PERTURBATIVE QCD CALCULATIONS FOR JET SUBSTRUCTURE

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2nd June 2014

Outline

- A very brief introduction to jets
- The jet mass distribution
- Grooming and Tagging
- Analytic calculations for jet substructure tools

Quarks, gluons and jets

- We usually discuss high-energy interactions in terms of quarks and gluons

- This is neither what we collide nor what we observe in the detectors

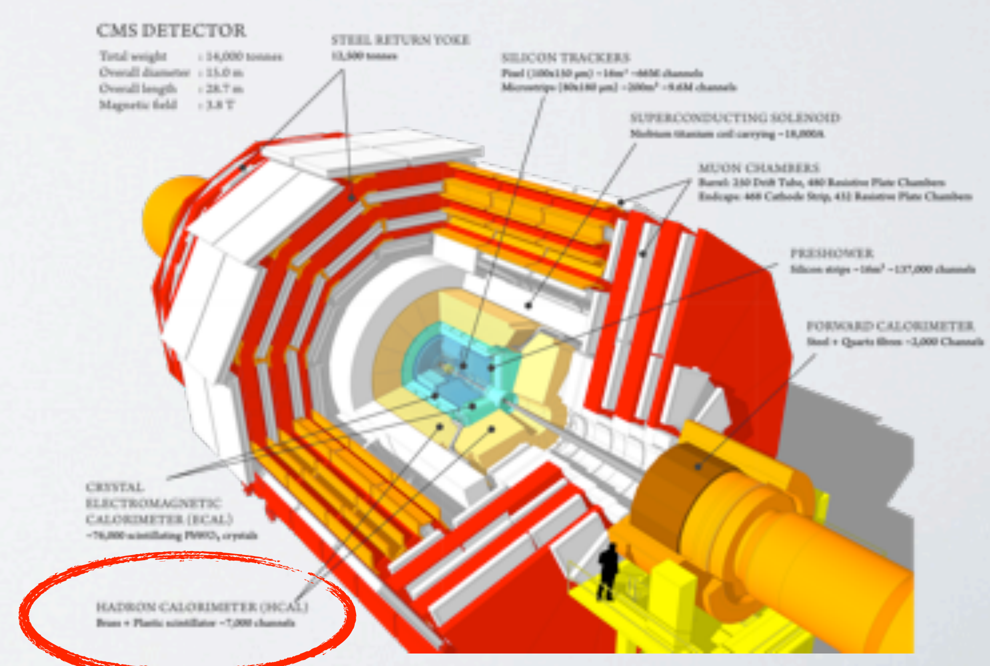
- Initial state: QCD factorisation theorems allow us to separate

    - long-distance physics (proton)
    - from hard interactions (partons)

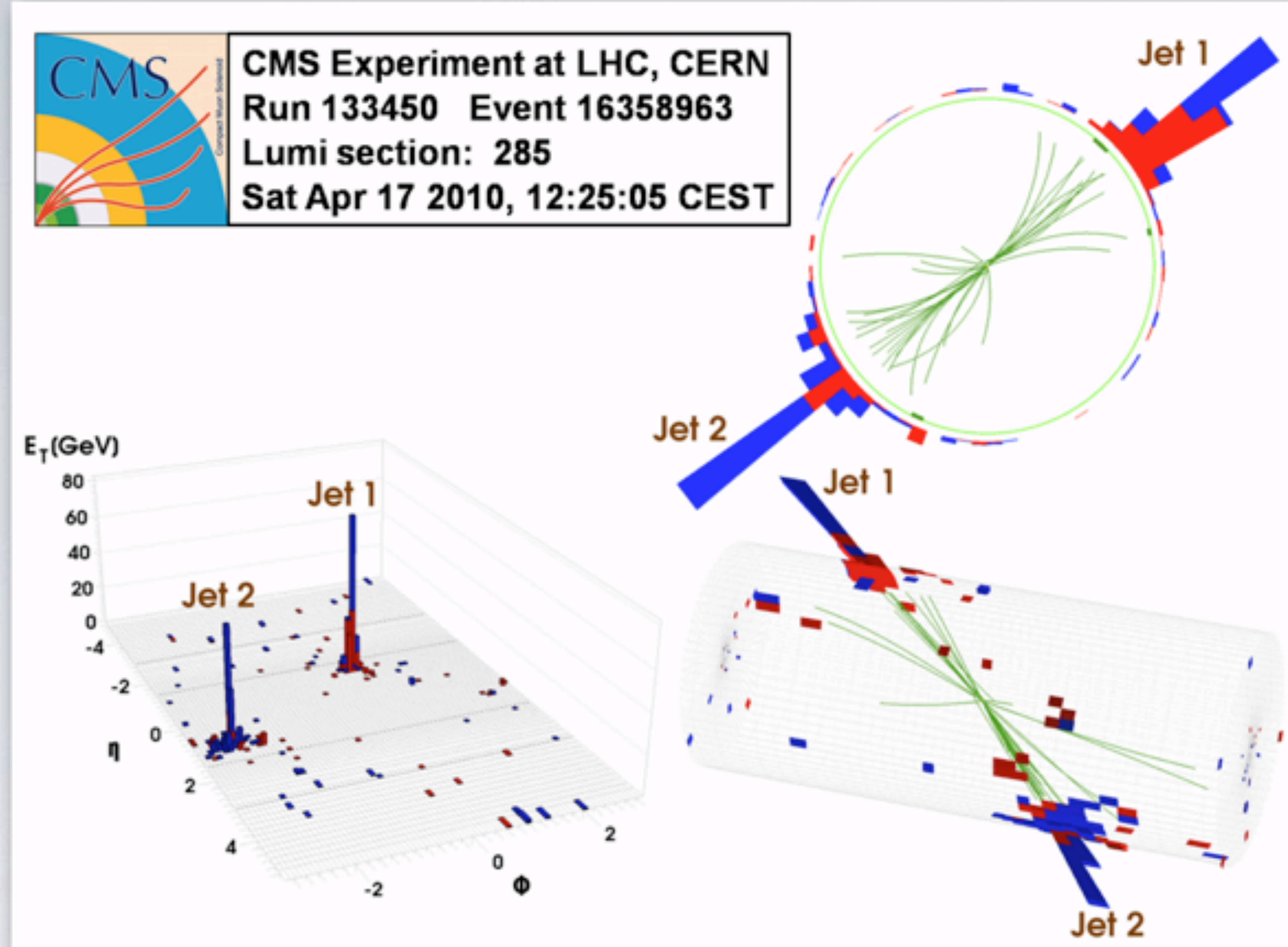
- Final states: QCD splittings are enhanced in the collinear region so we are left with collimated sprays of hadrons: jets

New physics searches and jets

- LHC energy (10^4 GeV) $\gg$ electro-weak scale (10^2 GeV)
- EW-scale particles (new physics, Z/W/H/top) are abundantly produced with a large boost
- their decay-products are then collimated
- if they decay into hadrons, we are left with collimated sprays of hadrons: jets
- jets are ubiquitous in collider phenomenology

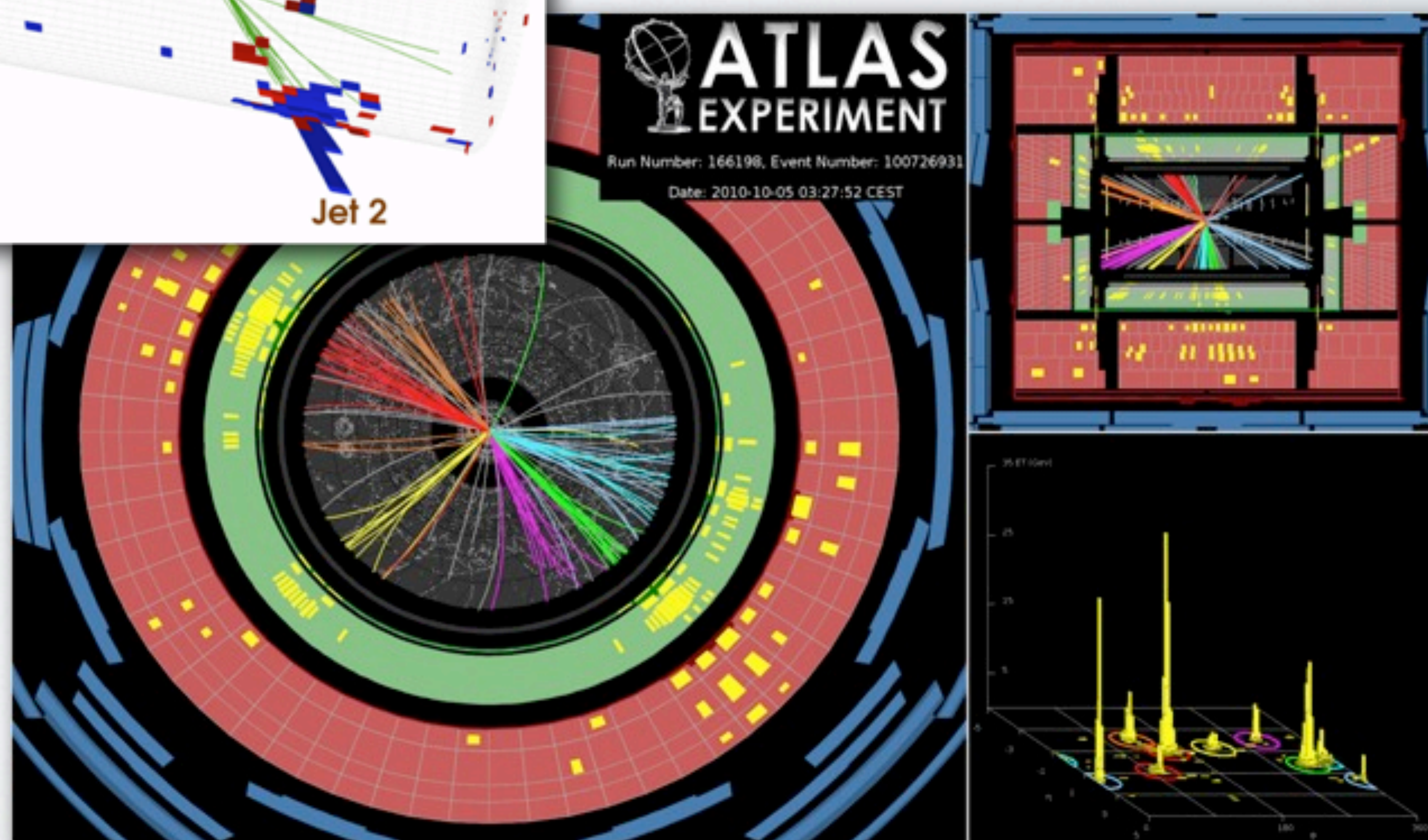


source CMS

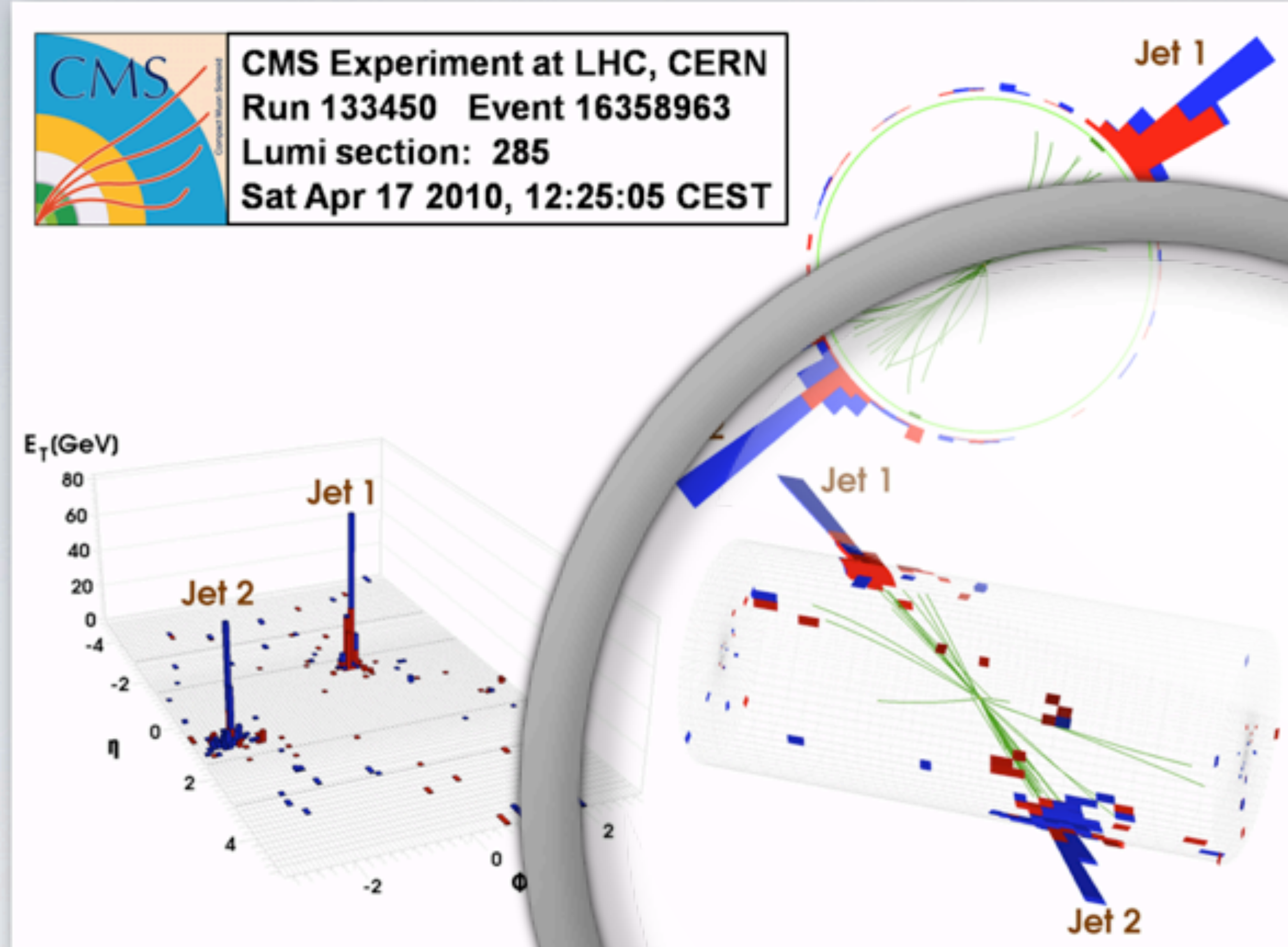


JETS

Collimated, energetic bunches of particles



Boundary between theory / experiment



JETS

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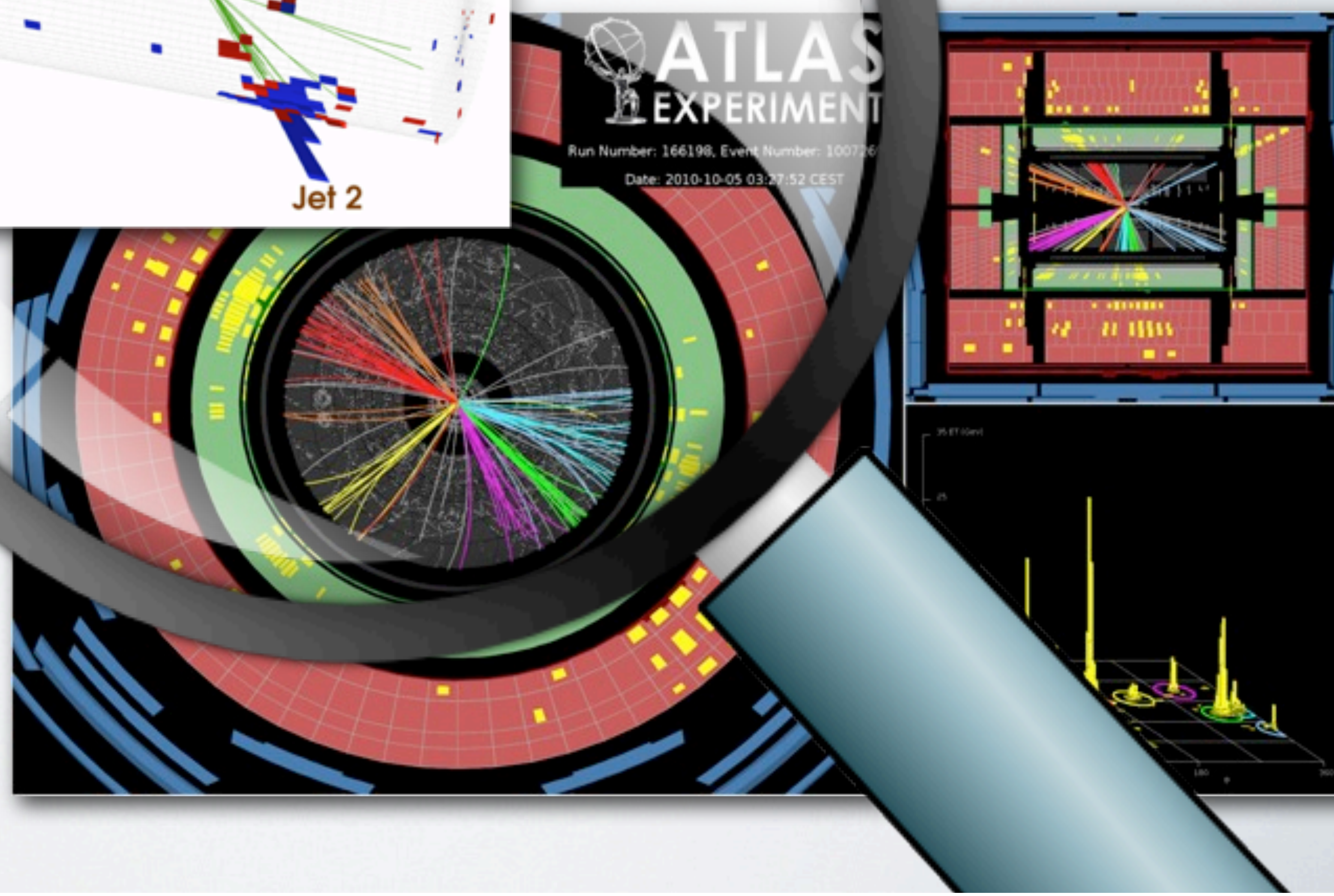
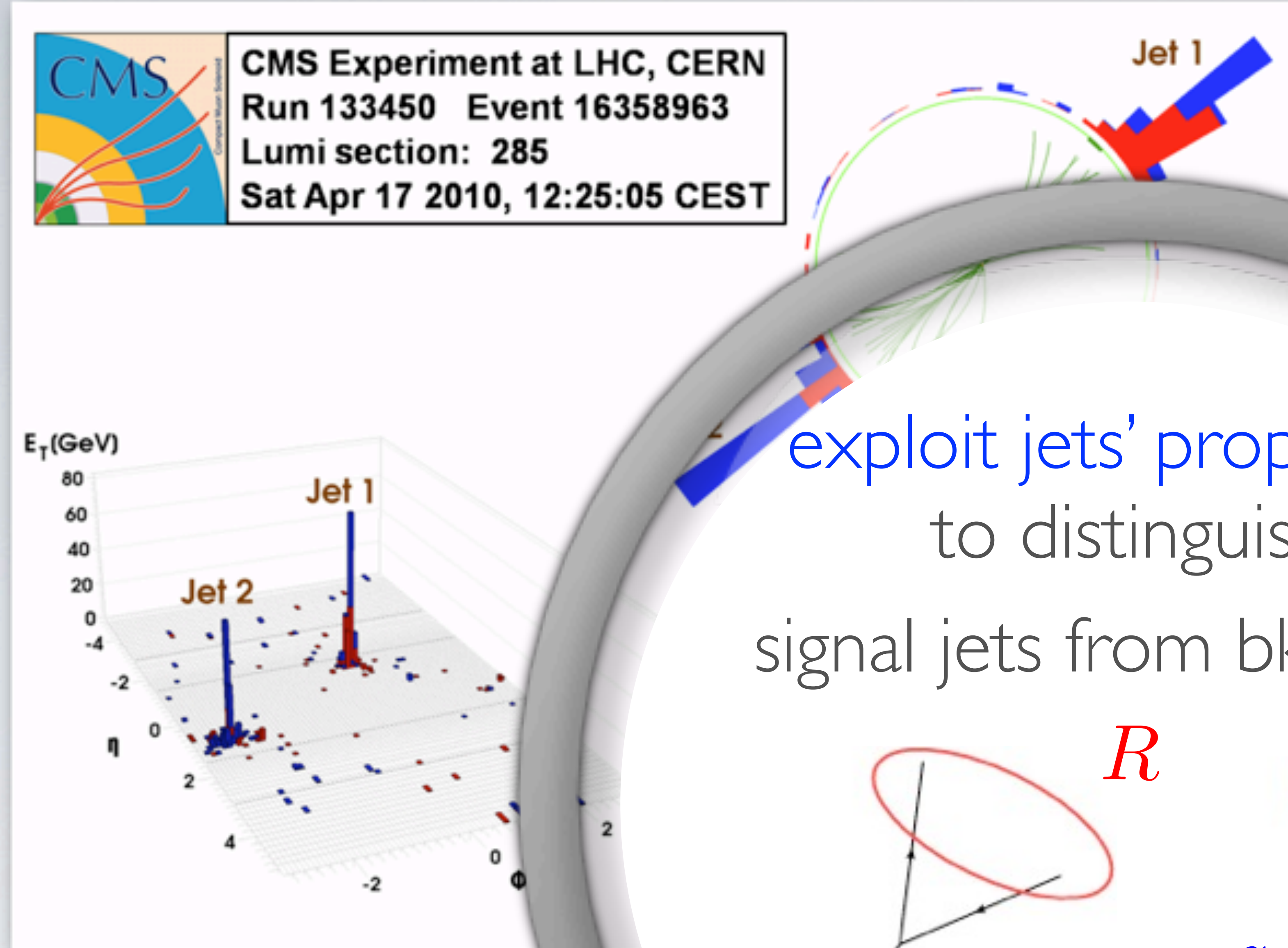
bu…es of particles



Boundary betwee…
theory / experime…

We want to look
inside a jet



JETS

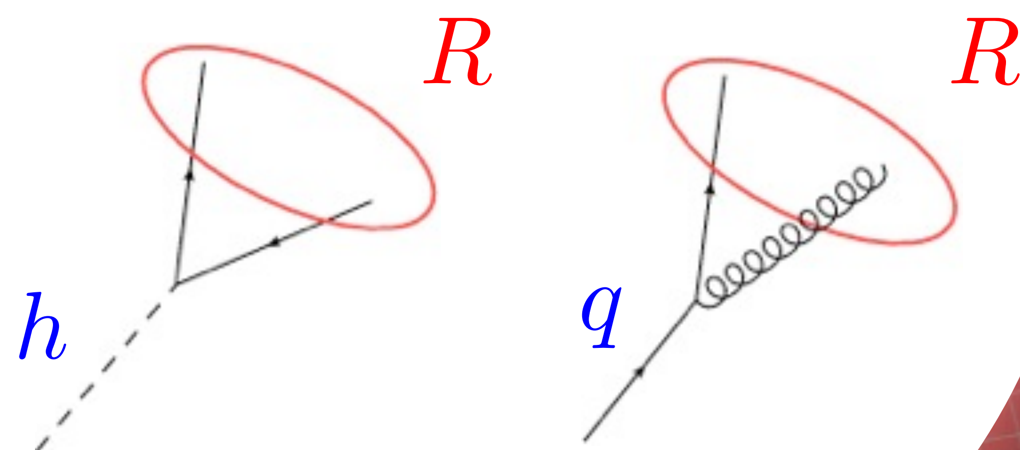
llimated, energetic

es of particles

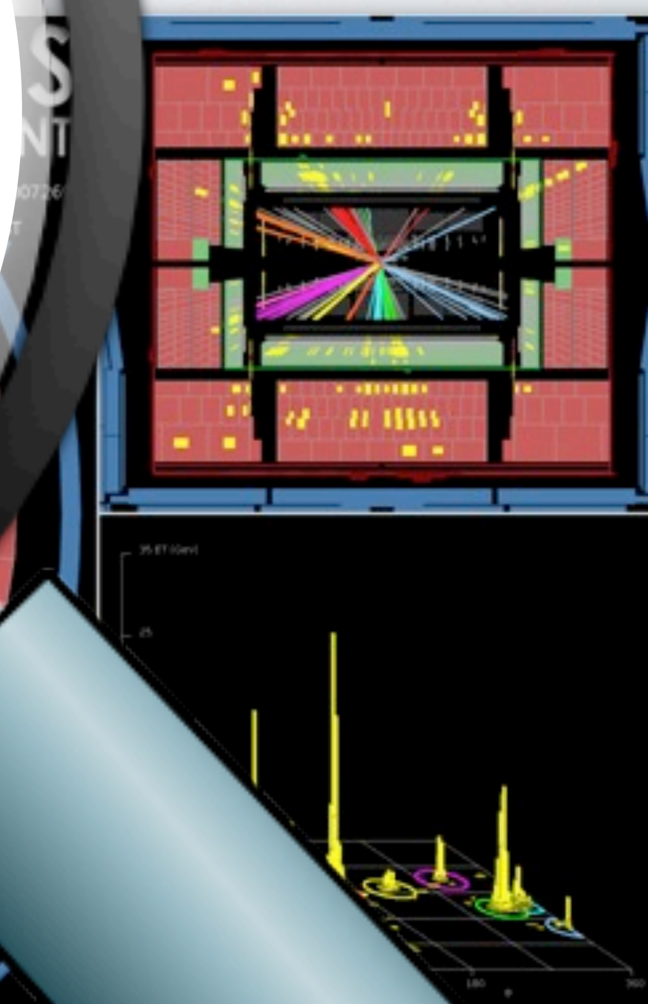
exploit jets' properties

to distinguish

signal jets from bkgd jets



$$p_t > 2m/R$$



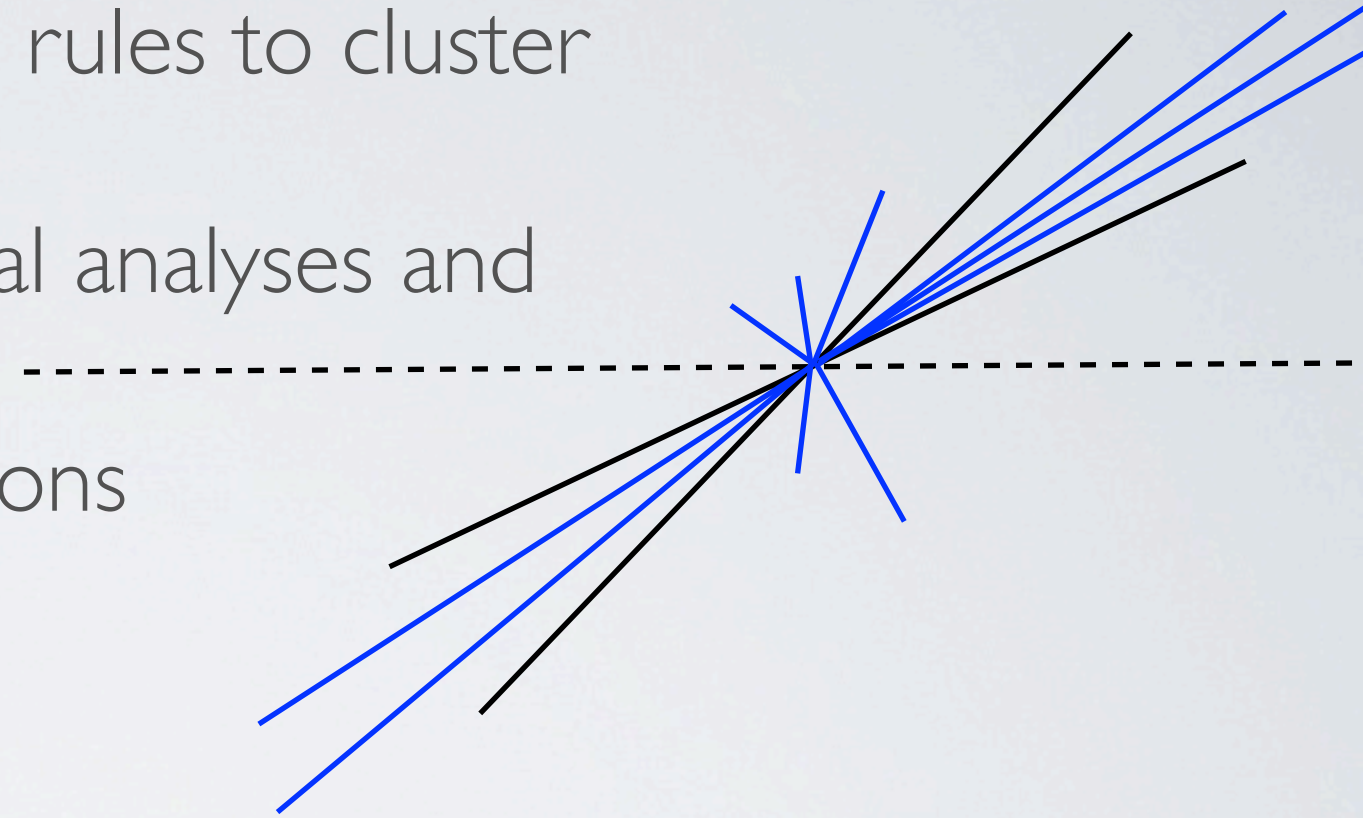
Boundary betwee

theory / experime

We want to look inside a jet

Jet definition(s)

- Jet algorithms: sets of (simple) rules to cluster particles together
- Implementable in experimental analyses and in theoretical calculations
- Must yield to finite cross sections
- First example :



> To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\varepsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\varepsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

Sterman and Weinberg,
Phys. Rev. Lett. 39, 1436 (1977):

Sequential recombination

- A large class of modern jet definitions is given by sequential recombination algorithms
- Starting with a list of particles, compute all distances d_{ij} and d_{iB}
- Find the minimum of all d_{ij} and d_{iB}

d_{iB} can be an external parameter (e.g. Jade algorithm), a distance from the beam ...

- If the minimum is a d_{ij}, recombine i and j and iterate
- Otherwise call i a final-state jet, remove it from the list and iterate

for a complete review see Salam, Towards jetography (2009)

Most common jet algorithms

Common choices for the distance are

$$d_{ij} = \min\left(p_{ti}^{2p}, p_{tj}^{2p}\right)\frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = p_{ti}^{2p}$$

with $\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

p = 1 k_t algortihm
(Catani *et al.*, Ellis and Soper)
p = 0 Cambridge / Aachen
(Dokshitzer *et al.*, Wobish and Wengler)
p = -1 anti-k_t algorithm
(Cacciari, Salam, Soyez)

- Different algorithms serve different purposes
- Anti-k_t clusters around hard particles giving round jets (default choice for ATLAS and CMS)
- Anti-k_t is less useful for substructure studies, while C/A reflects angular ordering

Jet mass

- It's the simplest variable describing the structure of a jet
- How well can we compute it ?
- Jet mass distributions are affected by *double* (soft & collinear) logarithms

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}m_J^2} = \frac{1}{m_J^2}\left[\alpha_s A_1 \ln\frac{m_J^2}{p_T^2} + \alpha_s^2 A_2 \ln^3\frac{m_J^2}{p_T^2} + \ldots\right]$$

- Reliable estimates of jet shapes should include:
  - fixed-order calculations at NLO (OK with public codes)
  - resummed (N)NLL predictions

Jet mass: all-order calculations

First NLL calculation of the jet mass in p-p collision

Banfi, Dasgupta, Khelifa-Kerfa and S.M. (2010)
Dasgupta, Khelifa-Kerfa, S.M. and Spannowsky (2012)

Calculations also in SCET

Chien, Kelley, Schwartz and Zhu (2012)
Jouttenus, Stewart, Tackmann and Waalewijn (2013)

For an isolated jet (small-R limit) the NLL cumulative distribution is

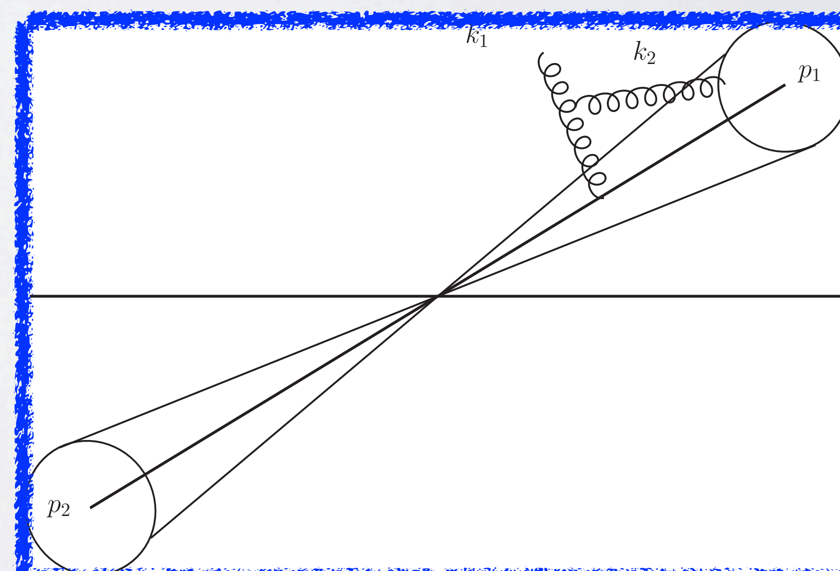
independent emissions   multiple emissions

$$\Sigma(\rho) \equiv \frac{1}{\sigma}\int^{\rho} d\,\rho' \frac{d\sigma}{d\rho'} = e^{-D(\rho)} \cdot \frac{e^{-\gamma_E D'(\rho)}}{\Gamma(1+D'(\rho))} \cdot \mathcal{N}(\rho)$$

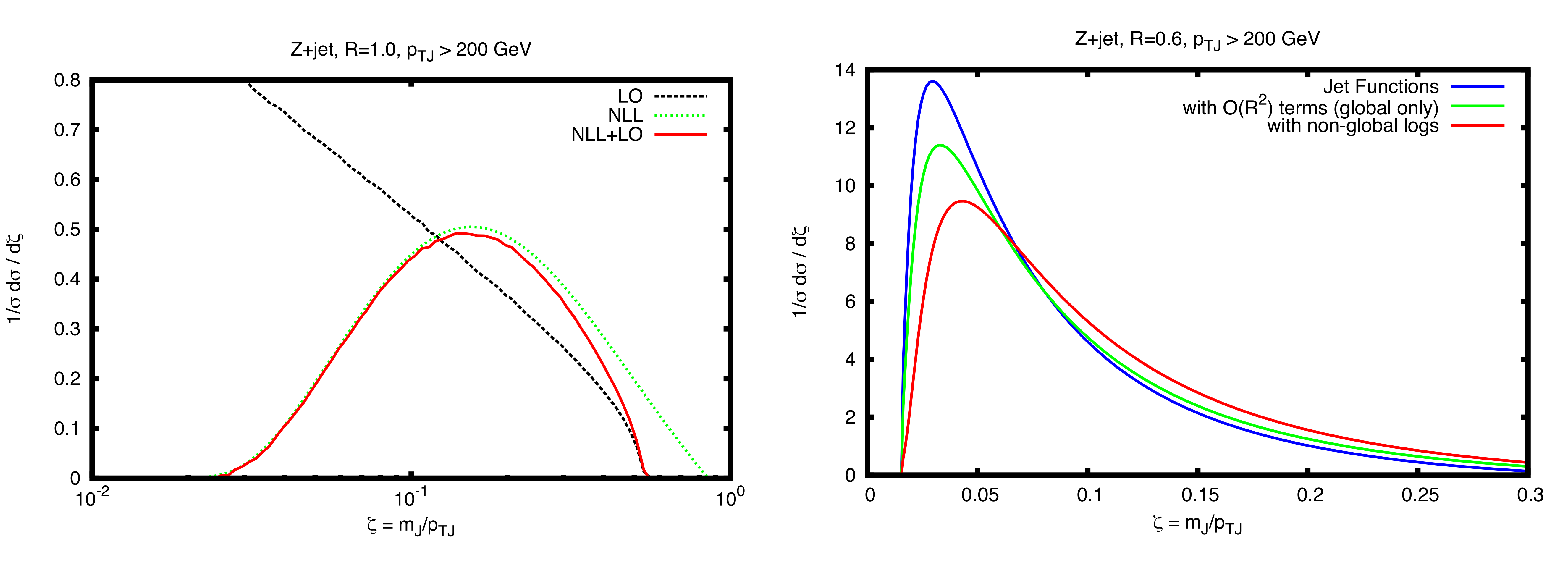
correlated emissions

$$\rho = \frac{m_j^2}{p_t^2 R^2}$$

non-global logs: difficult to resum
dependence on the jet algorithm



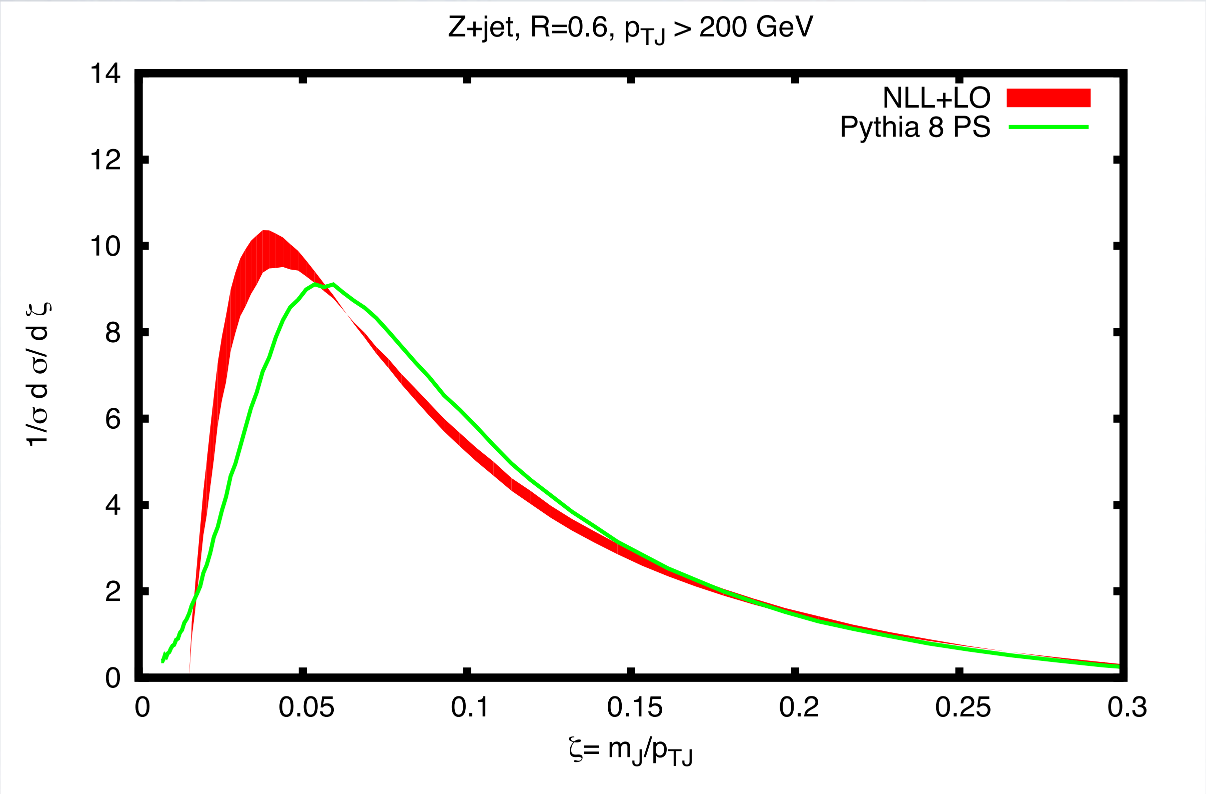
Z+jet



- Fixed-order QCD is inadequate: we need resummation
- Precision QCD requires resummation beyond the naive "isolated jet approximation": initial-state radiation and non-global logs

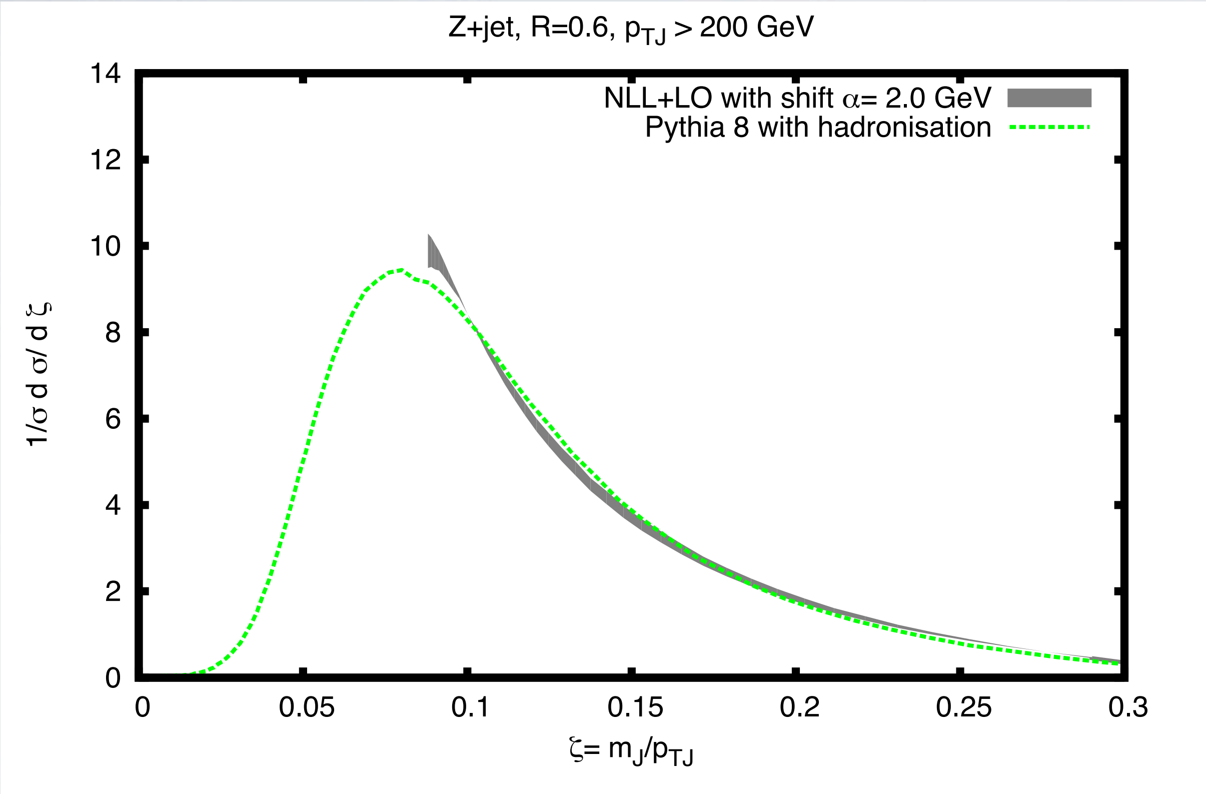
Dasgupta, Khelifa-Kerfa, S.M. and Spannowsky (2012)

Non-perturbative effects



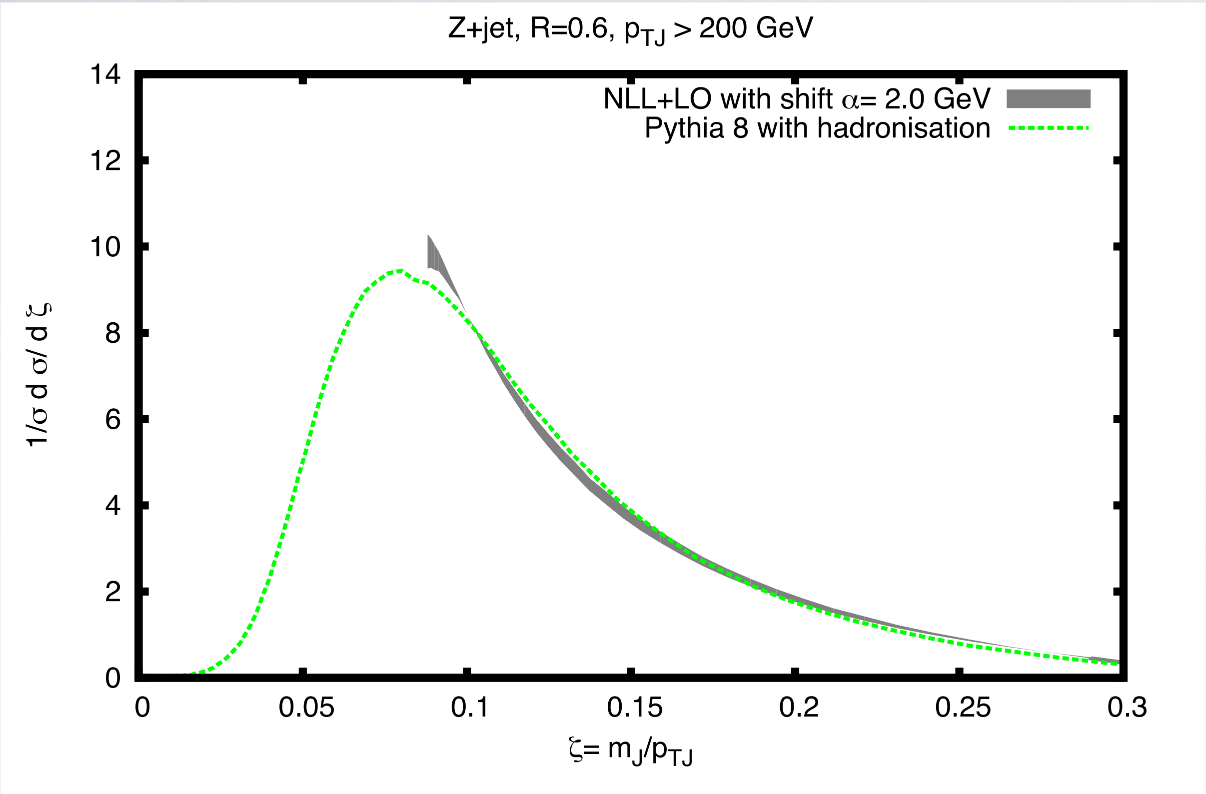
- We have to include the effect of NP physics

Non-perturbative effects



- We have to include the effect of NP physics
- First consider hadronisation: large effect but fairly understood

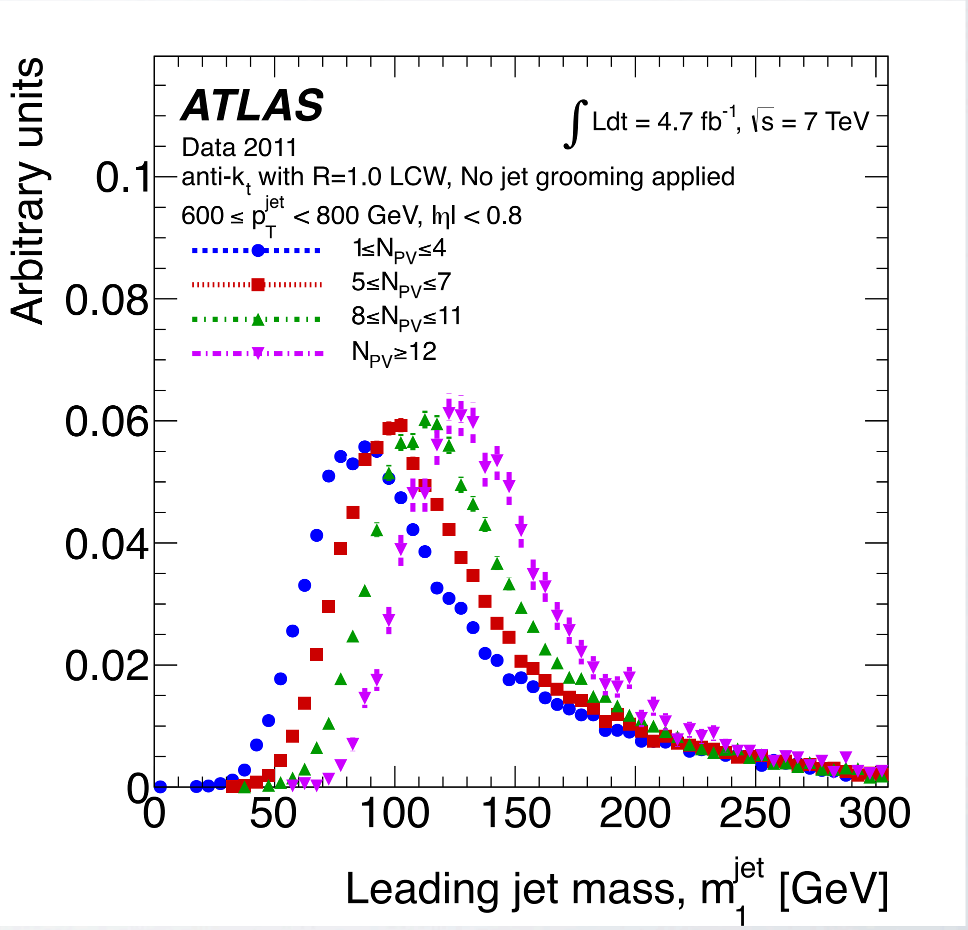
Non-perturbative effects



- We have to include the effect of NP physics
- First consider hadronisation: large effect but fairly understood

- But we also have to consider the possibility of multiple interactions: the underlying event and pile-up

- Theoretical control over these effects is less good

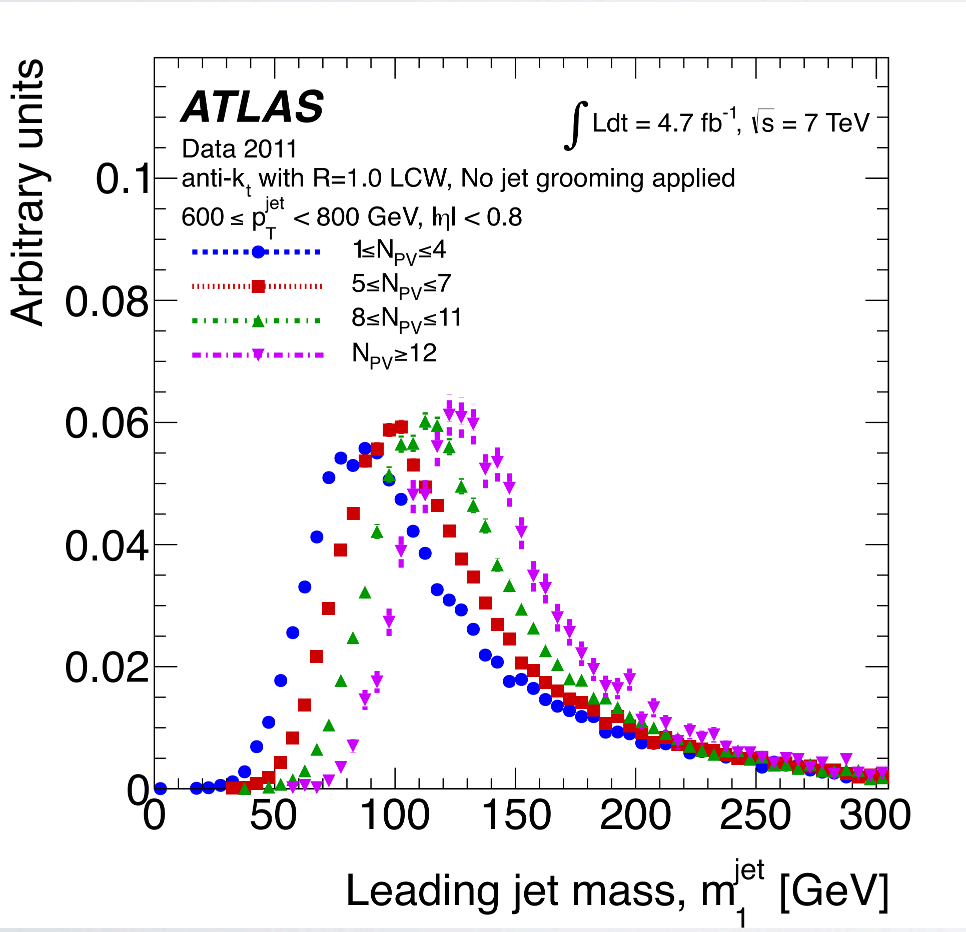


ATLAS, JHEP 1309 (2013) 076

Beyond the mass: substructure

- Let's have a closer look: background peaks in the EW region
- Need to go beyond the mass and exploit jet substructure
- Grooming and Tagging:

  1. clean the jets up by removing soft junk
  2. identify the features of hard decays and cut on them



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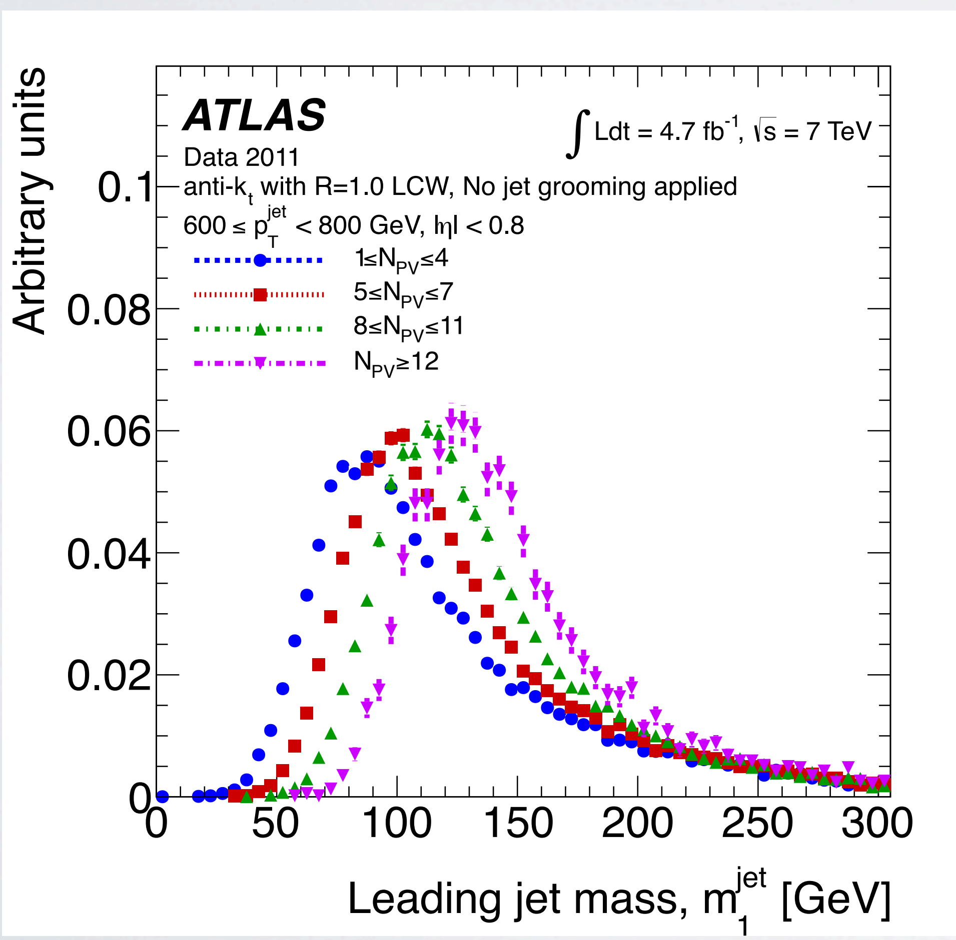
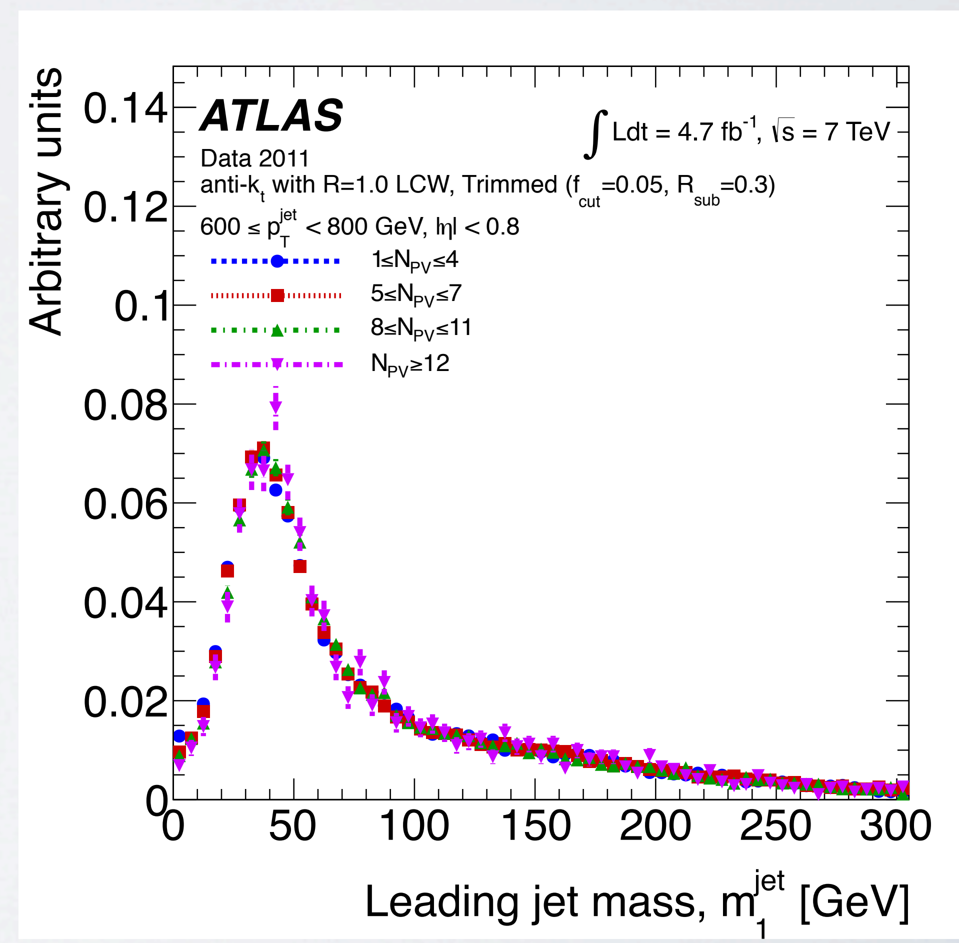
- Need to go beyond the mass and exploit jet substructure
- Grooming and Tagging:
  1. clean the jets up by removing soft junk
  2. identify the features of hard decays and cut on them
- Grooming provides a handle on UE and pile-up



ATLAS, JHEP 1309 (2013) 076

grooming



ATLAS, JHEP 1309 (2013) 076

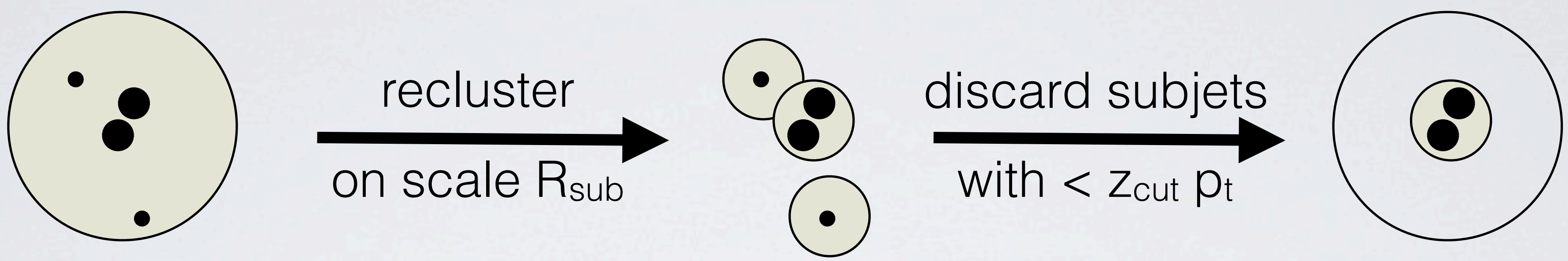
Grooming and tagging

- The last few years have seen a rapid development in substructure techniques:
- Very active (and growing) field with dedicated theory / experiment conference series (BOOST)
- O(10-20) powerful methods to tag jet substructure
- Substructure techniques are being used in searches and QCD measurements: *they will be crucial in the years to come*
- The existence of many methods can lead to some confusion
- Do we understand how / why they work ?
- Only analytic understanding can make this field *robust*

Where to start ?

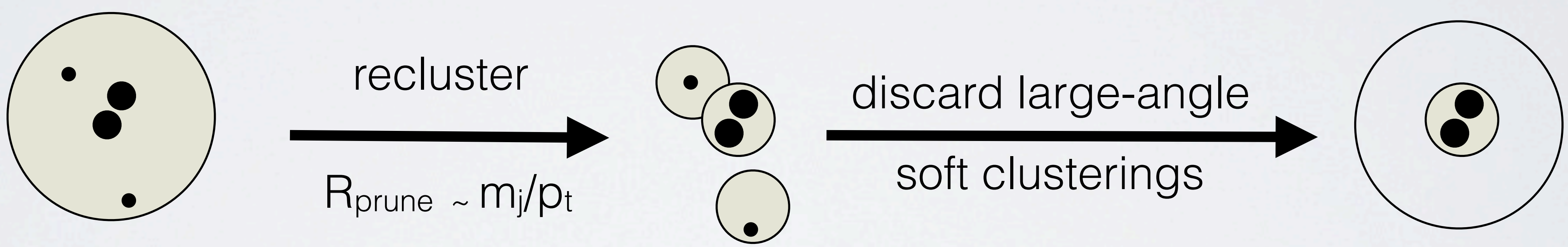
Cannot possibly study all tools
These 3 are widely used
We concentrate on background (QCD jets)

Trimming



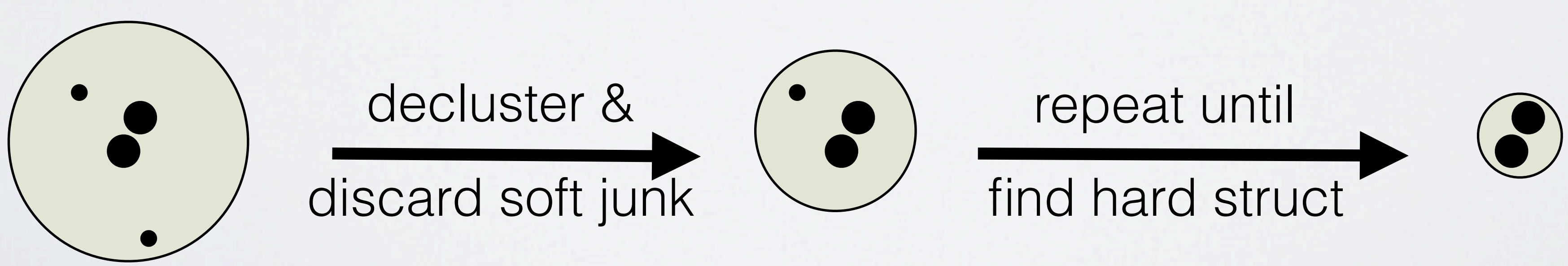
Krohn, Thaler and Wang (2010)

Pruning



Ellis, Vermillion and Walsh (2009)

Mass-drop tagger (MDT, aka BDRS)



Butterworth, Davison, Rubin and Salam (2008)

Our understanding so far

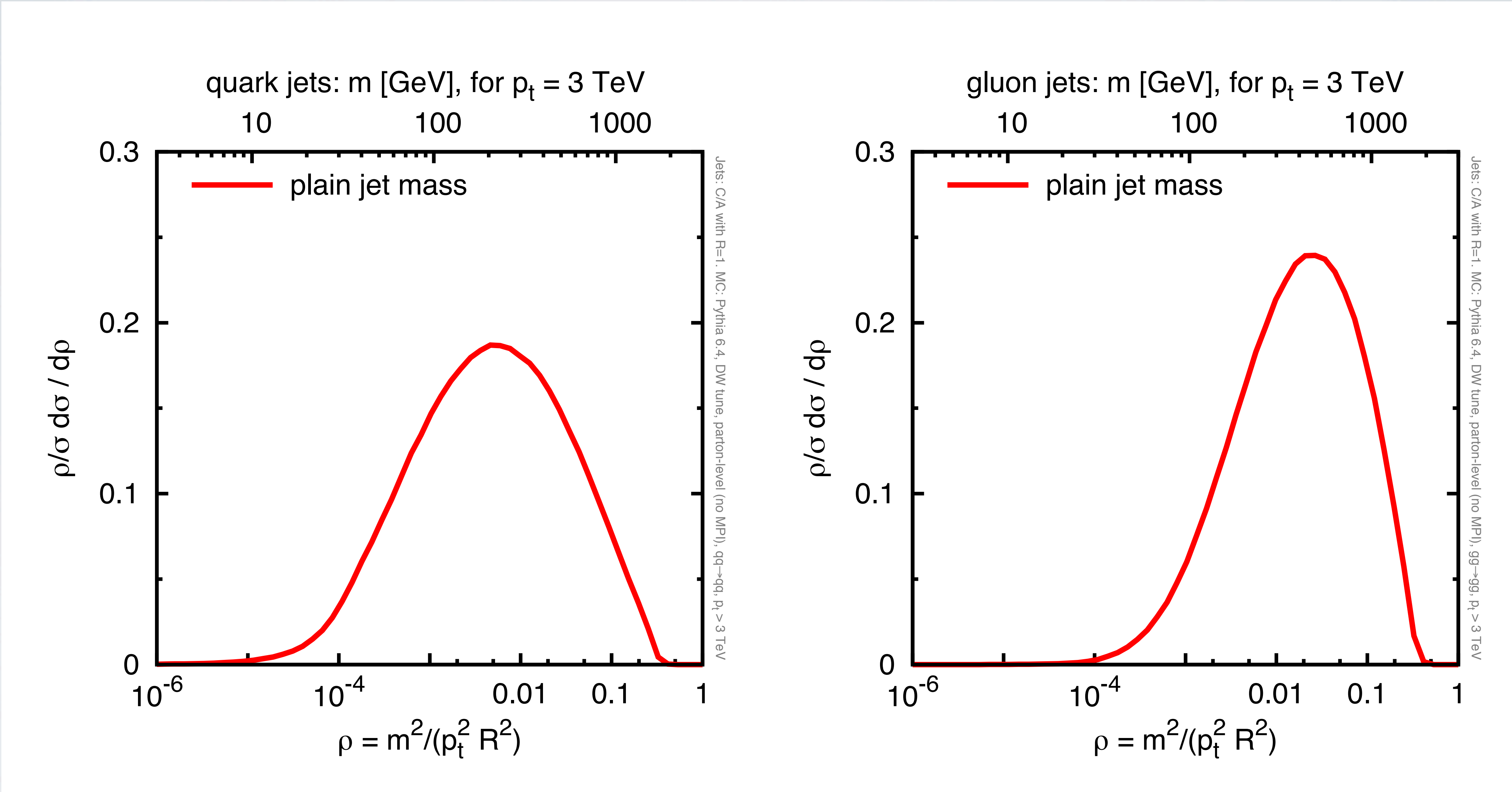
Boost 2010 proceedings:

The [Monte Carlo] findings discussed above indicate that while [pruning, trimming and filtering] have qualitatively similar effects, there are important differences. For our choice of parameters, pruning acts most aggressively on the signal and background followed by trimming and filtering.

- To what extent are the taggers above similar ?
- How does the statement of aggressive behaviour depend on the taggers' parameters and on the jet's kinematics ?

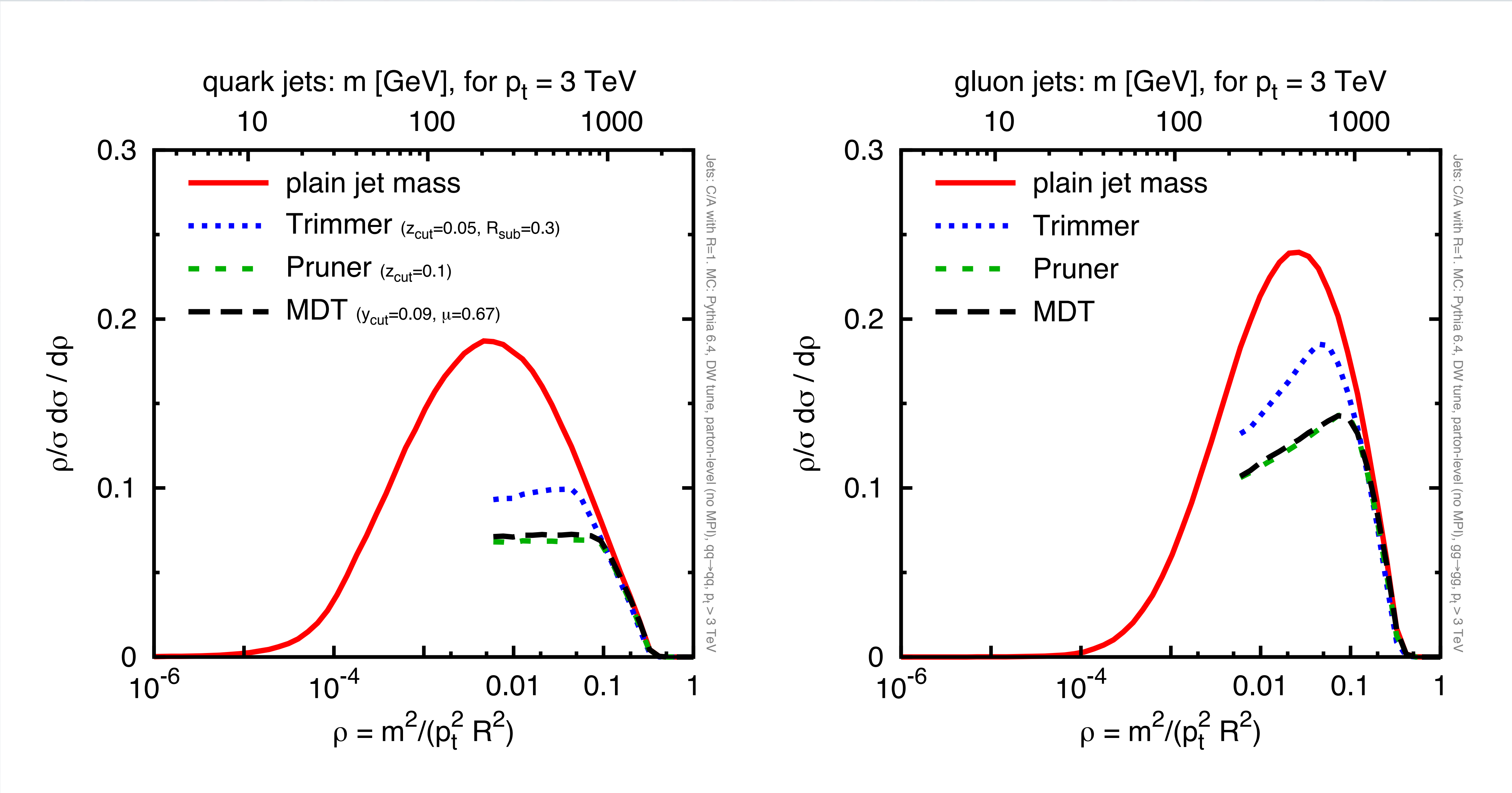
- Let's start with the "right" Monte Carlo study

Comparison of taggers



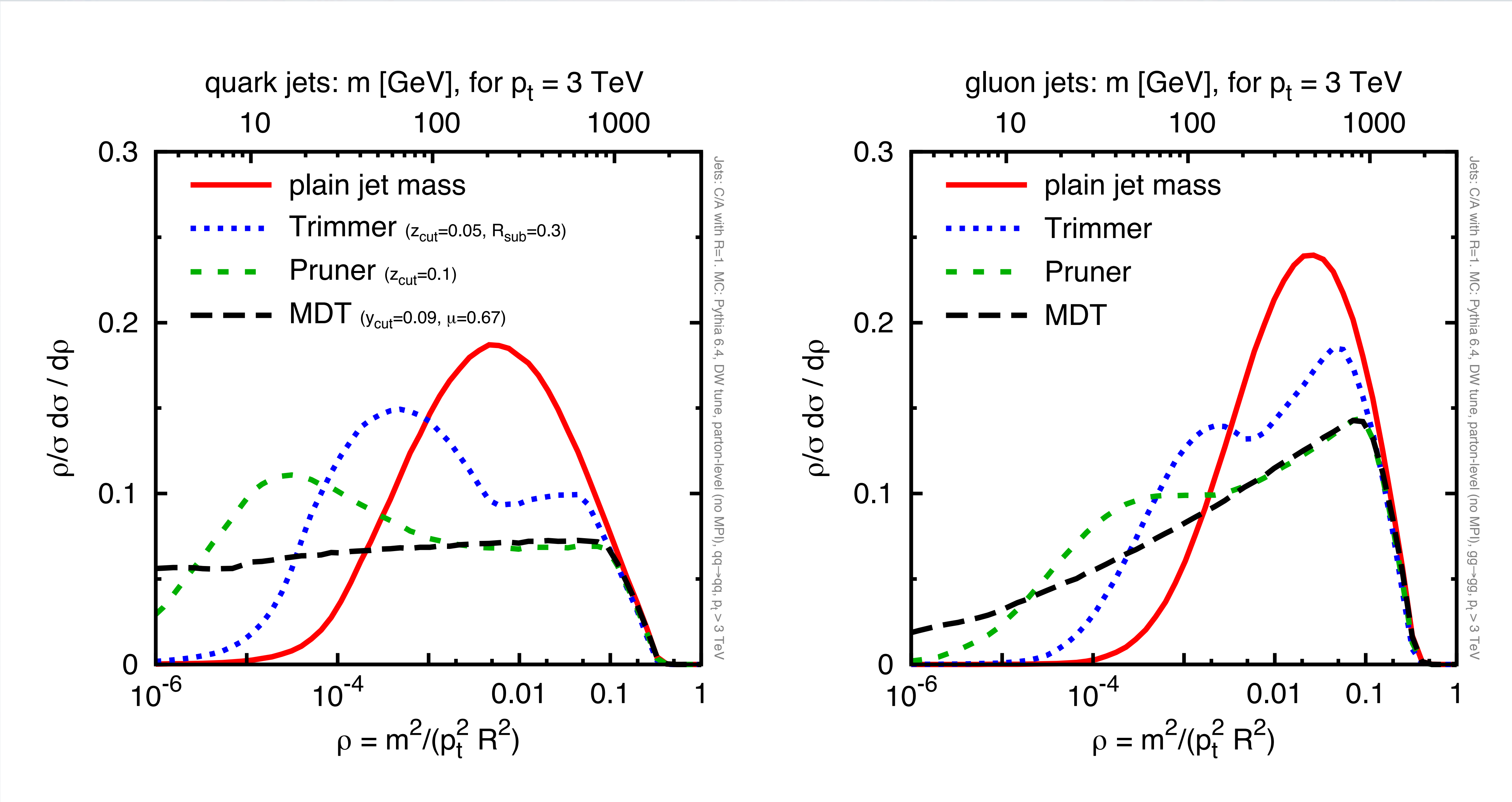
Plain jet mass: characteristic Sudakov peak

Comparison of taggers



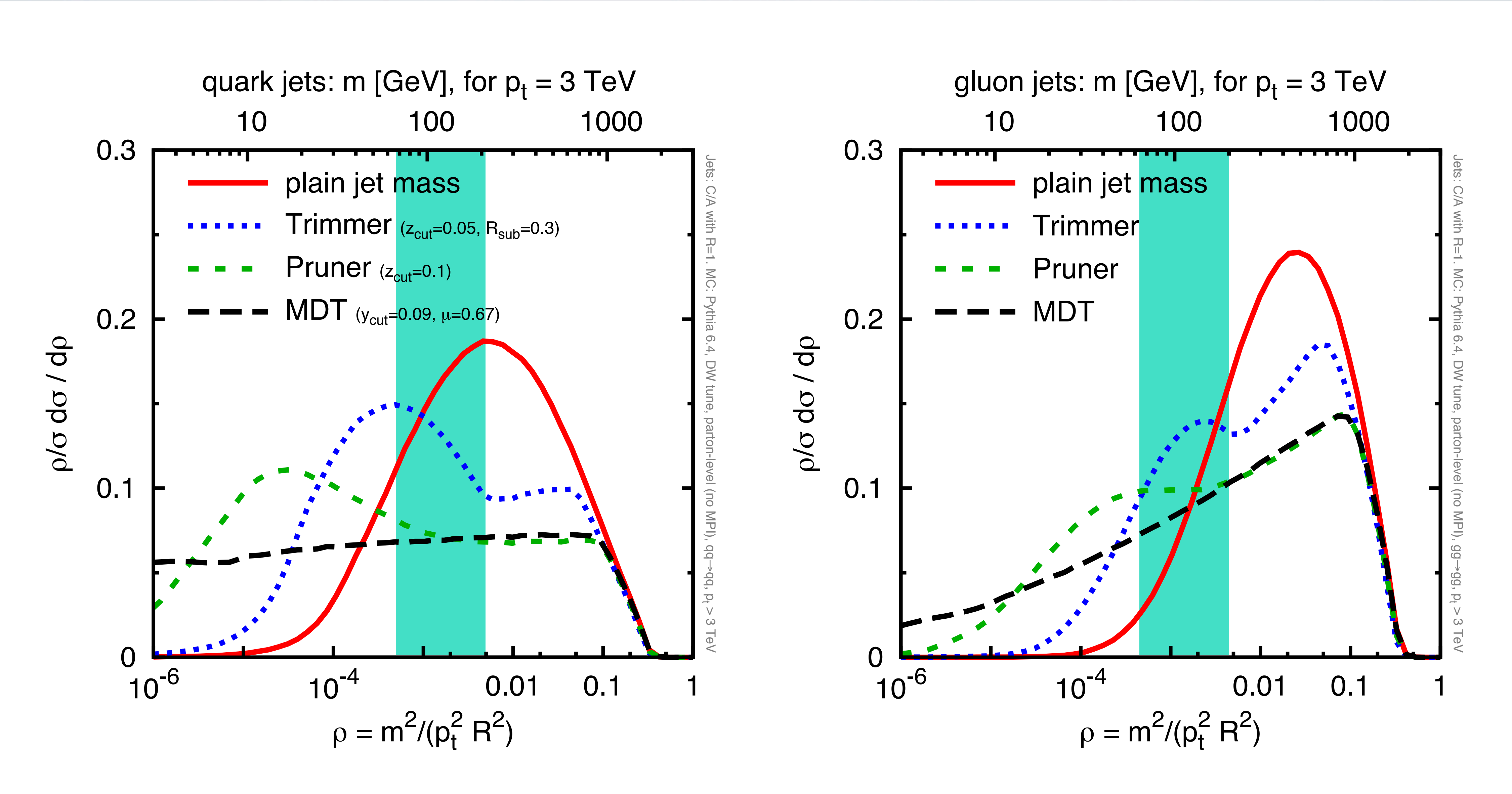
Different taggers appear to behave quite similarly

Comparison of taggers



But only for a limited kinematic region !

Comparison of taggers



Let's translate from QCD variables to ``search'' variables:

$\boldsymbol{\rho} \rightarrow m$, for $p_t = 3$ TeV, $R = 1.0$

Questions that arise

- to what extent are these tools similar ?
- how does the statement of aggressive behaviour depend on the parameters and on the jet's kinematics ?

- a theorist's worry: complicated algorithms with many parameters
- are we giving up on calculability / precision QCD ?
- first comprehensive field-theoretical understanding of these algorithms

M. Schwartz (Boost 2012)

Dasgupta, Fregoso, SM, Powling EPJ C (2013)
Dasgupta, Fregoso, SM, Salam, JHEP 1309 029 (2013)

Analytic understanding

1. explains features and reveal properties

2. indicates how to improve existing tools

3. enables us to develop better tools

4. provides valuable checks on MC parton showers

$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots]$$

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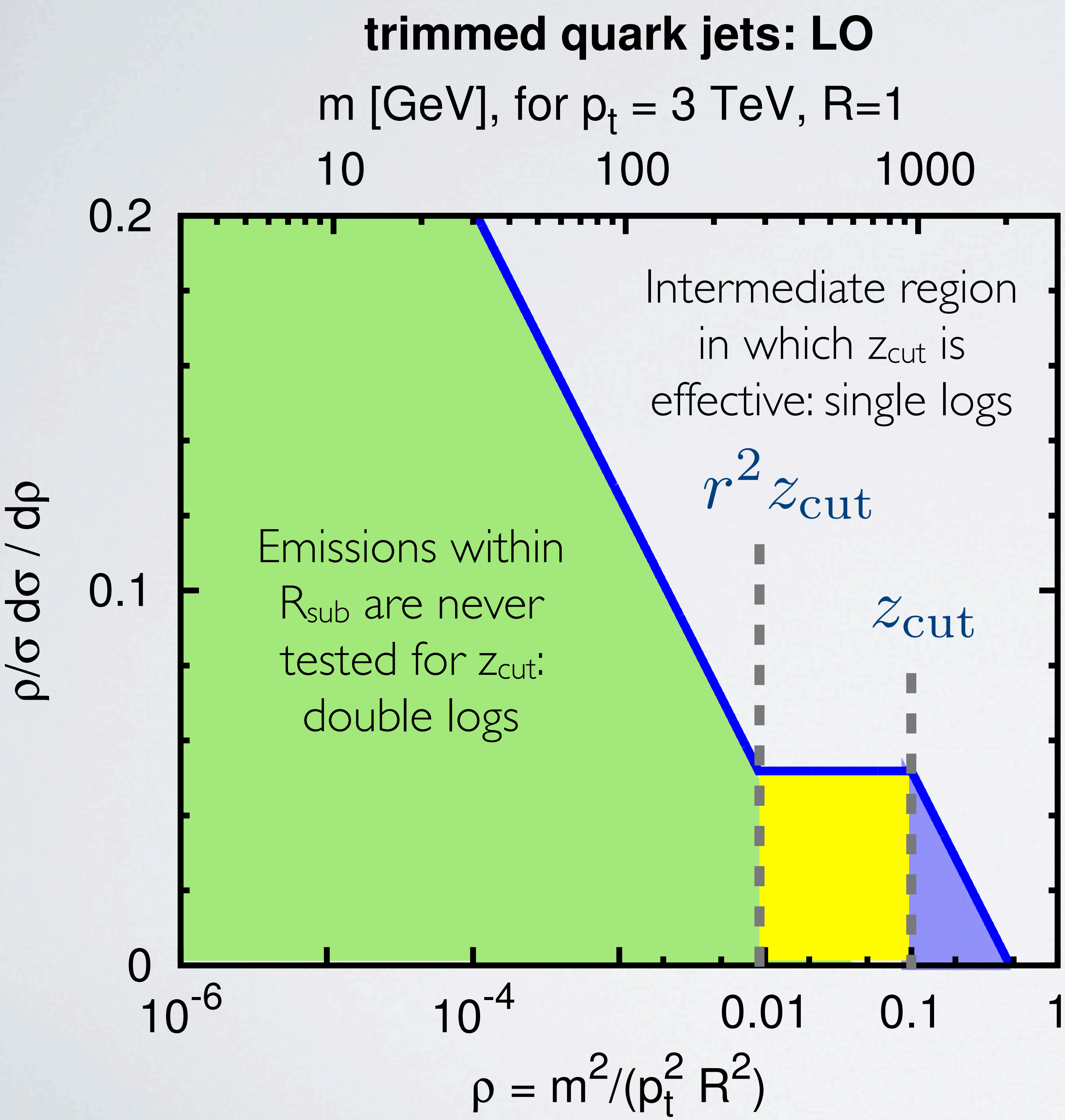
Trimming

1. Take all particles in a jet and re-cluster them with a smaller jet radius $R_{sub} < R$

2. Keep all subjets for which $p_t^{subjet} > z_{cut}\ p_t$

3. Recombine the subjets to form the trimmed jet

Trimming at LO



$$\boxed{\frac{\rho}{\sigma}\frac{d\sigma^{(\rm trim,LO)}}{d\rho} =}$$

$$\frac{\alpha_s C_F}{\pi}\left(\ln\frac{r^2}{\rho} - \frac{3}{4}\right)$$

$$\frac{\alpha_s C_F}{\pi}\left(\ln\frac{1}{z_{\rm cut}} - \frac{3}{4}\right)$$

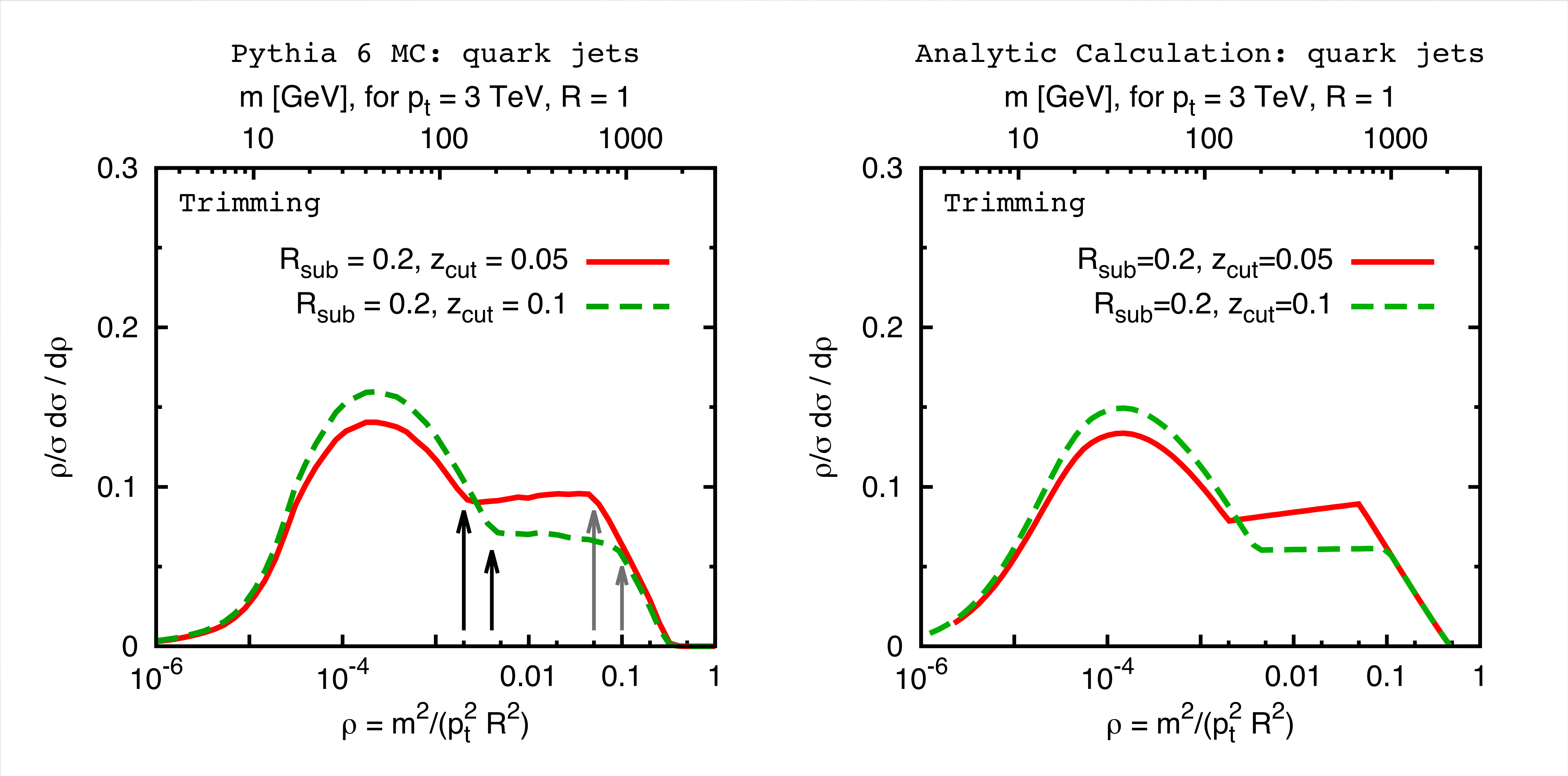
$$\frac{\alpha_s C_F}{\pi}\left(\ln\frac{1}{\rho} - \frac{3}{4}\right)$$

$$r = \frac{R_{\rm sub}}{R}$$

Trimming: all orders

One gets exponentiation of LO (+ running coupling)

$$\frac{d\sigma^{\text{trim,resum}}}{d\rho} = \frac{d\sigma^{\text{trim,LO}}}{d\rho} \exp\left[-\int_\rho d\rho' \frac{1}{\sigma}\frac{d\sigma^{\text{trim,LO}}}{d\rho'}\right]$$



All-order calculation done in the small-z_{cut} limit

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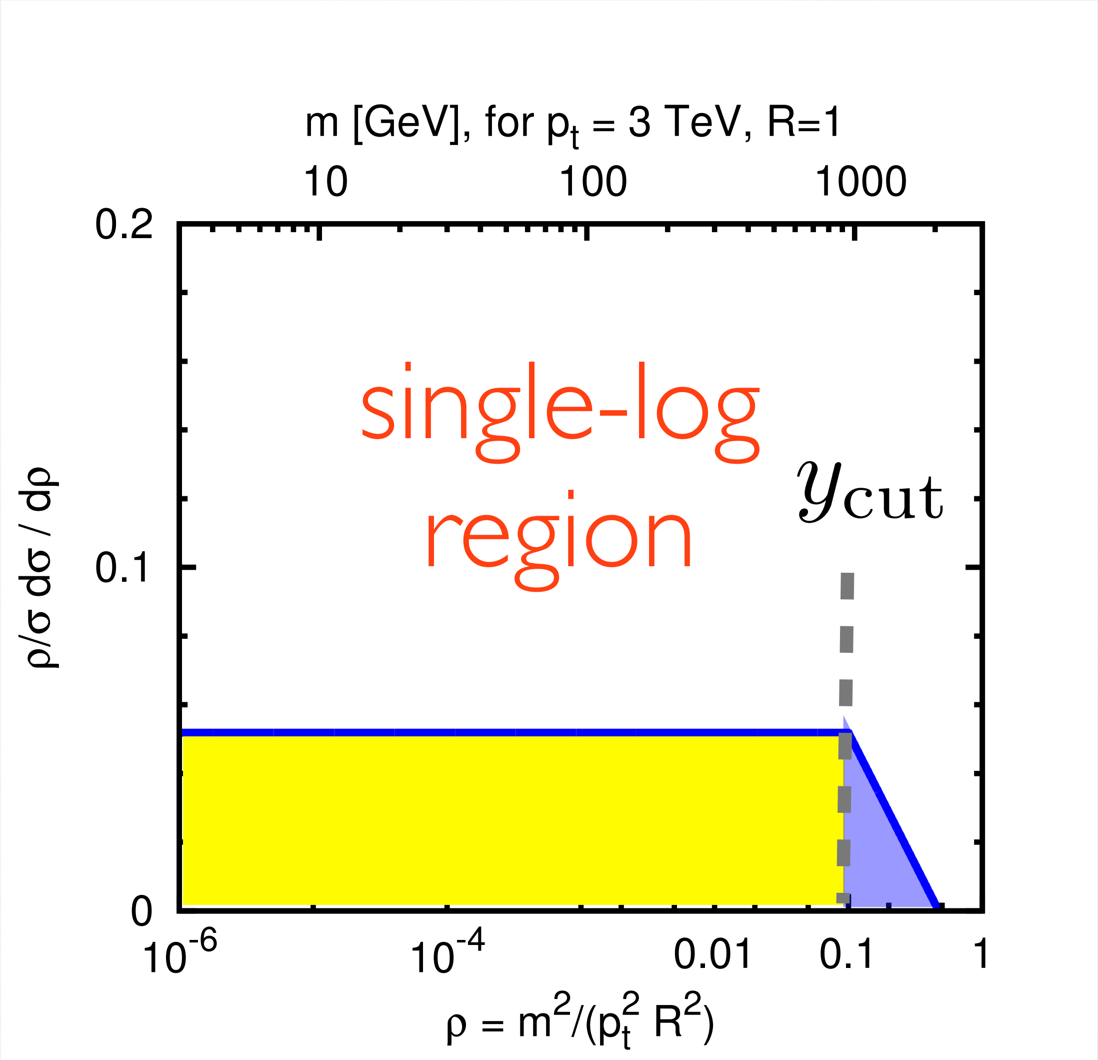
$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$$

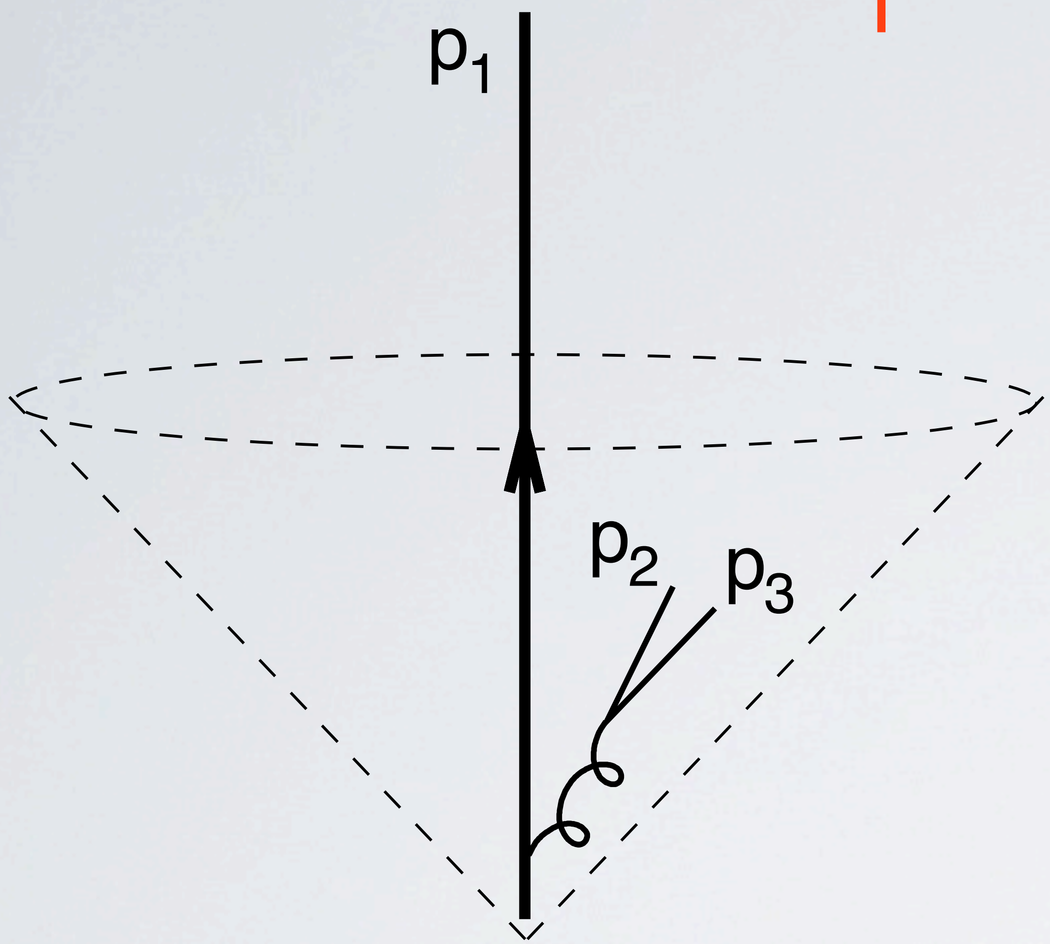
Mass Drop Tagger at LO

1. Undo the last stage of the C/A clustering. Label the two subjets j_1 and j_2 ($m_1 > m_2$)
2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{cut}$), tag the jet.
3. Otherwise redefine $j = j_1$ and iterate.

Interesting result: at LO we only have single logs!



Complications at NLO



What MDT does wrong:

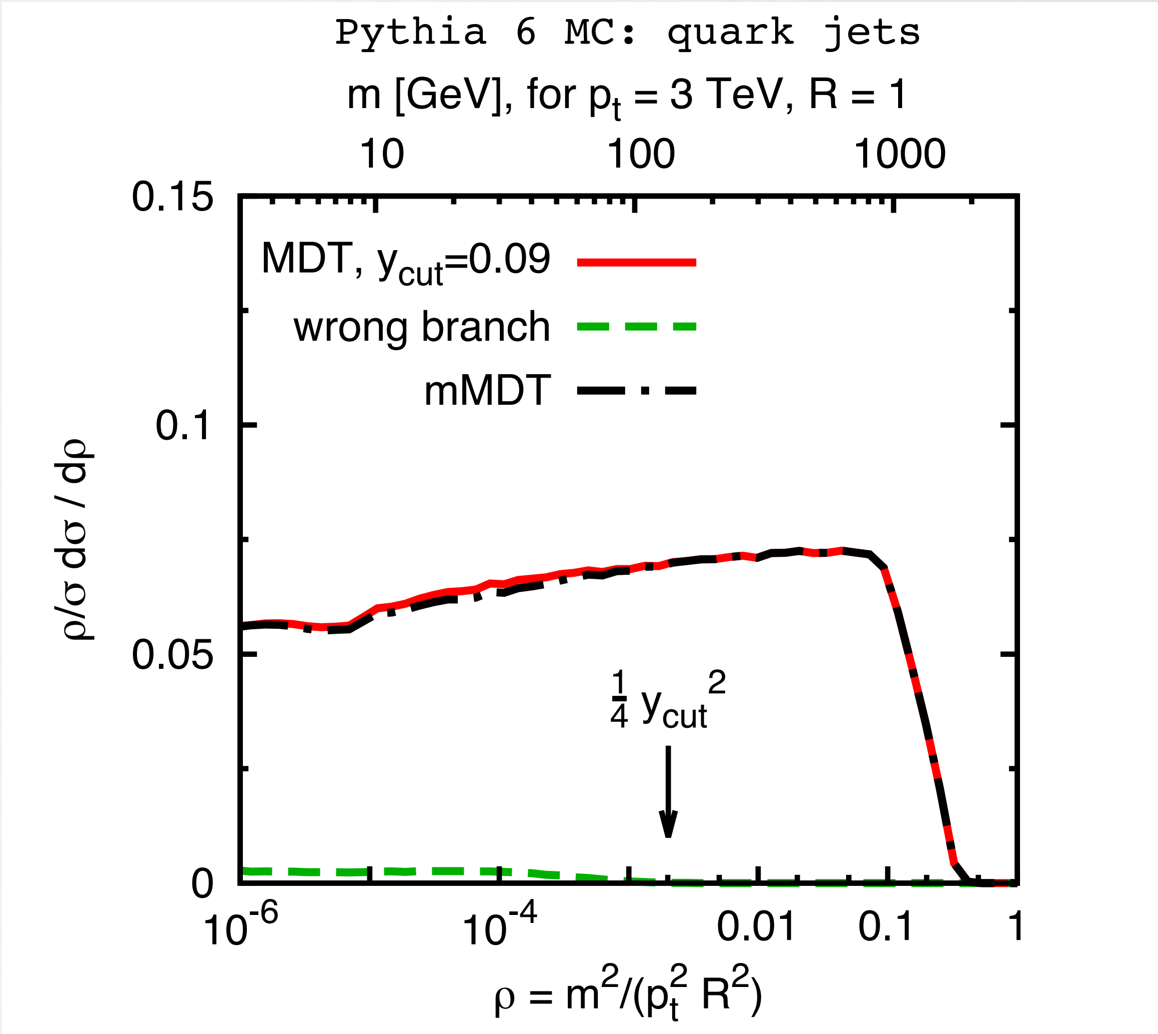
If the y_{ij} condition fails, MDT iterates on the more massive subjet. It can follow a soft branch ($p_2+p_3 < y_{cut}\ p_{tjet}$), when the "right" answer was that the (massless) hard branch had no substructure

- This can be considered a flaw of the tagger
- It worsens the logarithmic structure $\sim\alpha_s^2 L^3$
- The all-order distribution is NOT given by exponentiation of LO
- It calls for a modification

The modified Mass Drop Tagger

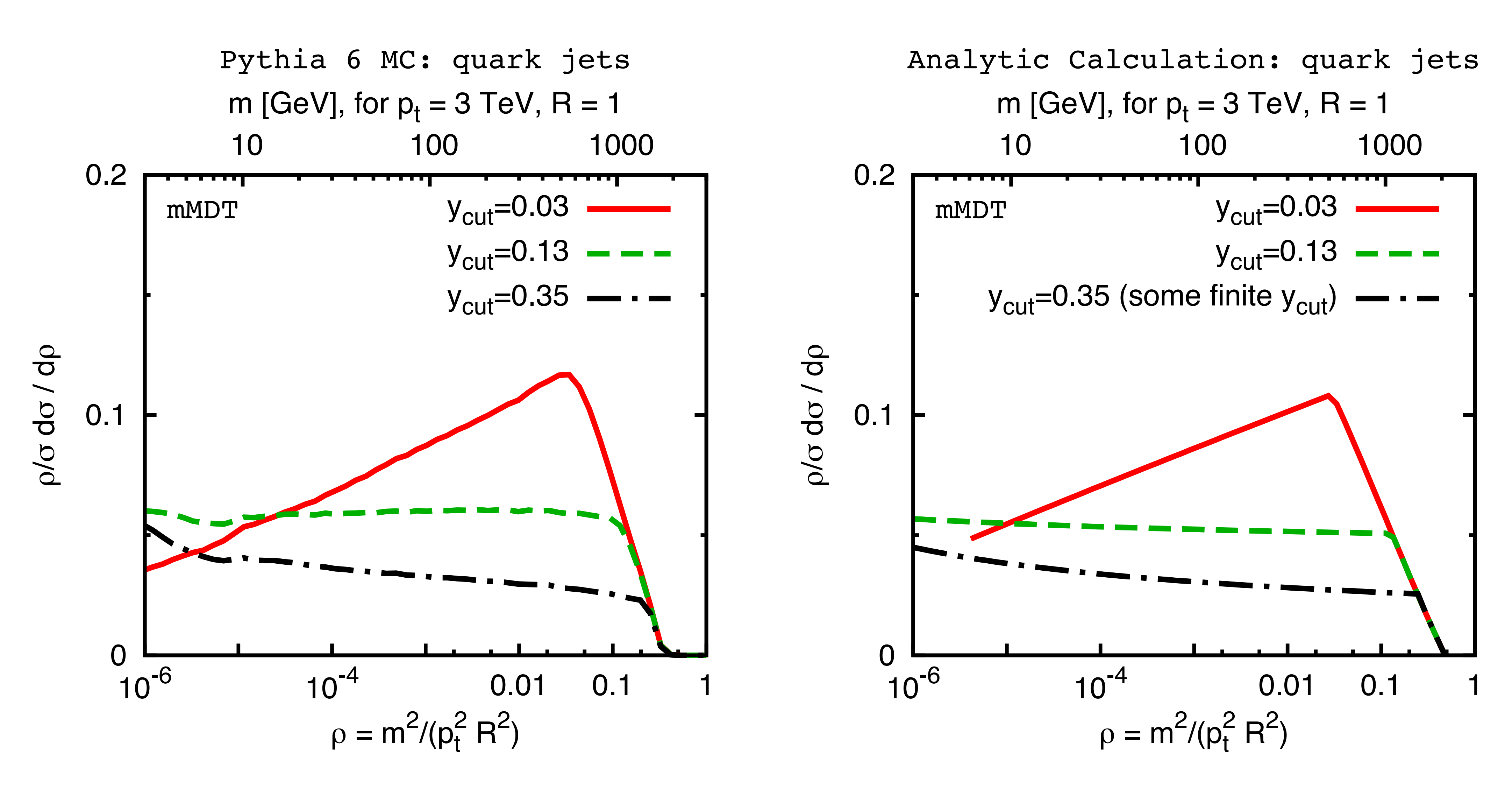
1. Undo the last stage of the C/A clustering. Label the two subjets j_1 and j_2 ($m_1 > m_2$)
2. If $m_1 < \mu m$ (mass drop) and the splitting was not too asymmetric ($y_{ij} > y_{cut}$), tag the jet.
3. Otherwise redefine j to be the subjet with highest transverse mass and iterate.

- In practice the soft-branch contribution is very small
- However, this modification makes the all-order structure particularly interesting



All-order structure of mMDT

- In the small y_{cut} limit, it is just the exponentiation of LO
- The mMDT has single logs to all orders (i.e. $\alpha_s^n L^n$)



Remarkable agreement !

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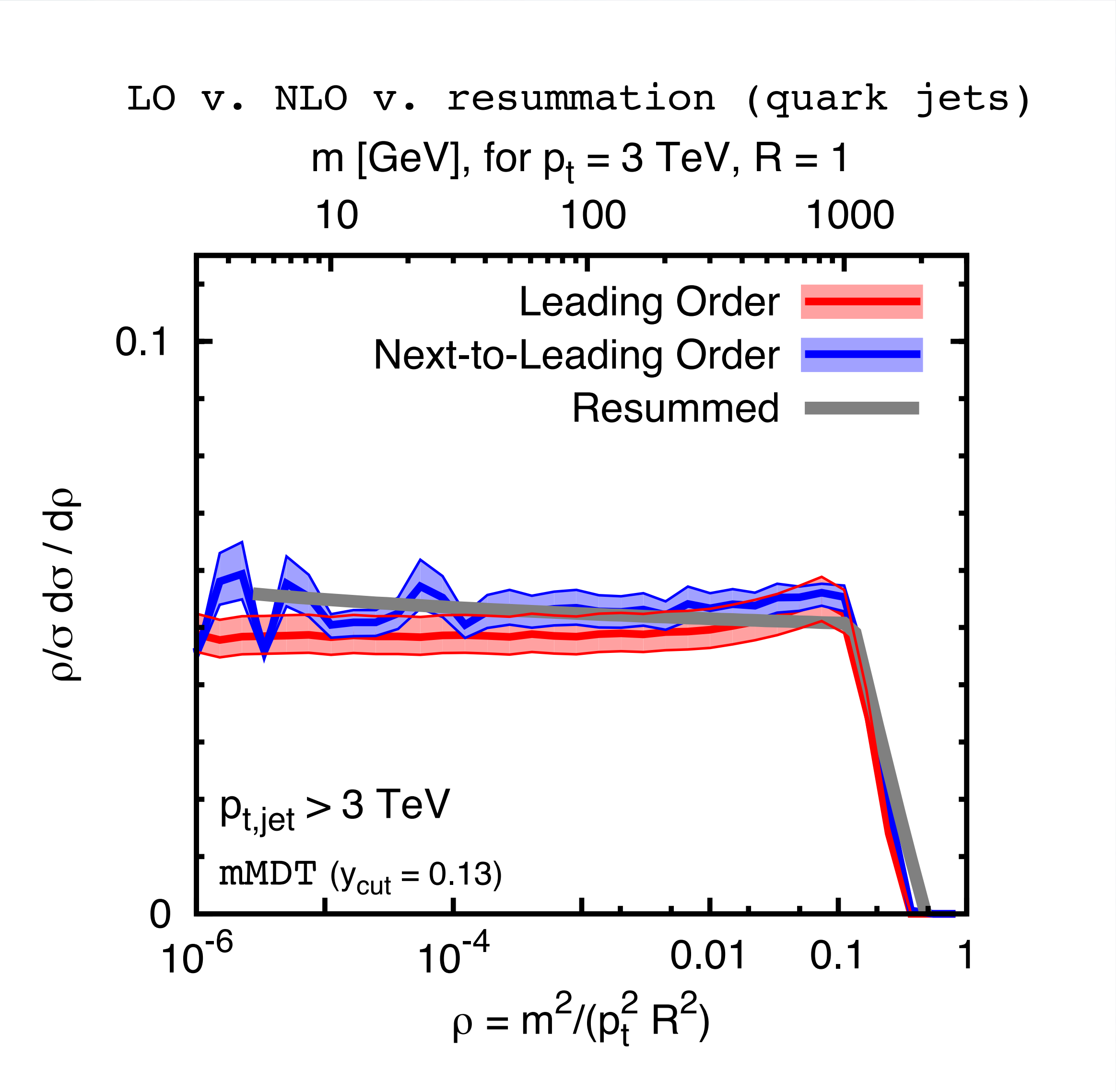
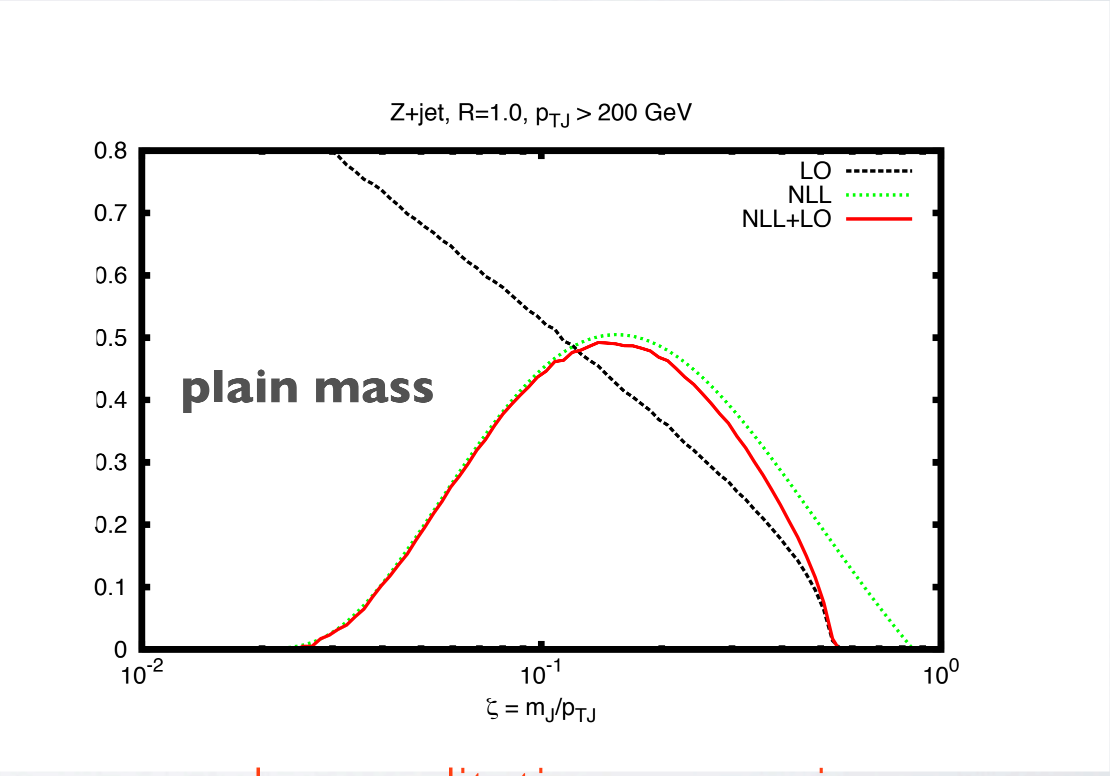


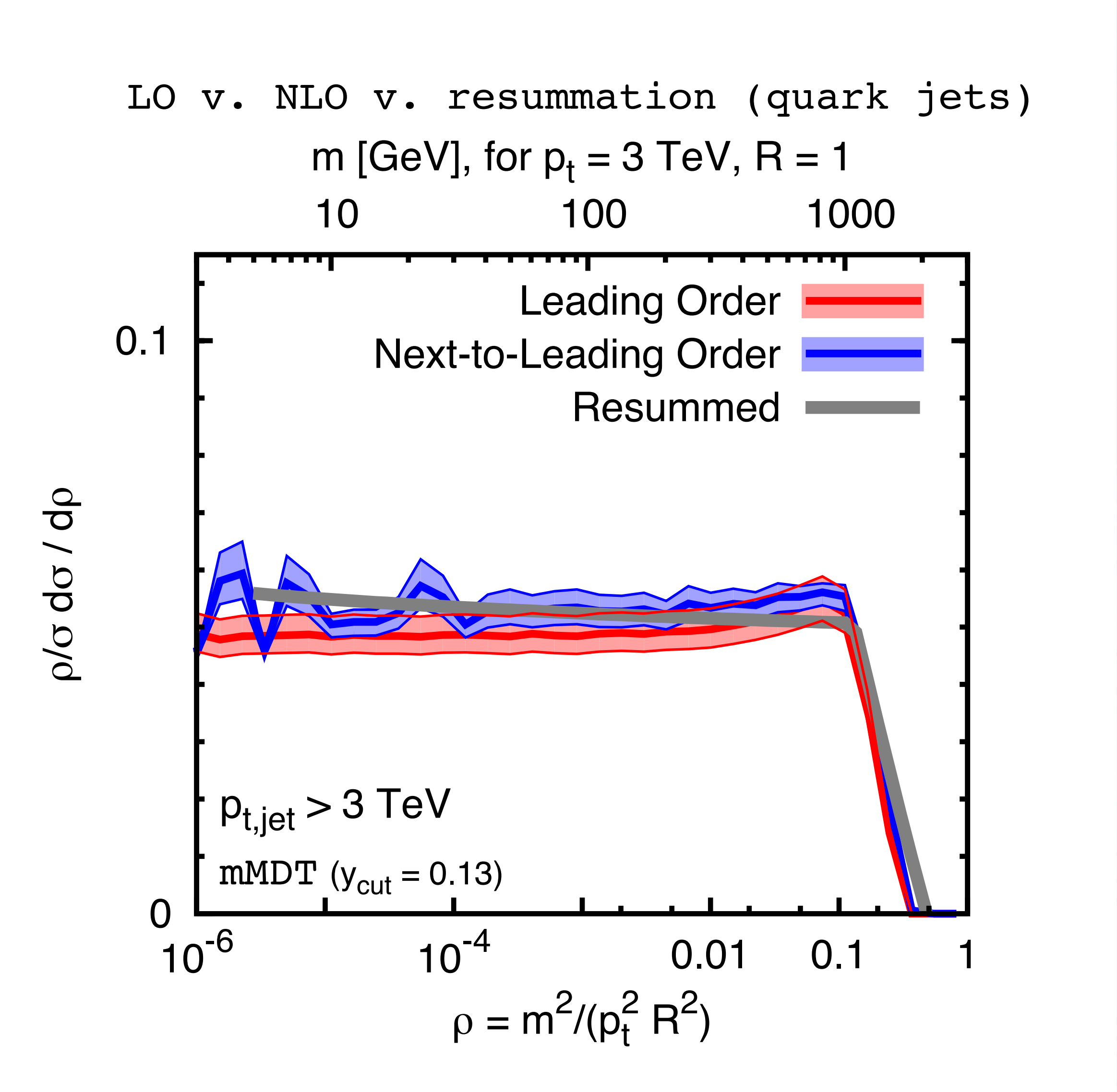
- Single logs: extended validity of FO calculations



only a qualitative comparison: different process/kinematics!

All-order structure of mMDT

- In the small y_{cut} limit, it is just the exponentiation of LO
- The mMDT has single logs to all orders (i.e. $\alpha_s^n L^n$)

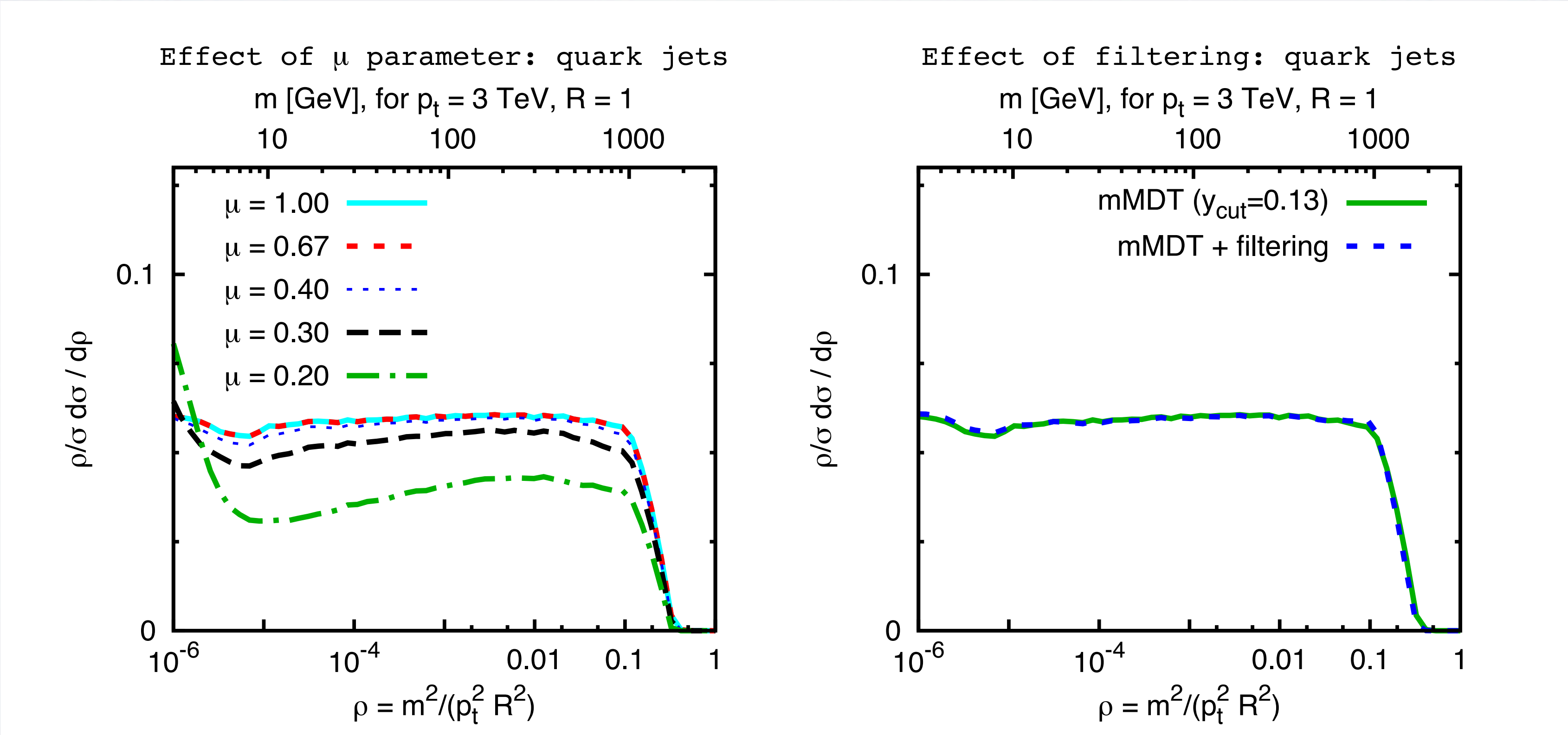


- Single logs: extended validity of FO calculations

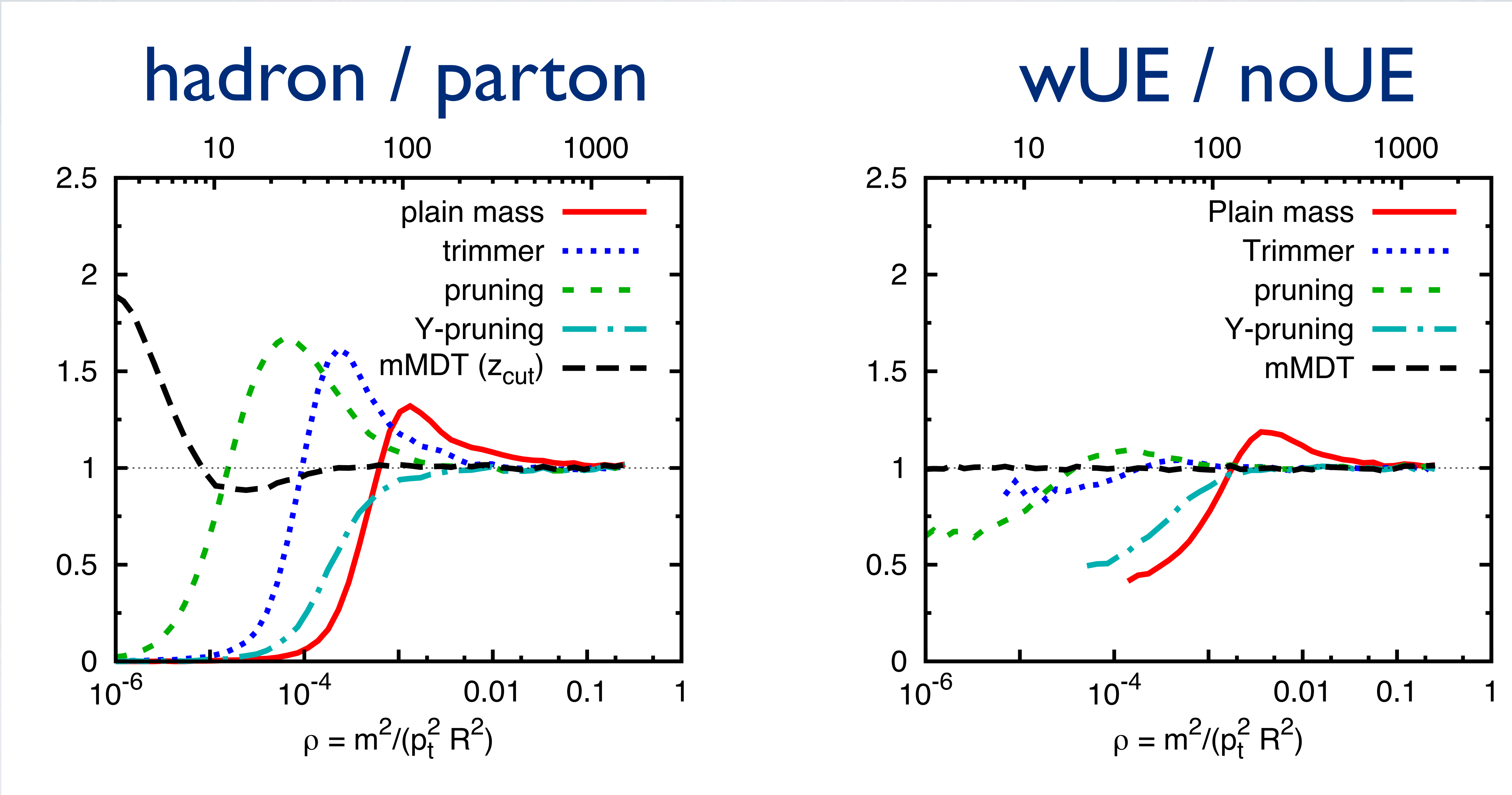
- Single logs of collinear origin
- Remarkable consequence: mMDT is free of non-global logs!

Other properties of mMDT

- Flatness of the background is a desirable property (data-driven analysis, side bands)
- y_{cut} can be adjusted to obtain it (analytic relation)
- Role of μ, not mentioned so far
- It contributes to subleading logs and has small impact if not too small (μ>0.4)
- Filtering only affects subleading ($N^{nfilt}LL$) terms



Non-perturbative effects



- Most taggers have reduced sensitivity to NP physics
- mMDT particularly so (it's the most calculable)

Analytic understanding

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4. provides valuable checks on MC parton showers

$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s\, g_3(\alpha_s L) + \ldots]$$

The crucial element

- Both trimming and mMDT cut on momentum fractions
- Trimming does that only above a certain resolution scale

$$\frac{\min(p_{T1}, p_{T2})}{p_{Tjet}} > z_{\rm cut} \quad (\text{if} \quad \Delta_{12} > R_{\rm sub})$$

VS

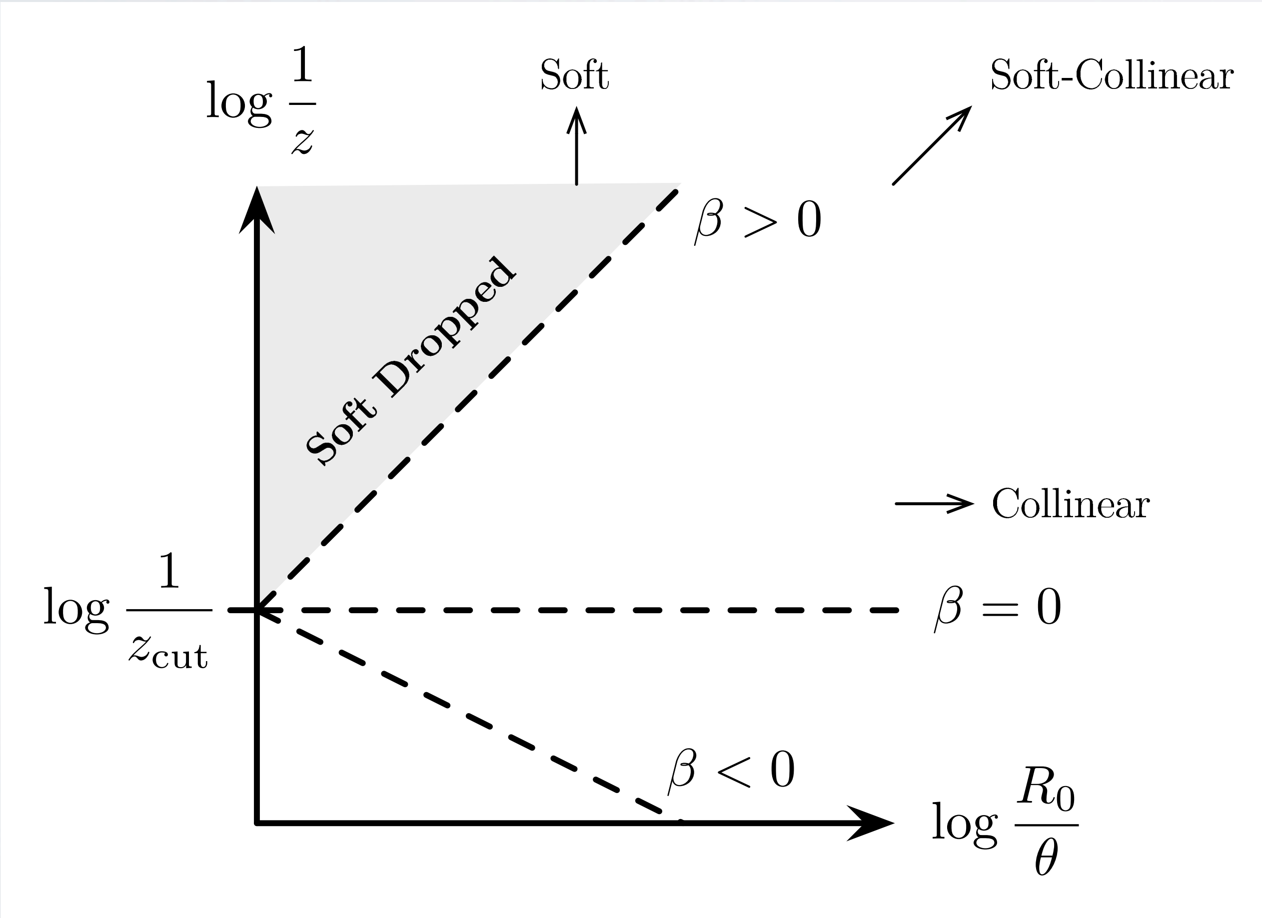
$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\rm cut}$$

- This difference is crucial: double logs vs single logs
- However, trimming is a groomer: any IRC safe observable on the starting jet is IRC safe on the trimmed jet
- Is this the case for mMDT ?
- mMDT mass-type observables are OK (that's what was designed for)
- but the jet energy of a mMDT jet is not collinear safe!

Soft drop

- Generalise mMDT procedure to obtain a more generic tool

$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\mathrm{cut}} \quad \longrightarrow \quad \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\mathrm{cut}} \left(\frac{\Delta_{12}}{R}\right)^{\beta}$$



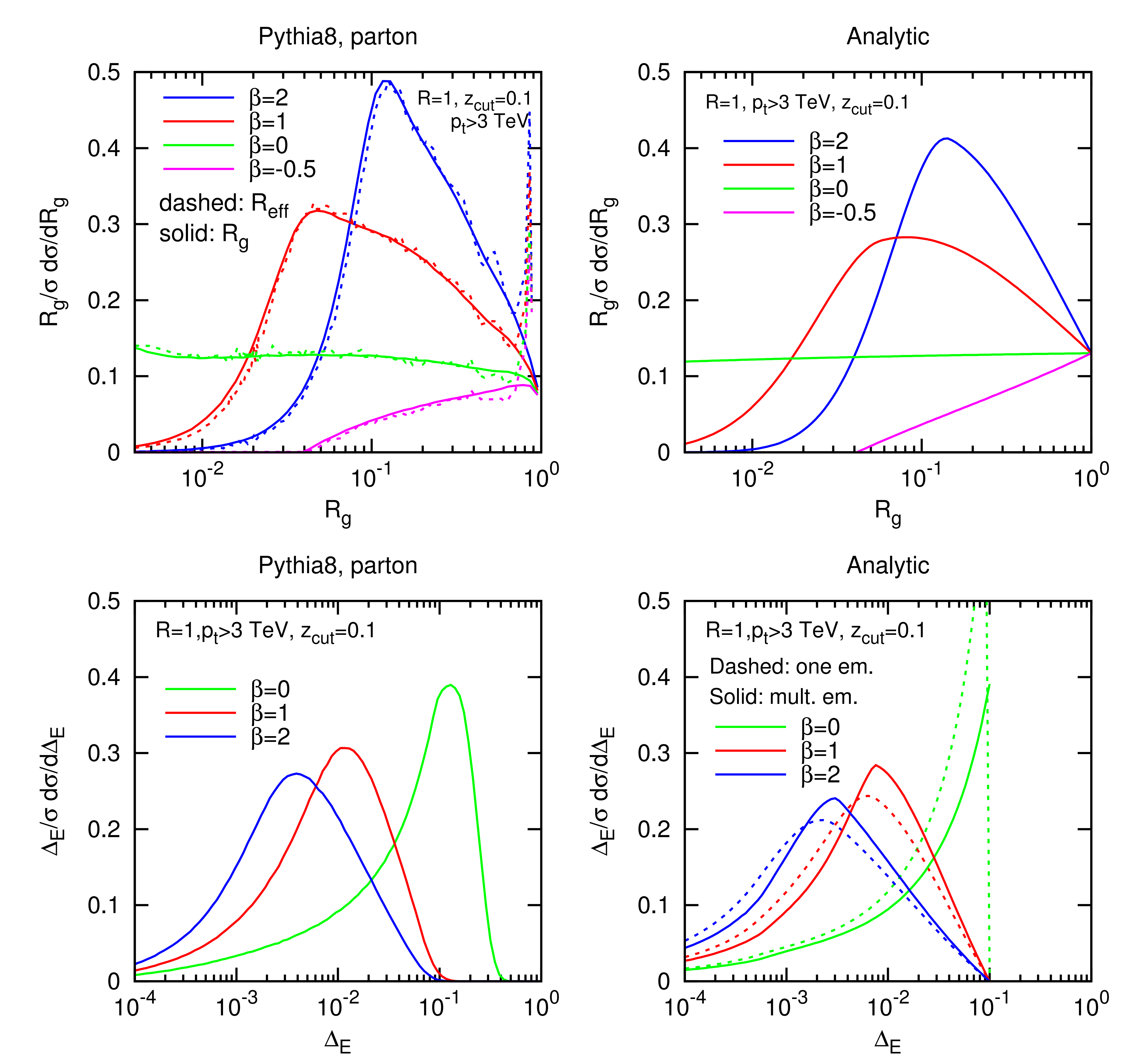
$\beta > 0$ grooming mode (dynamical trimmer)

$\beta = 0$ mMDT

$\beta < 0$ tagging mode (cut on pure collinear)

Larkoski, SM, Soyez and Thaler (2014)

Groomed jet properties



- Radius & energy of groomed jets
- relevant for pile-up and calibration

- Power of analytics: we know what's happening!

Δ_E for $\beta = 0$ particularly interesting: not IRC but finite!

Analytic understanding

1. explains features and reveal properties

2. indicates how to improve existing tools

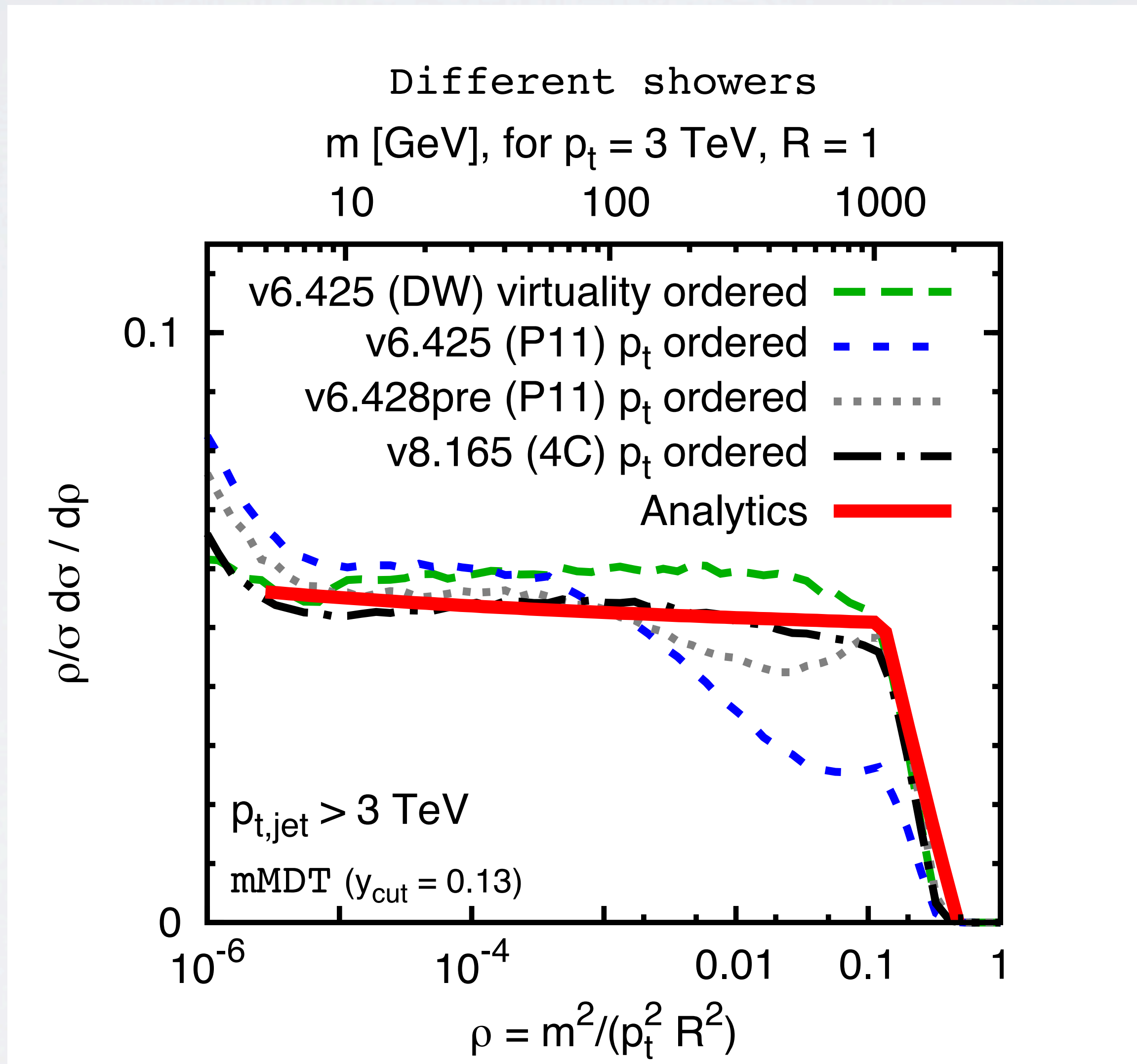
3. enables us to develop better tools

4. provides valuable checks on MC parton showers

$$\sigma_{res} = g_0 \exp[g_1(\alpha_s L)/\alpha_s + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots]$$

Different Monte Carlos

- mMDT is the most calculable (single-logs, small NP effects)
- Useful tool if MCs don't agree



In summary ...

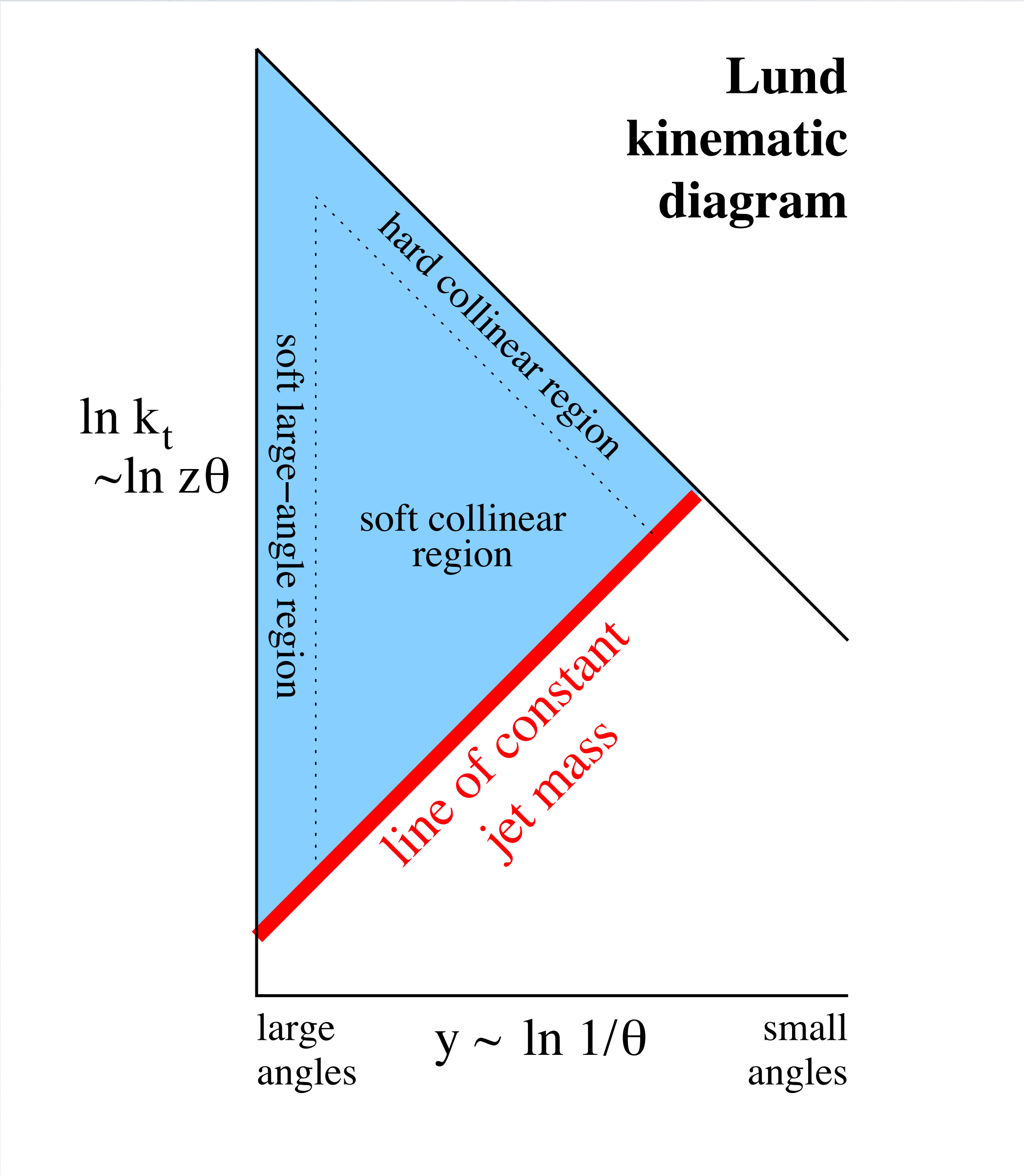
- Jet substructure is playing an increasingly important role in LHC phenomenology (searches and QCD)

- I have presented the first comprehensive field-theoretical understanding of these algorithms

- Analytic studies of groomers and taggers
  - reveal their properties
  - enable us to improve existing tools (mMDT)
  - indicate how to develop better tools (soft drop)
  - provide valuable checks on MCs

BACKUP SLIDES

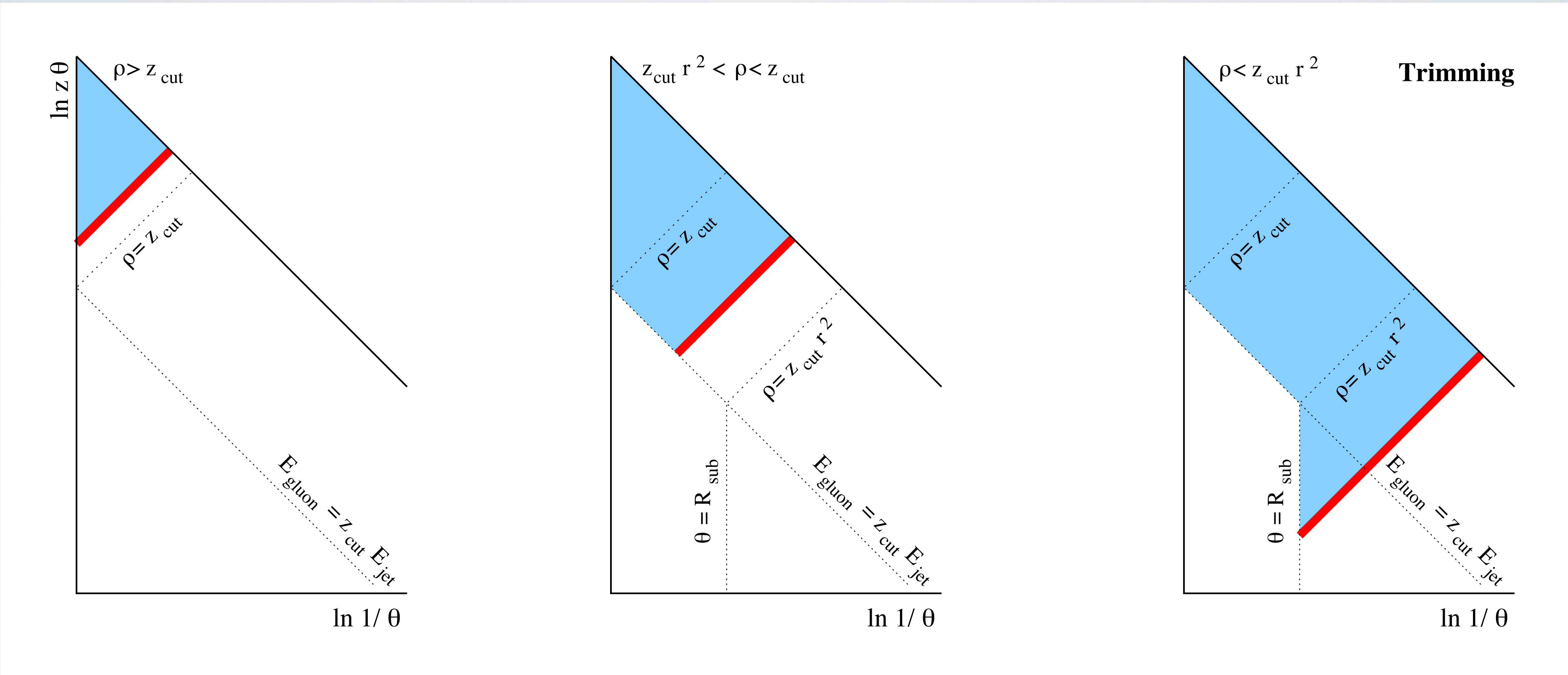
Lund diagrams for jet mass



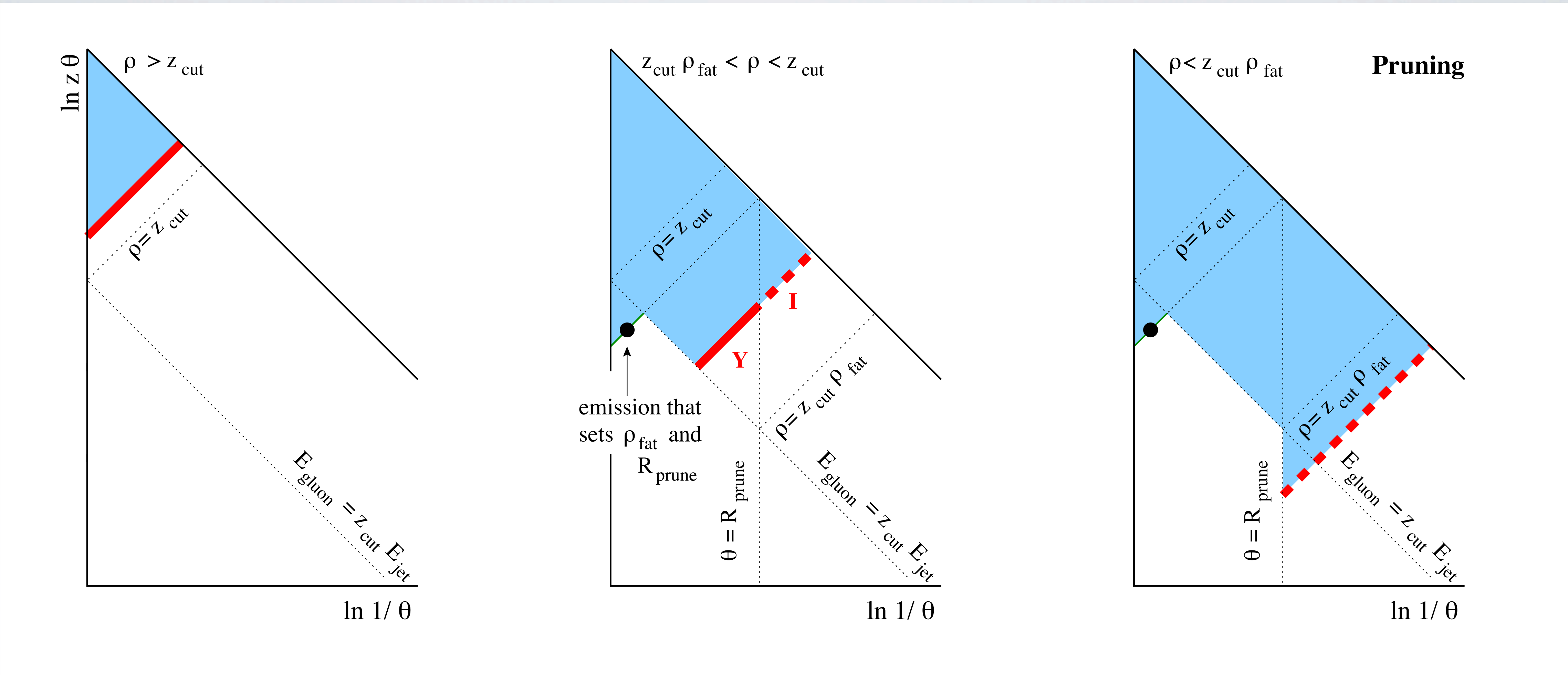
Summary table

	highest logs	transition(s)	Sudakov peak	NGLs	NP: $m^2 \lesssim$
plain mass	$\alpha_s^n L^{2n}$	—	$L \simeq 1/\sqrt{\bar{\alpha}_s}$	yes	$\mu_{\rm NP}\, p_t\, R$
trimming	$\alpha_s^n L^{2n}$	$z_{\rm cut},\ r^2 z_{\rm cut}$	$L \simeq 1/\sqrt{\bar{\alpha}_s} - 2\ln r$	yes	$\mu_{\rm NP}\, p_t\, R_{\rm sub}$
pruning	$\alpha_s^n L^{2n}$	$z_{\rm cut},\ z_{\rm cut}^2$	$L \simeq 2.3/\sqrt{\bar{\alpha}_s}$	yes	$\mu_{\rm NP}\, p_t\, R$
MDT	$\alpha_s^n L^{2n-1}$	$y_{\rm cut},\ \frac{1}{4}y_{\rm cut}^2,\ y_{\rm cut}^3$	—	yes	$\mu_{\rm NP}\, p_t\, R$
Y-pruning	$\alpha_s^n L^{2n-1}$	$z_{\rm cut}$	(Sudakov tail)	yes	$\mu_{\rm NP}\, p_t\, R$
mMDT	$\alpha_s^n L^n$	$y_{\rm cut}$	—	no	$\mu_{\rm NP}^2/y_{\rm cut}$

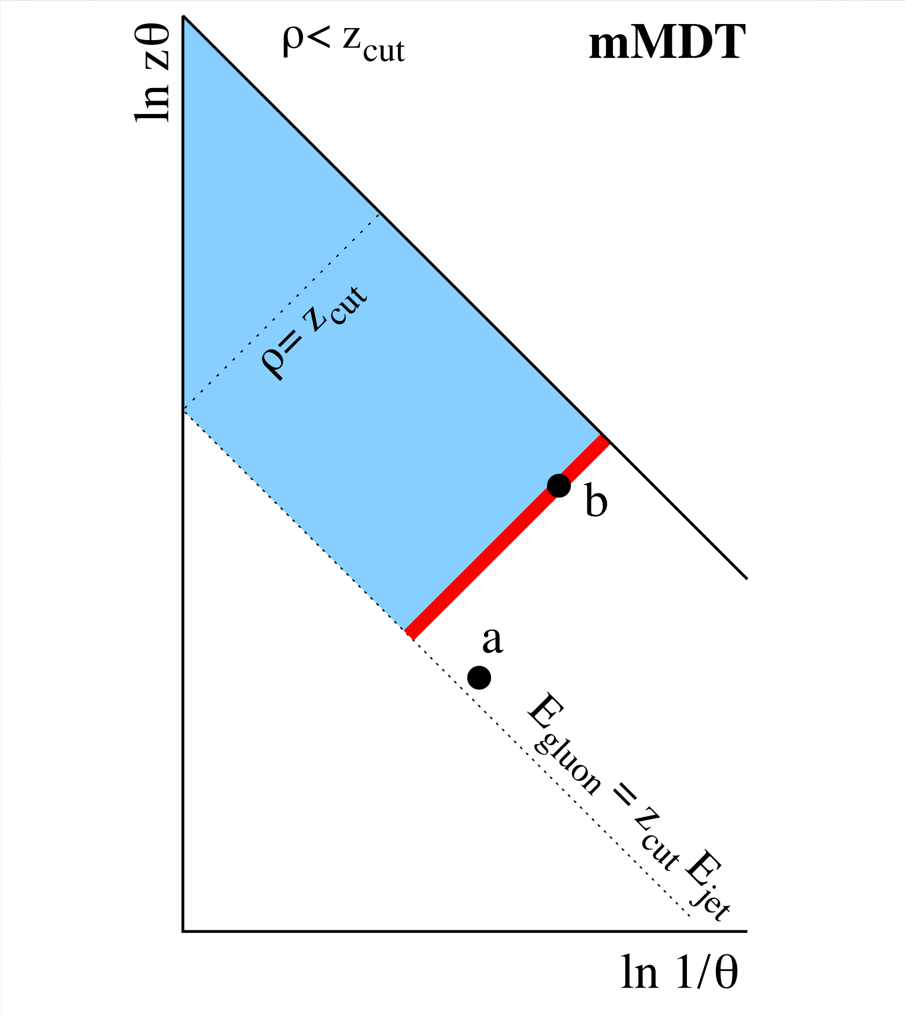
Lund diagrams for trimming



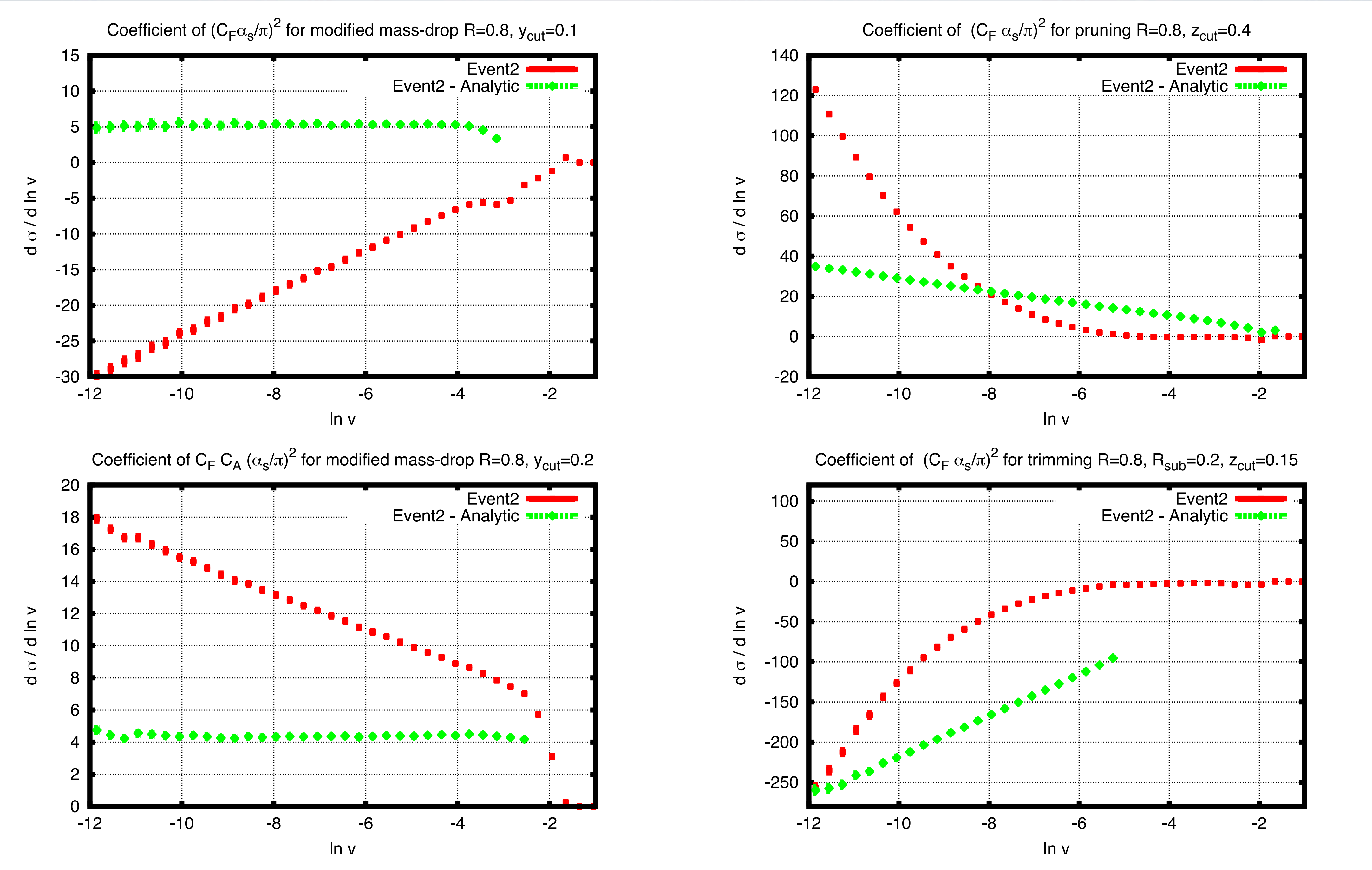
Lund diagrams for pruning



Lund diagrams for mMDT



Examples of NLO checks



$$\frac{\rho}{\sigma}\frac{d\sigma^{(\text{I-prune})}}{d\rho} = \int_{\rho}^{z_{\text{cut}}} \frac{d\rho_{\text{fat}}}{\rho_{\text{fat}}} \left(e^{-D(\rho_{\text{fat}})} \int_{\rho_{\text{fat}}}^{z_{\text{cut}}} \frac{dz'}{z'} \frac{\alpha_s(\rho_{\text{fat}} z'\, p_t^2 R^2)\, C_F}{\pi} \right) \times$$

$$\times\, e^{-S(\rho_{\text{fat}},\rho)} \int_{\rho/\rho_{\text{fat}}}^{1} dz\ p_{gq}(z) \frac{\alpha_s(\rho z\, p_t^2 R^2)\, C_F}{\pi} \left[\Theta\left(\frac{\rho}{\rho_{\text{fat}}} - z_{\text{cut}} \right) + \right.$$

$$\left. + \Theta\left(z_{\text{cut}} - \frac{\rho}{\rho_{\text{fat}}} \right) \exp\left(- \int_{\rho}^{z_{\text{cut}}\rho_{\text{fat}}} \frac{d\rho'}{\rho'} \int_{\rho'/\rho_{\text{fat}}}^{z_{\text{cut}}} \frac{dz'}{z'} \frac{C_F}{\pi} \alpha_s(\rho' z' p_t^2 R^2) \right) \right]$$

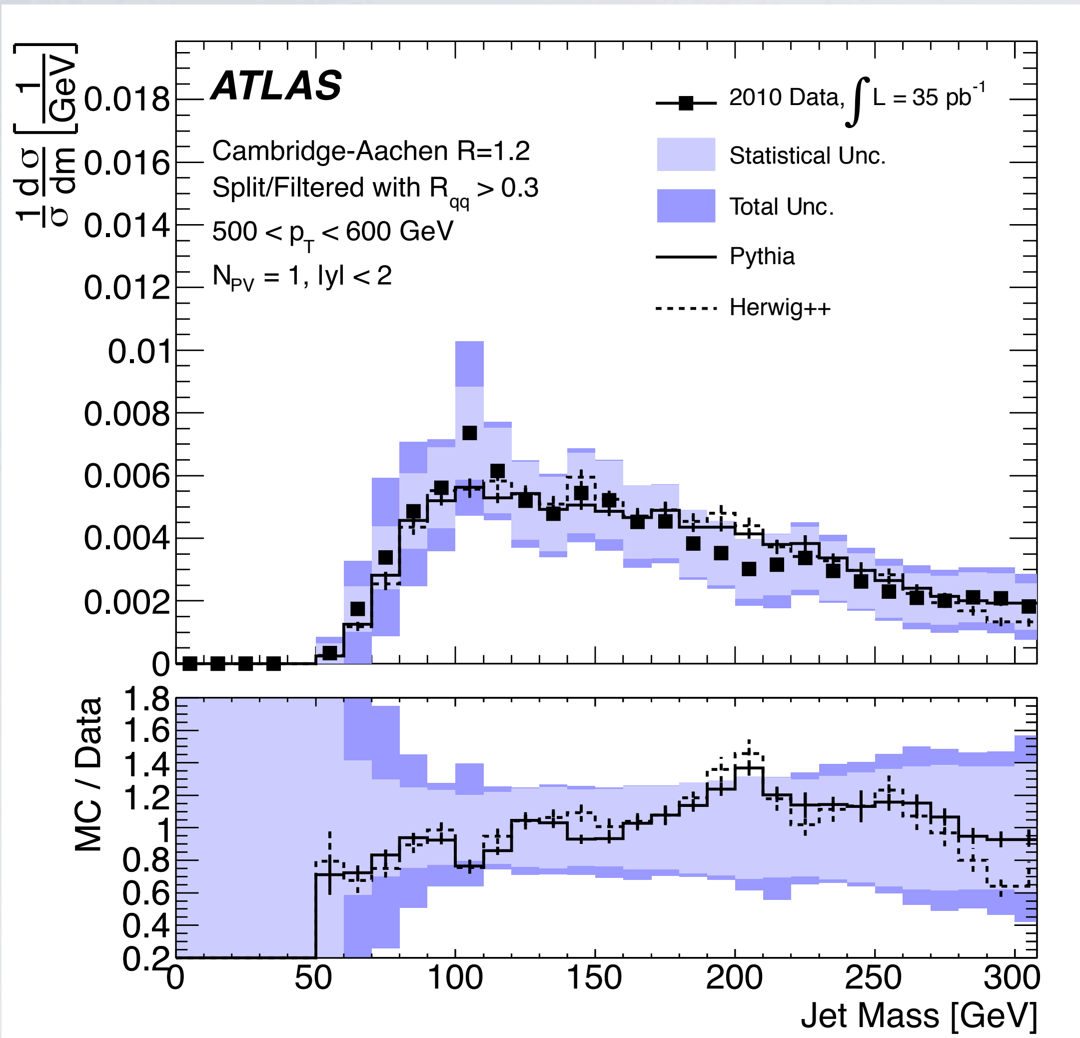
$$\frac{\rho}{\sigma}\frac{d\sigma^{(\text{Y-prune})}}{d\rho} = e^{-D(\rho)} \int_{z_{\text{cut}}}^{1} dz\ p_{gq}(z) \frac{\alpha_s(\rho z\, p_t^2 R^2)\, C_F}{\pi} +$$

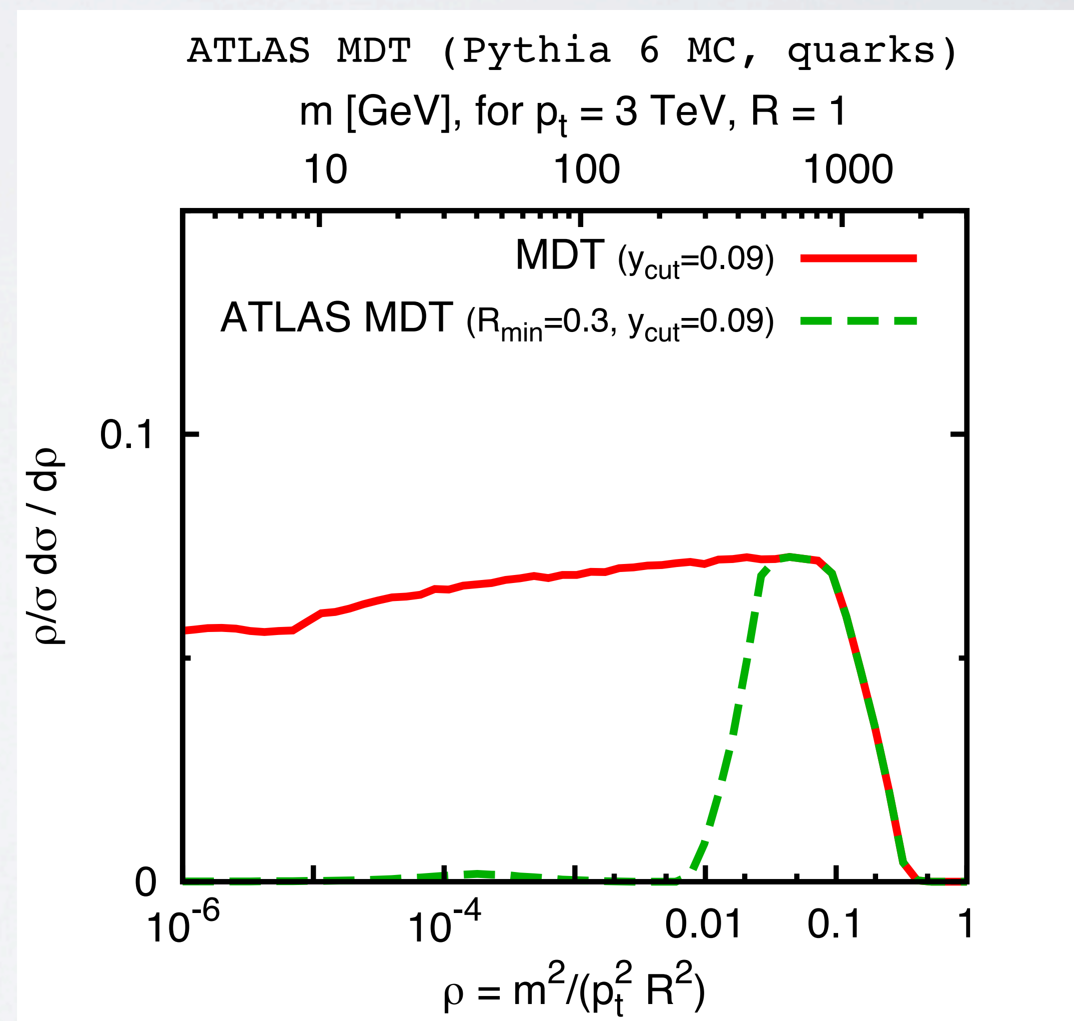
$$+ \int_{\rho}^{\min(z_{\text{cut}},\rho/z_{\text{cut}})} \frac{d\rho_{\text{fat}}}{\rho_{\text{fat}}} \left(e^{-D(\rho_{\text{fat}})} \int_{\rho_{\text{fat}}}^{z_{\text{cut}}} \frac{dz'}{z'} \frac{\alpha_s(\rho_{\text{fat}} z'\, p_t^2 R^2)\, C_F}{\pi} \right) \times$$

$$\times\, e^{-S(\rho_{\text{fat}},\rho)} \int_{z_{\text{cut}}}^{\rho/\rho_{\text{fat}}} dz\ p_{gq}(z) \frac{\alpha_s(\rho z\, p_t^2 R^2)\, C_F}{\pi}$$

ATLAS MDT

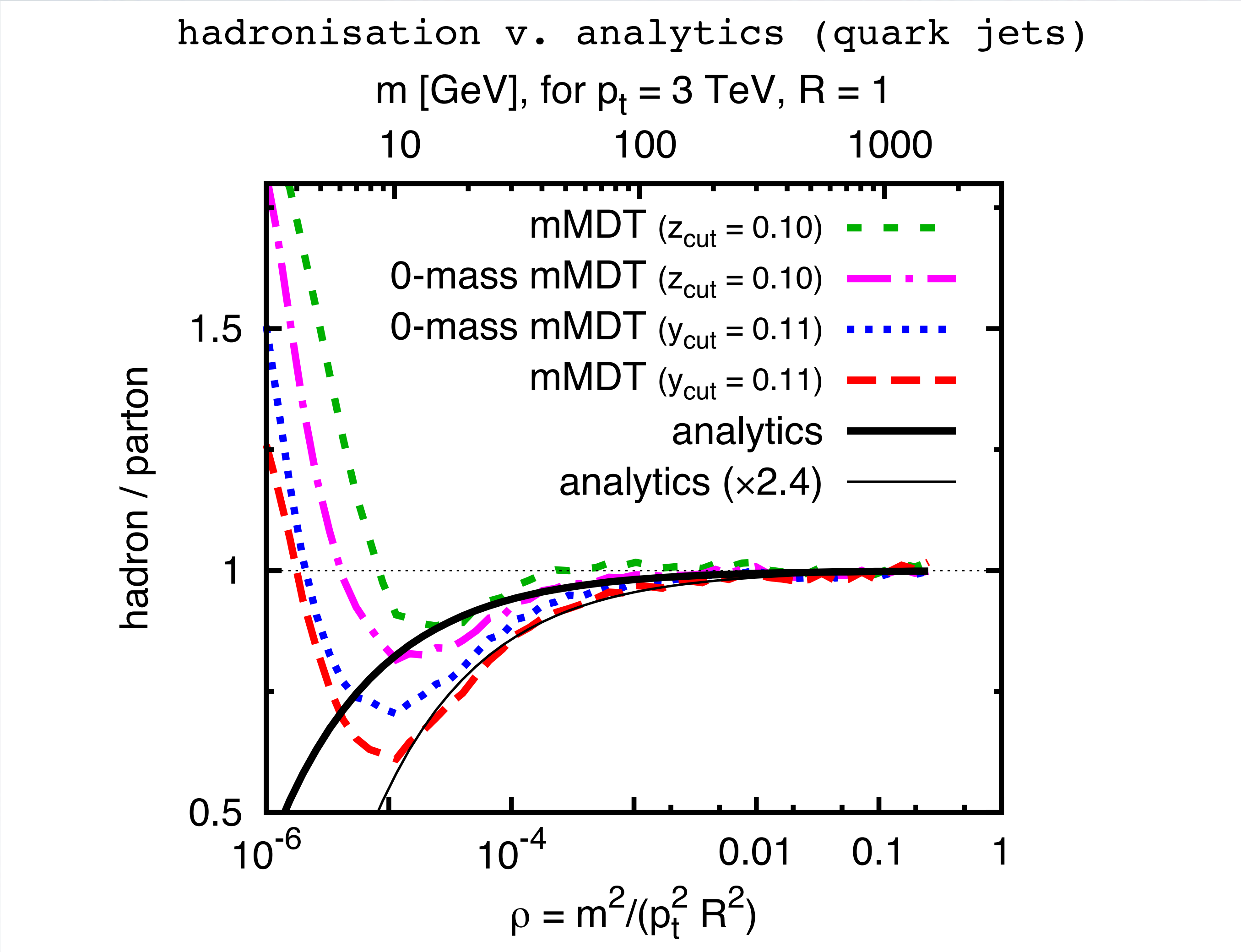
- ATLAS measured the jet mass with MDT
- Different version of the tagger with R_{min}=0.3 between the prongs





- This cut significantly changes the tagger's behaviour: mass minimum
- The single-log region is reduced (and can even disappear)
- We hope that future studies will be able to avoid this

Hadronisation effects for mMDT



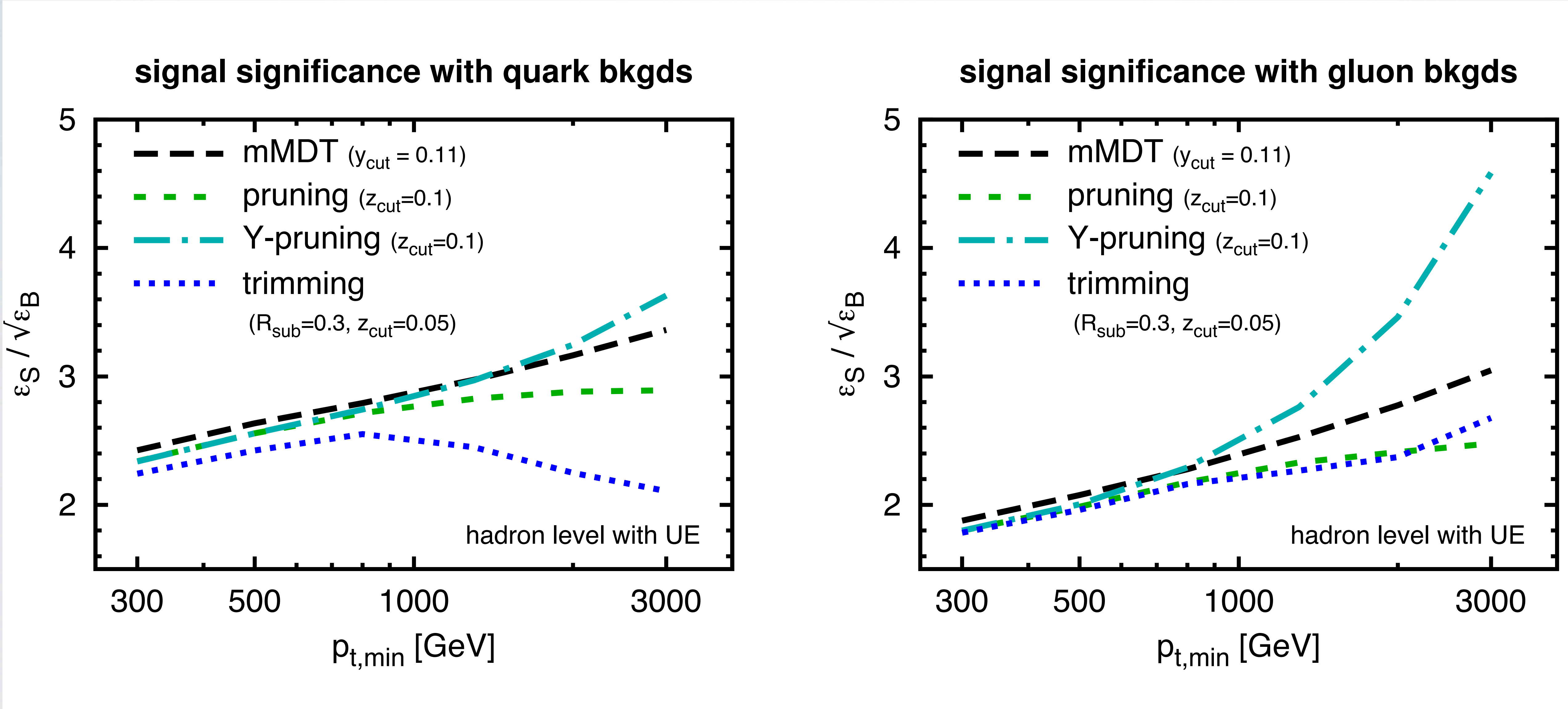
Hadronisation produces:

1. a shift in jet mass
2. a shift in the jet's (or prong's) momentum

Same power behaviour but with competing signs:

$$\frac{d\sigma^{\mathrm{NP}}}{dm} \simeq \frac{d\sigma^{\mathrm{PT}}}{dm}\left[1 + a\frac{\Lambda_{\mathrm{NP}}}{m}\right]$$

Performances for finding signals (Ws)



Y-pruning gives a visible improvement