A Silent black hole

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Outline

I. Introduction
II. Tunneling in black holes
III. Complex path method
IV. Thermodynamic relations
V. Conclusions

Outline

- Black Hole: is a region of space-time where the gravitational field is so strong that nothing, even the light, can ever escapes from it.

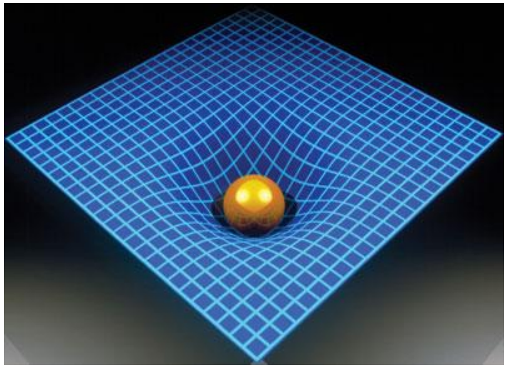
- S. XVIII – Michell, Laplace.

Escape velocity: $$\frac{1}{2}mv_e^2 = G\frac{mM}{R}$$

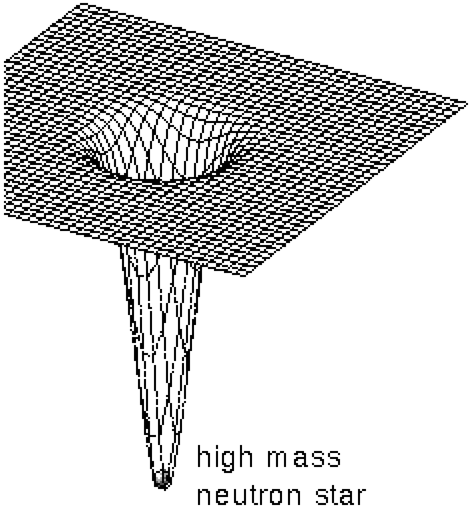
$$R_S \equiv \frac{2GM}{c^2}$$, if $R < R_S$ $\longrightarrow$ Black Star

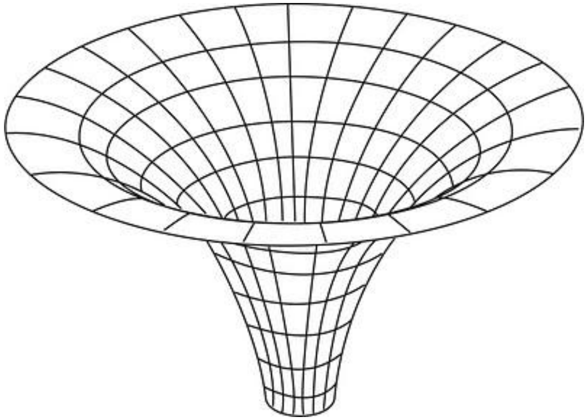
- General Relativity

Gravity ⇄ Geometry



Sun





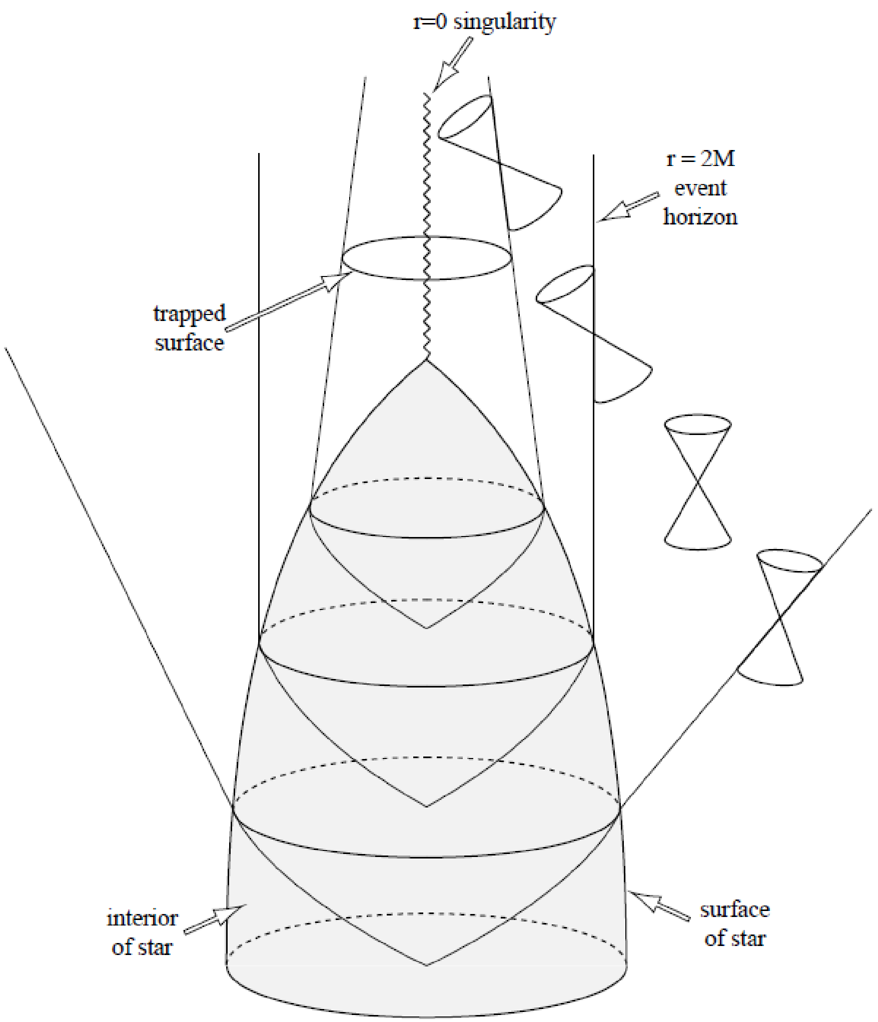
Black hole

- Cosmic Censorship

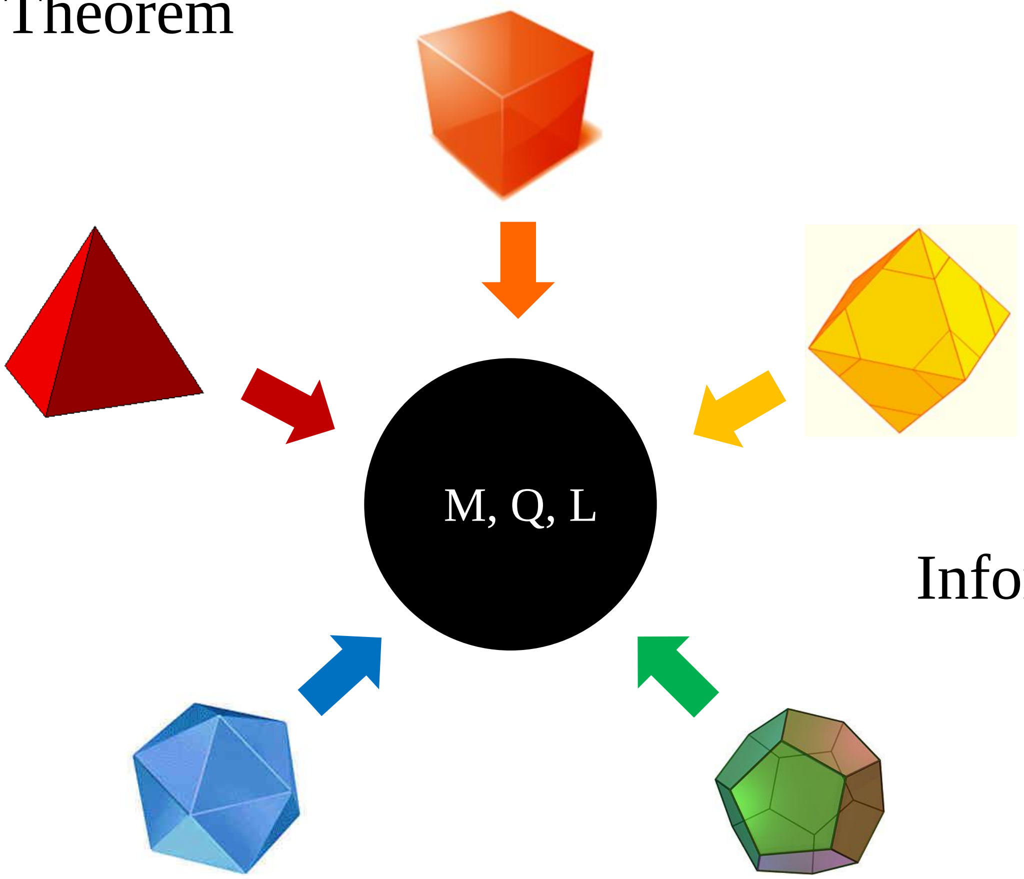
$R < R_S$

- Gravitational collapse

- The singularity is hidden behind the event horizon

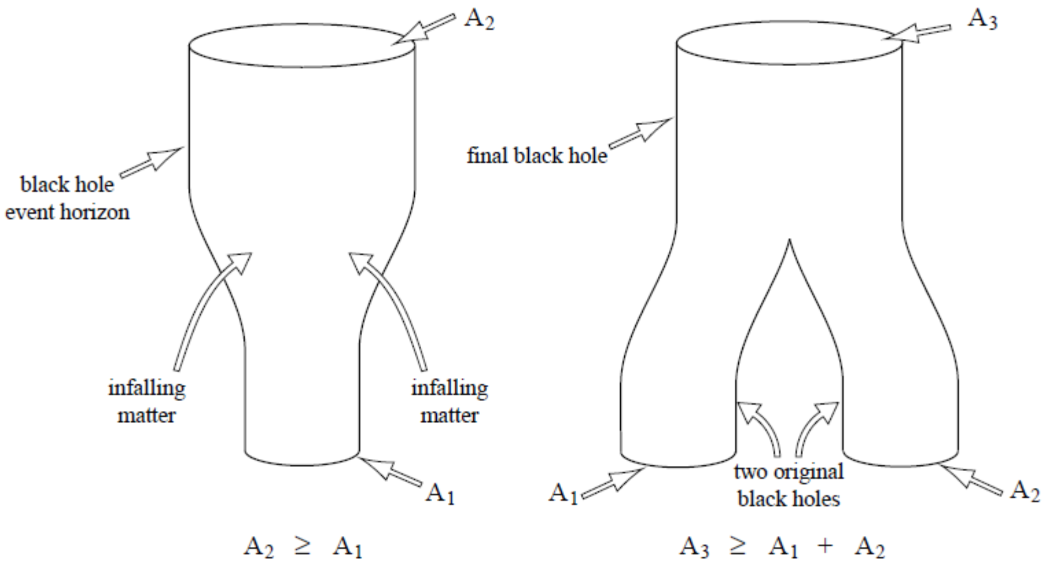


- No-Hair Theorem



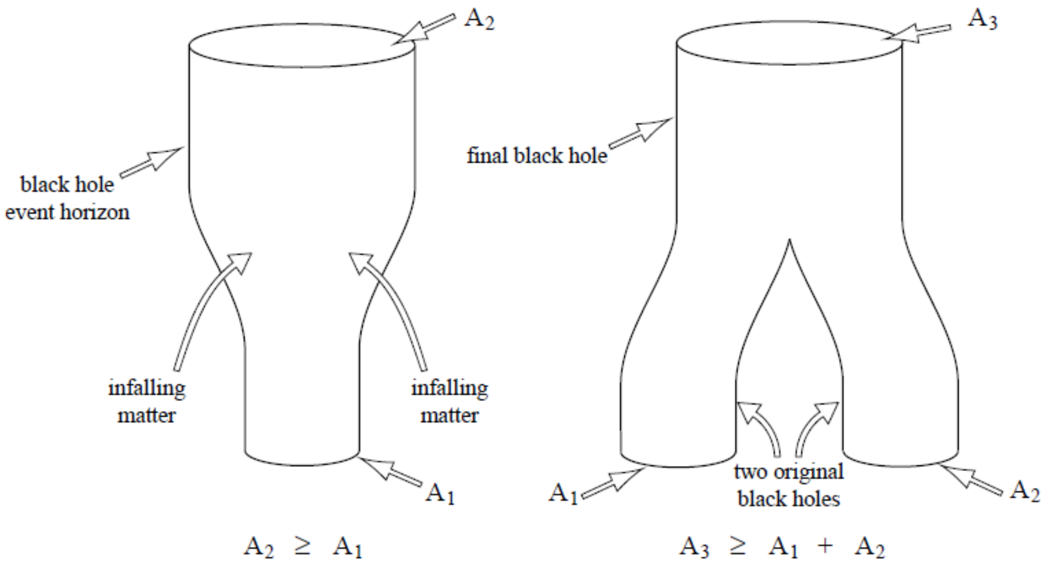
Information Loss

- Thermodynamics

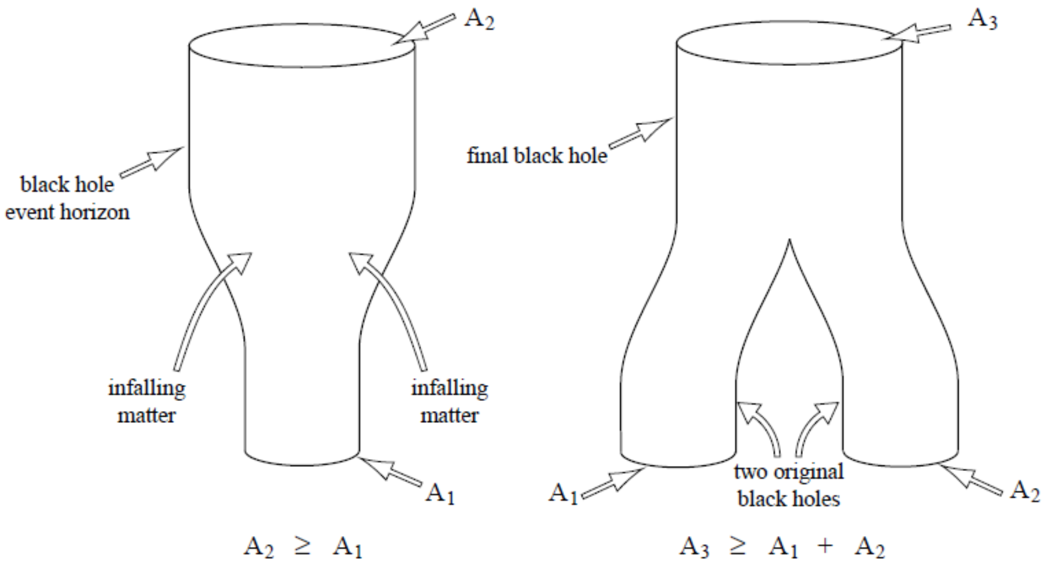


- Thermodynamics

  - Entropy : $S = A/4$
    (in Planck units)
  - Temperature: $T_H = \kappa/2\pi$



- Thermodynamics

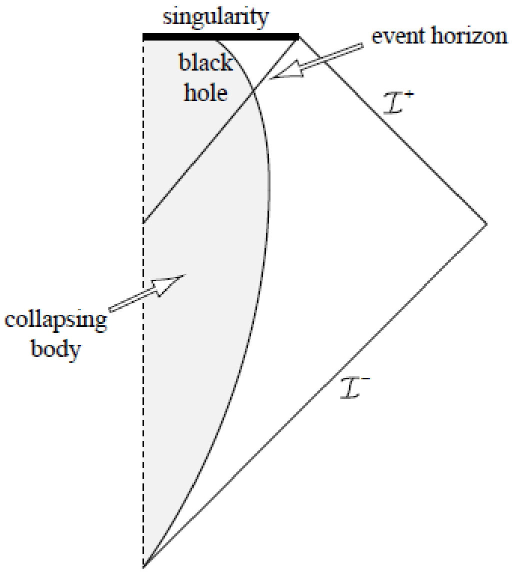


- Entropy : $S = A/4$
  (in Planck units)
- Temperature: $T_H = \kappa/2\pi$

Laws	**Thermodynamics**	**Black hole mechanics**
0th	T	κ
1st	$dE = TdS$ + Work terms	$dM = (\kappa/8\pi)\, dA$ + Work terms
2nd	$dS \geq 0$	$dA \geq 0$ Generalized: $d\{S_{BH} + S_{matter}\} \geq 0$

➢ Hawking radiation or the quantum mechanics of black holes

Semi-classical process ➡ Classical background + QM matter fields



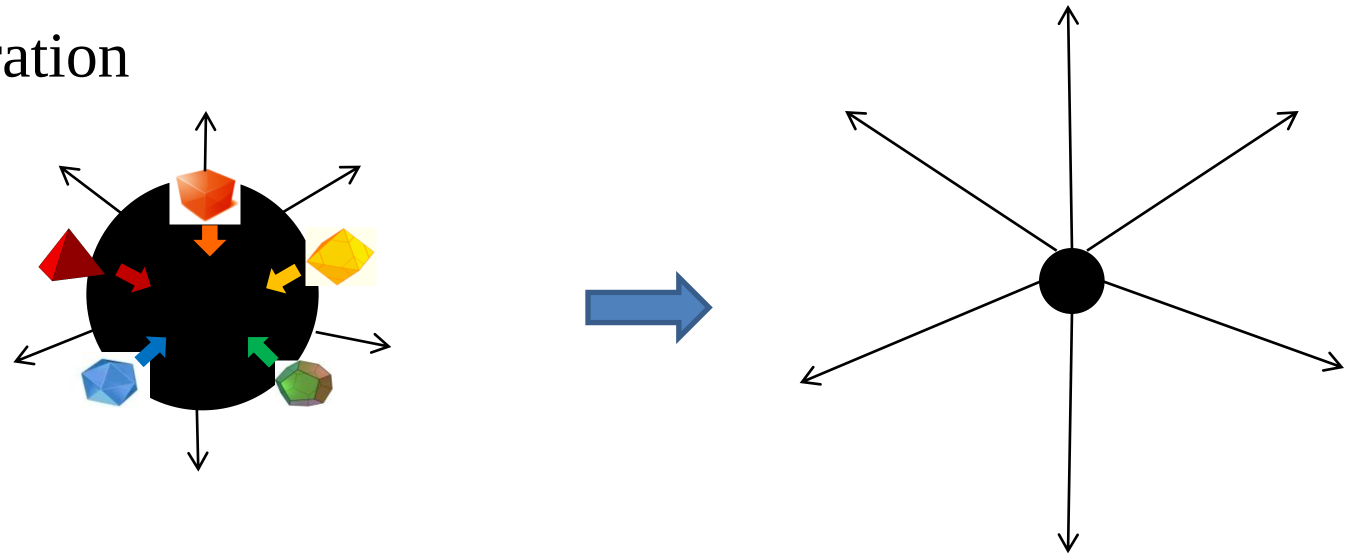
○ Creation and emission of particles at late times

○ Thermal flux ⇄ Blackbody spectrum

$$\langle N_i \rangle_{\Im^+} = \frac{1}{e^{2\pi\omega_i/\kappa} - 1}$$

[S. W. Hawking, "Particle Creation by Black Holes," Commun. Math. Phys. **43 (1975)** 199-220.]

- Evaporation



Initial pure states → Final mixed states

Information loss paradox

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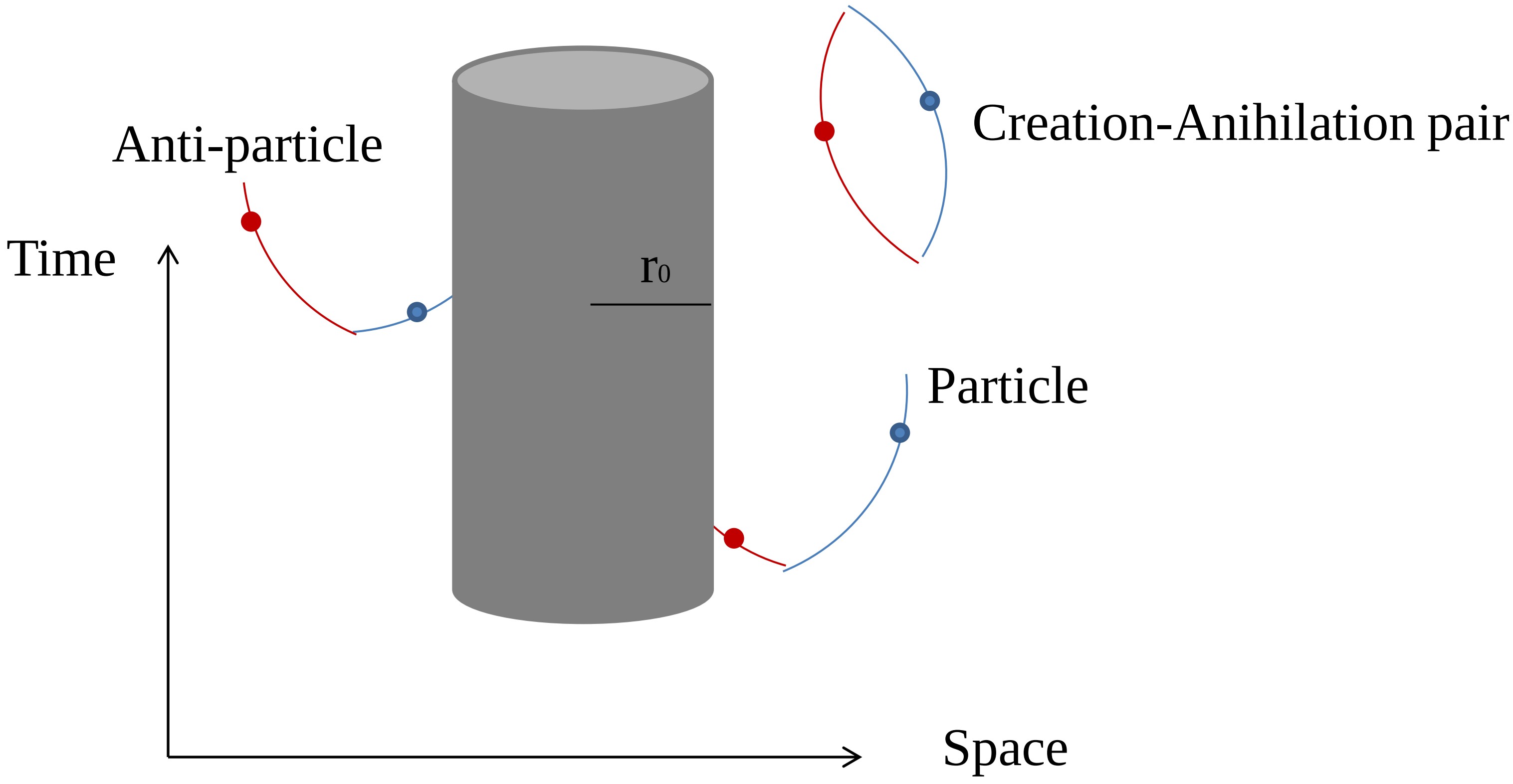

Anti-particle
Creation-Anihilation pair
Time
r0
Particle
Space

Tunneling in black holes

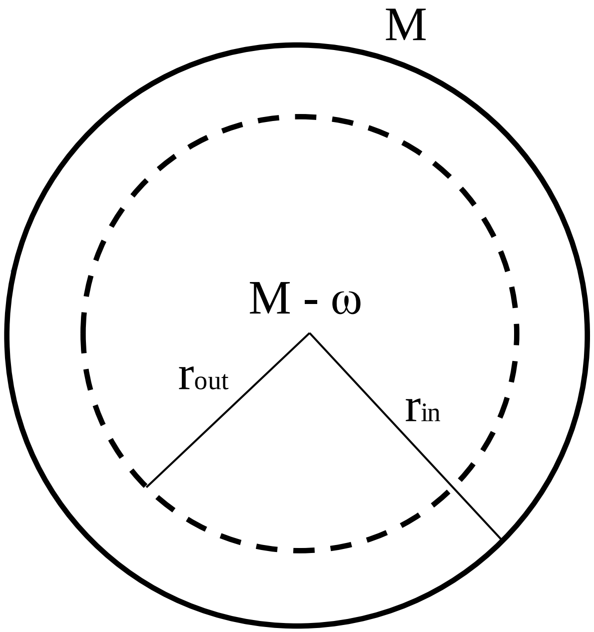
- Dynamical geometry

- Energy conservation

  $$M \longrightarrow M - \omega$$

- Shrinking of the event horizon

  $r_{in} > r_{out}$



[M. K. Parikh, F. Wilczek, "Hawking radiation as tunneling," Phys. Rev. Lett. **85 (2000)** 5042-5045. [hep-th/9907001].]

- Dynamical geometry
- Energy conservation

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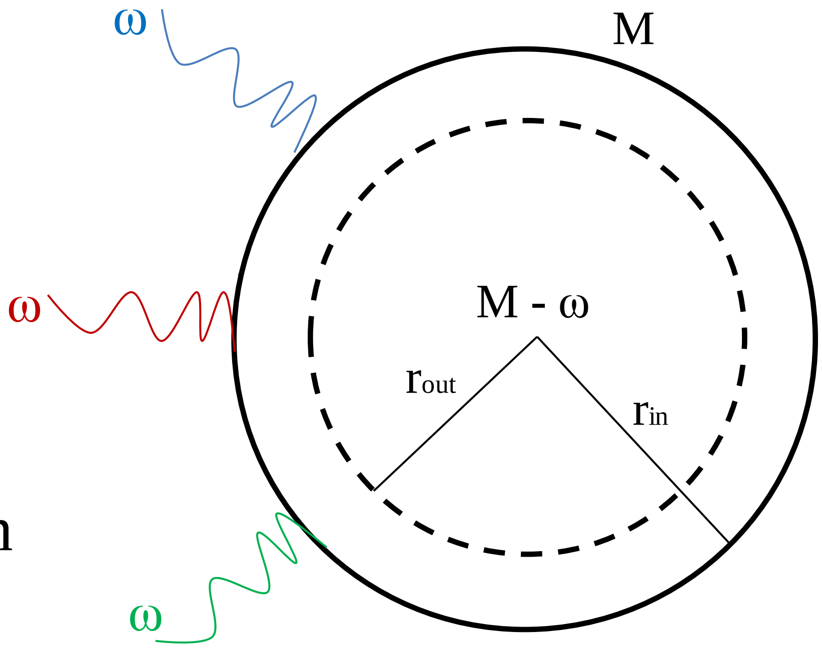
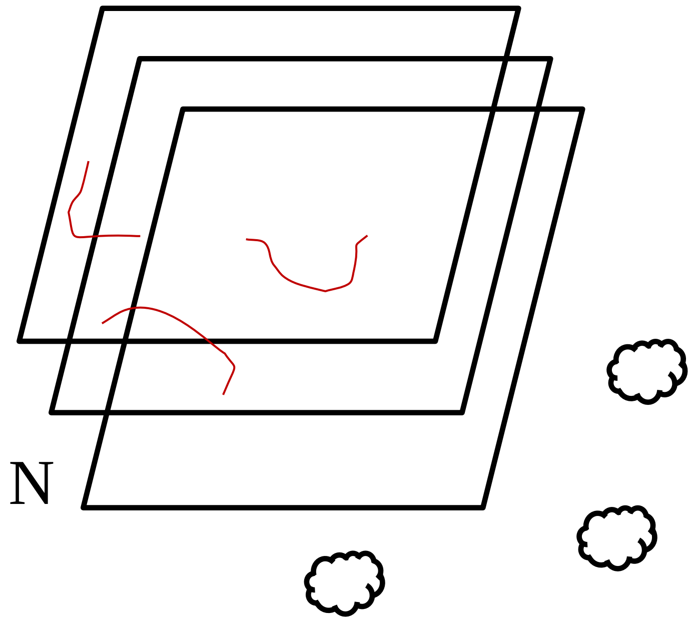


$$\Gamma \sim e^{-\frac{\omega}{T_H}+\mathcal{O}(\omega^2)}$$; emission rate

[M. K. Parikh, F. Wilczek, "Hawking radiation as tunneling," Phys. Rev. Lett. **85 (2000)** 5042-5045. [hep-th/9907001].]

Stack of N NS5 -branes



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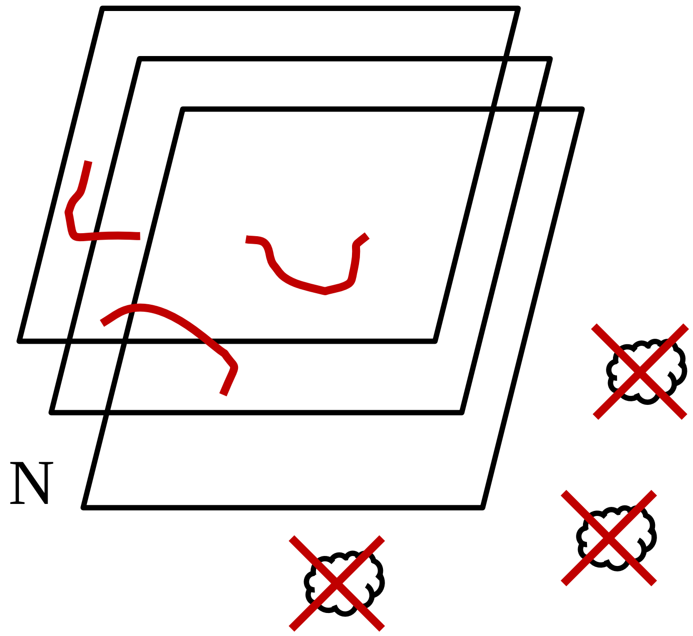
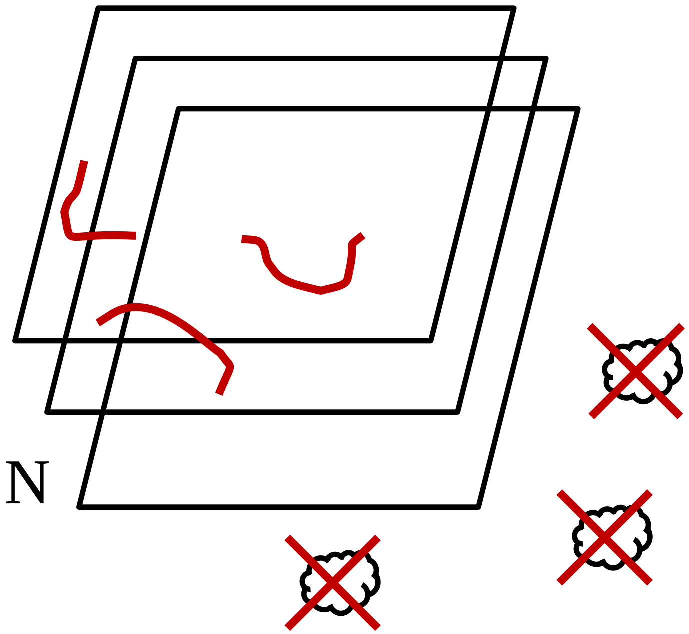


- $g_s \longrightarrow 0$
- $E/m_s =$ fixed

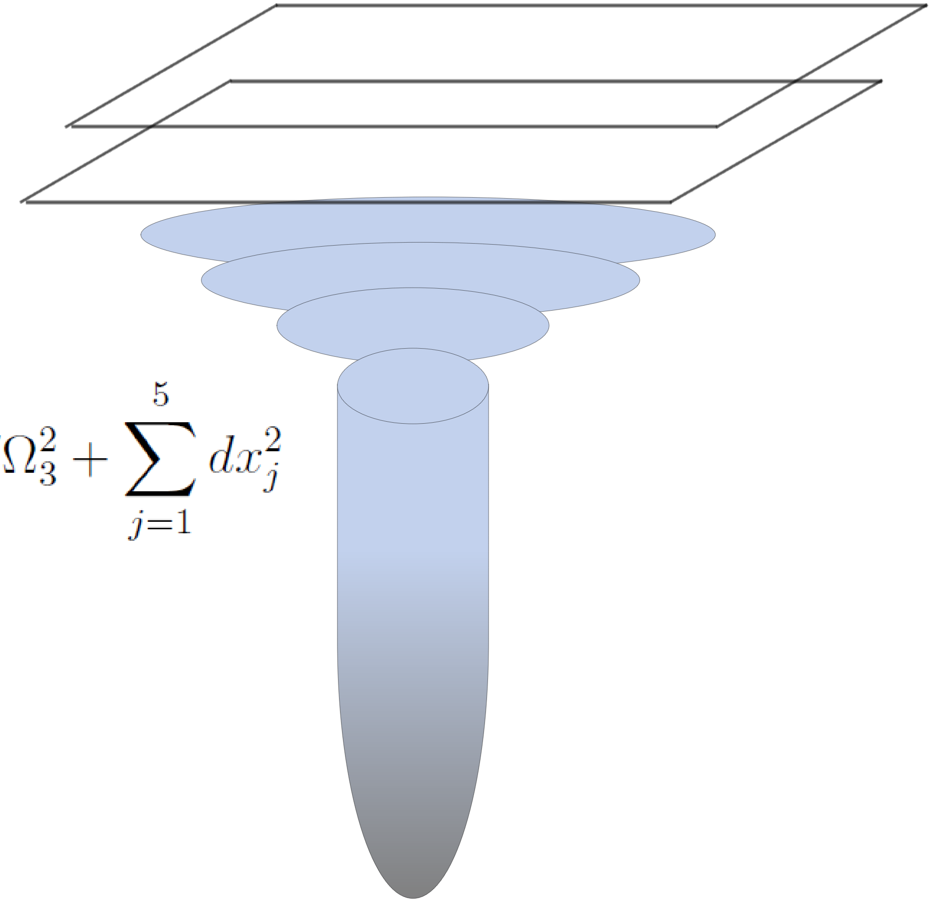
Tunneling in LST

Stack of N NS5 -branes



- $g_s \longrightarrow 0$
- $E/m_s =$ fixed
- Little String Theory
  - → (2,0) LST Type IIA NS5
  - → (1,1) LST Type IIB NS5

- Throat geometry of N coincident NS5-branes in string frame



$$ds^2 = -f(r)dt^2 + \frac{A(r)}{f(r)}dr^2 + A(r)r^2 d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2$$

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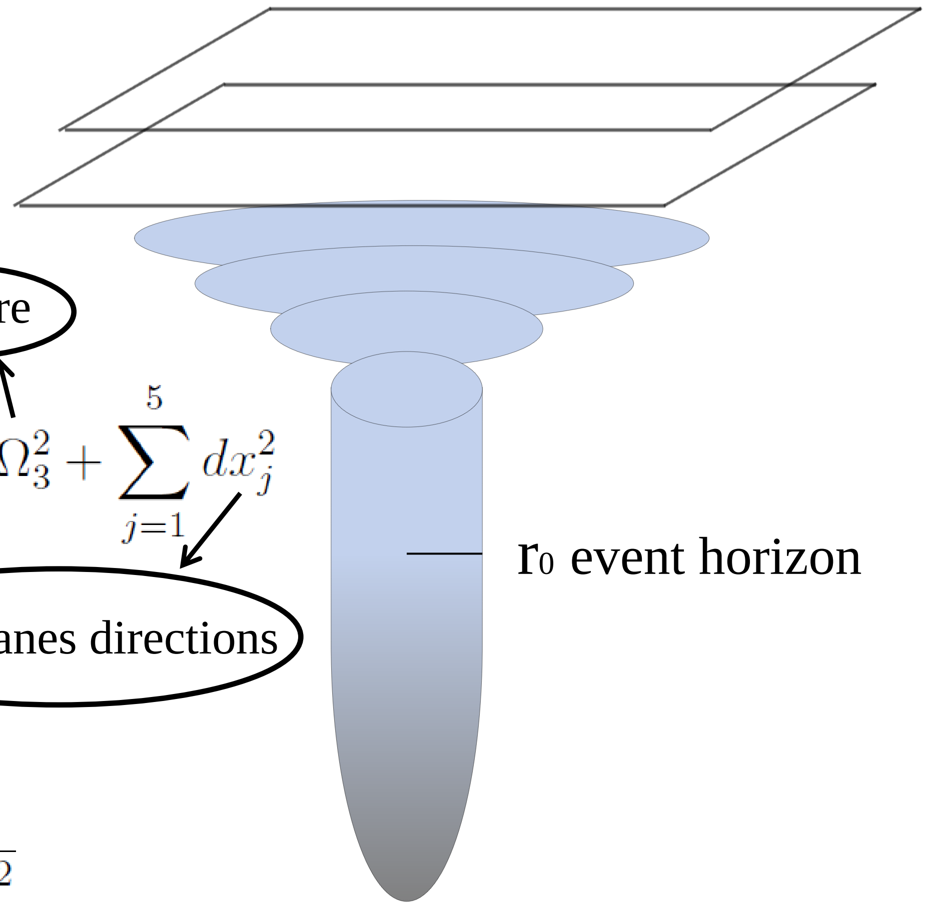


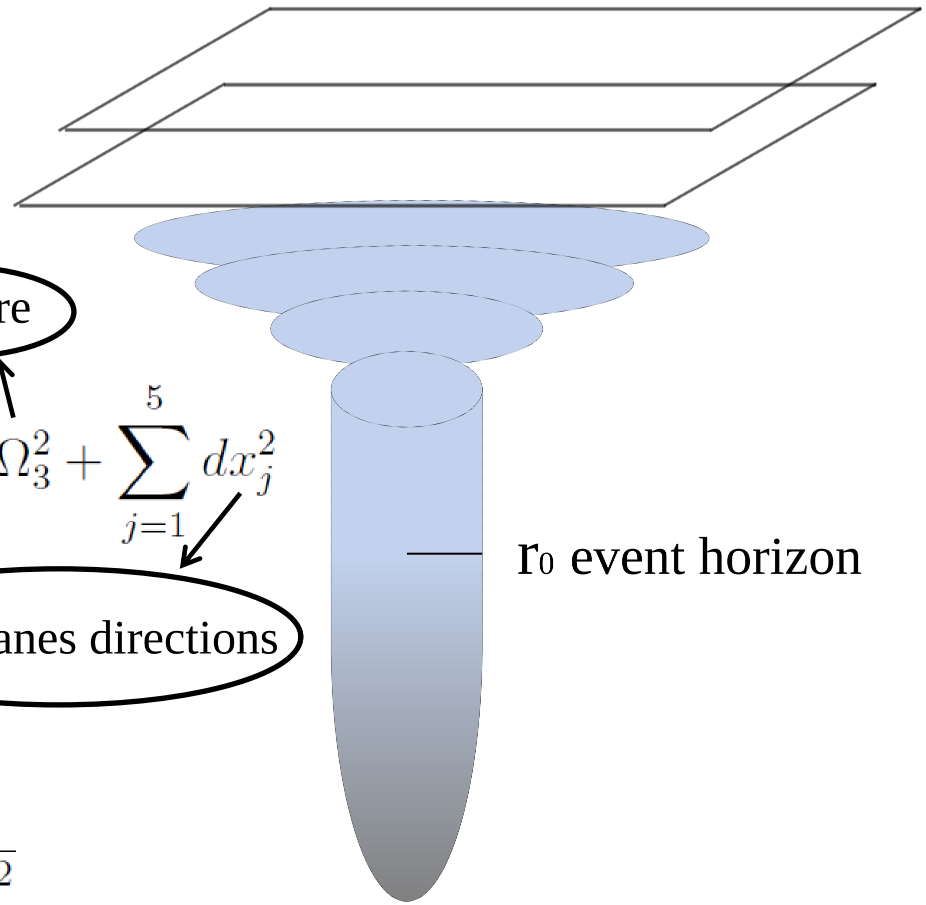
3-sphere

$$ds^2 = -f(r)dt^2 + \frac{A(r)}{f(r)}dr^2 + A(r)r^2 d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2$$

flat 5-branes directions

$$f(r) = 1 - \frac{r_0^2}{r^2} \quad , \quad A(r) = \chi + \frac{N}{m_s^2 r^2}$$

- Throat geometry of N coincident NS5-branes in string frame



3-sphere

$$ds^2 = -f(r)dt^2 + \frac{A(r)}{f(r)}dr^2 + A(r)r^2 d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2$$

flat 5-branes directions

$\chi = 1 \longrightarrow$ NS5
$\chi = 0 \longrightarrow$ LST

$$f(r) = 1 - \frac{r_0^2}{r^2} \quad , \quad A(r) = \chi + \frac{N}{m_s^2 r^2}$$

- Thermodynamics

$$M \sim r_0^2$$

- Temperature $T_H = \frac{\hbar}{2\pi\sqrt{\chi r_0^2 + \frac{N}{m_s^2}}}$
- Entropy $S_{BH} = \frac{A_H}{4G^{(10)}\hbar} = \frac{V_5\, \pi^2 r_0^2\, \sqrt{\chi m_s^2 r_0^2 + N}}{2G^{(10)}\hbar m_s}$
- In LST ($\chi = 0$) ⇒ $E = T_H S$ ⇒ $\mathcal{F} = E - TS = 0$

- Thermodynamics

$$M \sim r_0^2$$

- Temperature $$T_H = \frac{\hbar}{2\pi\sqrt{\chi r_0^2 + \frac{N}{m_s^2}}}$$

- Entropy $$S_{BH} = \frac{A_H}{4G^{(10)}\hbar} = \frac{V_5\, \pi^2 r_0^2\, \sqrt{\chi m_s^2 r_0^2 + N}}{2G^{(10)}\hbar m_s}$$

- In LST ($\chi = 0$) ⇨ $E = T_H S$ ⇨ $\mathcal{F} = E - TS = 0$

  $\downarrow$

  Hagedorn $\longrightarrow$ $\rho = e^{S(E)} \sim e^{E/T_H}$

- Metric in Painlevé coordinates

$$t \to \tau + g(r)$$

- Smooth at event horizon
- Proper time along the radial geodesic

metric stationary

$$ds^2 = -f(r)d\tau^2 - 2\sqrt{A(r)(1-f(r))}\,d\tau dr + A(r)dr^2 + A(r)r^2 d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2$$

- Radial null geodesic $\dot{r} = \dfrac{r \pm r_0}{\sqrt{\chi r^2 + \frac{N}{m_s^2}}}$

- Near horizon λ is blue-shifted ⇒ WKB

s-wave

+ ω << M

$$\text{Im } S = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} p_r \, dr = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^{p_r} dp'_r \, dr$$

- Radial null geodesic $\dot{r} = \frac{r \pm r_0}{\sqrt{\chi r^2 + \frac{N}{m_s^2}}}$

- Near horizon λ is blue-shifted ➡ WKB

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$$\dot{r} = + \left. \frac{dH}{dp_r} \right|_r$$

$$= \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} \int_M^{M-\omega} \frac{dH}{\dot{r}} dr = - \int_0^{\omega} d\omega \int_{r_{\mathrm{in}}}^{r_{\mathrm{out}}} \frac{dr}{\dot{r}}$$

Tunneling in LST

- Radial null geodesic $\dot{r} = \frac{r \pm r_0}{\sqrt{\chi r^2 + \frac{N}{m_s^2}}}$
- Near horizon λ is blue-shifted ⇨ WKB

s-wave

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$$\text{Im } S = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} p_r \, dr = \text{Im} \int_{r_{\text{in}}}^{r_{\text{out}}} \int_0^{p_r} dp'_r \, dr$$

$$\dot{r} = + \left. \frac{dH}{dp_r} \right|_r$$

Energy conservation ⇨ Back-reaction

$< r_{\text{in}}$

$$= \int_{r_{\text{in}}}^{r_{\text{out}}} \int_M^{M-\omega} \frac{dH}{\dot{r}} dr = - \int_0^{\omega} d\omega \int_{r_{\text{in}}}^{r_{\text{out}}} \frac{dr}{\dot{r}}$$

- Emission rate $\Gamma \sim e^{-2\,\mathrm{Im}\ S}$ ⇒

$$\begin{cases} \exp[-\frac{\pi}{\hbar}(\omega\sqrt{r_0^2+N} - \frac{\omega^2}{2\sqrt{r_0^2+N}} + \cdots)] & \text{if } \chi = 1 \text{ (NS5)} \\ \exp[-\frac{\pi}{\hbar}\omega\sqrt{N}] & \text{if } \chi = 0 \text{ (LST)}. \end{cases}$$

[O. Lorente-Espin, P. Talavera, "A Silence black hole: Hawking radiation at the Hagedorn temperature," JHEP **0804 (2008) 080. [arXiv:0710.3833 [hep-th]].]**

- Emission rate $\Gamma \sim e^{-2\,\mathrm{Im}\ S}$

Non-thermal spectrum

$$\begin{cases} \exp[-\frac{\pi}{\hbar}(\omega\sqrt{r_0^2+N} - \frac{\omega^2}{2\sqrt{r_0^2+N}} + \cdots)] & \text{if } \chi = 1 \text{ (NS5)} \\ \exp[-\frac{\pi}{\hbar}\omega\sqrt{N}] & \text{if } \chi = 0 \text{ (LST)} \end{cases}$$

$$e^{-\omega/T_H} \longrightarrow \frac{\hbar m_s}{2\pi\sqrt{N}}$$ Hagedorn temperature

[O. Lorente-Espin, P. Talavera, “A Silence black hole: Hawking radiation at the Hagedorn temperature,” JHEP **0804 (2008) 080. [arXiv:0710.3833 [hep-th]].]**

Outline

- Scalar field action

$$S=\frac{1}{2\kappa_{10}^{2}}\int_M d^{10}x\sqrt{-g}\left(R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{12}e^{-\Phi}H_{(3)}^{2}\right)$$

- s-wave
- Eigenstates of momentum parallel to the NS5-brane = 0
- Near horizon limit

Reduction to 2-dimensional r-t sector

$$ds_{eff}^{2}=-f(r)\,dt^{2}+\frac{A(r)}{f(r)}\,dr^{2}$$

➢ (1+1)-dimensional massless scalar field $\phi(t,r)$

$$\Box\phi = 0$$

EOM

$$-A(r)\frac{\partial^2}{\partial t^2}\phi(t,r) + \frac{f(r)}{r^3}\frac{\partial}{\partial r}\left[r^3 f(r)\frac{\partial}{\partial r}\phi(t,r)\right] = 0$$

Complex path method

[K. Srinivasan, T. Padmanabhan, "Particle production and complex path analysis," Phys. Rev. **D60 (1999) 024007. [gr-qc/9812028].]**

1) Ansatz solution

$$\phi(t,r) \sim e^{\frac{i}{\hbar}S(t,r)}$$

2) Expansion of the action

$$S(t,r) = S_0(t,r) + \left(\frac{\hbar}{i}\right)S_1(t,r) + \left(\frac{\hbar}{i}\right)^2 S_2(t,r) + \cdots$$

3) Hamilton-Jacobi (leading order)

$$-A(r)\left(\frac{\partial S_0(t,r)}{\partial t}\right)^2 + f(r)^2\left(\frac{\partial S_0(t,r)}{\partial r}\right)^2 = 0$$

Solution

$$S_0(r_2,t_2;r_1,t_1) = -\omega(t_2-t_1) \pm \omega \int_{r_1}^{r_2} \frac{\sqrt{A(r)}}{f(r)}\, dr$$

$$A(r) = \chi + \frac{N}{m_s^2 r^2}$$

$$f(r) = 1 - \frac{r_0^2}{r^2}$$

[O. Lorente-Espin, "Some considerations about NS5 and LST Hawking radiation," Phys.Lett. **B703 (2011) 627-632. [arXiv:1107.0713 [hep-th]].]**

Complex path method

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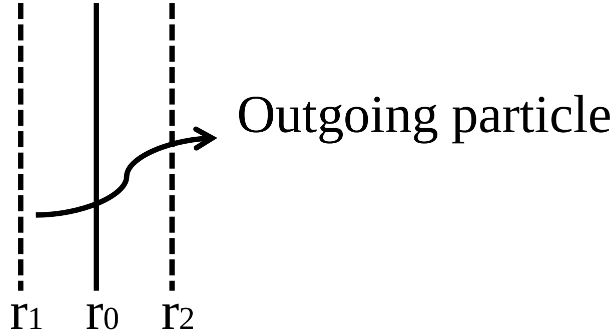
Complex integration around the pole r_0



Spatial emission action

$$S_0^e = \frac{i\pi\omega}{2} r_0 \sqrt{A(r_0)}$$

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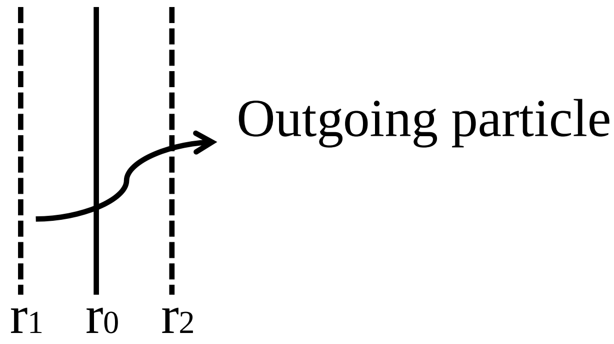


Spatial emission action

$$S_0^e = \frac{i\pi\omega}{2} r_0 \sqrt{A(r_0)} \Rightarrow S_0^e = \frac{i\pi}{2}\omega\sqrt{\chi\left(r_0^2-\omega\right)+N}$$

Back-reaction $r_0^2 \to r_0^2 - \omega$

- Emission rate

1) Semi-classical propagator:

$$K(r_2, t_2; r_1, t_1) = N \exp\left[\frac{i}{\hbar} S_0(r_2, t_2; r_1, t_1)\right]$$

2) Probability: $P = |K(r_2, t_2; r_1, t_1)|^2$

3) Emission probability at low energies

$$P_e \sim \begin{cases} \exp[-\frac{\pi}{\hbar}(\omega\sqrt{r_0^2 + N} - \frac{\omega^2}{2\sqrt{r_0^2+N}} + \cdots)] & \text{if } \chi = 1 \text{ (NS5)} \\ \exp[-\frac{\pi}{\hbar}\omega\sqrt{N}] & \text{if } \chi = 0 \text{ (LST)}. \end{cases}$$

1

$$P_e = e^{-\beta\omega} \quad \text{with} \quad \beta = \frac{2\pi}{\kappa}$$

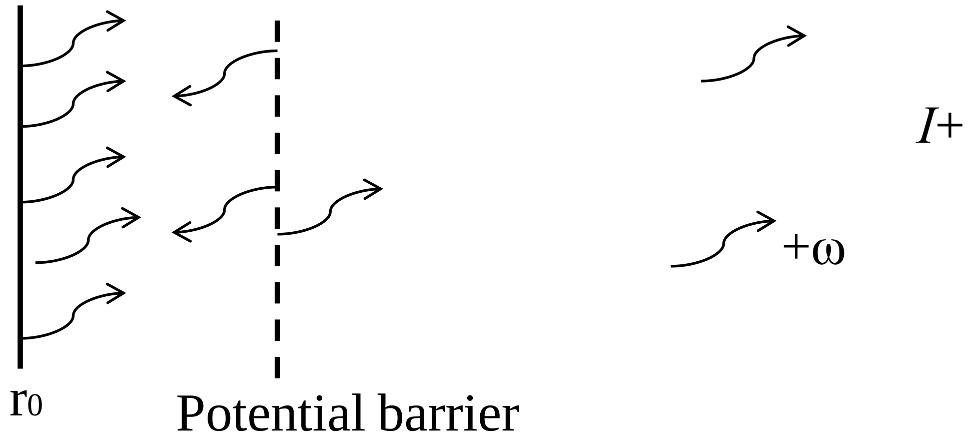
From Back-reaction we identify an effective temperature

$$\tilde{T} = \tilde{\beta}^{-1} = \frac{\hbar}{2\pi\sqrt{\chi(r_0^2 - \omega) + \frac{N}{m_s^2}}}$$ and verify that $$P_e = e^{-\omega/\tilde{T}}$$

Deviation from thermality ⇒ $$\frac{\tilde{T}}{T_H} = \left(\sqrt{1 - \frac{\chi\omega}{\chi r_0^2 + \frac{N}{m_s^2}}}\right)^{-1}$$

2 Greybody factor $\Gamma_\omega \longrightarrow \sigma = \frac{|F_h|}{|F_\infty|} = 1$ LST

$$< N_\omega >= \frac{\Gamma_\omega}{e^{2\pi\frac{\omega}{\kappa}} - 1}$$ Deviation from pure Planckian spectrum



3 Fermions $$\left[\gamma^a e_a^\mu (\partial_\mu + \Gamma_\mu) + m\right] \Psi = 0$$

Dirac equation in the r-t metric sector

$$\left[\gamma^0 \frac{1}{f(r)^{\frac{1}{2}}}\,\partial_t + \gamma^1 \left(\frac{f(r)'}{4A(r)^{\frac{1}{2}} f(r)^{\frac{1}{2}}} + \left(\frac{f(r)}{A(r)} \right)^{\frac{1}{2}} \partial_r \right) + m \right] \Psi = 0$$

Spin-up fermion field ➡ $$\Psi_\uparrow = \begin{pmatrix} \hat{A}(t,r)\xi_\uparrow \\ 0 \\ 0 \\ D(t,r)\xi_\uparrow \end{pmatrix} \exp\left[\frac{i}{\hbar} S_\uparrow(t,r)\right]$$

3

- Equation system

$$\left(-\frac{1}{f(r)^{\frac{1}{2}}}\,\partial_t S_\uparrow(t,r)+m\right)\hat{A}(t,r)-\left(\frac{f(r)}{A(r)}\right)^{\frac{1}{2}}\,\partial_r S_\uparrow(t,r)D(t,r)=0$$

$$\left(\frac{f(r)}{A(r)}\right)^{\frac{1}{2}}\,\partial_r S_\uparrow(t,r)\hat{A}(t,r)+\left(\frac{1}{f(r)^{\frac{1}{2}}}\,\partial_t S_\uparrow(t,r)+m\right)D(t,r)=0$$

- for non-vanishing values of the functions

$$\begin{vmatrix} -\frac{1}{f(r)^{\frac{1}{2}}}\,\partial_t S_\uparrow(t,r)+m & -\left(\frac{f(r)}{A(r)}\right)^{\frac{1}{2}}\,\partial_r S_\uparrow(t,r) \\ \left(\frac{f(r)}{A(r)}\right)^{\frac{1}{2}}\,\partial_r S_\uparrow(t,r) & \frac{1}{f(r)^{\frac{1}{2}}}\,\partial_t S_\uparrow(t,r)+m \end{vmatrix}=0$$

3

- Expanding the action

$$S_{\uparrow}(t,r) = S_{0\uparrow}(t,r) + \left(\frac{\hbar}{i}\right) S_{1\uparrow}(t,r) + \left(\frac{\hbar}{i}\right)^2 S_{2\uparrow}(t,r) + \dots$$

- At leading order

$$-A(r)\left(\partial_t S_{0\uparrow}(t,r)\right)^2 + f(r)^2\left(\partial_r S_{0\uparrow}(t,r)\right)^2 + A(r)f(r)m^2 = 0$$

- Solution $$S_{0\uparrow}(t,r) = \omega t \pm \omega \int_{r_{in}}^{r_{out}} \frac{\sqrt{A(r)}}{f(r)}\, dr$$

- LST $$P_e \sim \exp\left[-\frac{\pi}{\hbar}\omega\sqrt{N}\right]$$ and $$T_H = \frac{\hbar m_s}{2\pi\sqrt{N}}$$

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- High energy thermodynamics of LST

↓

Hagedorn density of states $\rho(E) = e^{S(E)} \sim e^{\beta_H E}$

Thermodynamic relations

- High energy thermodynamics of LST

free energy vanish

Hagedorn density of states $\rho(E) = e^{S(E)} \sim e^{\beta_H E}$

Hagedorn temperature

$$T_H = \frac{\hbar m_s}{2\pi\sqrt{N}}$$

Mass-independent

- One loop correction

[D. Kutasov, D. A. Sahakyan, "Comments on the thermodynamics of little string theory," JHEP **0102 (2001) 021. [hep-th/0012258].]**

$$\rho(E) \sim E^{\alpha} e^{\beta_H E} \left[1 + O\left(\frac{1}{E}\right)\right]$$

↓ Entropy-energy relation

$$S(E) = \beta_H E + \alpha \log \frac{E}{\Lambda} + O\left(\frac{1}{E}\right)$$

T Hagedorn reached at finite energy

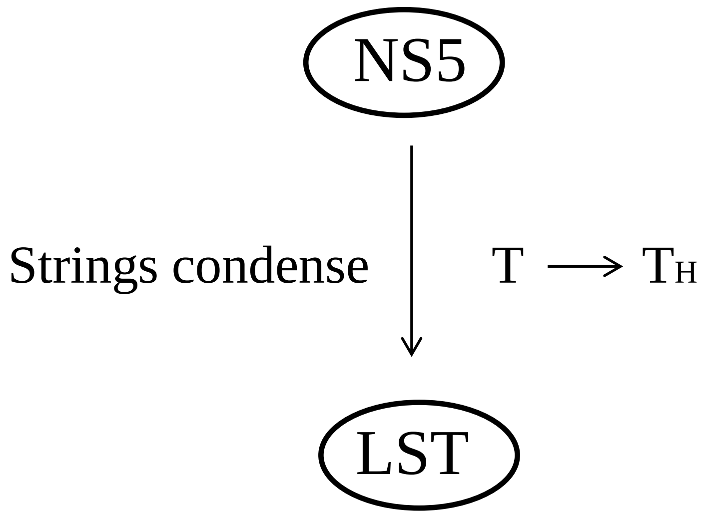
↓

Phase transition

with α < 0

- T > T Hagedorn
- C < 0
- Instability → tachyon

➢



One single state which radiates at the same temperature T_H, and contains NO information

[O. Lorente-Espin, P. Talavera, “A Silence black hole: Hawking radiation at the Hagedorn temperature,” JHEP **0804 (2008) 080. [arXiv:0710.3833 [hep-th]].]**

Thermodynamic relations

- Cluster decomposition

$$\ln\left|\Gamma(\omega_1+\omega_2)\right| - \ln\left|\Gamma(\omega_1)\Gamma(\omega_2)\right| = \begin{cases} \dfrac{\omega_1\omega_2}{2\sqrt{N+r_0^2}} & \text{NS5} \\ 0 & \text{LST} \end{cases}$$

NS5 ➡ correlated emission ⇄ recovery of information?

LST ➡ uncorrelated emission behind the horizon ⇆ information remains hidden

[O. Lorente-Espin, "Some considerations about NS5 and LST Hawking radiation," Phys.Lett. **B703 (2011) 627-632. [arXiv:1107.0713 [hep-th]].]**

Outline

I. Introduction
II. Tunneling in black holes
III. Complex path method
IV. Thermodynamic relations
V. Conclusions

Conclusions

- Tunneling approach and Complex path method overcomes, semi-classically, the information loss paradox. In virtue of the non-thermal results, obtained by the imposition of energy conservation, one could establish correlations between the emitted particles.
- Complex path lead us to the same conclusions as the tunneling picture; however avoiding a heuristic picture and change of coordinates.
- NS5 shows a non-thermal behaviour, whereas LST black hole keeps its thermal profile.

Conclusions

- NS5 reaches the Hagedorn temperature at finite energy density, suffering a phase transition to LST.
- LST shows a Hagedorn density of states. It consists in one single state that emits thermally at constant Hagedorn temperature, being this independent of the black hole mass.

- Introduction of quantum scalar perturbations into the r-t sector of the metric, using dimensional arguments (in natural units):

$$\tilde{l}_P^{d-2} = \frac{\hbar G^{(d)}}{c^3} \Rightarrow [\tilde{l}_P^{d-2}] = [\hbar]$$

Perturbed metric

Dimensionless parameter

$$\hat{ds}^2 = -f(r)\left(1+\sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{-1} dt^2 + \frac{g(r)}{f(r)}\left(1+\sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right) dr^2$$

$$\Rightarrow \hat{\kappa} = \frac{f'(r)}{2\sqrt{g(r)}}\Big|_{r\to r_0}\left(1+\sum_i \xi_i \frac{\hbar^i}{r_0^{(d-2)i}}\right)^{-1} \Rightarrow \text{Effective temperature}$$

❑ Back-reaction as a quantum correction with

$$\xi_i = \left(\frac{r_0^{d-2}}{\hbar}\right)^i \frac{\omega^i}{r_0^{(d-3)i} - \omega^i}$$

Effective temperature: $$\hat{T} = T_H \left(1 + \sum_i \frac{\omega^i}{r_0^{(d-3)i} - \omega^i}\right)^{-1}$$

Entropy: S ≈ M/T_H + Vol($\boldsymbol{R}^5$) log (M)

String one-loop

[O. Lorente-Espin **arXiv:1204.5756 [hep-th]**]

Thank you

Helpful discussions with:

- Pere Talavera (Universitat Politècnica de Catalunya , UPC)