A New Perspective on Quantum Field Theory

arXiv:1003.1366 [hep-th]

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Sussex U.

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Outline of this Lecture

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What is the Exact Renormalization Group?

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A microscope with variable resolving power

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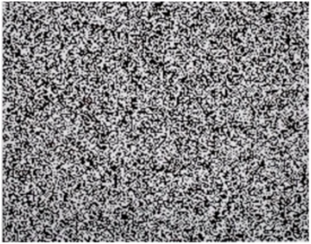

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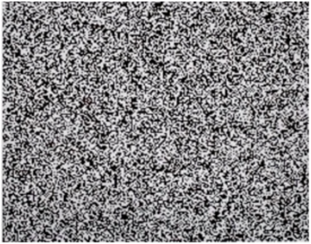

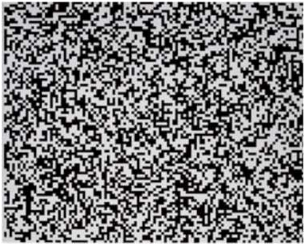

When should we use the Exact Renormalization Group?

The ERG is of use for systems with

- A large number of degrees of freedom per correlation length
- Local interactions

Easy Problems (no need for ERG)

- A small number of degrees of freedom per correlation length

- The subsystem is easy to understand
- The subsystem captures the behaviour of the whole system
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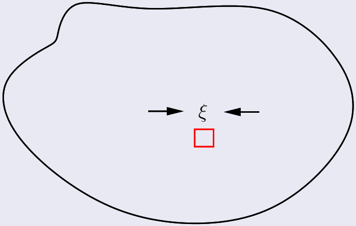

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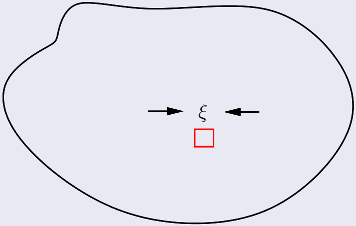

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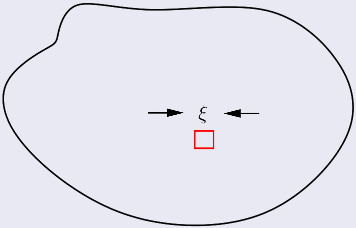

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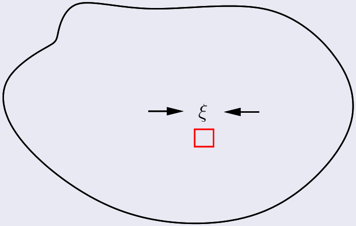

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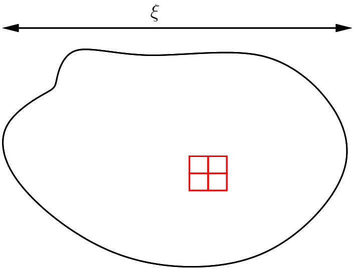

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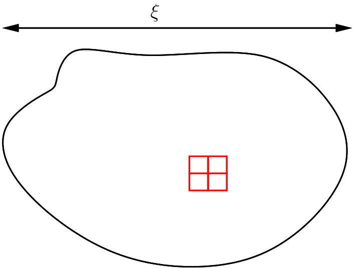

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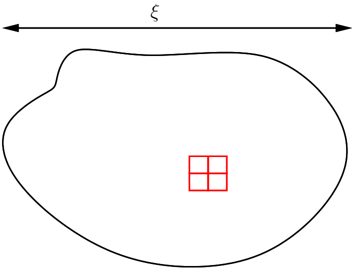

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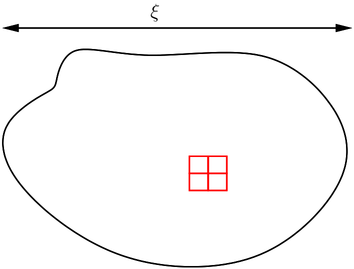

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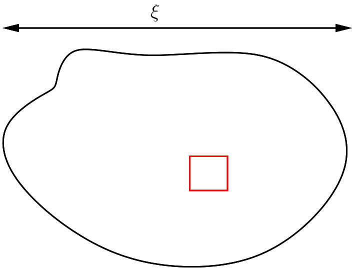

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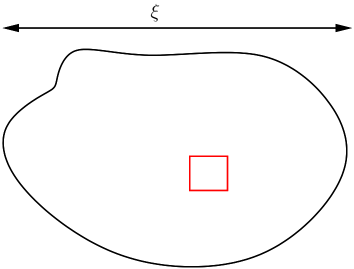

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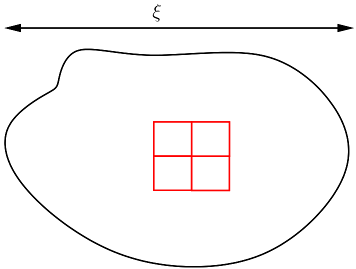

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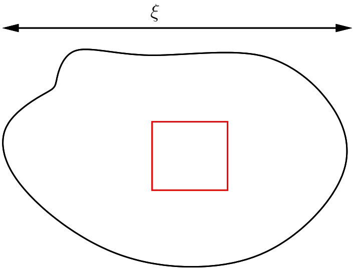

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Applications

- Quantum field theory
- Critical phenomena
- Kondo effect, ultra cold gases, nuclear physics

What has the ERG given us?

- A deep understanding of renormalization and universality
- A tool for performing real calculations

What's the catch?

- The coarse-graining procedure cannot be done exactly
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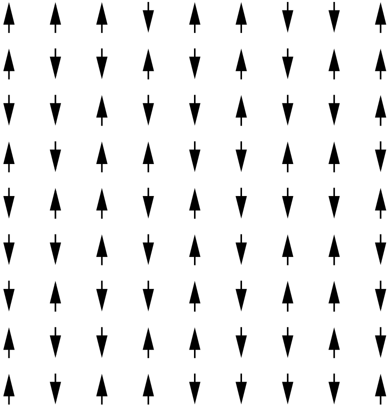

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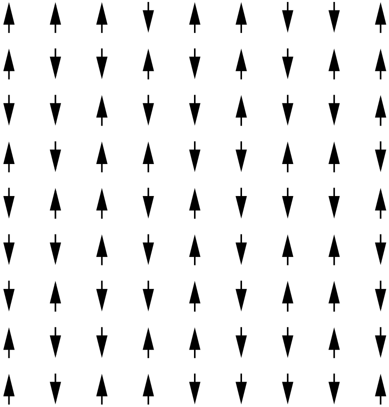

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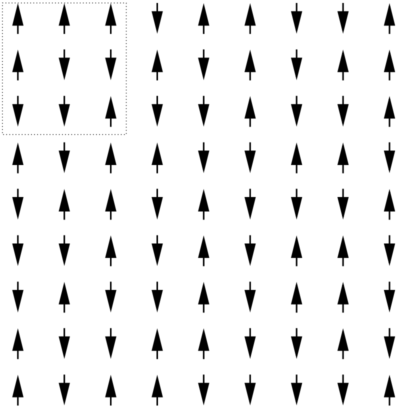

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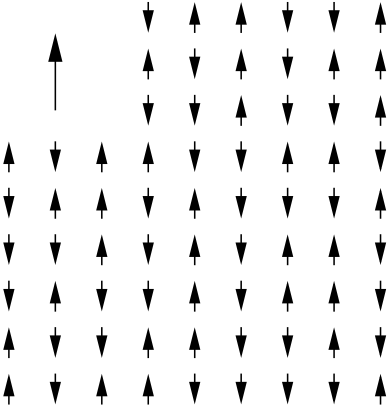

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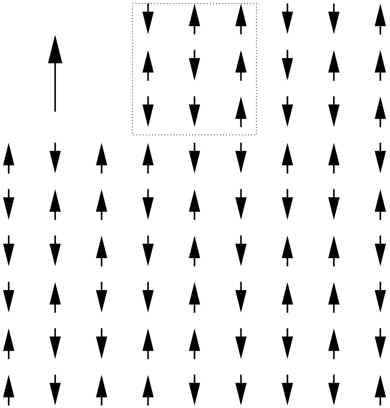

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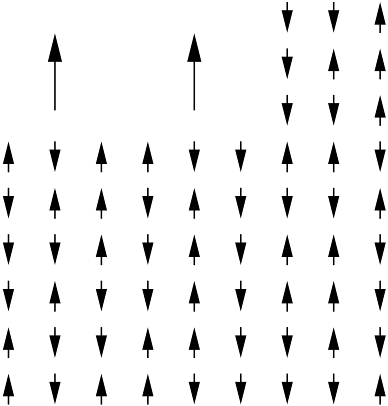

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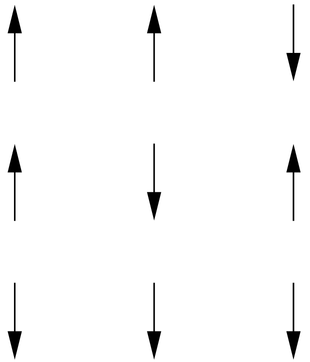

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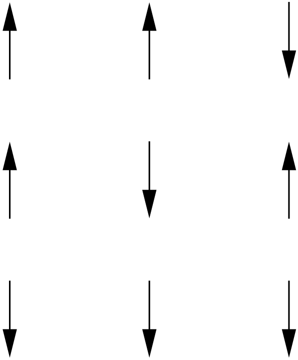

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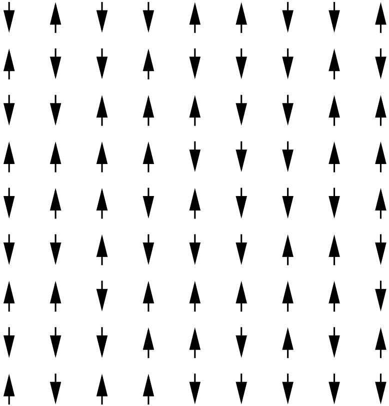

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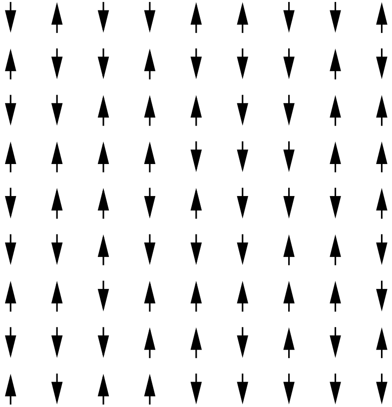

The ERG implements the continuous version of blocking

Flows in Parameter Space

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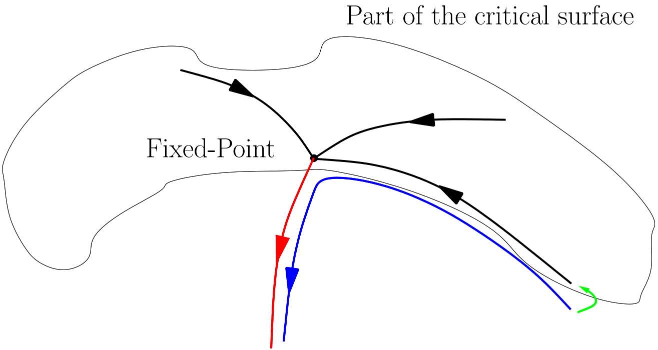

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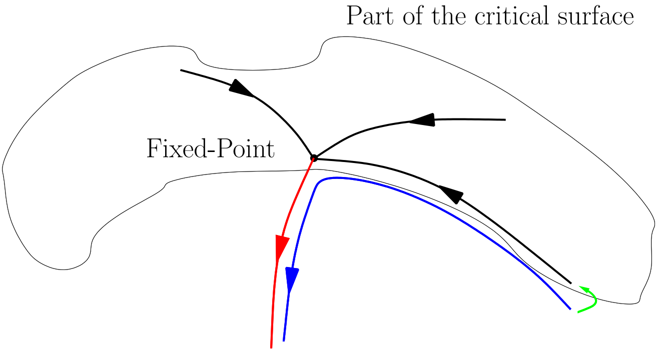

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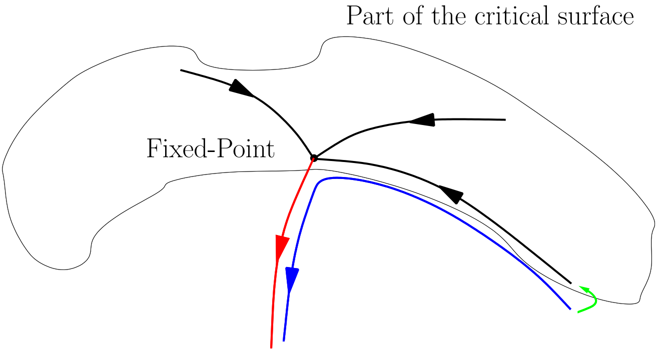

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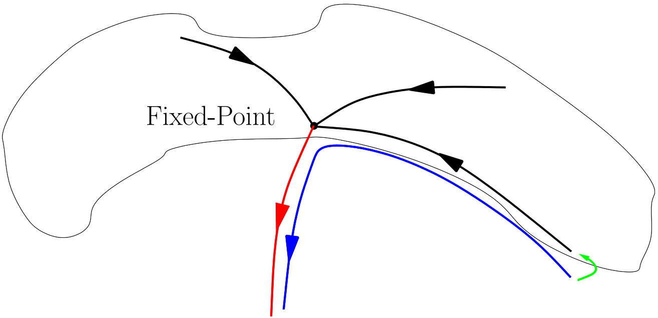

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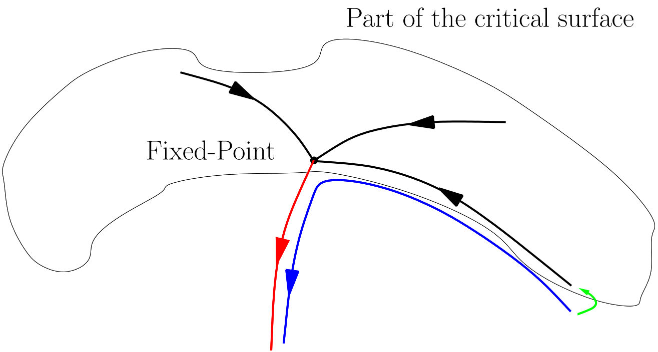

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- If there are *n* relevant directions, then we must tune *n* quantities to get on to the critical surface

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\phi e^{-S_{\Lambda_0}[\phi]} = \int_{\Lambda} \mathcal{D}\phi e^{-S_\Lambda[\phi]}$$

- The bare scale
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ
  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
  - The action evolves $\Rightarrow$ Wilsonian effective action

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- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ
  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
  - The action evolves $\Rightarrow$ Wilsonian effective action

The Wilsonian Effective Action

Start with the partition function

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Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\phi]} = \int_x \frac{\delta}{\delta \phi(x)} \left(\Psi_x[\phi] e^{-S[\phi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\phi \, e^{-S[\phi]}$, invariant under the flow
- Parametrizes blocking procedure

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \phi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \phi(x)}$$

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Rescaling

Ingredients of ERG Transformation

- Blocking (coarse graining)
- Rescaling

Implementing Rescaling

- Measure all dimensionful quantities in units of Λ
- Remember to take account of anomalous dimensions!

$$X \rightarrow X\Lambda^{\text{full scaling dimension}}$$

- Notation: ϕ dimensionful, φ dimensionless
- $-\Lambda\partial_\Lambda \rightarrow \partial_t$, with $t = \ln \mu/\Lambda$

What we need for this talk

- ERG Equation: $\partial_t S[\varphi] = \ldots$
- Fixed-points: $\partial_t S_\star[\varphi] = 0$

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Relevance/Irrelevance

- At a fixed-point we have $\partial_t S_\star = 0$
- Consider an infinitesimal perturbation

First order classification

- Operators that grow with t are relevant
- Operators that shrink with t are irrelevant
- Operators that stay the same are marginal

Marginal Operators

- $S_\star + \alpha \mathcal{O}_{\text{marginal}}$ is a fixed-point up to $\mathcal{O}(\alpha^2)$
- This might not be true beyond leading order
- Eg the four point coupling in $D = 4$ scalar field theory is marginally irrelevant
- An exactly marginal operator generates a line of fixed-points

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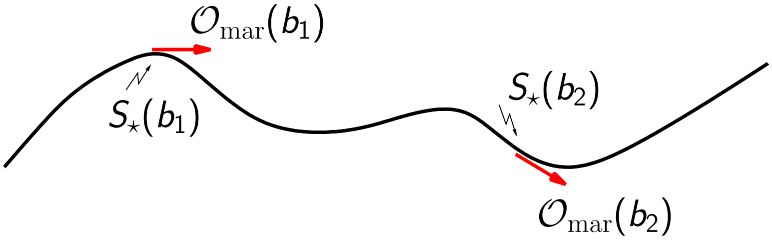

Redundant Operators

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The Fixed-Point Spectrum

The Anomalous Dimension of the Field

- The classical scaling dimension of the fundamental field is $(D-2)/2$
- In QFT, this becomes $(D-2+\eta_\star)/2$
- $\eta_\star$ is the anomalous dimension of the fundamental field

Quantization of the Anomalous Dimension

- Every critical fixed-point possesses an exactly marginal redundant direction
- This generates a line of equivalent fixed-points
- This implies that the spectrum of $\eta_\star$ is quantized

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- The classical scaling dimension of the fundamental field is $(D-2)/2$
- In QFT, this becomes $(D-2+\eta_\star)/2$
- $\eta_\star$ is the anomalous dimension of the fundamental field

Quantization of the Anomalous Dimension

- Every critical fixed-point possesses an exactly marginal redundant direction
- This generates a line of equivalent fixed-points
- This implies that the spectrum of $\eta_\star$ is quantized

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1. Qualitative Aspects of the ERG
2. Renormalizability
3. Correlation Functions in the ERG

Textbook renormalization

- Choose an action *e.g.*

$$S[\phi] = \int d^D x \left[\frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle \phi(x_1) \cdots \phi(x_n) \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, \phi(x_1) \cdots \phi(x_n) e^{-S[\phi]}$$

- Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

- If δS has the same form as S, the theory is renormalizable

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Wilsonian Renormalization

The Question

Are there effective actions $S_{\Lambda,\Lambda_0}[\varphi]$ for which we can safely send $\Lambda_0 \to \infty$?

The Simplest Affirmative Answer

- Fixed-points of the ERG correspond to renormalizable theories!

- Recall: all dimensionful quantities have been rescaled using Λ
- Therefore, S_* is independent of all scales, including Λ_0
- Trivially, we can send $\Lambda_0 \to \infty$

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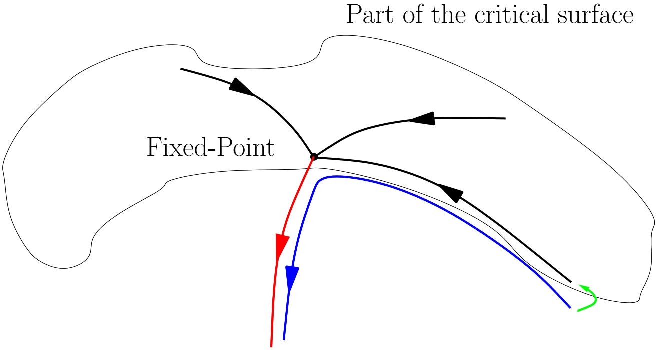

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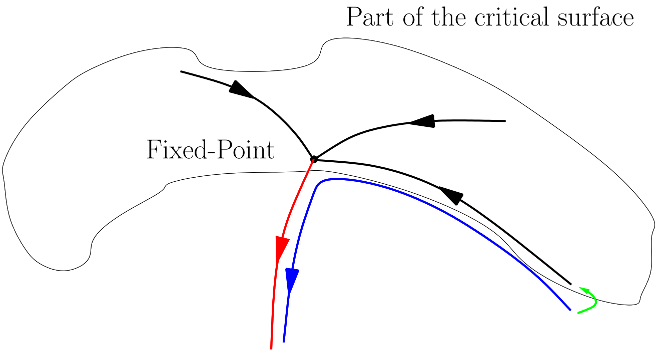

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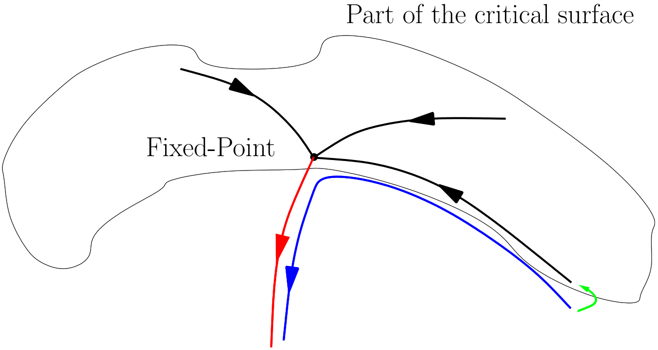

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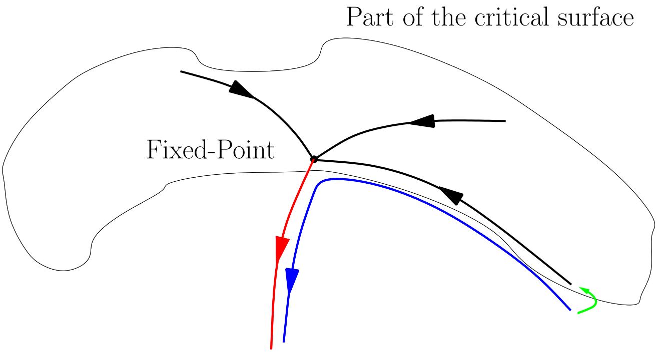

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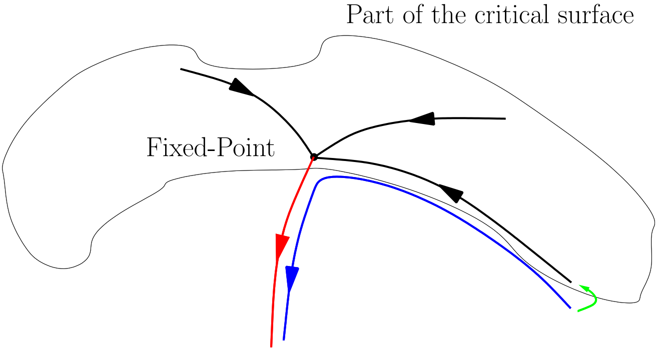

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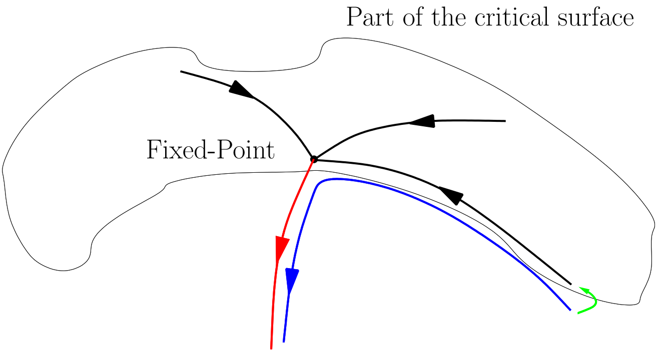

- Tune the trajectory towards the critical surface, as $\Lambda_0 \to \infty$
- The trajectory splits in two:
  - One part sinks into the fixed-point
  - One part emanates out
- Actions on the RT are renormalizable

The Key Point

Nonperturbatively renormalizable theories follow from fixed-points

- Either directly
- Or from the renormalized trajectories emanating from them

Theory Space

- QFTs should be understood in terms of theory space
- Renormalizable QFTs follow from the solutions of a functional equation

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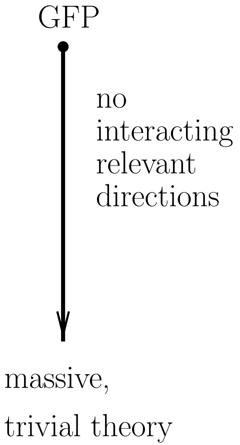

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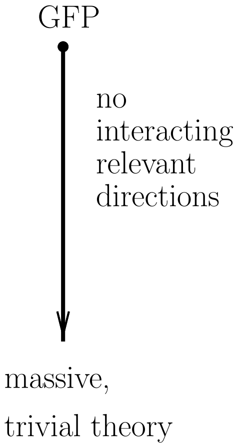

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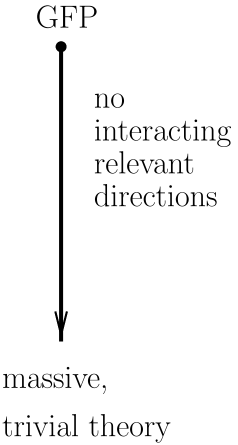

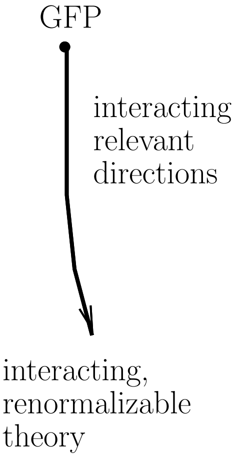

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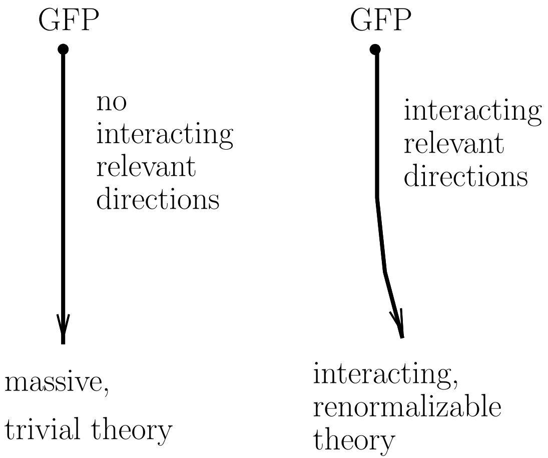

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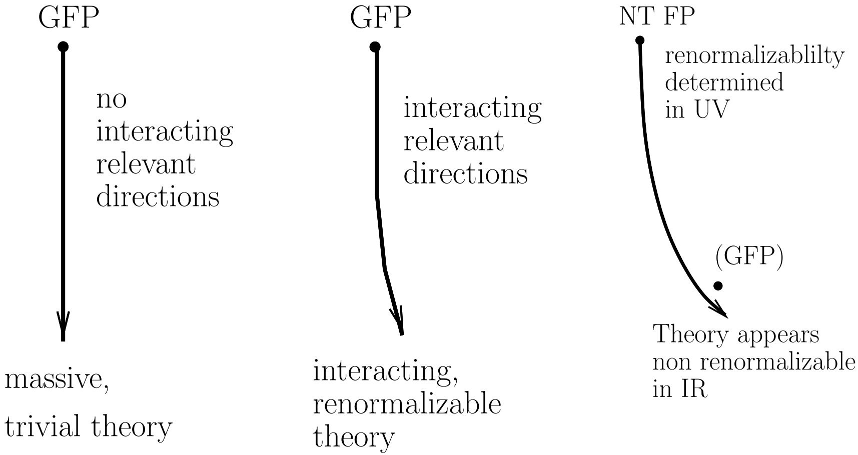

Scalar Field Theory: Four Dimensions

- Only the Gaussian FP exists
- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant
- The only nonperturbatively renormalizable scalar field theories in four dimensions are trivial!

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Gaussian Fixed-point

- The mass term is relevant
- The four-point coupling is relevant
- Non-trivial renormalizable theories exist along the $\lambda\varphi^4$ direction!

Wilson-Fisher Fixed-point

- In addition to the Gaussian FP, there is a non-trivial FP
- The W-F FP possesses a single relevant direction
- This can also be used to construct an RT

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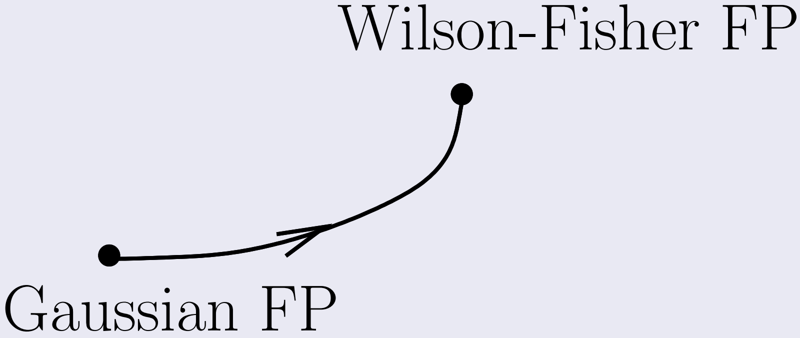
Example of a Continuum Limit in D=3



Textbook versus Wilsonian

Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of S_Λ

My aims in the rest of this talk

- To convince you that the question is profound

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Polchinski's Equation

- Polchinski made a particular choice

$$\Psi = \Psi_{\mathrm{Pol}}$$

Pros

- The flow equation is simple
- The correlation functions can be extracted from $S_{\Lambda \to 0}$
- Renormalizability of $S \Rightarrow$ renormalizability of $\langle \varphi(x_1) \cdots \varphi(x_n) \rangle$

Cons

- It is inconvenient for finding fixed-points

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A modified version of Polchinski's equation

- Allow for an extra field redefinition along the flow

$$\Psi = \Psi_{\mathrm{Pol}} + \psi$$

- Choose

$$\psi = -\frac{1}{2}\eta\phi, \qquad \eta \equiv \Lambda\frac{d\ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action

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A modified version of Polchinski's equation

- Allow for an extra field redefinition along the flow

$$\Psi = \Psi_{\text{Pol}} + \psi$$

- Choose

$$\psi = -\frac{1}{2}\eta\phi, \qquad \eta \equiv \Lambda \frac{d \ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
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The Standard Correlation Functions

- Introduce a source term in the bare action
- Extract the connected correlation functions from

Composite Operators

- Add additional source terms e.g.
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- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for J-dependence of the action

$$S_\Lambda[\phi] \to T_\Lambda[\phi, J]$$

- The flow equation follows as before

$$-\Lambda \partial_\Lambda e^{-T_\Lambda[\phi,J]} = \int d^D x \frac{\delta}{\delta \phi(x)} \left\{ \Psi(x) e^{-T_\Lambda[\phi,J]} \right\}$$

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The game

- Search for renormalizable, source-dependent solutions

The strategy

Nonperturbatively renormalizable solutions follow from fixed-points

- Either directly: $\partial_t \mathcal{T}_\star[\varphi, j] = 0$
- Or from relevant (source-dependent) perturbations

Notation

- ϕ, J dimensionful, $\quad \varphi, j$ dimensionless

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- Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

- Then there is always a source-dependent f-p

$$T_\star[\varphi, j] = S_\star[\varphi] + \left[e^{-\tilde{j}\cdot\varrho\cdot\delta/\delta\varphi} - 1\right]\left[S_\star[\varphi] + \frac{1}{2}\varphi \cdot f \cdot \varphi\right]$$

- $\tilde{j}(p) = j(p)/p^2$
- $\varrho = \varrho(p^2)$, $f = f(p^2)$

Two crucial points

- The solution only works if $d_J = (D + 2 - \eta_\star)/2$
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And more...

- For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

- Every eigenperturbation, $\mathcal{O}_i$, has a source-dependent extension

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- Every critical f-p has a particular source-dependent extension
- Every renormalized trajectory has a source-dependent extension
- This source-dependence corresponds to the boundary condition

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If we use the modified Polchinski equation with $\psi = -\eta\varphi/2$

Renormalizability of S_Λ implies renormalizability of the standard correlation functions

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A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions
- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

Philosophy

- The Wilsonian effective action is fundamental
- QFT determines which quantities we should compute

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- QFT determines which quantities we should compute

A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions
- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

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Additional Source-Dependent Fixed-Points?

- Additional fixed-points correspond to renormalizable composite operators
- For a scalar source, solutions correspond to the standard eigenperturbations, $\mathcal{O}_i$
- For a finite number of counter-terms, require $d_j > 0$

Theory Space & its Extension

Do source-dependent extensions of a single fixed-point probe global features of the standard theory space??

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Further Directions

Modified Polchinski Equation $\psi = -\eta\varphi/2$

- How does the OPE play a role?
- Can a link be made with methods of CFT?
- What is implied for asymptotic safety?

Other flow equations

- What happens for other flow equations?
- What does this imply for gauge theories?

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Gauge Theory

- It is possible to construct a gauge invariant cutoff, using
  - Covariant higher derivatives
  - Pauli-Villars fields
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- No gauge fixing is required at any stage!
- The formalism is very complicated

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- How do manifestly gauge invariant operators renormalize?
- Throw in sources and let the ERG tell us!

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Wilsonian

- Renormalizable theories follow from fixed points
- Fixed-points can be found by solving an equation

Extension

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- After finding a solution, we deduce the correlation functions to which it corresponds

The criterion of renormalizability

- Determines the interactions of QFTs
- Determines the objects we should be computing

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