Identifying Conformal Gauge Theories (CGT)

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17.1.2011, University of Sussex

Overview

- ☆ BSM, Technicolor, Walking-TC $\Rightarrow$ study strongly coupled gauge theories (3)
- ☆ General remarks gauge theories - conformal window SUSY & non-SUSY (4)
- ☆ conformal gauge theories (CGT) -- observables? (1)
- ☆ observables in mass-deformed CGT (8)
  - hyperscaling laws from RG
  - mass scaling from Feynmann-Hellmann thm
  - another look at β-function from trace anomaly
  - trajectory mass & decay constants
  - remarks on S-parameter

Del Debbio & RZ
PRD'10 & arXiv:1009.2894

- ☆ Lattice results (3)

$$\Delta_{\bar{q}q} = 3 - \gamma_*$$
where γ_* mass anomalous dimension

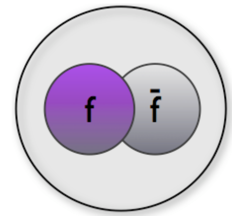
- ☆ Epilogue

Beyond the SM

centered around the Higgs mechanism of SSB
⇒ W,Z masses; technical hierarchy problem?*

Is the Higgs (object that unitarizes $W_L W_L$-scattering) fundamental or composite?

fundamental particle small width		composite particle large width

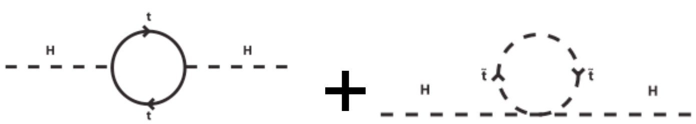


strong dynamics

Supersymmetry
opposite statistics partner

prototype

Technicolour
Higgs sector ⇒ Gauge theory



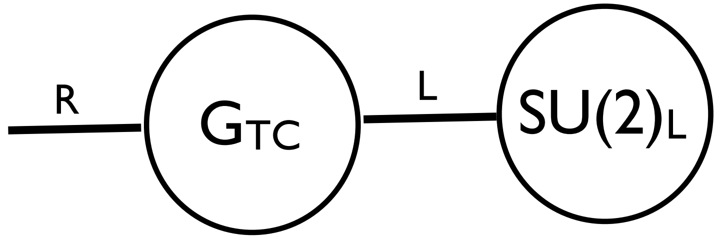
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_{TC}$

* Flavour sector, where real hierarchies are present, harder for model building

Technicolor

Susskind'79 Weinberg'79

- Higgs sector →strongly coupled gauge theory



moose notation

- χ-symmetry breaking: $\langle \bar{Q}_L Q_R \rangle \sim N_{TC} \Lambda_{TC}^3$

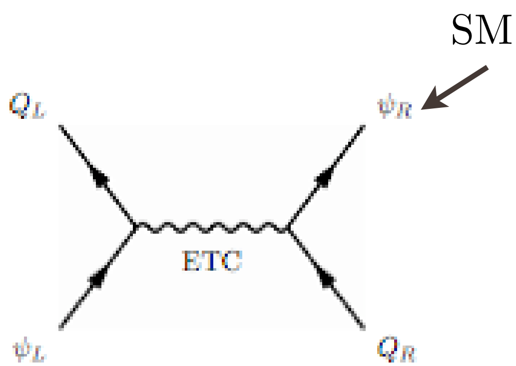
$\Lambda_{TC} \sim 4\pi F_T$

$F_T = v$

masses W,Z bosons through SSB (as in SM!)

- Fermion masses -> Extended TC $G_{\rm SM} \times G_{TC} \subset G_{ETC}$

Dimopoulus Susskind'79 Eichten Lane'80



$breaking: \; G_{ETC} \to G_{SM} \times G_{TC}$

$$\mathcal{L}^{\rm eff} = \alpha_{ab} \frac{\bar{Q} T^a Q \bar{\psi} T^b \psi}{\Lambda_{ETC}^2} + \beta_{ab} \frac{\bar{Q} T^a Q \bar{Q} T^b Q}{\Lambda_{ETC}^2} + \gamma_{ab} \frac{\bar{\psi} T^a \psi \bar{\psi} T^b \psi}{\Lambda_{ETC}^2} + \ldots$$

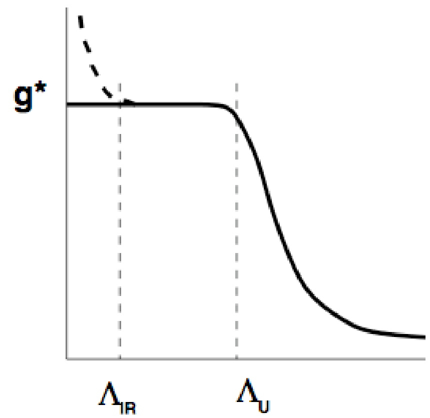
↑ *SM fermion masses* ↑ *FCNC*

* QCD breaks SU(2)L spontaneously and gives mass to W boson (orders of mag. too small)

Pheno improvement via Walking TC*

- Issues: 1. electroweak precision parameter S ~'95 Lep
  2. dynamical generation of fermion masses and FCNC Extended-TC

- 'Improvement' Walking-TC: almost reaches IR fixed-point



1. $|S_{WTC}| << |S_{TC}|$ 2. $\gamma_{mass}{}^{*}$ large

- Walking results enhancement of $\dfrac{\langle \bar{Q}Q \rangle_{TC}}{f^3_{\pi(TC)}}$
  No parametric definition (challenge)

⇒ *need to know more about strongly coupled near-conformal gauge theories ..*

* Also a scheme called Conformal Technicolor on the market (Luty'04)

Gauge theories

- what theorists can adjust:

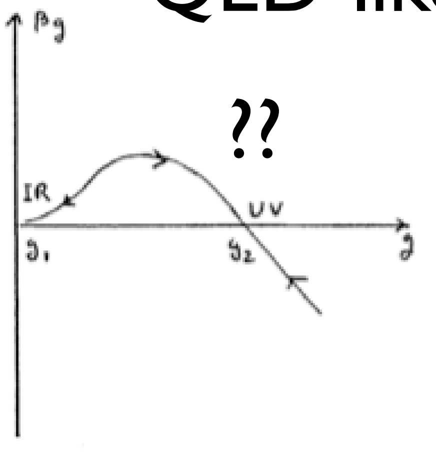
  

N_c N_F representation

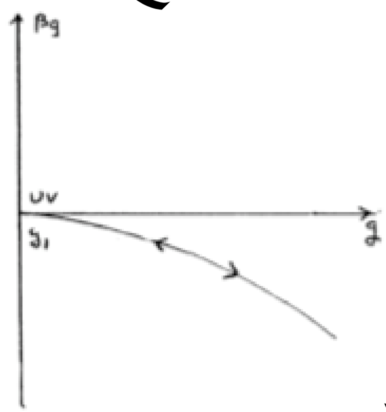
- one coupling theory:
  $g^* = 0$ either IR (QED-like) or UV (QCD-like asymptotic freedom) fixed-point

- focus AF-theories ($-\beta_0 < 0$) we know how to handle

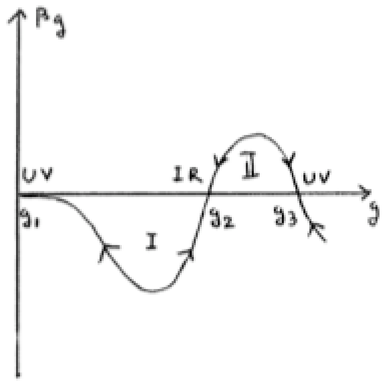
QED-like

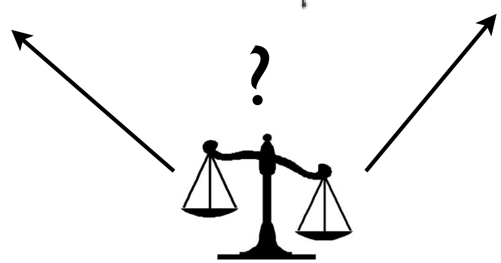


QCD-like



IR-conformal





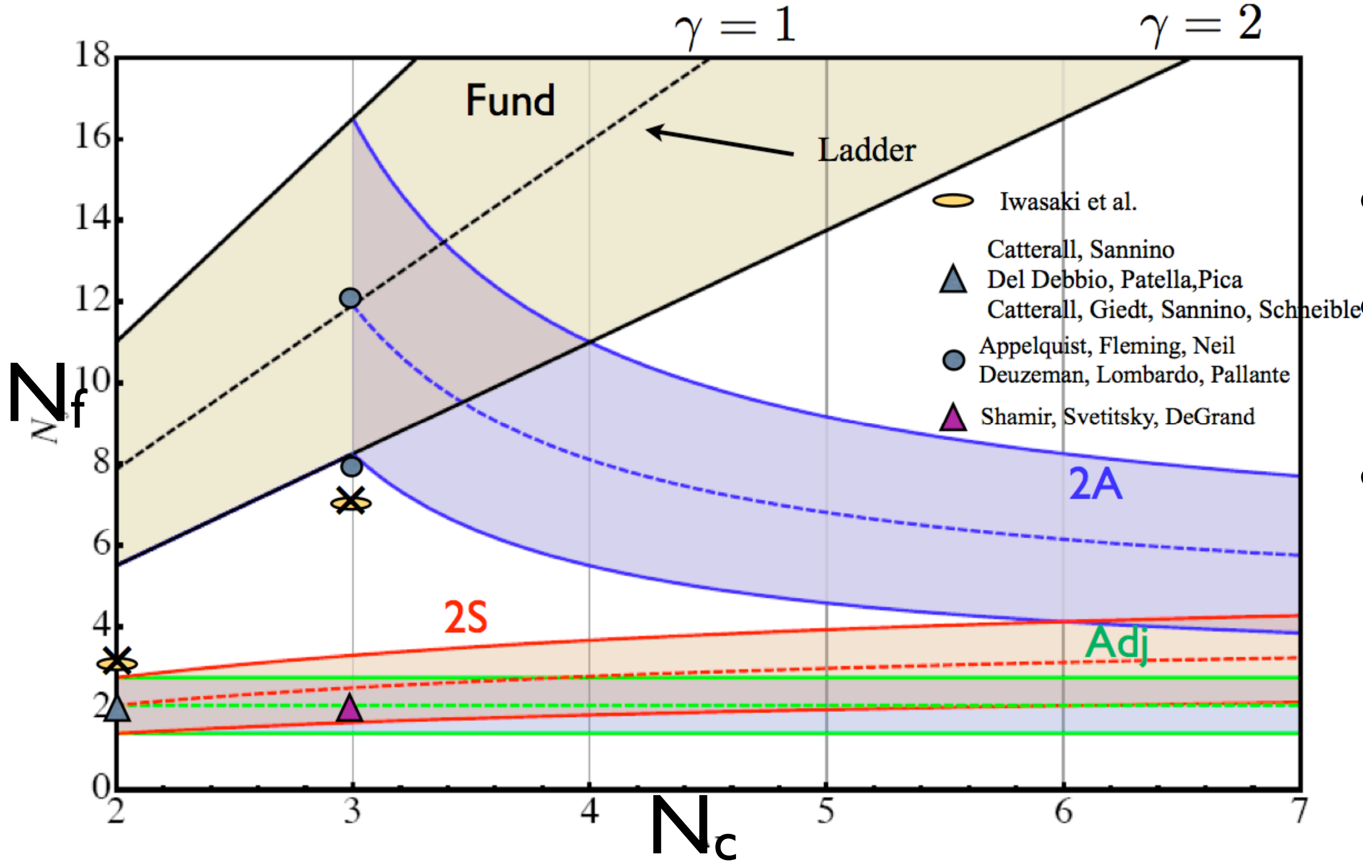
Facts non-SUSY Conformal Window

proof of principle

- ⭐ QCD chiral symmetry is broken (empirical) ⇒ not in CW!
- ⭐ Banks-Zaks'82 (Belavin-Migdal'76) perturbative IR fixed point (conformal)

If β_0 tuned small $\frac{\alpha_s^*}{2\pi} = \frac{\beta_0}{-\beta_1} \ll 1$ $\qquad \beta(\alpha_s) = -\beta_0 \frac{\alpha_S^2}{2\pi} - \beta_1 \frac{\alpha_S^3}{(2\pi)^2} + \ldots$

/ $\alpha_s^* \sim 0.02$



- upper line AF (ok)
- dashed Dyson-Schwinger $\Delta_{\bar{q}q} = 3 - \gamma \simeq 2$
- lower unitary bound $\Delta_{\bar{q}q} \geq 1$ via conjectured β-fct

SUSY Conformal Window

☆ Exact NSVZ'83 β-fct: $\beta(g) = -\frac{1}{16\pi^2}\frac{3t_2(A) - \sum_i t_2(i)(1-\gamma_i)}{1 - t_2(A)g^2/8\pi^2}.$

from $\beta = 0$ get γ^*

1. Unitarity bound on squark-bound state $\Delta_{QQ} = 2 - \gamma^* \geq 1 \;\Rightarrow \gamma^* \leq 1$

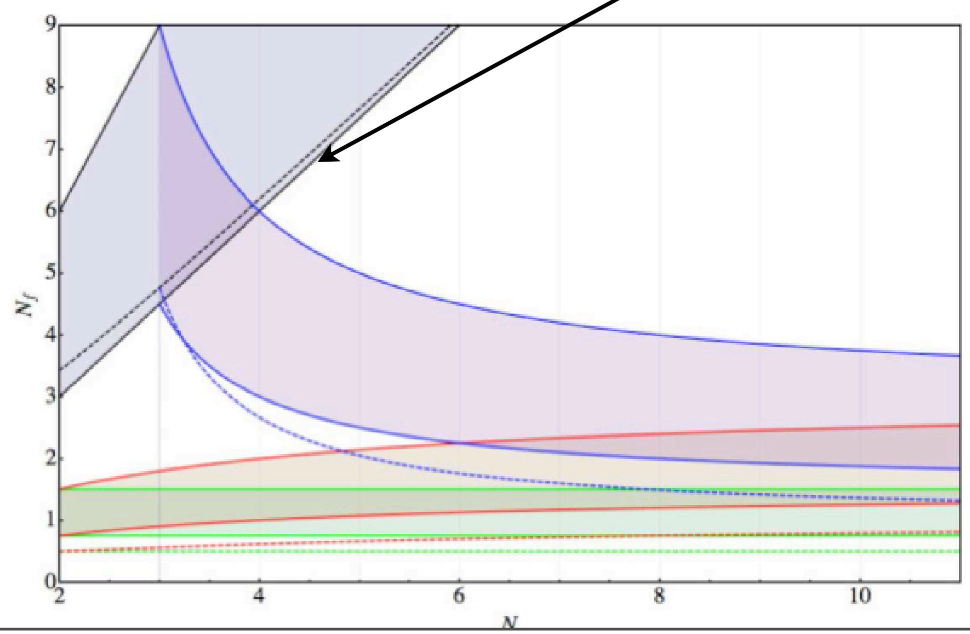
2. Electric magnetic duality $N^{dual} = N_F - N$

perturbative electric BZ-fixed point upper boundary (like QCD)

perturbative magnetic BZ-fixed point lower boundary !!

⇒ weak-strong coupling duality ⇒ exist strongly coupled CGT (also from γ^*)

lower bdry unitarity bound



Two objectives (almost repetition)

☆ AF gauge theories ≈ 1/2 non-CFT + 1/2 CFT (SUSY)

(N.B. only known CFT in 4D are GT, coheres with Coleman-Gross Thm)

size of conformal window?

☆ strong coupling -- value of γ_*

strongly coupled? size γ_*

- SUSY $\mathcal{N} = 1$ tells $\Delta_{\bar{q}q} = 3 - \gamma_* \geq 2$
- Dyson Schwinger eqn: chiral symmetry breaks $\Delta_{\bar{q}q} \simeq 2$
- unitarity bound (Mack'77) $\Delta_{\bar{q}q} \geq 1$

Is the unitarity bound ever reached?

1. SUSY its because of the squark $\Delta_{QQ} = 2 - \gamma$
2. DS-eqs. truncation -- ladder approximation ... NJL
3. N.B. Δ=1 free field (very strong force ....)

$\Rightarrow$ *we want answers* $\Rightarrow$ *lattice simulations*

Observables in a CFT?

Or how to identify a CFT

1. Observables: vanishing β-function & <O(x)O(0)> ~ $(x^2)^{-\Delta}$;$\Delta = d + \gamma_*$
2. Lattice computation finite m_{quark} (& volume anyway)

⇒ look mass-deformed conformal gauge theories (mCGT)*

$$\mathcal{L} = \mathcal{L}_{\rm CGT} - m\bar{q}q$$

* hardly related to 2D CFT mass deformation a part of algebra and 'therefore' integrability is maintained

Observables in mCGT

- Goal: analytic guidance for lattice (parametric laws)
- finite m_q quarks decouple ⇒ pure YM confines (string tension confirmed lattice)

  ⇒ hadronic spectrum ⇒beloved hadronic observables

signature of such a theory: each hadronic observable

$$\mathcal{O} \sim m^{\eta_{\mathcal{O}}} \, , \qquad \eta_{\mathcal{O}} > 0 \, , \eta = f(\gamma_*)$$

- Let's settle some notation:

$$\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}} \, , \quad \text{scaling} = \text{physical} + \text{anomalous dimension}$$

$$\gamma_m = -\gamma_{\bar{q}q} \, , \quad \text{denoted by } \gamma_* \text{ at fixed-point}$$

$$\Rightarrow \Delta_{\bar{q}q} = 3 - \gamma_*$$

Hyperscaling laws

Consider matrix element: $\mathcal{O}_{12}(g, \hat{m}, \mu) \equiv \langle \varphi_2 | \mathcal{O} | \varphi_1 \rangle$

physical states
no anomalous dim.

1. $\mathcal{O}_{12}(g, \hat{m}, \mu) = b^{-\gamma_{\mathcal{O}}} \mathcal{O}_{12}(g', \hat{m}', \mu')$,

$$g' = b^{y_g} g \quad \hat{m}' = b^{y_m} \hat{m} \, , \quad y_m = 1 + \gamma_* \, , \quad y_g < 0 \text{ (irrelevant)}$$

RG -transformation*
$\mu = b\mu'$

2. $\mathcal{O}_{12}(\hat{m}', \mu') = b^{-(d_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})} \mathcal{O}_{12}(\hat{m}', \mu)$

change
physical units

3. Choose b s.t. $\hat{m}' = 1$

Hyperscaling
relations

$$\Rightarrow \quad \mathcal{O}_{12}(\hat{m}, \mu) \sim (\hat{m})^{(\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m}$$

* From Weinberg-like RNG eqs on correlation functions (widely used in critical phenomena)

Applications: $\eta_{\mathcal{O}_{12}} = (\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m$

- vacuum condensates: $\langle \bar{q}q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$, $\langle G^2 \rangle \sim m^{\frac{4}{1+\gamma_*}}$ more later on...

- decay constants:
  $|\varphi\rangle = |H(\text{adronic})\rangle$

  N.B. ($\Delta_H = d_H = -1$ choice)

$\mathcal{O}$	def	$\langle 0\|\mathcal{O}\|J^{P(C)}(p)\rangle$	$J^{P(C)}$	$\Delta_{\mathcal{O}}$	$\eta_{G[F]}$
S	$\bar{q}q$	G_S	0^{++}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
S^a	$\bar{q}\lambda^a q$	G_{S^a}	0^+	$3-\gamma_*$	$(2-\gamma_*)/y_m$
P^a	$\bar{q}i\gamma_5 q$	G_{P^a}	0^-	$3-\gamma_*$	$(2-\gamma_*)/y_m$
V	$\bar{q}\gamma_\mu q$	$\epsilon_\mu(p)M_V F_V$	1^{--}	3	$1/y_m$
V^a	$\bar{q}\gamma_\mu\lambda^a q$	$\epsilon_\mu(p)M_V F_{V^a}$	1^-	3	$1/y_m$
A^a	$\bar{q}\gamma_\mu\gamma_5\lambda^a q$	$\epsilon_\mu(p)M_A F_{A^a}$	1^+	3	$1/y_m$
		$ip_\mu F_{P^a}$	0^-	3	$1/y_m$

- masses from **trace anomaly**: $\theta_\alpha{}^\alpha|^{q\neq 0}_{\text{on-shell}} = \frac{1}{2}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$

  Adler et al, Collins et al
  N.Nielsen '77 Fujikawa '81

  $\beta = 0 \quad \& \quad \langle H(p)|H(k)\rangle = 2E_p\delta^{(3)}(p-k) \Rightarrow$

$$2M_h^2 = N_f(1+\gamma_*)m\langle H|\bar{q}q|H\rangle$$

$\sim m^{\frac{2}{(1+\gamma_*)}}$

relation reminiscent GMOR-relation

...

☆ Summarizing:

scaling laws for entire spectrum, decay constants & condensates

No SSB of χ-symmetry breaking (no goldstone boson)

since condensate triggered by explicit χ-breaking

There is no chiral perturbation theory

☆ Credits (presentation focused last paper):

lowest mass state Miransky ’98

quark condensate (just stated) DeGrand’09

all lowest state results DelDebbio RZ’10 May (large time euclidian correlators)

all state results DelDebbio RZ’10 Sep

☆ A point that can be clarified:

$M_H \sim m^{1/(1+\gamma^*)}$ looks a bit like heavy quark physics

The definite signature is $f_{P(B\text{-meson})} \sim m^{-1/2}$ whereas $f_{P(mCGT)} \sim m^{(2-\gamma^*)/(1+\gamma^*)}$

Mass scaling without RG

Del Debbio, RZ Sep'10

Hellmann-Feynman-Thm

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial \hat{H}(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle$$

idea: $\frac{\partial \langle \psi(\lambda) | \psi(\lambda) \rangle}{\partial \lambda} = 0$

- applied to our case: $m \frac{\partial M_h^2}{\partial m} = N_f m \langle H | \bar{q} q | H \rangle$

- combined with GMOR-like .. $m \frac{\partial M_H}{\partial m} = \frac{1}{1+\gamma_*} M_H$

  $2M_H^2 = N_f (1+\gamma_*) m \langle H | \bar{q} q | H \rangle$

$$M_H \sim m^{\frac{1}{1+\gamma_*}}$$

scaling law
without using RG!

Generalized Banks-Casher relation

- Banks & Casher '80 a la Leutwyler & Smilga 92':

Green's function: $\langle q(x)\bar{q}(y)\rangle = \sum_n \frac{u_n(x)u_n^\dagger(y)}{m-i\lambda_n}$, where $\not{D} u_n = \lambda_n u_n$

$$\langle \bar{q}q\rangle_V = \frac{1}{V}\int dx\, \langle \bar{q}(x)q(x)\rangle = -\frac{2m}{V}\sum_{\lambda_n>0}\frac{1}{m^2+\lambda_n^2} \overset{V\to\infty}{=} -2m\int_0^\infty d\lambda \frac{\rho(\lambda)}{m^2+\lambda^2}$$

- UV-divergences later -- focus IR-physics

$$\langle \bar{q}q\rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}}$$

- QCD : $\eta_{\bar{q}q} = 0 \Rightarrow \rho(0) = -\pi\langle\bar{q}q\rangle$

mCGT: another way to measure anomalous dimension

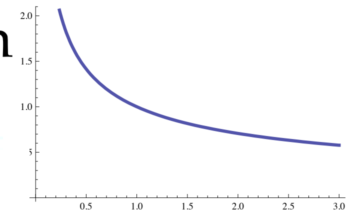
Banks, Casher'80

DeGrand'09
DelDebbio RZ'10 May

Heuristic look

- Deconstruct the continuous spectrum of a two point function Stephanov'07

  Infinite sum of adjusted particles can mimick continuous spectrum



$$\bar{q}q(x) \sim \sum_n f_n \varphi_n(x)\,; \qquad \langle \varphi_n | \bar{q}q | 0 \rangle \sim f_n\,, \qquad \begin{cases} f_n^2 = \delta^2 \, (M_n^2)^{\Delta_{\bar{q}q}-2} \\ M_n^2 = n\delta^2 \end{cases}$$

- Adding mass term looks like tadpole.

  ⇒ find new minimum -- add M_n to potential

$$\mathcal{L} = -m \sum_n f_n \varphi_n - 1/2 \sum_n M_n^2 \varphi_n^2$$

Delgado, Espinoso, Quiros'07

- Solve $\quad m f_n + M_n^2 \varphi_n = 0 \quad \Rightarrow \quad \langle \varphi_n \rangle = -m f_n / M_n^2$ and reinsert:

$$\langle \bar{q}q \rangle \sim \sum_n f_n \langle \varphi_n \rangle = -m \sum_n \frac{f_n^2}{M_n^2} \overset{\delta \to 0}{\to} -m \int_{\Lambda_{\rm IR}^2}^{\Lambda_{\rm UV}^2} s^{\Delta_{\bar{q}q}-3} ds$$

  - Λ_{UV}: $\Delta_{qq} = 3$ find quadratic divergence known from Leutwyler-Smilga rep.

  - Λ_{IR}: 1) $\Lambda_{IR} \sim M_H \sim m^{1/(1+\gamma)}$ or use $(M_{dyn})^{\Delta_{qq}} \sim$ <qq> generalizing Politzer OPE.
    and confirm $\eta_{qq} = \Delta_{qq}/(1+\gamma)$!

A few additional topics

Another look at the β-function

- Consider the again the trace (scale) anomaly:

$$\theta_\alpha{}^\alpha|_{\mathrm{on-shell}} = \frac{1}{2g}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$$

- Evaluate it on any hadronic state |H> and solve for β:

$$\beta = \frac{A_H + \gamma_m B_H}{G_H}$$

$$A_H = 2M_H^2 - mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$B_H = mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$G_H = \langle H|G^2|H\rangle\,.$$

- Ratios of A_H/G_H & B_H/G_H independent

- Form β-function close to NSVZ β (for N=1 SUSY gauge theories)

$$\beta(g) = -\frac{1}{16\pi^2}\frac{3t_2(A) - \sum_i t_2(i)(1-\gamma_i)}{1 - t_2(A)g^2/8\pi^2}.$$

Mass & decay constant trajectory

- At large-N_c neglect width → $g_{H_n} \equiv \langle 0|\mathcal{O}|H_n\rangle$ (decay constant)

$$\Delta(q^2) \sim \int_x e^{ixq}\langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle = \sum_n \frac{|g_{H_n}|^2}{q^2+M_{H_n}^2}$$

- In limit m → 0 (scale invariant correlator)

$$\Delta(q^2) = \int_0^\infty \frac{ds\, s^{1-\gamma_*}}{q^2+s} + \text{s.t} \propto (q^2)^{1-\gamma_*}$$

- Solution are given by:

$$M_{H_n}^2 \sim \alpha_n m^{\frac{2}{1+\gamma_*}} \;, \quad g_{H_n}^2 \sim \alpha_n'(\alpha_n)^{1-\gamma_*} m^{\frac{2(2-\gamma_*)}{1+\gamma_*}}$$

where α_n arbitrary function (corresponds freedom change of variables in ∫)

- QCD expect $\alpha_n \sim n$ (linear radial Regge-trajectory) (few more words)

- For those who know: resembles deconstruction Stephanov'07
  difference physical interpretation of spacing due to scaling spectrum

remarks S-parameter

Analytical guidance S-parameter: $S = 4\pi\Pi_{V-A}(0) - \text{pion pole}$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\delta_{ab}\Pi_{V-A}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\left(V_a^\mu(x)V_b^\nu(0) - (V \leftrightarrow A)\right)|0\rangle$$

$$\Pi_{V-A}(q^2) \simeq \frac{f_V^2}{m_V^2 - q^2} - \frac{f_A^2}{m_A^2 - q^2} - \frac{f_P^2}{m_P^2 - q^2} + \dots .$$

$$\Pi_{V-A}^{\mathrm{W-TC}}(0) \sim O(m^{-1})$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(0) \sim O(m^0)$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(q^2) \sim \frac{m^{2/y_m}}{q^2} \quad \text{for } -q^2 \gg (\Lambda_U)^2$$

Sannino'10 free theory

module (conspiracy) cancellations improve on ...

$\Rightarrow$ lattice determination coming soon (already some market)

Lattice simulations (generic remarks):

- Ca 7(2) groups UK Swansea/Edbgh), Finnland, Holland, Lin & Onugi
  USA (LSD,deGRand, Knuti,Fodor, Caterall & Sannino .....)

- IR mass is relevant; coupling irrelevant *(principal no tuning necessary)*

- Measure β-fct (stepsize scaling)
  problem: $m \neq 0$ so not fixed-pt β-fct not physical
  measuring zero (cancellations)

- measure enhancement <QQ>/f_π^3 (LSD) parametric control?

- It would seem longterm mass/decay constant parametric scaling should help

Summary of results:

- ☆ See reasonable results scaling in $0^{-+}, 1^{--}$ channels
  0^{++} more noisy (as usual)

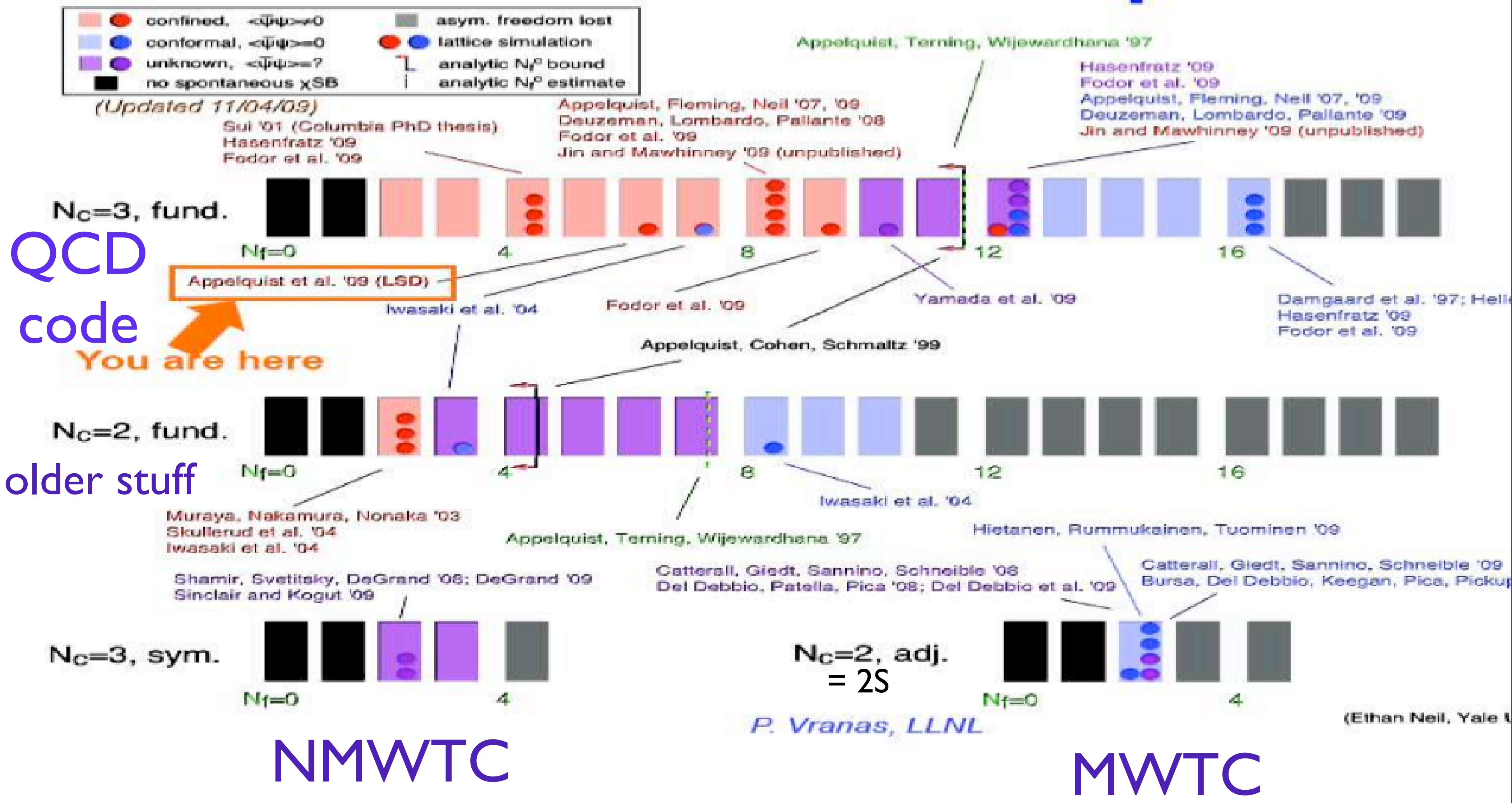
- ☆ typically $\gamma^{*} \sim 0.4(3?)$ not too large (upper bound difficult)

- ☆ so-called MinimalWTC looks conformal ⇒ conformal-TC model building

- ☆ Why & What is simulated: next slide ....

Not Quite the

Current landscape



Epilogue

- People accept it will take more time to establish CW than foreseen

- Major goals:
  1) size of conformal window
  2) how large anomalous dimension $1 \leq \Delta_{qq}$ .....
     will unitary-bound be reached? Is $\Delta_{qq} \cong 2$ border DS?
  3) measurement S-parameter

- Conformal window studies open up theory space model building CTC...

- More theoretical questions: trajectories etc

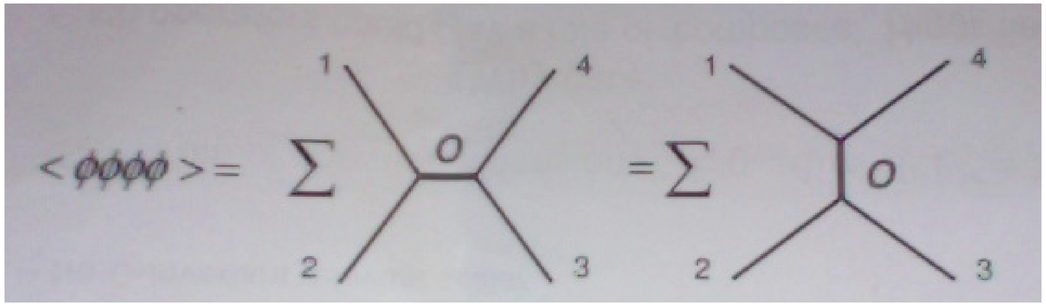
- Maybe by understanding mCGT better we learn sthg about QCD

Thanks for your attention!

Backup slides ...

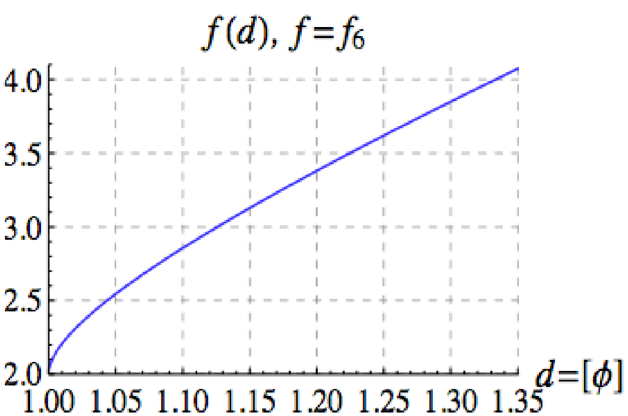
Addendum (bounds scaling dimension)

- assume add L = mqq (N.B. not a scalar under global flavour symmetry!)
- using bootstrap ('associative' OPE on 4pt function) possible to obtain <u>upper</u>-bound on scaling dimension Δ of lowest operator in OPE



non-singlet

Rattazzi, Rychkov Tonni & Vichi'08



1.35 still rather close to unitarity bound

singlet $\Delta \leq 4$

allows for Δ_{qq} to be:

Rattazzi, Rychkov & Vichi '10

G	$U(1) \equiv SO(2)$	$SO(3)$	$SO(4)$	$SU(2)$	$SU(3)$
d_*	1.063 $(k=2)$ 1.12 $(k=4)$	1.032 $(k=2)$ 1.08 $(k=4)$	1.017 $(k=2)$ 1.06 $(k=4)$	1.016 $(k=2)$	1.003 $(k=2)$

very close to unitarity bound!

good news for Luty's conformal TC