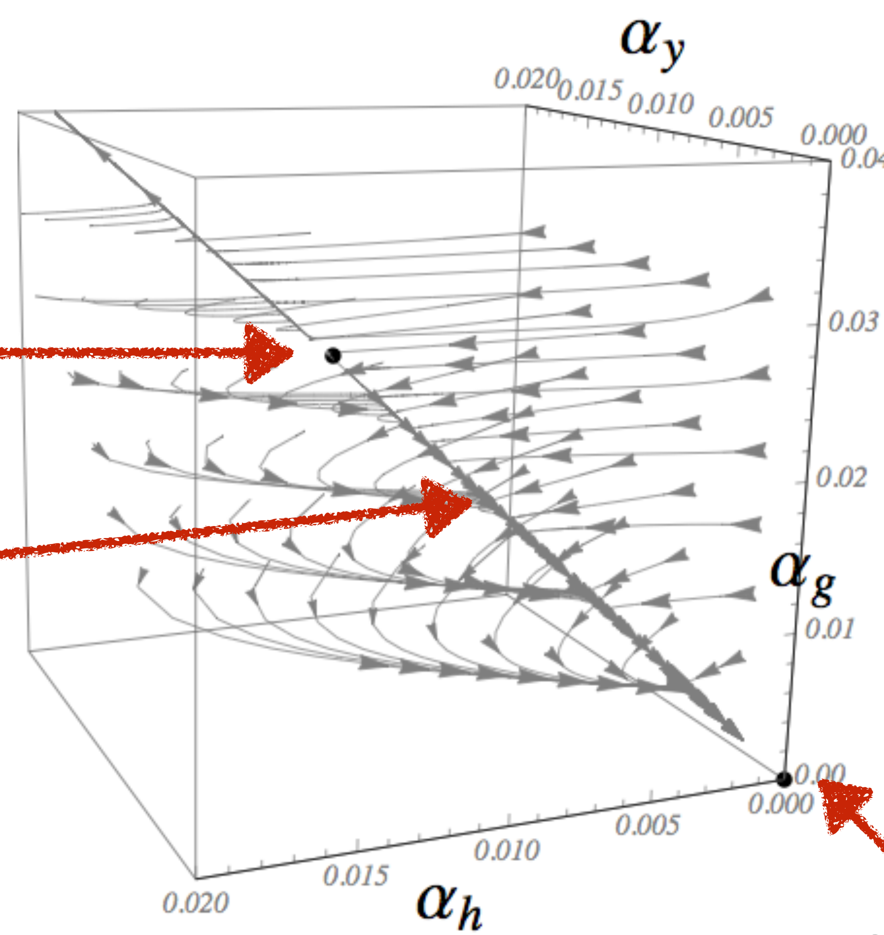
Four ’t Hooft-like couplings - flow could in principle be four dimensional

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}, \quad \alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}$$

but …

UV fixed point

1D exactly renormalisable trajectory!



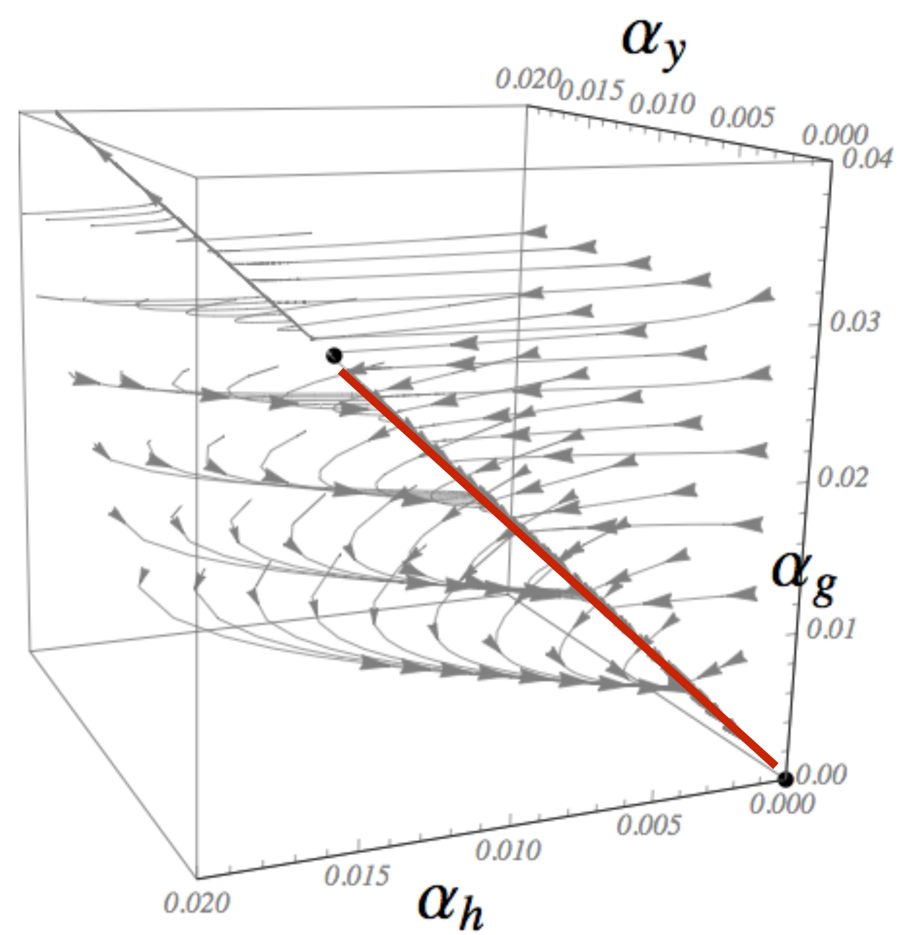
Gaussian IR fixed point

Along the critical-curve/exact-trajectory can parameterise the flow in terms of $\alpha_g(t)$

$$\alpha_y(t) = \frac{6}{13}\alpha_g(t) \ ,$$

$$\alpha_h(t) = 3\frac{\sqrt{23}-1}{26}\alpha_g(t) \ ,$$

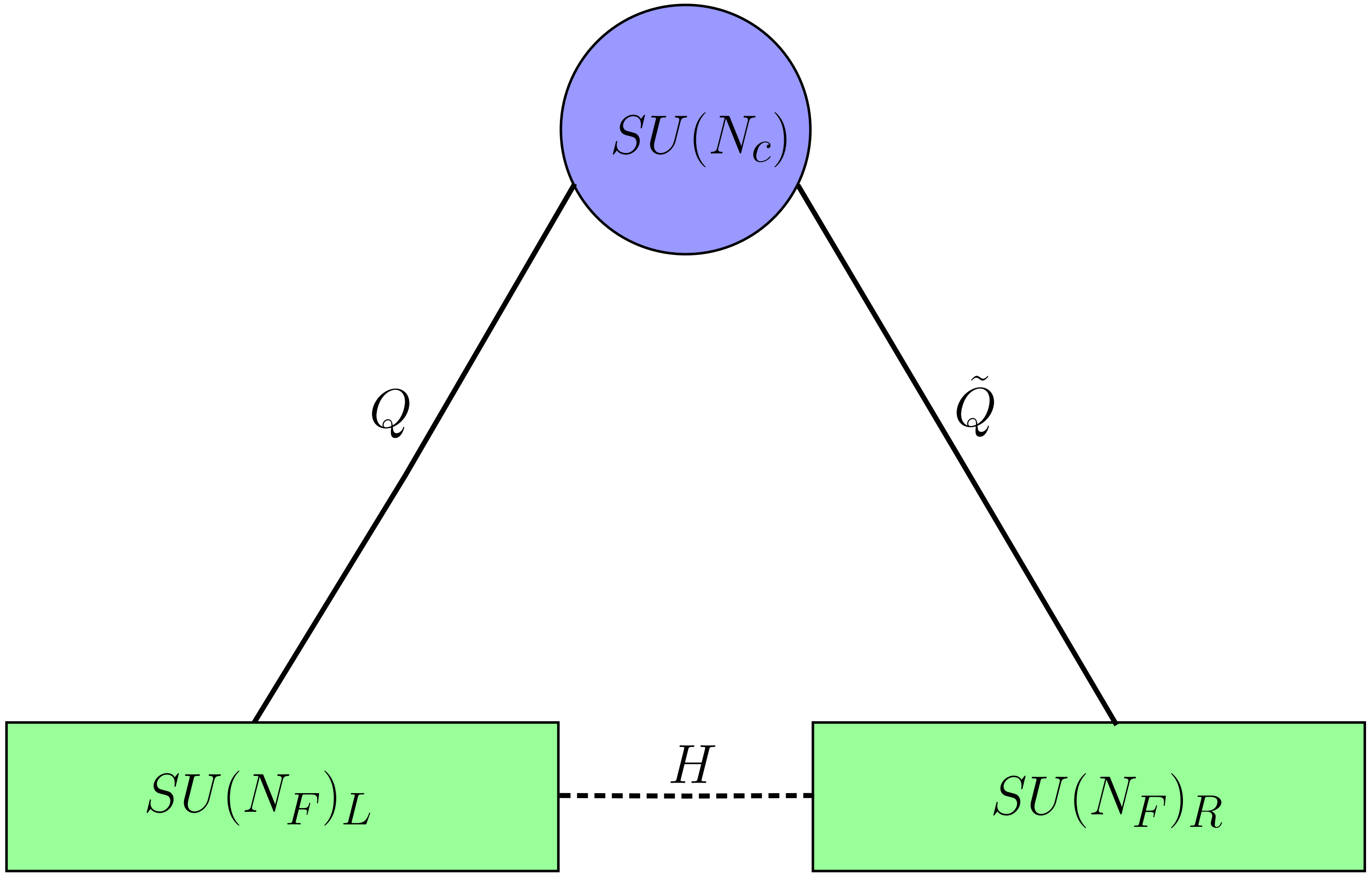
$$\alpha_v(t) = \frac{3\sqrt{20+6\sqrt{23}}-6\sqrt{23}}{26}\alpha_g(t) \ ,$$



At the fixed point it is arbitrarily weakly coupled, $\alpha_g^* = \ 0.4561\,\epsilon\,,$ where $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$

Quiver diagram for this model:

	$SU(N_C)$	$SU(N_F)_L$	$SU(N_F)_R$	spin
Q_{ai}	$\square$	$\square$	1	1/2
$\tilde{Q}^{ia}$	$\tilde{\square}$	1	$\tilde{\square}$	1/2
H^i_j	1	$\tilde{\square}$	$\square$	0



Adding relevant operators (mass-squareds)

Adding relevant operators (mass-squareds)

The non-predictive free parameters

Organize relevant operators in terms of the $U(N_F) \times U(N_F)$ flavour symmetry that we break with the mass-squareds (closed under RG):

$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a) T_a$$

$$\mathcal{L}_{Soft} \;=\; -m_{h_0}^2 \mathrm{Tr}\left[H^\dagger H\right] - \sum_{a=1}^{N_F^2 - 1} \Delta_a^2 \mathrm{Tr}\left[H T^a\right] \mathrm{Tr}\left[H^\dagger T^a\right]$$

Organize relevant operators in terms of the $U(N_F) \times U(N_F)$ flavour symmetry that we break with the mass-squareds (closed under RG):

$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a) T_a$$

$$\mathcal{L}_{Soft} = -m_{h_0}^2 \mathrm{Tr}\left[H^\dagger H\right] - \sum_{a=1}^{N_F^2-1} \Delta_a^2 \mathrm{Tr}\left[H T^a\right] \mathrm{Tr}\left[H^\dagger T^a\right]$$

Then solve Callan Symanzik eqn for them as usual =>

$$\bar{\beta} = \frac{d\lambda^{(n)}(t)}{dt} = \frac{\partial \lambda_{eff}^{(n)}}{\partial t} + n\bar{\gamma}\lambda^{(n)}$$

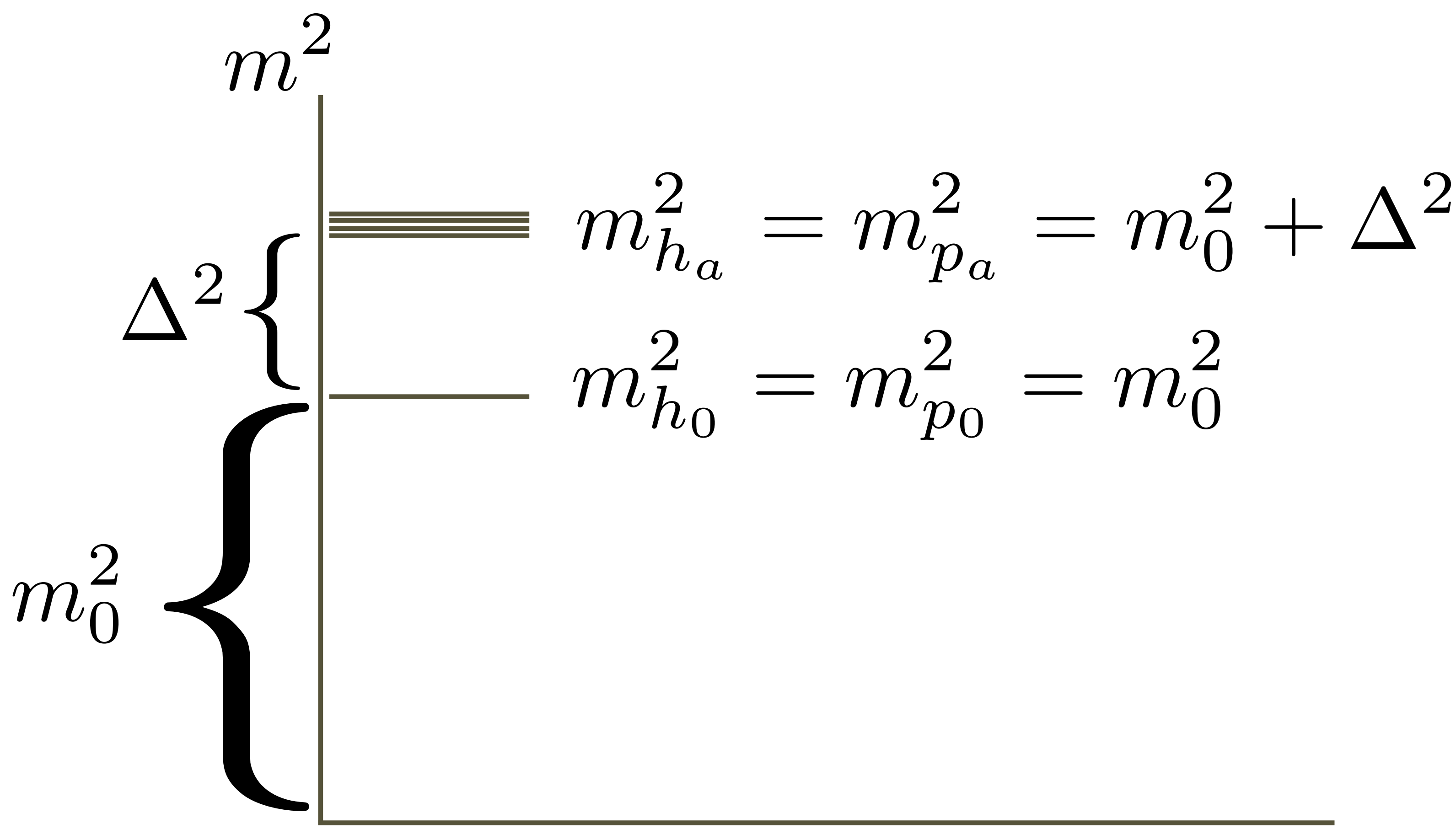
Anomalous dimension of fields

t-dependence in one-loop calculation of V

Non-trivial simple example...

Consider case where the trace component has a slightly smaller mass-squared:

$$V^{(2)}_{class} \;=\; m_0^2 \mathrm{Tr}(H^\dagger H) + 2\Delta^2 \sum_a \mathrm{Tr}(T_a H^\dagger)\mathrm{Tr}(T_a H)$$



Non-trivial simple example...

After some work find the following answer in terms of two RG invariants, one for each independent (non-predicted) relevant operator (where ν=(1-1/NF^2)):

$$m_0^2 = \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_{m_0}}{4\epsilon}} - \Delta_*^2 \nu \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_\Delta}{4\epsilon}},$$

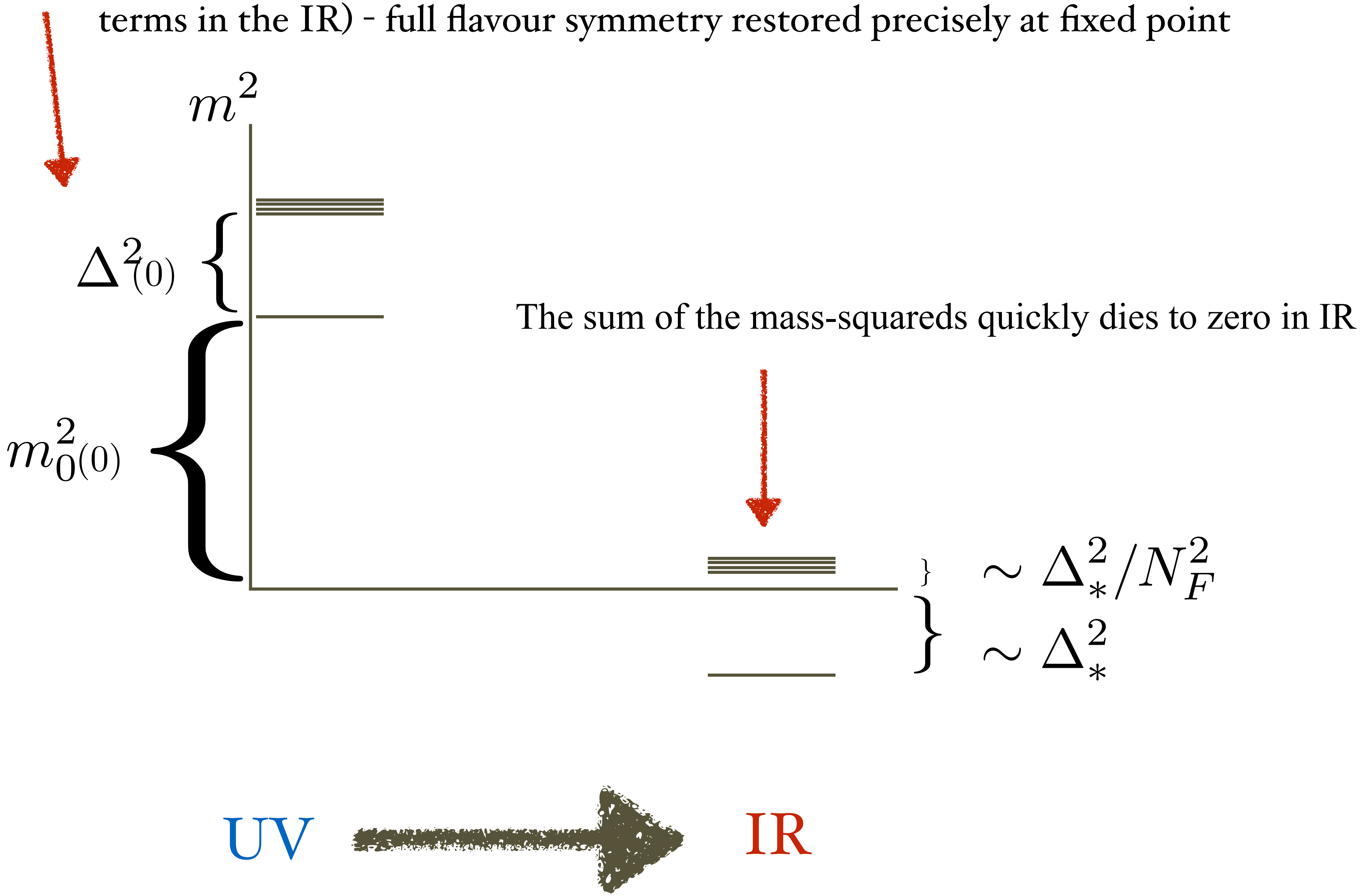
$$m_{a=1\dots N_F^2-1}^2 = \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_{m_0}}{4\epsilon}} + \Delta_*^2 (1-\nu) \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_\Delta}{4\epsilon}}$$

$$f_{m_0} > f_\Delta$$

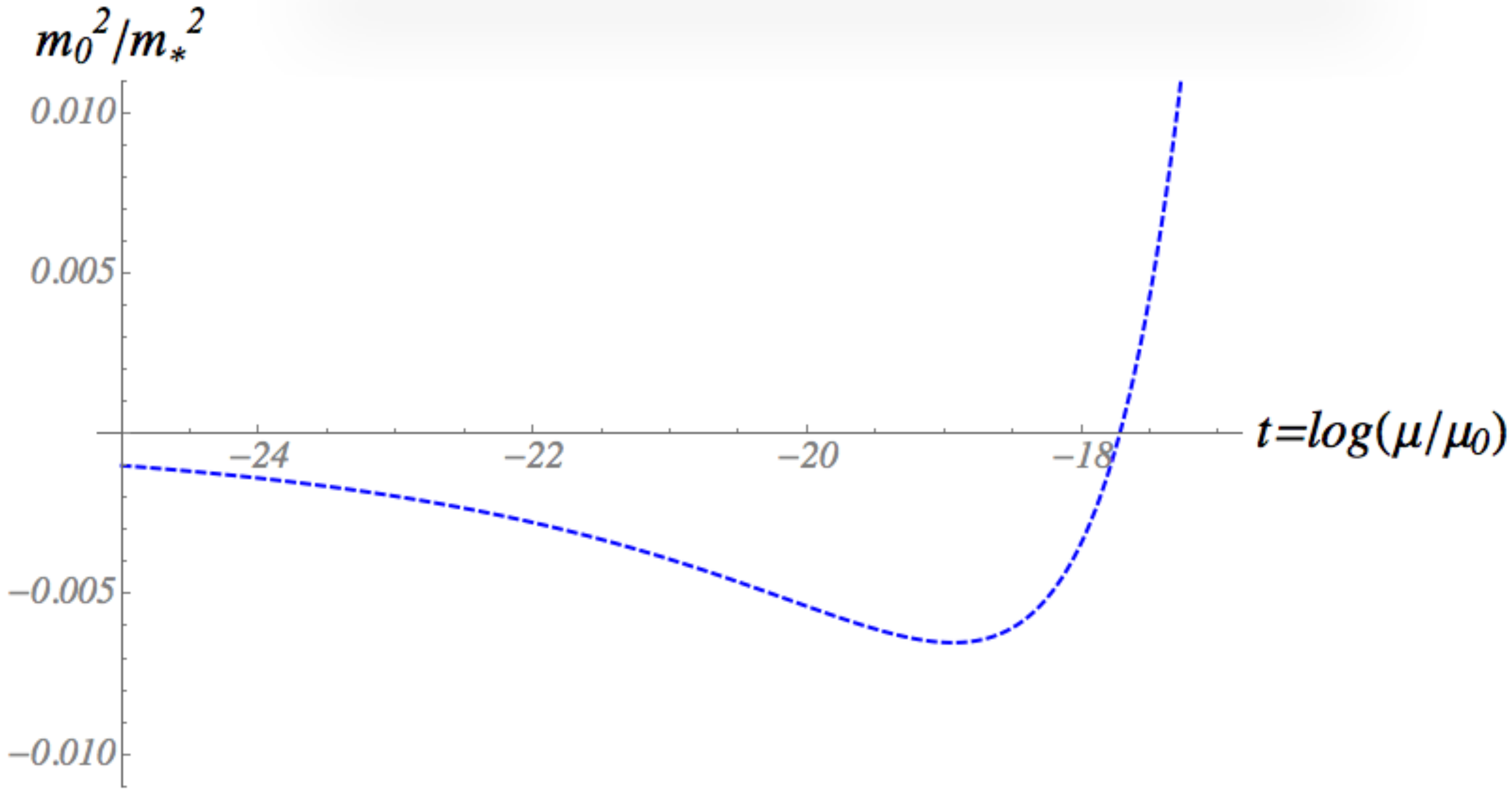
Dies away quickly *in the IR*

Dies away slowly *in the IR*

Starting values get relatively closer in UV (note the masses are all shrinking in absolute terms in the IR) - full flavour symmetry restored precisely at fixed point



Induces radiative breaking...



$$\alpha_{g,min} \xrightarrow{\epsilon \to 0} \frac{1}{2}\alpha_g^*$$

$$m_{0,min}^2 \sim -\tilde{m}_*^2$$

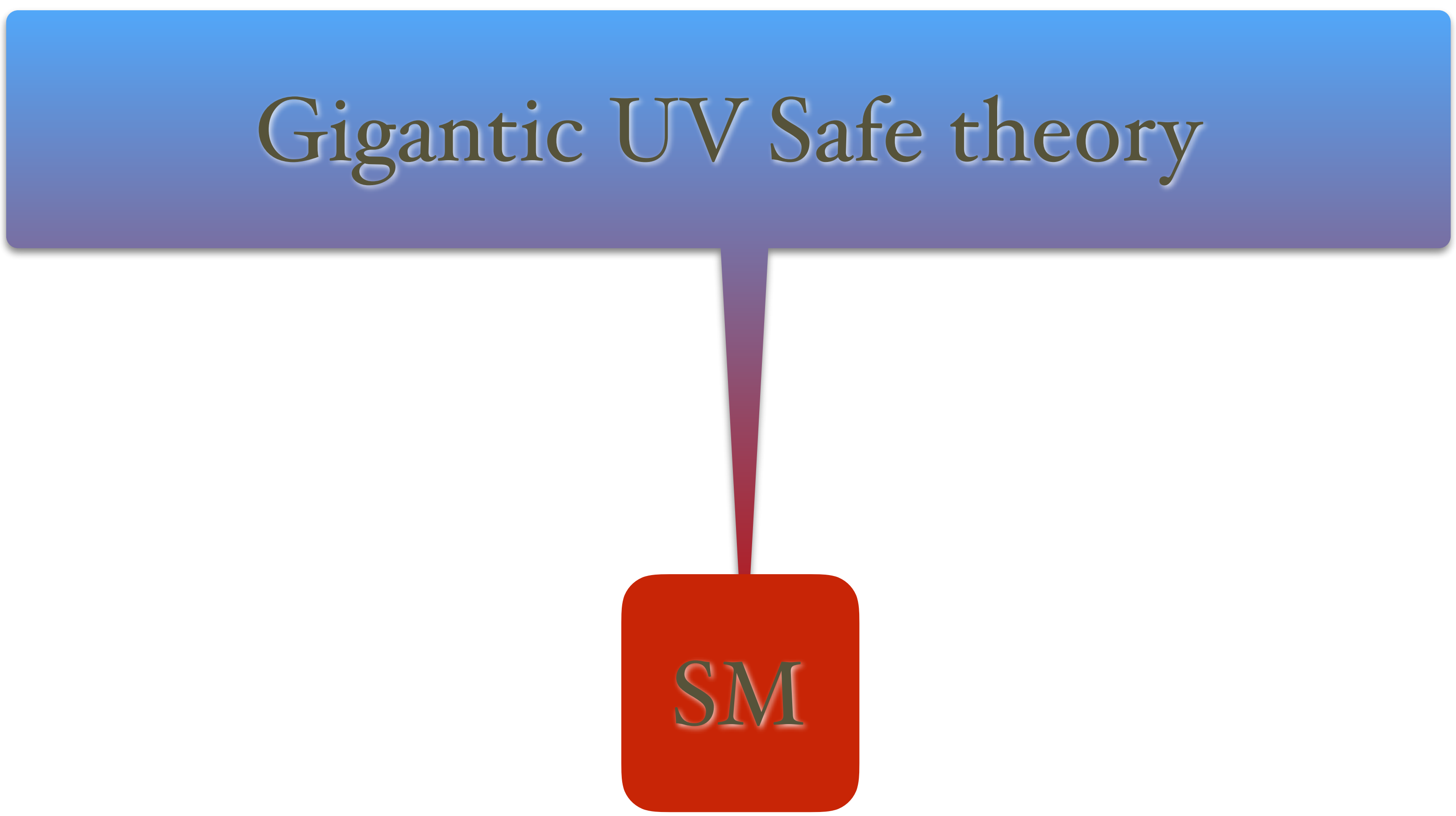
Generally in IR find flavour hierarchies grow ...

$$V \to \sum_{n>1} \Delta_n^2 \left[\mathrm{Tr}_n \left(h^2 + p^2 \right) - n \left(\left(\mathrm{Tr}_n h \right)^2 + \left(\mathrm{Tr}_n p \right)^2 \right) \right]$$

where Tr_n is the trace over the SU(n) sub-matrix

The ASSM via radiative breaking...

The ASSM via radiative breaking...



The ASSM via radiative breaking...

SM

The ASSM via radiative breaking...

SM

- **To embed the SM - focus on breaking SU(Nc) to SU(3) colour with new scalars …**

c.f. Pelaggi, Sannino Strumia Vigiani; Bond, Litim; Bond, Hiller, Kowalska, Litim

	$SU(N_C)$	$SU(N_F)_L \supset$ $SU(2)_L \otimes SU(n_g)_L$	$SU(N_F)_R \supset$ $SU(2)_r \otimes SU(n_g)_r$	$SU(N_S) =$ $SU(N_C-4)_R \oplus SU(2)_S$	spin
Q_{ai}	$\square$	$\square \supset (\square, \square)$	1	1	$1/2$
$\tilde{Q}^{ia}$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1	$1/2$
H^i_j	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1	0
$\tilde{S}_{a,\ell=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	0
$\tilde{q}^i_\ell$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1	$\square = \square_{N_C-4} \oplus \square_2$	$1/2$
q^ℓ_j	1	1	$\square \supset (\square, \square)$	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	$1/2$

Tab. 2: *Fields in the Asymptotically Safe SM, where $N_S = N_C - 2$. The top $2n_g = 6$ components of flavour $SU(N_F)$ correspond to $SU(2)$ multiplets, where n_g is the generation number. There is a mass-term $m_q q\tilde{q}$ that respects the $SU(N_C-4)$ in addition to the gauging for the usual Pati-Salam $SU(2)_R$, given by $SU(2)_R = [SU(2)_r \otimes SU(2)_S]_{diag}$.*

- **To embed the SM - focus on breaking SU(Nc) to SU(3) colour with new scalars …**

c.f. Pelaggi, Sannino Strumia Vigiani; Bond, Litim; Bond, Hiller, Kowalska, Litim

	$SU(N_C)$	$SU(N_F)_L \supset$ $SU(2)_L \otimes SU(n_g)_L$	$SU(N_F)_R \supset$ $SU(2)_r \otimes SU(n_g)_r$	$SU(N_S) =$ $SU(N_C-4)_R \oplus SU(2)_S$	spin
Q_{ai}	$\square$	$\square \supset (\square, \square)$	1	1	1/2
$\tilde{Q}^{ia}$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1	1/2
H^i_j	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1	0
$\tilde{S}_{a,\ell=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	0
$\tilde{q}^i_\ell$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1	$\square = \square_{N_C-4} \oplus \square_2$	1/2
q^ℓ_j	1	1	$\square \supset (\square, \square)$	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	1/2

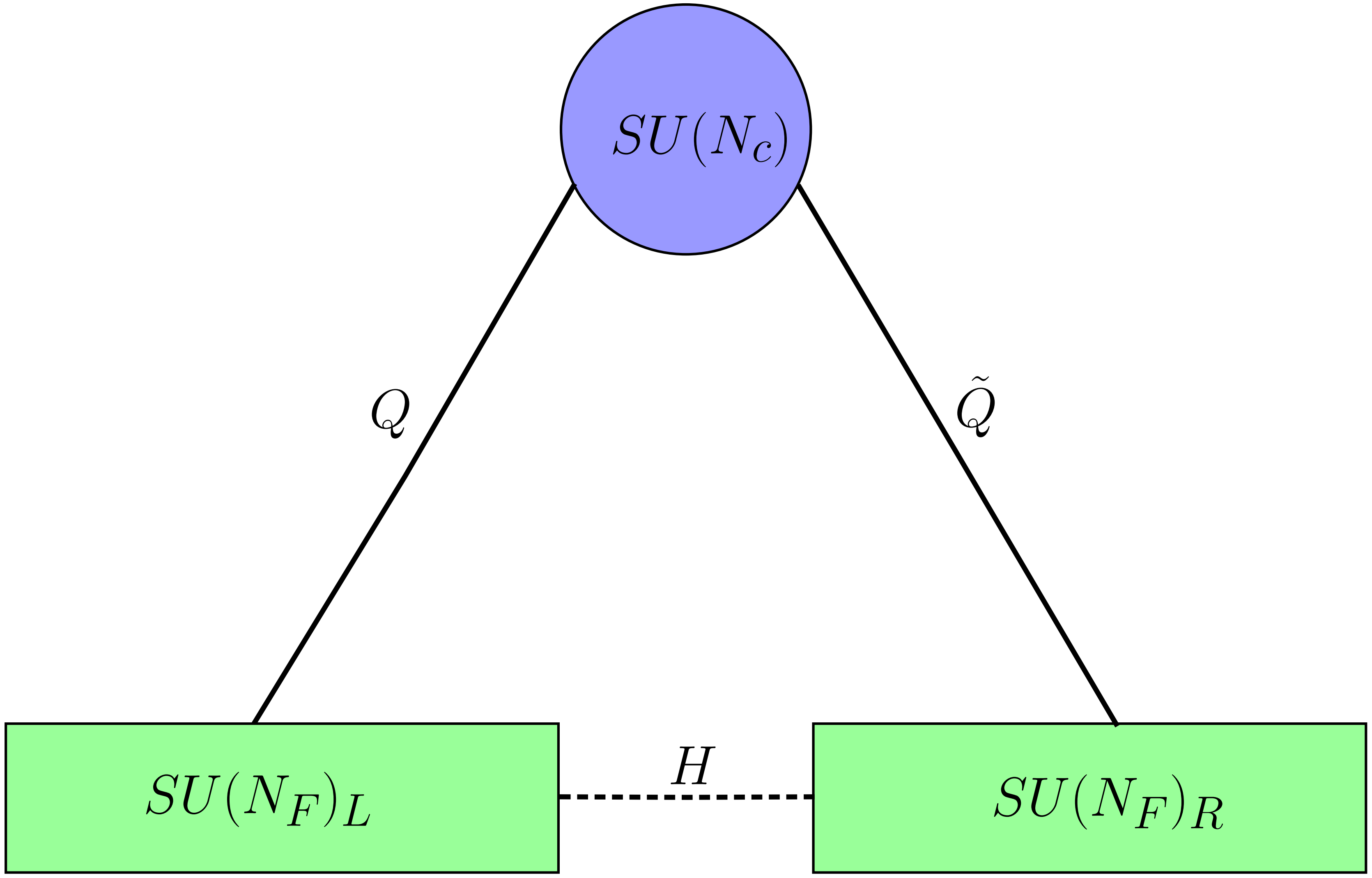
Tab. 2: *Fields in the Asymptotically Safe SM, where $N_S = N_C - 2$. The top $2n_g = 6$ components of flavour $SU(N_F)$ correspond to $SU(2)$ multiplets, where n_g is the generation number. There is a mass-term $m_q q\tilde{q}$ that respects the $SU(N_C-4)$ in addition to the gauging for the usual Pati-Salam $SU(2)_R$, given by $SU(2)_R = [SU(2)_r \otimes SU(2)_S]_{diag}$.*

Extension of Pati-Salam - breaks to SU(3) if we choose $N_S = N_C - 2$

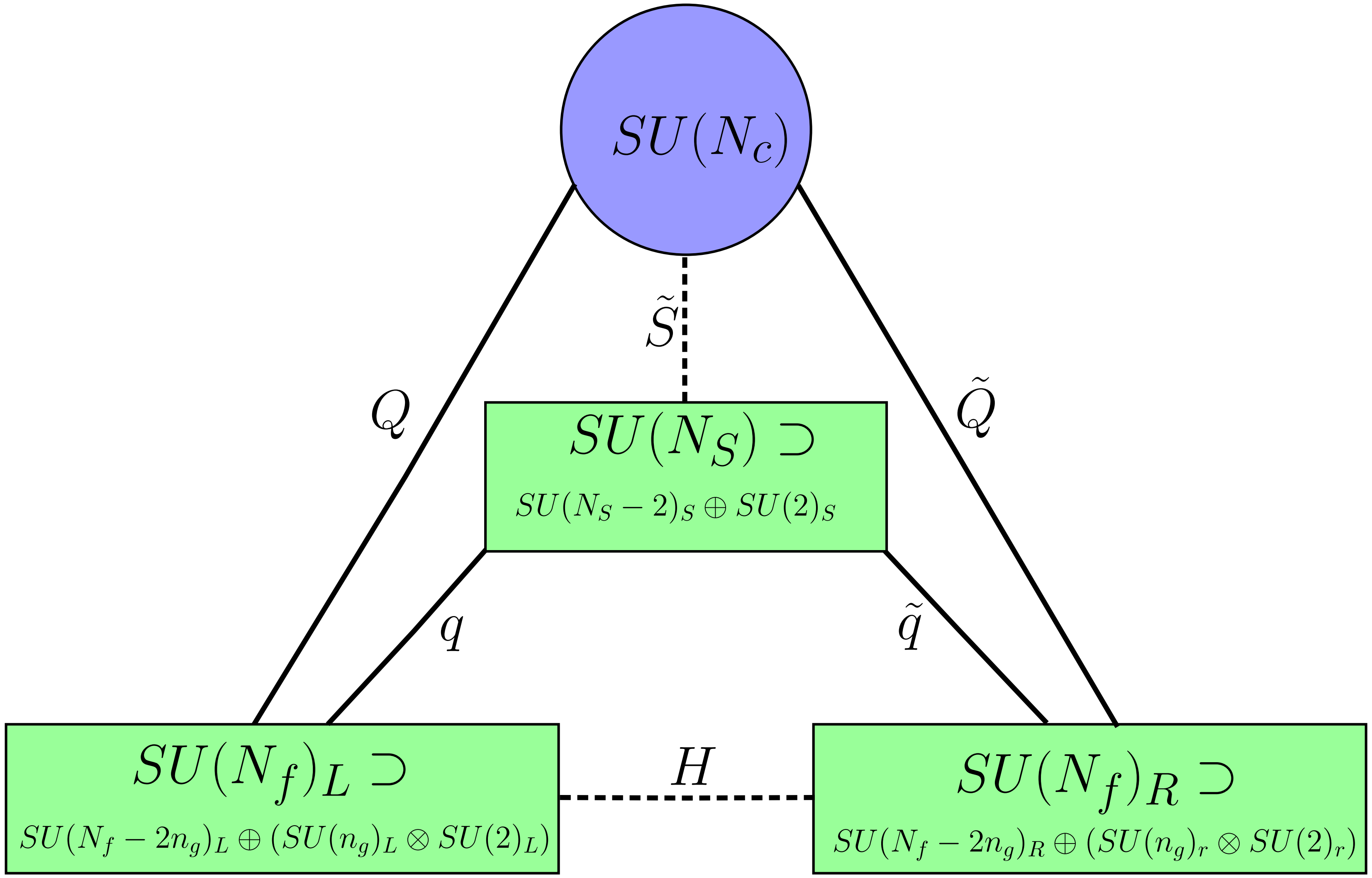
$$\frac{N_S}{N_C} \to 1\,; \quad \frac{N_F}{N_C} \to \frac{21}{4} + \epsilon$$

$$\tilde{S} = \left(\overbrace{\begin{matrix} \begin{pmatrix} \tilde{d}^c \\ \tilde{u}^c \end{pmatrix} & \begin{pmatrix} \tilde{e}^c \\ \tilde{\nu}^c \end{pmatrix} & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \end{matrix}}^{N_C} \right) \Bigg\} N_S = N_C - 2$$

Before:



After:


$SU(N_c)$
Q
$\tilde{S}$
$\tilde{Q}$
$SU(N_S) \supset$
$SU(N_S-2)_S \oplus SU(2)_S$
q
$\tilde{q}$
$SU(N_f)_L \supset$
$SU(N_f-2n_g)_L \oplus (SU(n_g)_L \otimes SU(2)_L)$
H
$SU(N_f)_R \supset$
$SU(N_f-2n_g)_R \oplus (SU(n_g)_r \otimes SU(2)_r)$

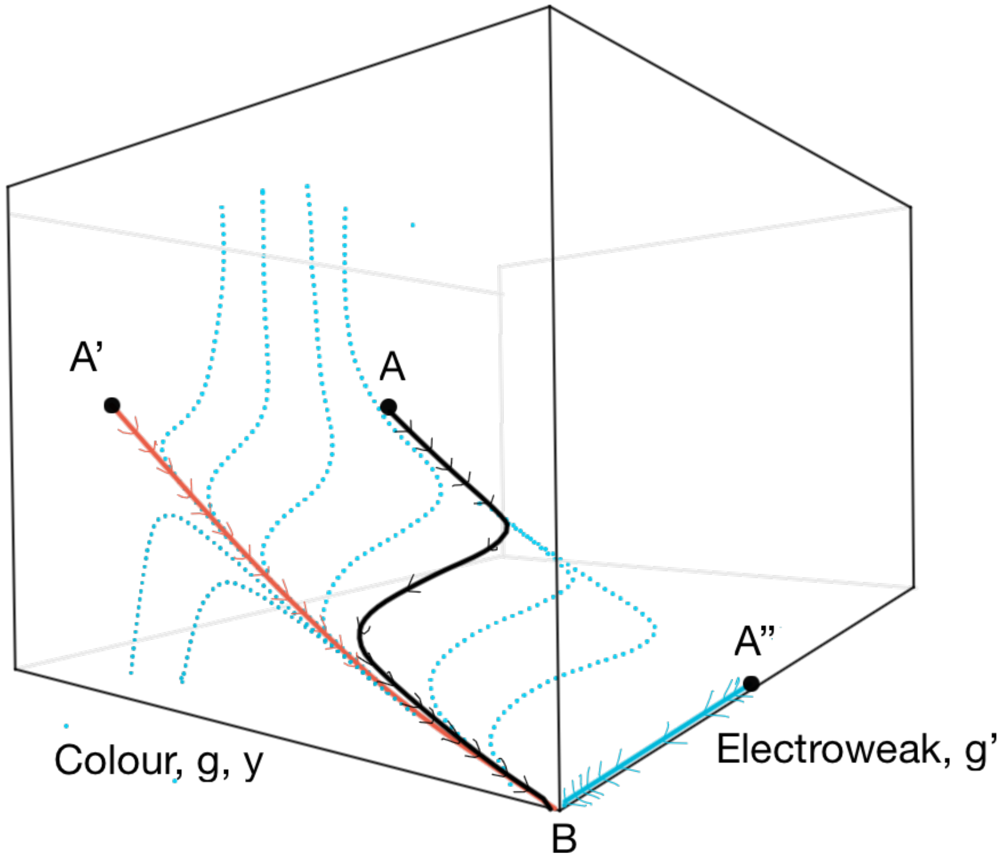
- Explicit embedding looks like P-S

$$Q = \left(\overbrace{\begin{array}{ccccc} q_1 & \ell_1 & \cdots & \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} & \cdots \\ q_2 & \ell_2 & \cdots & \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} & \cdots \\ q_3 & \ell_3 & \cdots & \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} & \cdots \\ \vdots & \vdots & & \ddots & \end{array}}^{N_C}\right)\Bigg\} N_F \quad ; \quad \tilde{Q} = \begin{pmatrix} \begin{pmatrix} u^c \\ d^c \end{pmatrix} & \begin{pmatrix} \nu_e^c \\ e^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} s^c \\ c^c \end{pmatrix} & \begin{pmatrix} \nu_\mu^c \\ \mu^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} b^c \\ t^c \end{pmatrix} & \begin{pmatrix} \nu_\tau^c \\ \tau^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

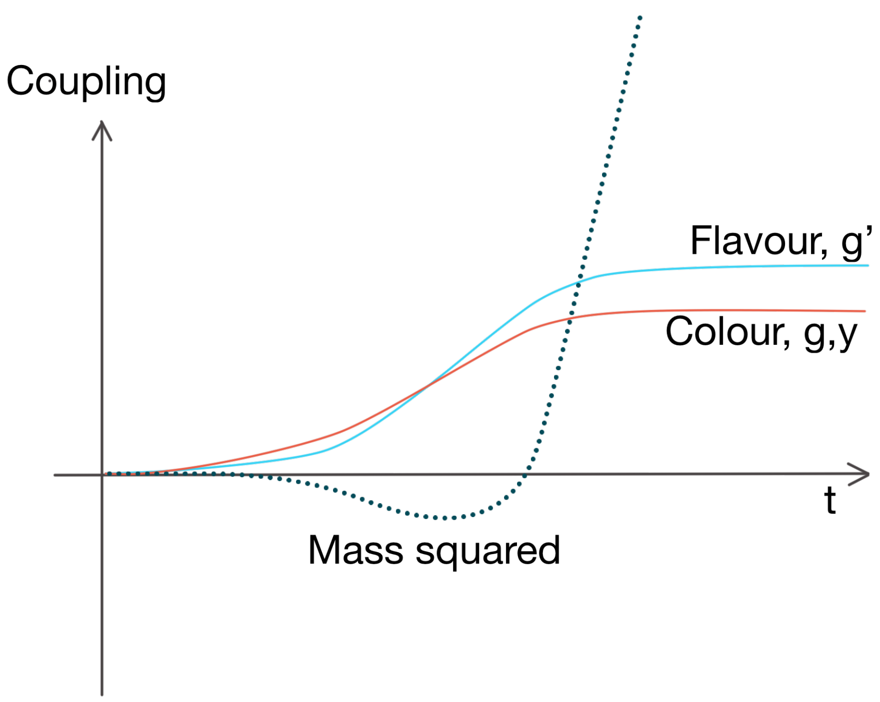
$$H = \begin{pmatrix} \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{11} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{12} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{13} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{21} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{22} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{23} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{31} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{32} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{33} & \cdots \\ \vdots & \vdots & \vdots & H_0 \end{pmatrix}$$

- Assignment implies 9 pairs of Higgses one for each Yukawa coupling

- **Need …**



A'
A
A"
Colour, g, y
Electroweak, g'
B



Coupling
Flavour, g'
Colour, g,y
t
Mass squared

- **What about AS for the SU(2)xSU(2) electroweak gauge groups?**

These see a large number of flavours (Nf (small f) of order order Nc)?

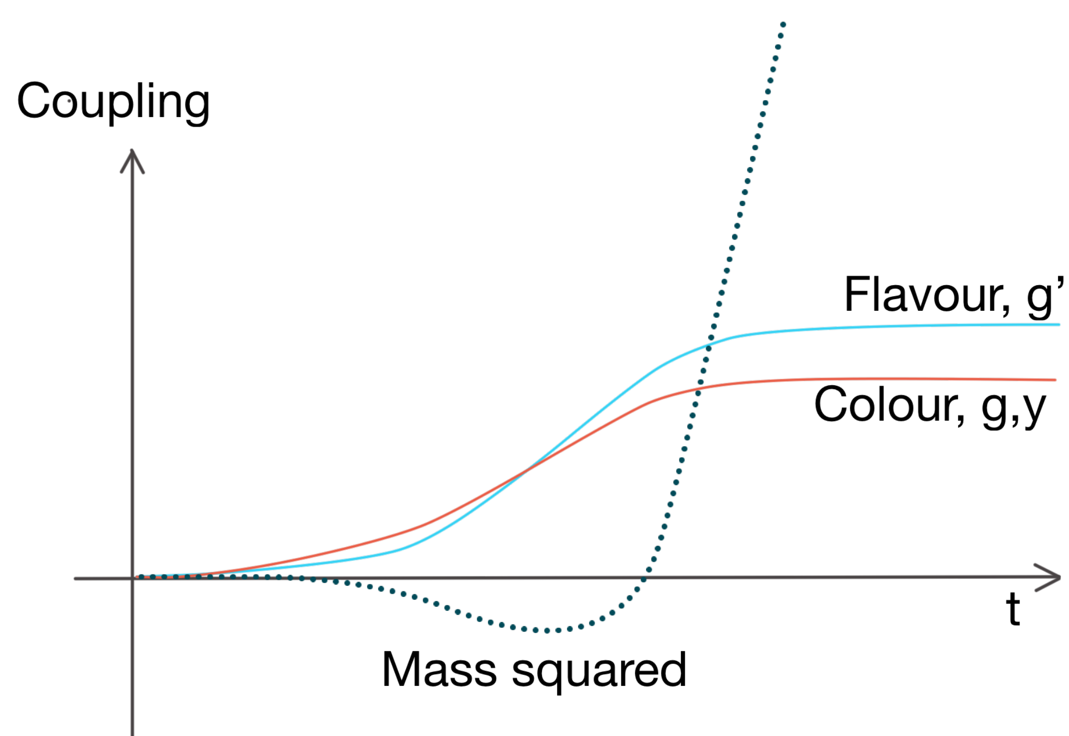
- **This gives UVFP behaviour with a fixed point at ’t Hooft couple ~ 1 … if Nf >>16:**

Gracey, Holdom, Shrock, Antipin, Pica, Sannino

Resum first terms gives

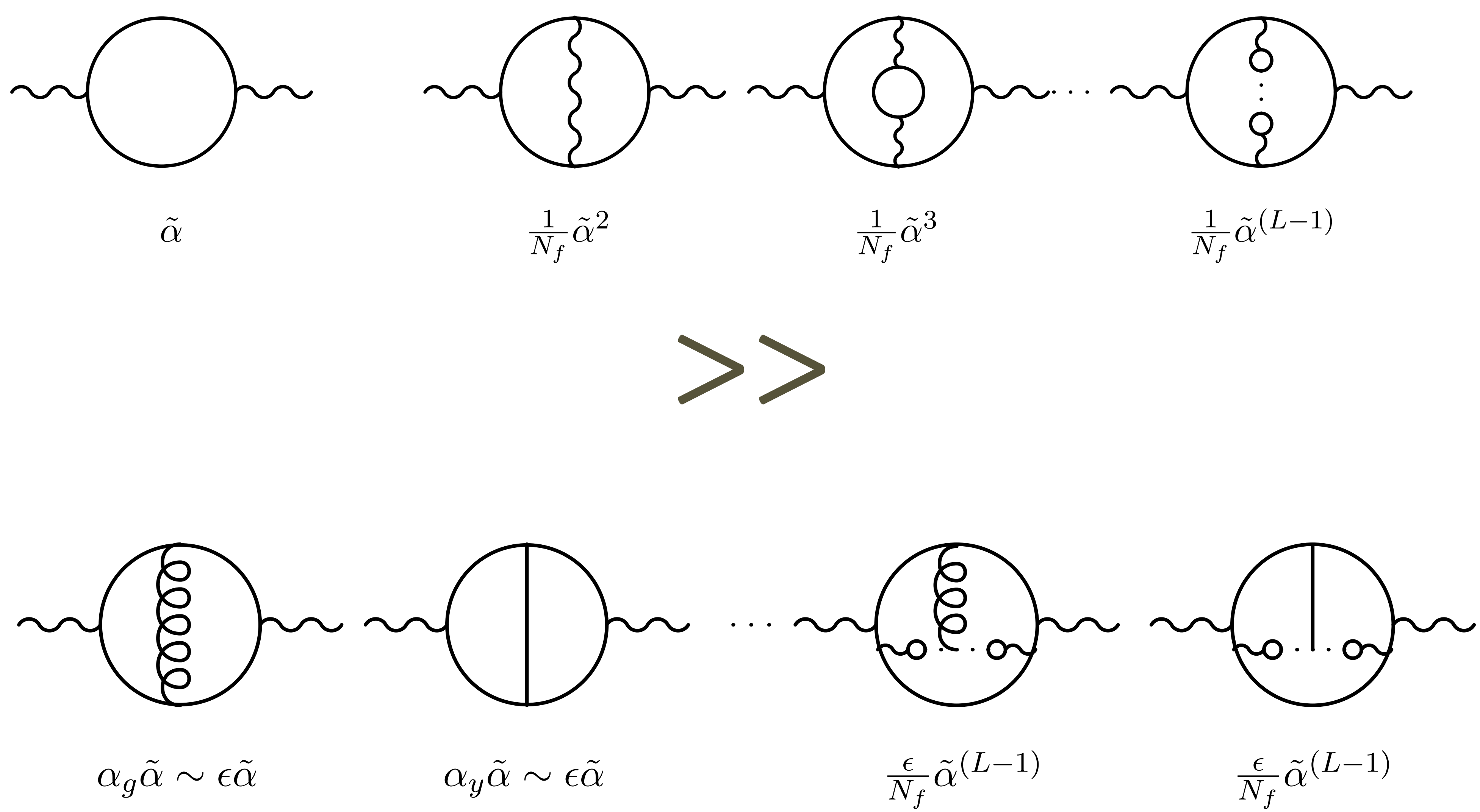
$$\frac{3}{4}\frac{\beta_{\tilde{\alpha}}}{\tilde{\alpha}^2} = 1 + \frac{H(\tilde{\alpha})}{N_f} + \mathcal{O}(N_f^{-2})$$

$$H(\tilde{\alpha}) = \frac{1}{4}\log|3-2\tilde{\alpha}| + \text{constant}$$

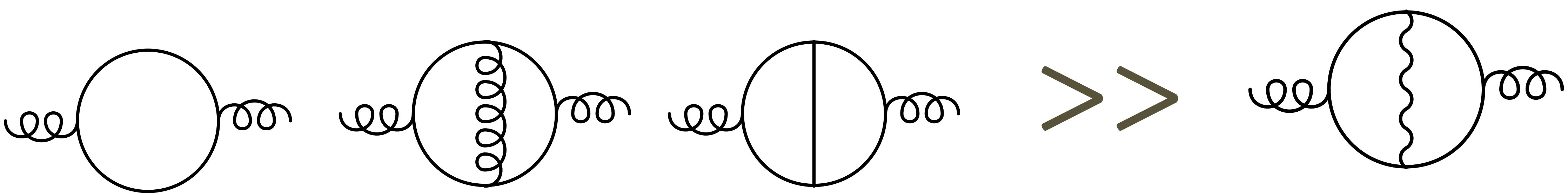


$$\tilde{\alpha}_* = \tfrac{3}{2} - Ce^{-4N_f}$$

- **The difficult part: Showing that the two kinds of UVFP decouple.**
- **Can show in the Veneziano limit the corrections to the weak FP go like epsilon. Can neglect everything but gauge couplings when determining the SU(2) fixed points.**



- **By simple power-counting, the SU(2) gauge couplings are subdominant (by 1/Nc) in the original “strong” UVFP ==> Can neglect the weak gauging for this UVFP.**
- **Likewise …**



$x_F\alpha_g$ $\quad$ $x_F\,\alpha_g^2 \sim \epsilon^2$ $\quad$ $x_F\,\alpha_y\alpha_g \sim \epsilon^2$ $\quad$ $\frac{1}{N_f N}\tilde{\alpha}\alpha_g \sim \frac{\epsilon}{N_f^2}\tilde{\alpha} \lesssim \epsilon^3\frac{\tilde{\alpha}}{25}$

Does this make sense without gravity?

What can we learn from string theory?

Asymptotic safety in strings

Asymptotic safety in strings

String theory is famously scale invariant, but not normally considered asymptotically safe. Why? It has ***two scales*** (String tension and Planck Scale - or in a compactification, the radius R)?

Asymptotic safety in strings

String theory is famously scale invariant, but not normally considered asymptotically safe. Why? It has ***two scales*** (String tension and Planck Scale - or in a compactification, the radius R)?

- But at E>>MPl effectively only one scale: the theory must be asymptotically safe even though it has an infinite number of degrees of freedom.

Asymptotic safety in strings

String theory is famously scale invariant, but not normally considered asymptotically safe. Why? It has ***two scales*** (String tension and Planck Scale - or in a compactification, the radius R)?

- But at E>>MPl effectively only one scale: the theory must be asymptotically safe even though it has an infinite number of degrees of freedom.
- This is hard to show above the string scale: must determine beta function w/o breaking important symmetries that give the theory its finiteness - what do we do about cut-offs (very non-Wilsonian). Cannot do what we would normally do …

Asymptotic safety in strings

String theory is famously scale invariant, but not normally considered asymptotically safe. Why? It has ***two scales*** (String tension and Planck Scale - or in a compactification, the radius R)?

- But at E>>MPl effectively only one scale: the theory must be asymptotically safe even though it has an infinite number of degrees of freedom.
- This is hard to show above the string scale: must determine beta function w/o breaking important symmetries that give the theory its finiteness - what do we do about cut-offs (very non-Wilsonian). Cannot do what we would normally do …

$$\delta\mathcal{L} = -\frac{1}{4}\tilde{C}_1(p^2,\mu^2)F_{\mu\nu}F^{\mu\nu}$$

$$\beta_g = \frac{g}{2}\frac{\partial\tilde{C}_1}{\partial\log\mu}.$$

Consider what happens in field theory

C1 given by 2-point function in background-field method:

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\text{gauge}}(s) = -\frac{22C_A}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(-\frac{\mu^2}{s}\right)\right) \ ,$$

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\text{ferm}}(s) = \frac{4N_f}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_f^2} + \left(1 + \frac{2m_s^2}{s}\right)\Lambda(s; m_f, m_f)\right) \ ,$$

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\text{scalar}}(s) = \frac{2N_s}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_s^2} + \left(1 - \frac{4m_s^2}{s}\right)\Lambda(s; m_s, m_s)\right) \ ,$$

Consider what happens in field theory

C1 given by 2-point function in background-field method:

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm gauge}(s) = -\frac{22C_A}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(-\frac{\mu^2}{s}\right)\right) \ ,$$

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm ferm}(s) = \frac{4N_f}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_f^2} + \left(1 + \frac{2m_s^2}{s}\right)\Lambda(s; m_f, m_f)\right) \ ,$$

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm scalar}(s) = \frac{2N_s}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_s^2} + \left(1 - \frac{4m_s^2}{s}\right)\Lambda(s; m_s, m_s)\right) \ ,$$

When s>>m get extra imaginary part
==> running beta function

Consider what happens in field theory

C1 given by 2-point function in background-field method:

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm gauge}(s) = -\frac{22C_A}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(-\frac{\mu^2}{s}\right)\right) ,$$

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm ferm}(s) = \frac{4N_f}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_f^2} + \left(1 + \frac{2m_s^2}{s}\right)\Lambda(s; m_f, m_f)\right) ,$$

$$(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm scalar}(s) = \frac{2N_s}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_s^2} + \left(1 - \frac{4m_s^2}{s}\right)\Lambda(s; m_s, m_s)\right) ,$$

$$\beta_{\frac{16\pi^2}{g^2}}(s) = -\frac{1}{\pi}\left[\frac{16\pi^2}{g^2}{\rm Im}\tilde{\mathcal{A}}^{(2)}(s)\right]$$

When s>>m get extra imaginary part
==> running beta function

Example: KK Towers

$$m^2 = \frac{\vec{m}\cdot\vec{m}}{R^2}$$

- Where Δb is the coefficient from a single KK level - doesn't care about level splitting
- Note A(s) is able to preserve all stringy symmetries that may be needed for finiteness
- This is an entirely *non-Wilsonian* definition
- Conjecture — string theory multiplets will fall into "misaligned" N=4 towers

Example: KK Towers

$$m^2 = \frac{\vec{m}\cdot\vec{m}}{R^2}$$

$$\beta_{\frac{16\pi^2}{g^2}}(s) = -\frac{1}{\pi}\left[\frac{16\pi^2}{g^2}\mathrm{Im}\tilde{\mathcal{A}}^{(2)}(s)\right]$$

$$= \frac{\Delta b}{\Gamma(3+d/2)}\frac{\pi^{(d+3)/2}}{2^{d+1}}\left(R\sqrt{s}\right)^d$$

- Where Δb is the coefficient from a single KK level - doesn't care about level splitting
- Note A(s) is able to preserve all stringy symmetries that may be needed for finiteness
- This is an entirely *non-Wilsonian* definition
- Conjecture — string theory multiplets will fall into "misaligned" N=4 towers

Summary

- Considered perturbative asymptotically safe QFTs (gauge-Yukawa theories)
- Positive mass-squareds can be driven negative in the IR, akin to radiative symmetry breaking in MSSM => radiative symmetry breaking
- A minimal embedding of the SM within this set-up relatively straightforward
- Overall now has the "feel of" other RG systems with large numbers of degrees of freedom in the UV such as duality cascade.
- Can this picture be "blended" in to a stringy version of AS— Misaligned *N=4 SUSY.*