Towards holography in vacuum Einstein gravity

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STAG RESEARCH CENTRE

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't Hooft '93, Susskind '94

- **Gravity is believed to be holographic:** it should be described by a non-gravitational theory in one dimension less

'**t Hooft** '93, **Susskind** '94

- **This is well understood for asymptotically anti-de Sitter spacetimes:** AdS/CFT correspondence

Maldacena '97, **Gubser Klebanov Polyakov** '98, **Witten** '98, ...

- **Original arguments for holography are insensitive to asymptotics**

- **Decoupling argument extends to nonconformal branes:**

Kanitscheider et al '08, **Wiseman & Withers** '08

- non-trivial dilaton, non-AdS asymptotics
- generalized dimensional reduction

Kanitscheider & **Skenderis** '09

We want to present a generalized dimensional reduction linking Ricci-flat and AdS solutions, to develop holography for Ricci-flat spacetimes

Holography

in

anti-de Sitter spacetimes

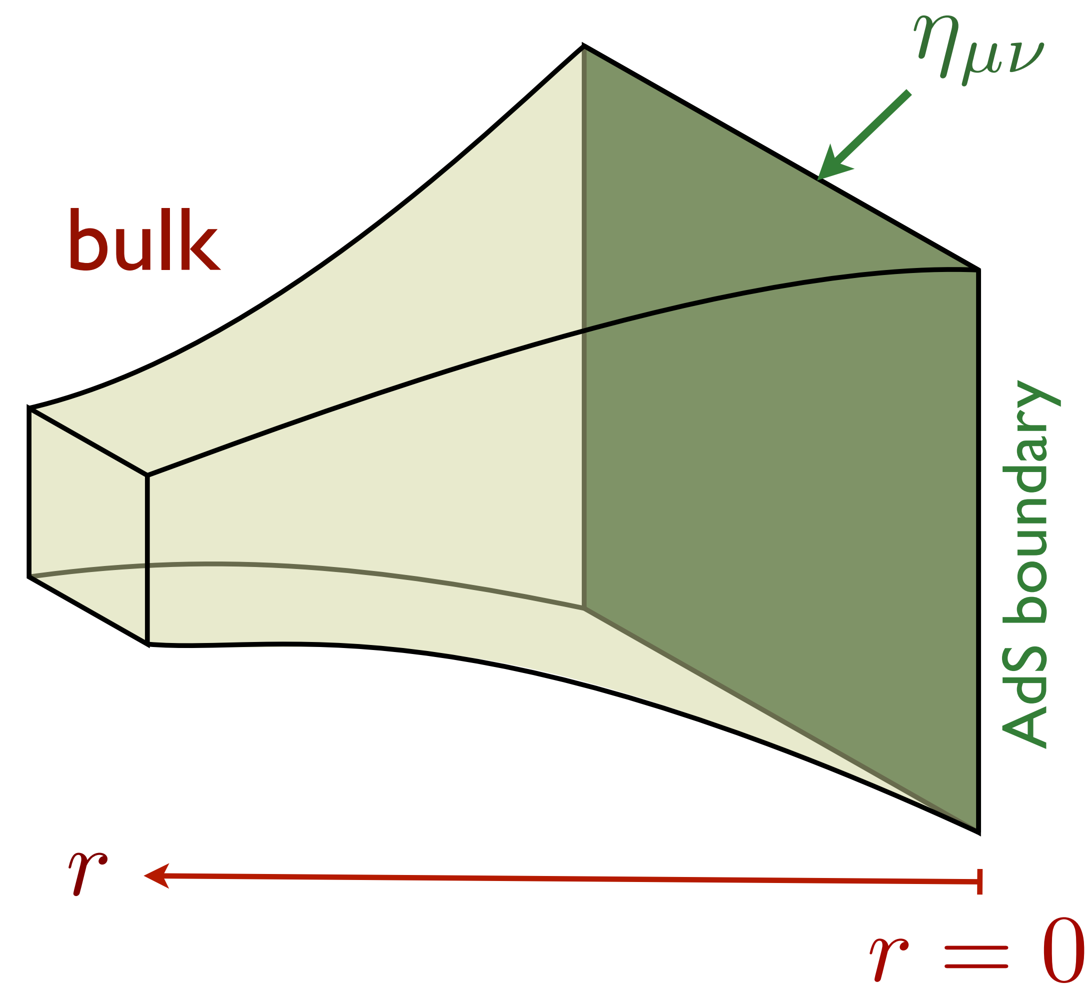
~ a lightning review ~

AdS Holography

anti-de Sitter (AdS)

$$ds^2_{d+1} = \frac{\ell^2}{r^2}\left(\mathrm{d}r^2 + \eta_{\mu\nu}\, dx^\mu dx^\nu\right)$$

$$\Lambda = -\frac{d(d-1)}{2\ell^2}$$



- ✧ Conformal **boundary** in $r = 0$, Minkowski in d dimensions (M_d)
- ✧ AdS isometry group is the **conformal group** of M_d
- ✧ AdS gravity is dual to a **conformal field theory** (CFT) on M_d
- ✧ The AdS solution represents the **vacuum** of the CFT

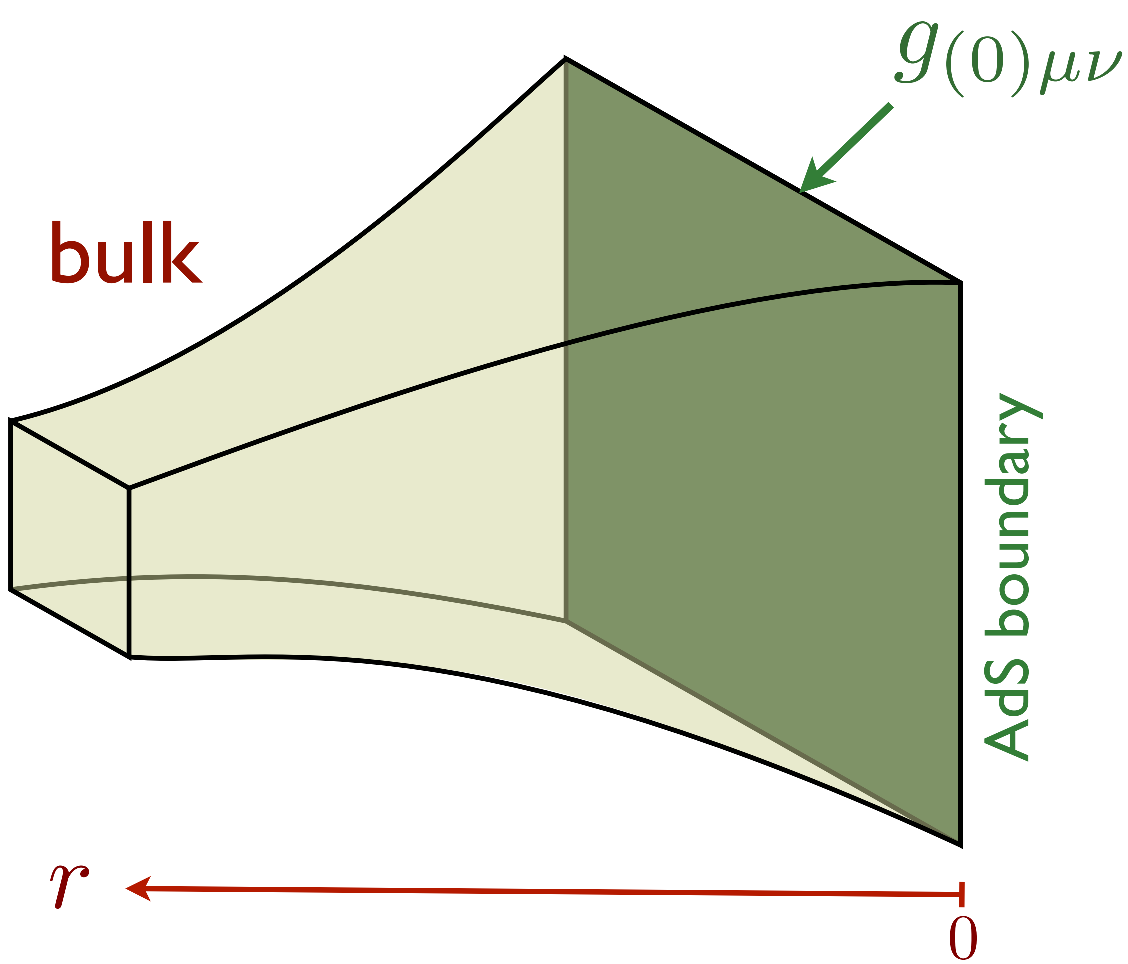
AdS Holography

Fefferman-Graham expansion near the boundary

$$ds^2 = \frac{\ell^2}{r^2}\left[\mathrm{d}r^2 + \left(g_{(0)\mu\nu} + r^2 g_{(2)\mu\nu} + \cdots + r^d g_{(d)\mu\nu} + \cdots\right) dx^\mu dx^\nu\right]$$

$g_{(0)\mu\nu}$ boundary metric

$g_{(d)\mu\nu}$ traceless and conserved, otherwise free



Dirichlet problem in AdS: fix the boundary metric (conformal class)

$$g_{(0)ij} \sim e^{2\sigma(x)} g_{(0)ij}(x)$$

AdS Holography

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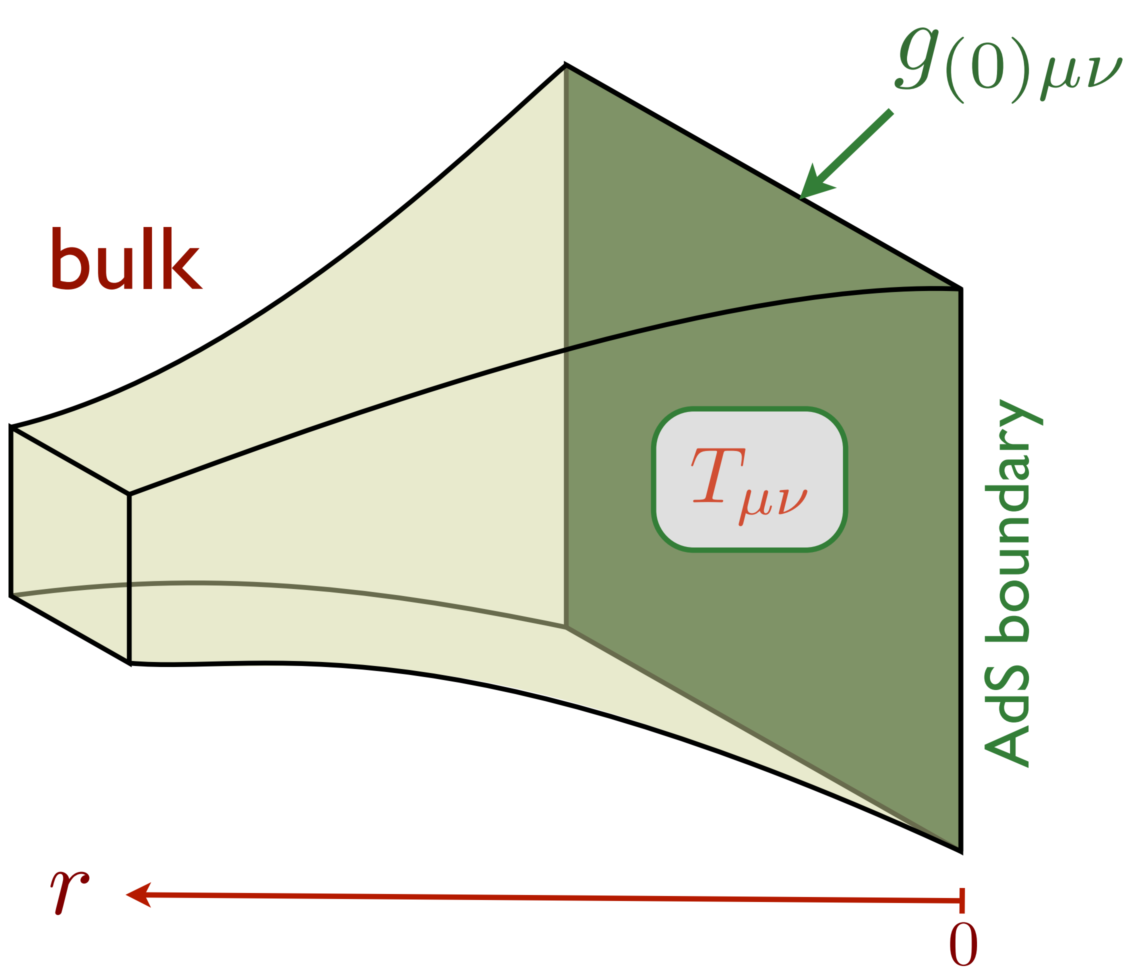
$$ds^2 = \frac{\ell^2}{r^2}\left[\mathrm{d}r^2 + \left(g_{(0)\mu\nu} + r^2 g_{(2)\mu\nu} + \cdots + r^d g_{(d)\mu\nu} + \cdots\right) dx^\mu dx^\nu\right]$$

$g_{(0)\mu\nu}$ source for the CFT stress energy tensor $T_{\mu\nu}$

$g_{(d)\mu\nu}$ expectation value of dual stress energy tensor

$$\langle T_{\mu\nu}\rangle \propto g_{(d)\mu\nu}$$



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Correlation functions

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$g_{(0)\mu\nu}$ source for the stress energy tensor $T_{\mu\nu}$

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Observables: correlators of local operators in dual CFT

Find the regular solution in the bulk satisfying appropriate Dirichlet boundary conditions. Perturbatively, expand $g_{(0)\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$g_{(d)\mu\nu} = g^{\text{bg}}_{(d)\mu\nu} + \mathcal{T}_{\mu\nu\rho\sigma} h^{\rho\sigma} + \frac{1}{2}\mathcal{T}_{\mu\nu\rho\sigma\alpha\beta} h^{\rho\sigma} h^{\alpha\beta} + \cdots$$

$\mathcal{T}_{\mu\nu\rho\sigma} \to \langle T_{\mu\nu} T_{\rho\sigma}\rangle$

$\mathcal{T}_{\mu\nu\rho\sigma\alpha\beta} \to \langle T_{\mu\nu} T_{\rho\sigma} T_{\alpha\beta}\rangle$

An example: 2-point function

Find the **regular linear** perturbation around AdS,

$$h_{\mu\nu}(k) = h_{(0)\mu\nu}(k)\frac{1}{2^{d/2-1}\Gamma(d/2)}\underbrace{(kr)^{d/2}K_{d/2}(kr)}_{1+\cdots+r^d k^d+\cdots}$$

Extract the 2-point function from the asymptotic expansion

$$\langle T_{\mu\nu}(k)T_{\rho\sigma}(-k)\rangle = \Pi_{\mu\nu\rho\sigma}k^d$$

projector to transverse traceless tensors

This is the correct 2-point function for the stress energy tensor of a CFT in d dimensions (d odd)

Can this construction be extended to asymptotically flat spacetimes?

A straightforward extension of this holographic procedure **fails** in asymptotically flat spacetimes!

WHY?

1. The fields that parametrize the boundary conditions are constrained
2. The infinities of the on-shell action are non local in these fields

We shall see that the holographic data is encoded in a different way!

AdS/Ricci-flat correspondence

~ a map linking AdS gravity and vacuum Einstein gravity ~

A map relating AdS and Ricci-flat solutions

MC, Camps, Goutéraux & Skenderis '12

1. Solutions to **AdS gravity** in d+1 dimensions of the form:

$$ds^2_\Lambda = d\hat{s}^2_{p+2}(x) + e^{\frac{2\phi(x)}{d-p-1}}\, d\vec{y}^2_{d-p-1}$$

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

$$\Lambda = -\frac{d(d-1)}{2\ell^2}$$

2. Extract (p+2)-dim metric $\hat{g}(x)$ and the scalar $\phi(x)$

3. Substitute $d \to -n$ in $\hat{g}(x)$ and $\phi(x)$

4. Insert back in $ds^2_0 = e^{\frac{2\phi(x)}{n+p+1}} \left(d\hat{s}^2_{p+2}(x) + \ell^2\, d\Omega^2_{n+1}\right)$ (unit $\mathcal{S}^{n+1}$)

Then, the metric ds^2_0 is **Ricci-flat** $\tilde{R}_{\mu\nu} = 0$

It solves **vacuum Einstein** equations in (n+p+3) dimensions

Trading curvatures: from AdS to Ricci-flat

AdS gravity

$D = d+1$

$$S = \frac{1}{16\pi G_N}\int_{\mathcal{M}} d^{d+1}x\,\sqrt{-g}\,(R-2\Lambda)$$

Reduction on $\mathcal{T}^{d-p-1}$

$$ds^2_\Lambda = d\hat{s}^2_{p+2} + e^{\frac{2\phi(x)}{d-p-1}}\,d\vec{y}^{\,2}_{d-p-1}$$

$\hat{D} = p+2$

$$\alpha = \frac{d-p-2}{d-p-1}$$

$$\beta = -2\Lambda$$

$$\hat{S} = \frac{1}{16\pi \hat{G}_N}\int_{\mathcal{M}} d^{p+2}x\,\sqrt{-\hat{g}}\,e^{\phi}\left(\hat{R} + \alpha\,(\partial\phi)^2 + \beta\right)$$

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Vacuum Einstein gravity

$\tilde{D} = n+p+3$

$$\tilde{S} = \frac{1}{16\pi \tilde{G}_N}\int_{\mathcal{M}} d^{n+p+3}x\,\sqrt{-\tilde{g}}\,\tilde{R}$$

Reduction on $\mathcal{T}^{d-p-1}$

$$ds^2_\Lambda = d\hat{s}^2_{p+2} + e^{\frac{2\phi(x)}{d-p-1}}\,d\vec{y}^2_{d-p-1}$$

Reduction on $\mathcal{S}^{n+1}$

$$ds^2_0 = e^{\frac{2\phi(x)}{n+p+1}}\left(d\hat{s}^2_{p+2} + \ell^2\,d\Omega^2_{n+1}\right)$$

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$$\alpha = \frac{d-p-2}{d-p-1}$$

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$$\alpha = \frac{n+p+2}{n+p+1}$$

$$\beta = \mathcal{R}_{\mathcal{S}^{n+1}}$$

Trading curvatures: from AdS to Ricci-flat

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Reduction on $\mathcal{T}^{d-p-1}$

$$ds^2_\Lambda = d\hat{s}^2_{p+2} + e^{\frac{2\phi(x)}{d-p-1}}\,d\vec{y}^2_{d-p-1}$$

Reduction on $\mathcal{S}^{n+1}$

$$ds^2_0 = e^{\frac{2\phi(x)}{n+p+1}}\left(d\hat{s}^2_{p+2} + \ell^2\,d\Omega^2_{n+1}\right)$$

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$$\beta = -2\Lambda$$

$$\alpha = \frac{n+p+2}{n+p+1}$$

$$\beta = \mathcal{R}_{\mathcal{S}^{n+1}}$$

$d \leftrightarrow -n$ $\qquad -2\Lambda \leftrightarrow \mathcal{R}_{\mathcal{S}^{\tilde{n}+1}}$

Dimension d (and n) enters analytically as a parameter in the equations of motion

Some remarks

1. Requires knowing the solution for any d (or n): we are mapping families of AdS solutions to families of Ricci-flat solutions

2. Analytical continuation $d \to -n$ on the lower dimensional theory: d and n should not be thought of as spacetime dimensions

3. We are trading the curvature of AdS with the curvature of the sphere

   $(-2\Lambda \leftrightarrow \mathcal{R}_{\mathcal{S}^{\tilde{n}+1}})$

4. This is an example of *Generalized Dimensional Reduction*

   (cf. Kanitscheider & Skenderis '09, Goutéraux, Smolic, Smolic, Skenderis & Taylor '11, Goutéraux & Kiritsis '11)

The resulting Ricci-flat class of solutions has an underlying holographic structure and hidden conformal symmetry inherited from the locally asymptotically AdS class of solutions.

Some simple examples

~ what happens to simple known solutions under this map? ~

First example: **AdS$_{d+1}$ on a Torus**

1. AdS spacetime in d+1 dimensions:

$\mathcal{T}^{d-p-1}$

$$\mathrm{d}s^2_\Lambda = \frac{\ell^2}{r^2}\left(\mathrm{d}r^2 + \eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + \mathrm{d}\vec{y}^2\right)$$

2. Extract the metric and scalar:

$$ds^2_\Lambda = d\hat{s}^2_{p+2} + e^{\frac{2\phi}{d-p-1}}\, d\vec{y}^2_{d-p-1} \Rightarrow \begin{cases} d\hat{s}^2_{p+2} = \dfrac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}\, dx^a dx^b\right) \\ \phi(x) = -(d-p-1)\ln\dfrac{r}{\ell} \end{cases}$$

3. Substitute $d \to -n$

$$\Rightarrow \begin{cases} d\hat{s}^2_{p+2} = \dfrac{\ell^2}{r^2}\left(dr^2 + \eta_{ab}\, dx^a dx^b\right) \\ \phi(x) = (n+p+1)\ln\dfrac{r}{\ell} \end{cases}$$

4. Lift to n+p+3 dimensions:

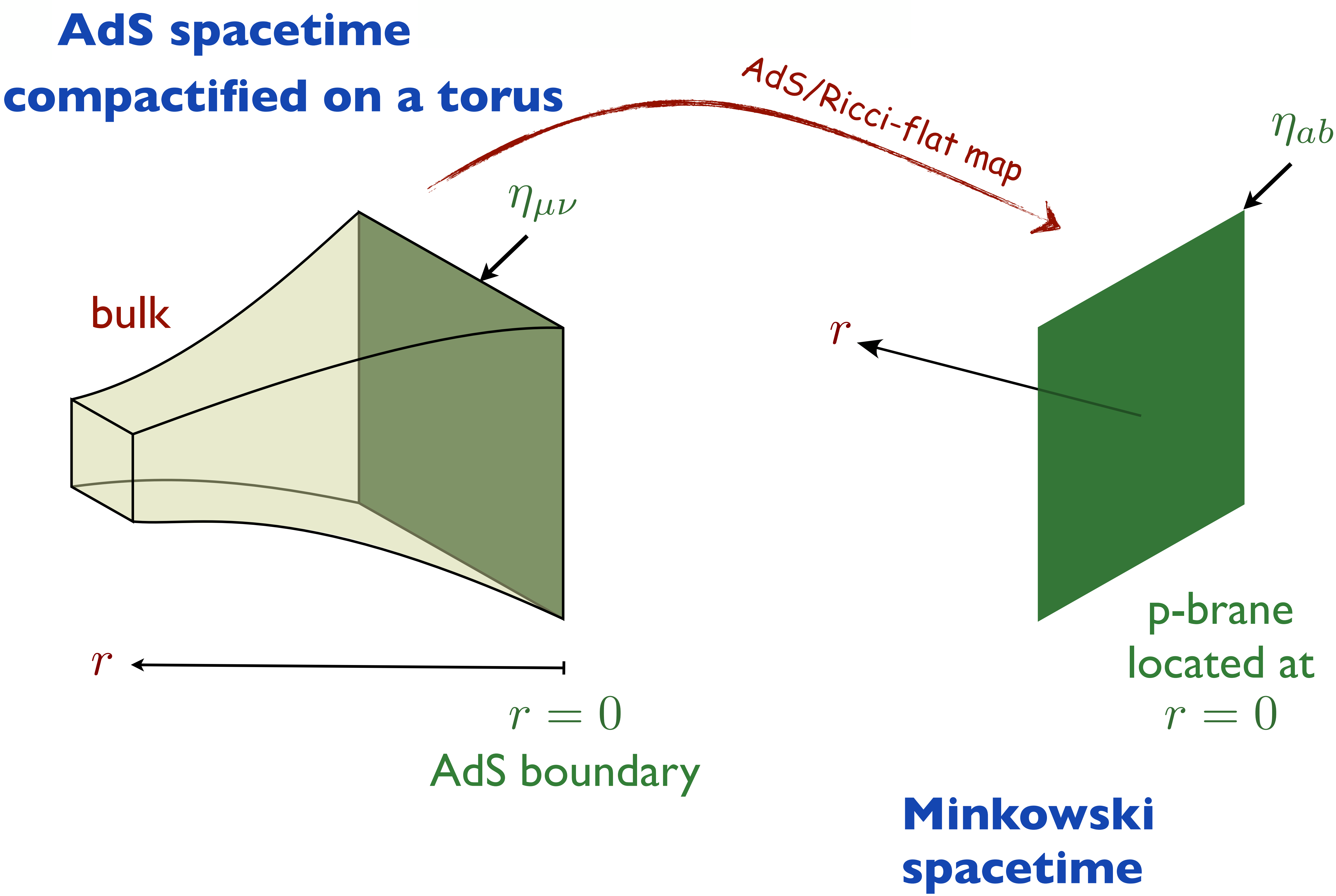
$\mathcal{S}^{n+1}$

$$ds^2_0 = e^{\frac{2\phi}{n+p+1}}\left(d\hat{s}^2_{p+2} + \ell^2\, d\Omega^2_{n+1}\right)$$

$$\Rightarrow \quad ds^2_0 = \underbrace{\eta_{ab}\mathrm{d}x^a\mathrm{d}x^b}_{\mathbb{R}^{1,p}} + \underbrace{\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{n+1}}_{\mathbb{R}^{n+2}}$$

Minkowski in n+p+3 dim.

First example: **AdS$_{d+1}$ on a Torus**



Second example: **Excitations on top of AdS**

1. Fefferman-Graham coordinates for Einstein-AdS solutions: $(\rho = r^2)$

$$\mathrm{d}s_\Lambda^2 = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}\left(\eta_{\mu\nu} + \rho^{d/2} g_{(d)\mu\nu} + \cdots\right)\mathrm{d}z^\mu \mathrm{d}z^\nu$$

Flat boundary metric

$$T_{\mu\nu} = \frac{d}{16\pi G_N} g_{(d)\mu\nu},$$

Expectation value of the dual stress tensor

The stress tensor satisfies:

$$\partial^a T_{ab} = 0\,, \qquad T_a{}^a = 0$$

as a consequence of the gravitational field equations (Ward identities for the CFT on flat background)

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$$\mathrm{d}s^2_\Lambda = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}\left(\eta_{\mu\nu} + \rho^{d/2} g_{(d)\mu\nu} + \cdots\right) \mathrm{d}z^\mu \mathrm{d}z^\nu$$

Flat boundary metric ← compactify (d-p-1) of these flat directions

2. Reduced theory: $\mathrm{d}\hat{s}^2 = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}\left(\eta_{ab} + \rho^{d/2}(\hat{g}_{(d)ab} + \rho\hat{g}_{(d+2)ab} + \ldots)\right) \mathrm{d}x^a \mathrm{d}x^b$

$$\phi = \rho^{d/2}\hat{\phi}_{(d)} + \rho^{d/2+1}\hat{\phi}_{(d+2)} + \ldots$$

Holographic dictionary for nonconformal branes: **Kanitscheider** & **Skenderis** '09

$$\hat{T}_{ab} = \frac{d}{16\pi\hat{G}_N}\hat{g}_{(d)ab}, \quad \hat{\mathcal{O}}_\phi = -\frac{d(d-p-1)}{32\pi\hat{G}_N}\hat{\phi}_{(d)}$$

expectation values of the dual stress energy tensor and of the scalar operator

Ward identities: $\partial^a \hat{T}_{ab} = 0\,, \qquad \hat{T}_a{}^a = (d-p-1)\hat{\mathcal{O}}_\phi$

the expectation value of the scalar operator breaks conformal invariance

Second example: **Excitations on top of AdS**

3. & 4. Analytical continuation and uplift to n+p+3 dimensions: $(\rho = 1/r^2)$

$$
\begin{aligned}
\mathrm{d}s_0^2 = &\left(1 - \frac{16\pi \hat{G}_N}{n\, r^n}(1+ \frac{r^2}{2(n-2)}\Box)\hat{\mathcal{O}}_\phi(x)\right)\left(\mathrm{d}r^2 + \eta_{ab}\mathrm{d}x^a\mathrm{d}x^b + r^2\mathrm{d}\Omega_{n+1}^2\right) \\
&- \frac{16\pi \hat{G}_N}{n\, r^n}(1 + \frac{r^2}{2(n-2)}\Box)\hat{T}_{ab}(x)\,\mathrm{d}x^a\mathrm{d}x^b + \dots \\
= &\left(\eta_{AB} + h_{AB} + \dots\right)\mathrm{d}x^A\mathrm{d}x^B
\end{aligned}
$$

As a perturbation of flat spacetime it verifies:

$$\bar{h}_{AB} = h_{AB} - \frac{h}{2}\eta_{AB} \qquad \Box\bar{h}_{AB} = 16\pi\hat{G}_N\Omega_{n+1}\delta_A{}^a\delta_B{}^b\hat{T}_{ab}\delta^{n+2}(r)$$

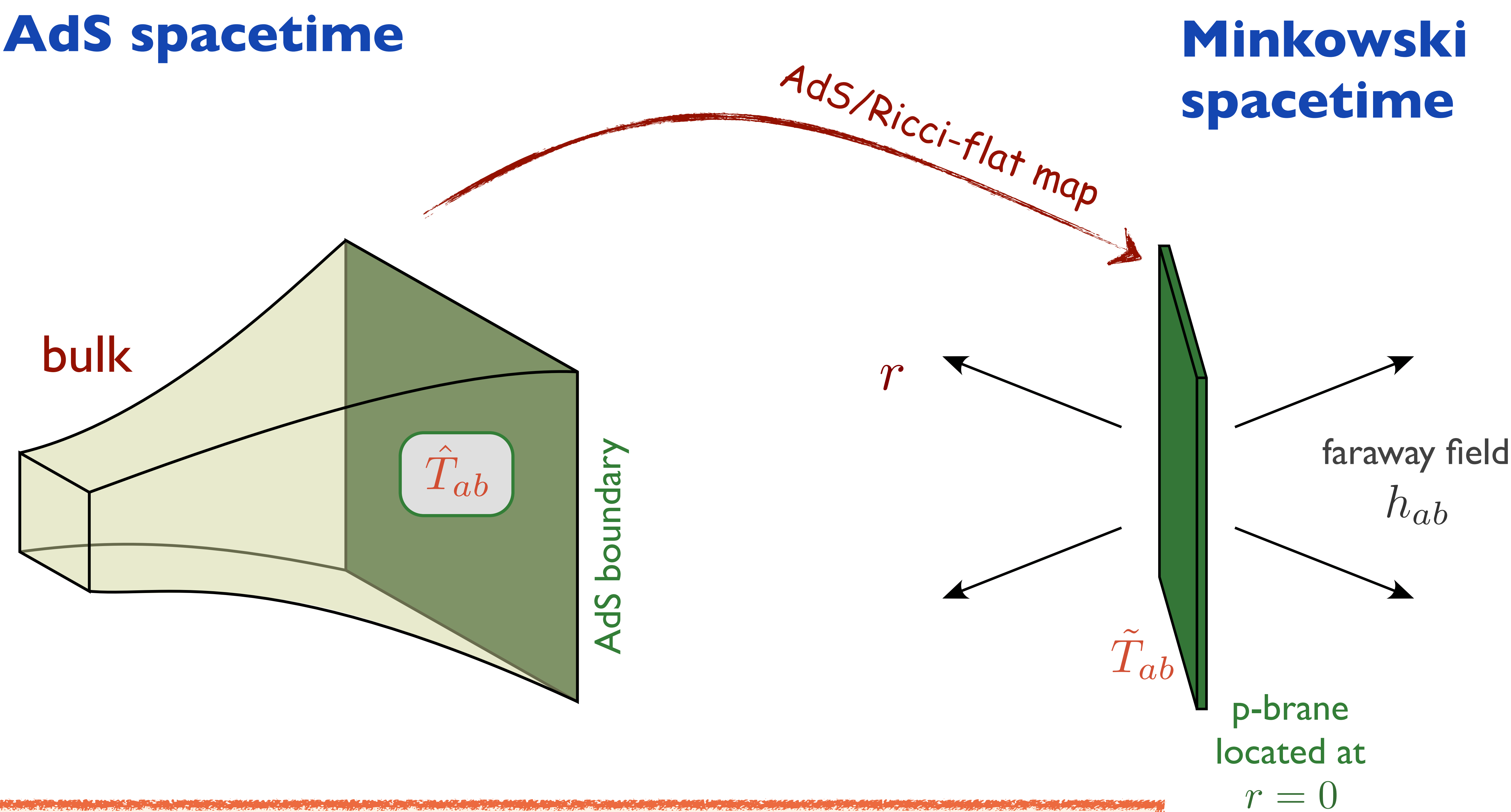
i.e. it solves linearized Einstein eqns $\Box\bar{h}_{AB} = -16\pi\tilde{G}_N\tilde{T}_{AB}$

with $\tilde{T}_{ab} = -\frac{\hat{G}_N}{\tilde{G}_N}\Omega_{n+1}\hat{T}_{ab}\delta^{n+2}(r)$

(stress tensor of a p-brane located at r=0)

Holographic stress tensor sources the faraway grav. field

Second example: **Excitations on top of AdS**



The holographic stress tensor is (minus) the stress tensor of a brane, located at the origin of Minkowski, that sources the linearized gravitational field h_{ab}

Correlation functions

To compute correlation functions we set $g_{(0)\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

and find **regular**, linear transverse traceless fluctuation in AdS

$$h^{\text{AdS}}_{\mu\nu}(k) = h_{(0)\mu\nu}(k) \frac{1}{2^{d/2-1}\Gamma(d/2)} (kr)^{d/2} K_{d/2}(kr)$$

Apply AdS/Ricci flat correspondence, with $d \to -n$

$$h^{\text{Mink}}_{\mu\nu}(k) = h_{(0)\mu\nu}(k) \frac{2^{n/2+1}}{\Gamma(-n/2)} \frac{K_{n/2}(kr)}{(kr)^{n/2}}$$

Linearized gravitational field produced by a **p-brane** with **worldvolume metric** $\eta_{\mu\nu} + h_{\mu\nu}$

Exponential fall-off at infinity: the metric is **asymptotically flat**

First entries in the holographic dictionary

On AdS, the boundary condition was to choose a metric on the boundary

This translates on the Ricci-flat side into a **choice of a metric at the location of a *p*-brane**

At linear order, the holographic stress energy tensor becomes the **stress energy tensor due to this *p*-brane**, that sources the linearized gravitational field

The regularity in the bulk of AdS becomes the requirement that **the Ricci-flat perturbation preserves asymptotic flatness**

Generalized conformal structure

The AdS metric $ds^2_\Lambda = \frac{\ell^2}{r^2}\left(\mathrm{d}r^2 + \eta_{\mu\nu}\, dx^\mu dx^\nu\right)$ is invatiant under isometries forming the boundary **conformal group**

Dilatations $\delta_\lambda x^M = \lambda x^M$

Special conformal transformations $\begin{cases} \delta_b z^\mu = b^\mu z^2 - 2z^\mu(z\cdot b) + r^2 b^\mu \\ \delta_b r = -2(z\cdot b) r \end{cases}$

Holographic stress energy tensor traceless and conserved

Compactification over a torus **breaks** these symmetries: the scalar now transforms as $\delta_\lambda \phi = (p+1-d)\lambda$

$$\delta_b \phi = -2(p+1-d)(x\cdot b)$$

The stress energy tensor is still conserved, but has a **non vanishing trace** $\partial^a \hat{T}_{ab} = 0\,, \qquad \hat{T}_a{}^a = (d-p-1)\hat{\mathcal{O}}_\phi$

Hidden symmetries and solution generating transformations

On Minkowski side they act as **conformal transformation**

$$\delta g_{0AB} = 2\sigma(x) g_{0AB}$$

with $\sigma(x) = \lambda$ for dilatations

$\sigma(x) = -2(x \cdot b)$ for special conformal transformations

They are not isometries of Minkowski, but the resulting metric is still Ricci-flat: they act as **solution generating transformations**

The underlying generalized conformal structure constrains the physics of these Ricci-flat spacetimes

Third example: **black branes**

Planar AdS black holes:

$\mathcal{T}^{d-p-1}$

$$\mathrm{d}s_\Lambda^2 = z^2(-f(z)\mathrm{d}\tau^2 + \mathrm{d}\vec{x}^2 + \mathrm{d}\vec{y}^2) + \frac{\mathrm{d}z^2}{z^2 f(z)},$$

$$f(z) = 1 - \frac{1}{(bz)^d}$$

$$d \leftrightarrow -n \qquad \Updownarrow \qquad z = \frac{1}{r}, \quad b = r_0$$

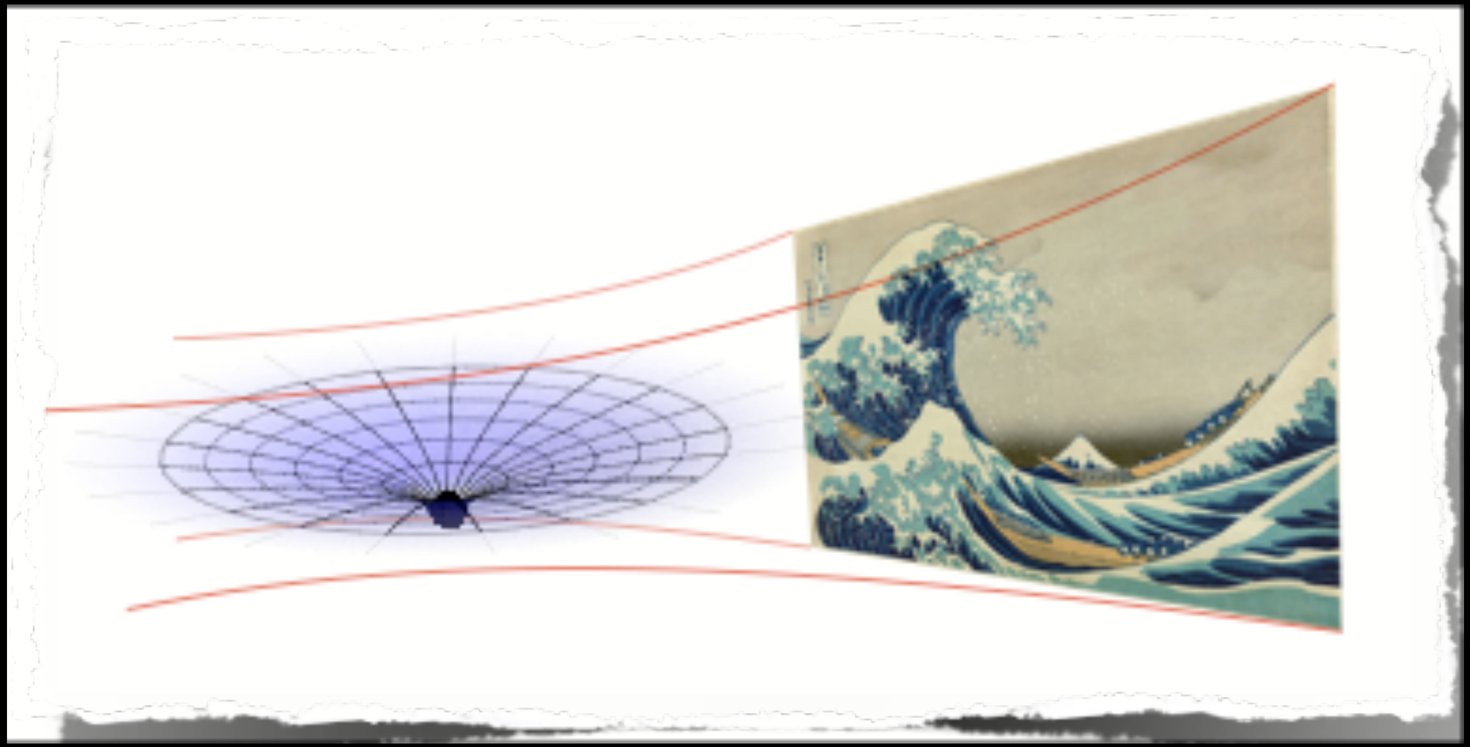
Schwarzschild black *p*-branes:

$\mathcal{S}^{n+1}$

$$\mathrm{d}s_0^2 = \underbrace{-f(r)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega_{n+1}^2}_{\text{Schwarzschild}} + \mathrm{d}\vec{x}^2$$

$$f(r) = 1 - \frac{r_0^n}{r^n}$$

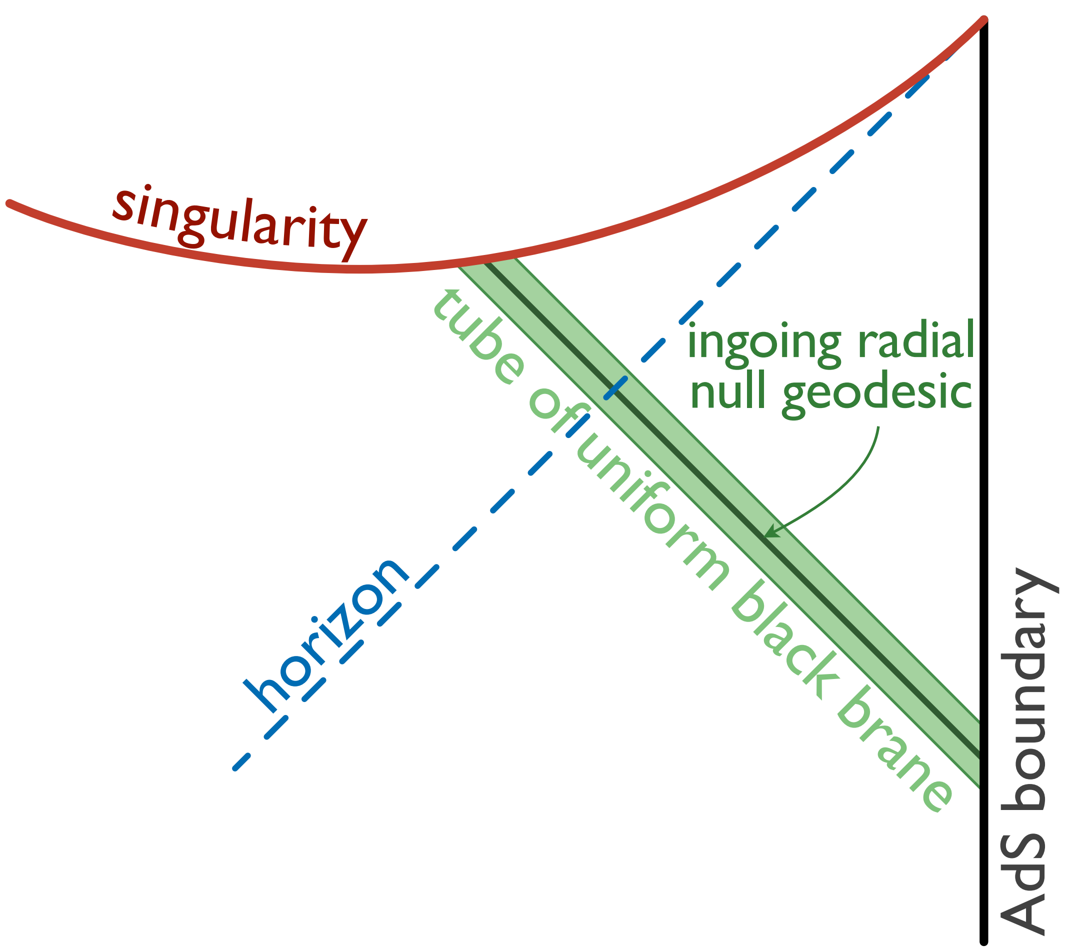
The Gravity/Fluid correspondence



~ applying the map to hydrodynamic perturbations of bhs ~

Fluid/gravity metrics in AdS

- Field theories are expected to equilibrate locally at high enough density $\Rightarrow$ **hydrodynamic** description in $\omega \to 0,\ \lambda \to \infty$ limit
- Hydrodynamic limit in **AdS/CFT**:
  Einstein eqns $\Leftrightarrow$ **Navier-Stokes eqns**
- Fluid/gravity metric can be built perturbatively from a black hole by varying slowly its temperature and boost



Patch-wise construction in Eddington-Finkelstein coords.

Bhattacharyya, Hubeny, Minwalla & Rangamani '07

The solution is corrected order by order in a derivative expansion by viscous corrections

The AdS fluid/gravity metric and ST...

Bhattacharyya, Loganayagam, Mandal, Minwalla & Sharma '08

AdS/fluid metric: $$ds^2 = -2u_\mu\, dx^\mu \left(dr + \mathcal{V}_\nu dx^\nu\right) + \mathcal{G}_{\mu\nu} dx^\mu dx^\nu$$

$$\mathcal{V}_\mu = r\mathcal{A}_\mu + \frac{1}{d-2}\left[\mathcal{D}_\lambda \omega^\lambda{}_\mu - \mathcal{D}_\lambda \sigma^\lambda{}_\mu + \frac{\mathcal{R}u_\mu}{2(d-1)}\right] - \frac{2L(br)}{(br)^{d-2}} P_\mu^\nu \mathcal{D}_\lambda \sigma^\lambda{}_\nu$$
$$- \frac{u_\mu}{2(br)^d}\left[r^2\left(1-(br)^d\right) - \frac{1}{2}\omega_{\alpha\beta}\omega^{\alpha\beta} - (br)^2 K_2(br)\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{(d-1)}\right]$$

$$\mathcal{G}_{\mu\nu} = r^2 P_{\mu\nu} - \omega_{\mu\lambda}\omega^\lambda{}_\nu + 2br^2 F\sigma_{\mu\nu} + 2(br)^2\sigma_{\mu\lambda}\sigma^\lambda{}_\nu\left[F^2 - H_1\right] + 2(br)^2\left[H_1 - K_1\right]\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1}P_{\mu\nu} +$$
$$+ 2(br)^2 u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu}\left[H_2 - H_1\right] + 4(br)^2 H_2\, \omega_{(\mu|\lambda|}\sigma^\lambda{}_{\nu)}$$

Holographic ST: (conformal fluid)

$$T_{\mu\nu} = P\left(g_{\mu\nu} + d u_\mu u_\nu\right) - 2\eta\sigma_{\mu\nu} - 2\eta\tau_\omega\left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\mu\lambda}\right]$$
$$+ 2\eta b\left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1}P_{\mu\nu}\right]$$

$$b \equiv \frac{d}{4\pi T} \qquad P = \frac{1}{16\pi G_N b^d} \qquad \epsilon = (d-1)P \qquad c_s^2 = \frac{1}{d-1}$$

$$\eta = \frac{s}{4\pi} = \frac{1}{16\pi G_N b^{d-1}} \qquad \zeta = 0 \qquad \tau_\omega = b\int_1^\infty \frac{\xi^{d-2}-1}{\xi(\xi^d-1)}d\xi$$

... are mapped to the blackfold metric / ST

MC, J Camps, B Goutéraux, K Skenderis

- AdS/Ricci-flat map gives the metric describing **long wavelength** perturbations of a **black *p*-brane**, whose (intrinsic) dynamics is captured by an **effective fluid** (**blackfold** Emparan, et al '09):

$$\tilde{T}_{\alpha\beta} = \tilde{P}\left(\eta_{\alpha\beta} - \tilde{n}\tilde{u}_\alpha\tilde{u}_\beta\right) - 2\tilde{\eta}\tilde{\sigma}_{\alpha\beta} - \tilde{\zeta}\tilde{\theta}\tilde{P}_{\alpha\beta}$$
$$+ 2\tilde{\eta}\tilde{\tau}_\omega\left[\tilde{P}_\alpha{}^\gamma\tilde{P}_\beta{}^\delta\tilde{u}^\epsilon\partial_\epsilon\tilde{\sigma}_{\gamma\delta} - \frac{\tilde{\theta}\tilde{\sigma}_{\alpha\beta}}{\tilde{n}+1} + 2\tilde{\omega}_{(\alpha}{}^\gamma\tilde{\sigma}_{\beta)\gamma}\right] + \tilde{\zeta}\tilde{\tau}_\omega\left[\tilde{P}_{\alpha\beta}\tilde{u}^\lambda\partial_\lambda\tilde{\theta} - \frac{1}{\tilde{n}+1}\tilde{\theta}^2\,\tilde{P}_{\alpha\beta}\right]$$
$$- 2\tilde{\eta}\tilde{b}\left[\tilde{P}_\alpha{}^\gamma\tilde{P}_\beta{}^\delta\tilde{u}^\epsilon\partial_\epsilon\tilde{\sigma}_{\gamma\delta} + \left(\frac{2}{p} + \frac{1}{\tilde{n}+1}\right)\tilde{\theta}\tilde{\sigma}_{\alpha\beta} + \tilde{\sigma}_\alpha{}^\gamma\tilde{\sigma}_{\gamma\beta} + \frac{\tilde{\sigma}^2}{\tilde{n}+1}\tilde{P}_{\alpha\beta}\right]$$
$$- \tilde{\zeta}\tilde{b}\left[\tilde{P}_{\alpha\beta}\tilde{u}^\gamma\partial_\gamma\tilde{\theta} + \left(\frac{1}{p} + \frac{1}{\tilde{n}+1}\right)\tilde{\theta}^2\tilde{P}_{\alpha\beta}\right]$$

Pressure

$$\tilde{P} = -\frac{\tilde{b}^{\tilde{n}}}{16\pi\tilde{G}_N}$$

Energy density

$$\tilde{\epsilon} = -(\tilde{n}+1)\tilde{P}$$

Speed of sound

$$\tilde{c}_s^2 = -\frac{1}{\tilde{n}+1}$$

Shear viscosity

$$\tilde{\eta} = \frac{\tilde{s}}{4\pi} = \frac{\tilde{b}^{\tilde{n}+1}}{16\pi\tilde{G}_N}$$

Bulk viscosity

$$\tilde{\zeta} = 2\tilde{\eta}\left(\frac{1}{p} - \tilde{c}_s^2\right)$$

Relaxation time

$$\tilde{\tau}_\omega = \frac{\tilde{b}}{\tilde{n}}\mathrm{Harmonic}\left(-\frac{2}{\tilde{n}} - 1\right)$$

Some checks...

$$d \to -n$$

	conformal fluid	black brane fluid	
Equation of state	$\epsilon = (d-1)P$	$\tilde{\epsilon} = -(n+1)\tilde{P}$	
Speed of sound	$c_s^2 = \frac{1}{d-1}$	$\tilde{c}_s^2 = -\frac{1}{n+1}$	**(GL instability)**

Bulk viscosity: saturation of the Buchel bound explained by the **conformal origin** of the effective black brane fluid

$$\tilde{\zeta} = 2\tilde{\eta}\left(\frac{1}{p} - \tilde{c}_s^2\right)$$

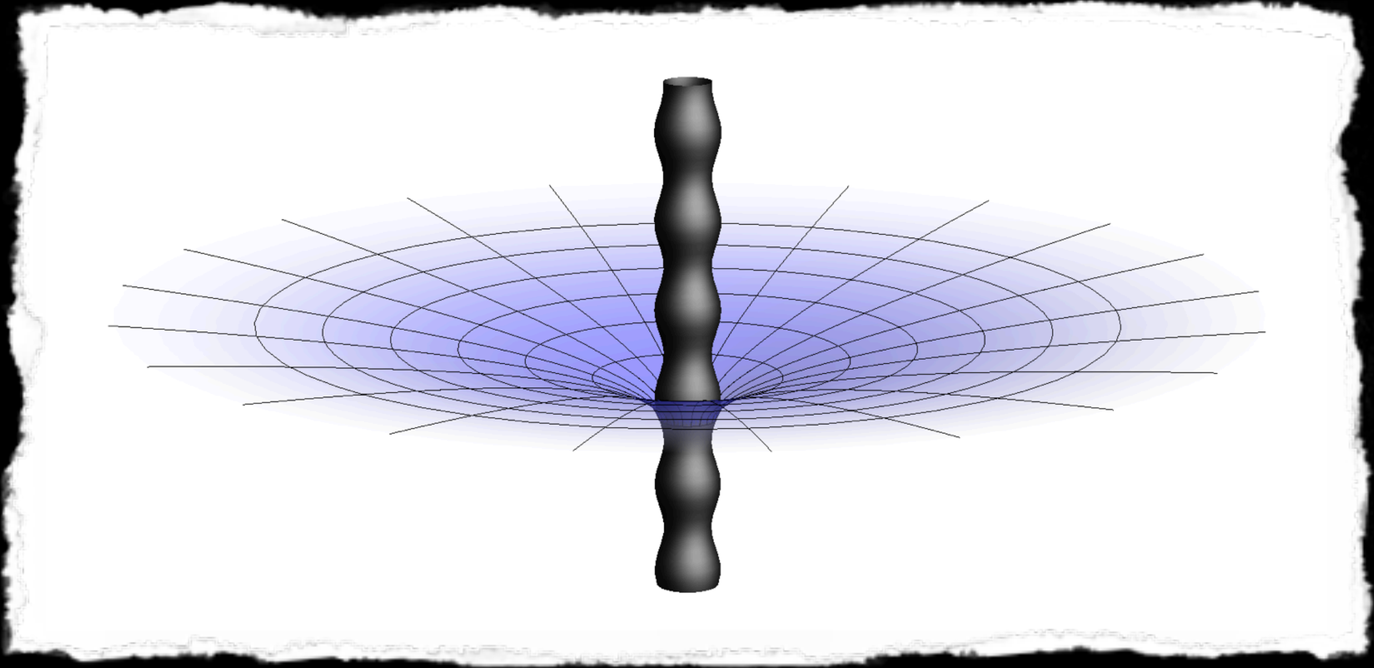
Exact agreement of the AF metric to first order in derivatives with the first order corrections of the blackfold metric computed by Camps Emparan & Haddad (2010)

... and some new results

In addition the AdS/Ricci-flat map provides us with the **second order corrections** in a derivative expansion to the **black *p*-brane metric** and its effective fluid **stess tensor**.

Next, we will see two applications of these results...

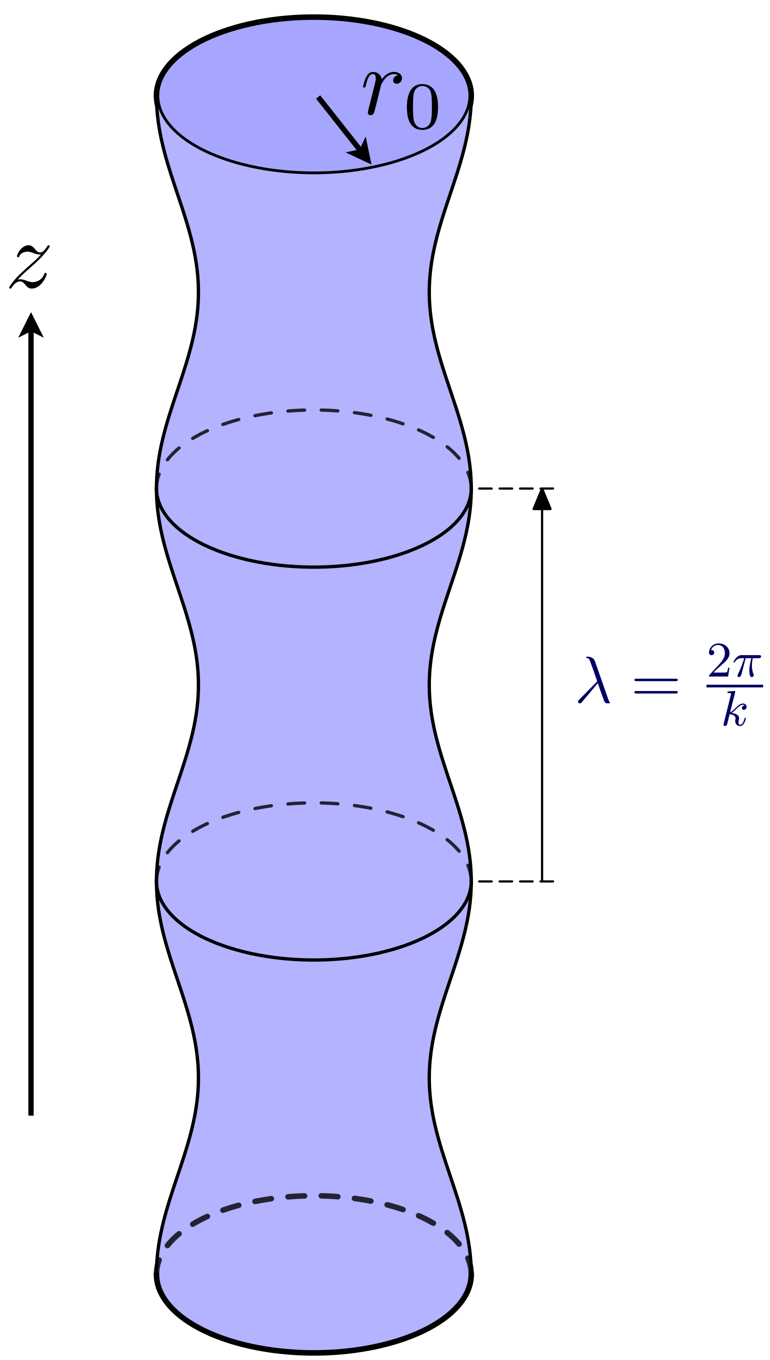
And two applications to conclude



~ GL instabilities and Rindler fluids ~

Gregory-Laflamme instability of black strings

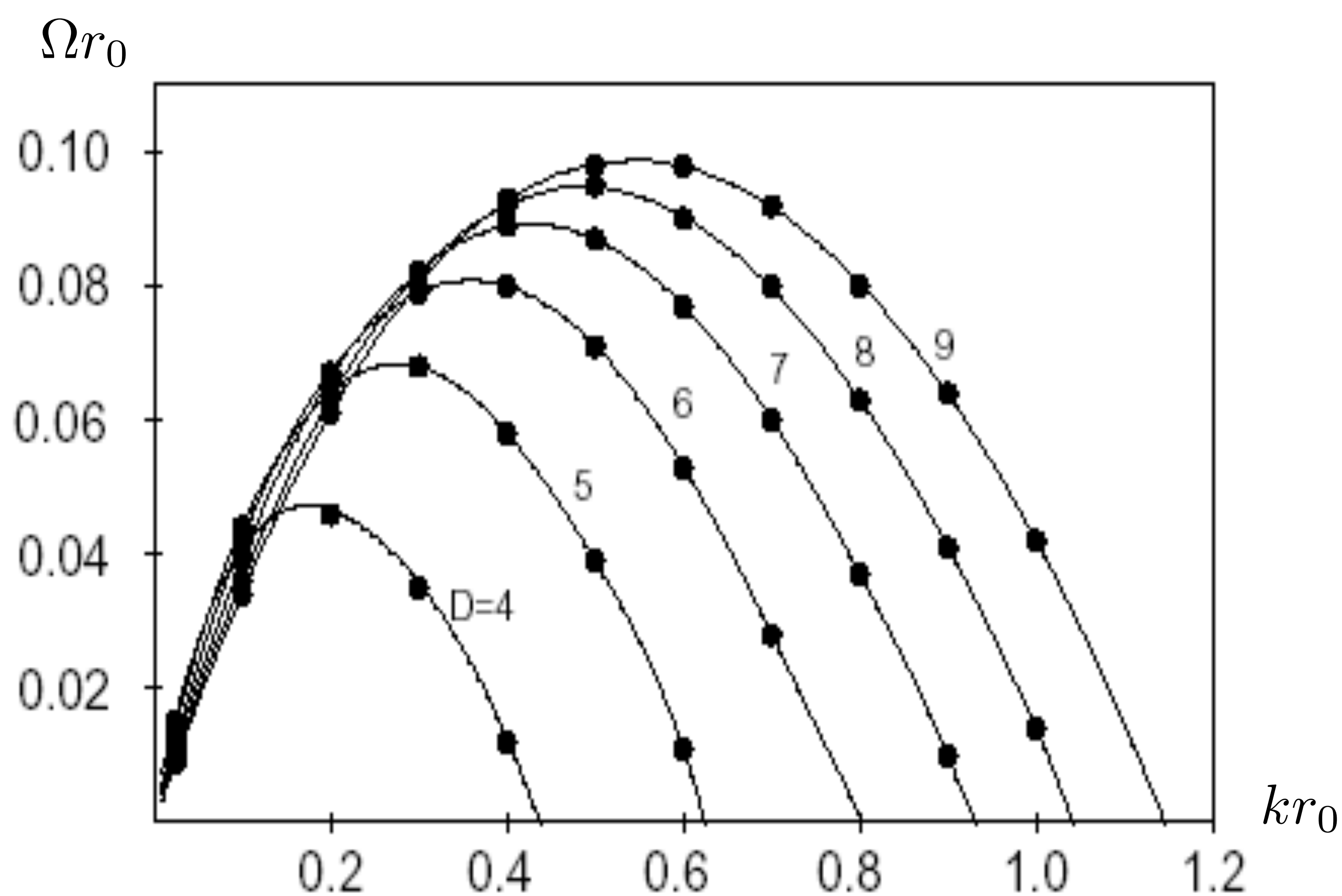
Gregory & Laflamme '94



Linearized perturbations: $g_{\mu\nu} + \epsilon h_{\mu\nu}$

$$\delta r_0 \sim e^{\Omega t + ikz}$$

Instability for $\lambda \gtrsim r_0$



Sound waves on a black string/brane

Intrinsic fluctuations δr_0 ➡ pressure/density fluctuations

*** sound waves ***

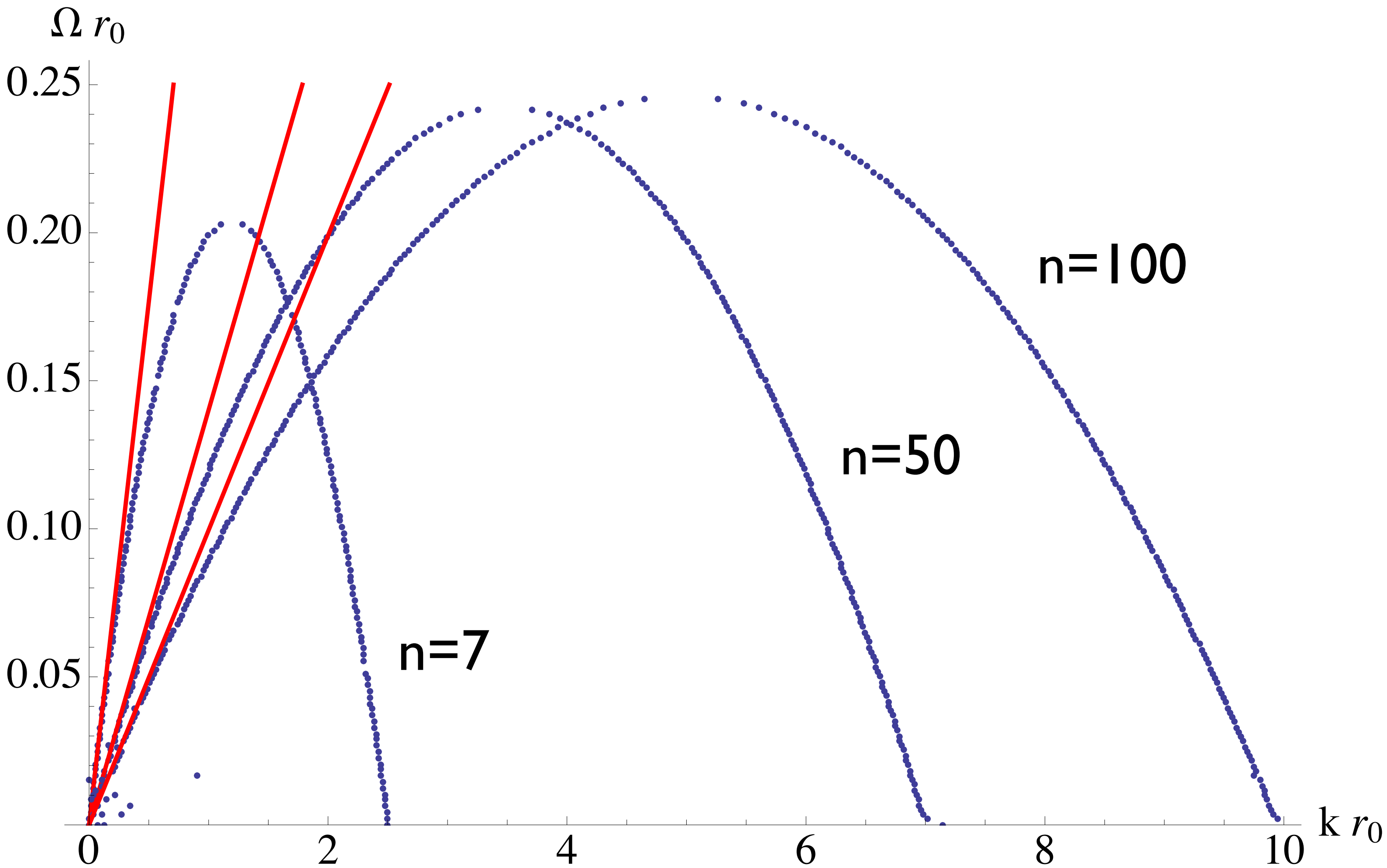
$$c_s^2 = \frac{dP}{d\epsilon} = -\frac{1}{n+1} < 0$$

➡ **unstable modes: the inhomogeneities tend to grow**

$$\delta r_0 \sim e^{\Omega t + ikz}$$

$$\Omega = \frac{k}{\sqrt{n+1}} + O\left(k^2\right)$$

captures the slope of the curve near the origin

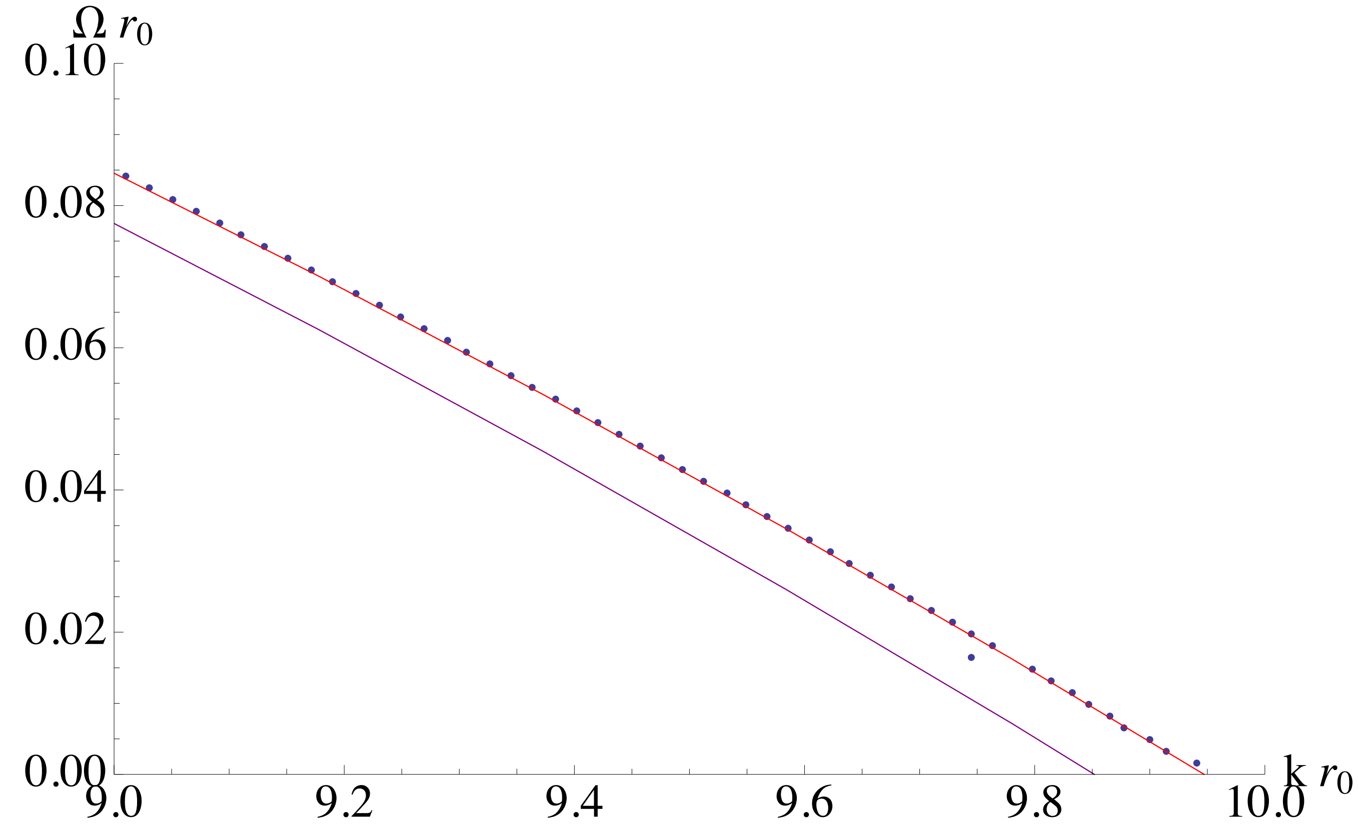


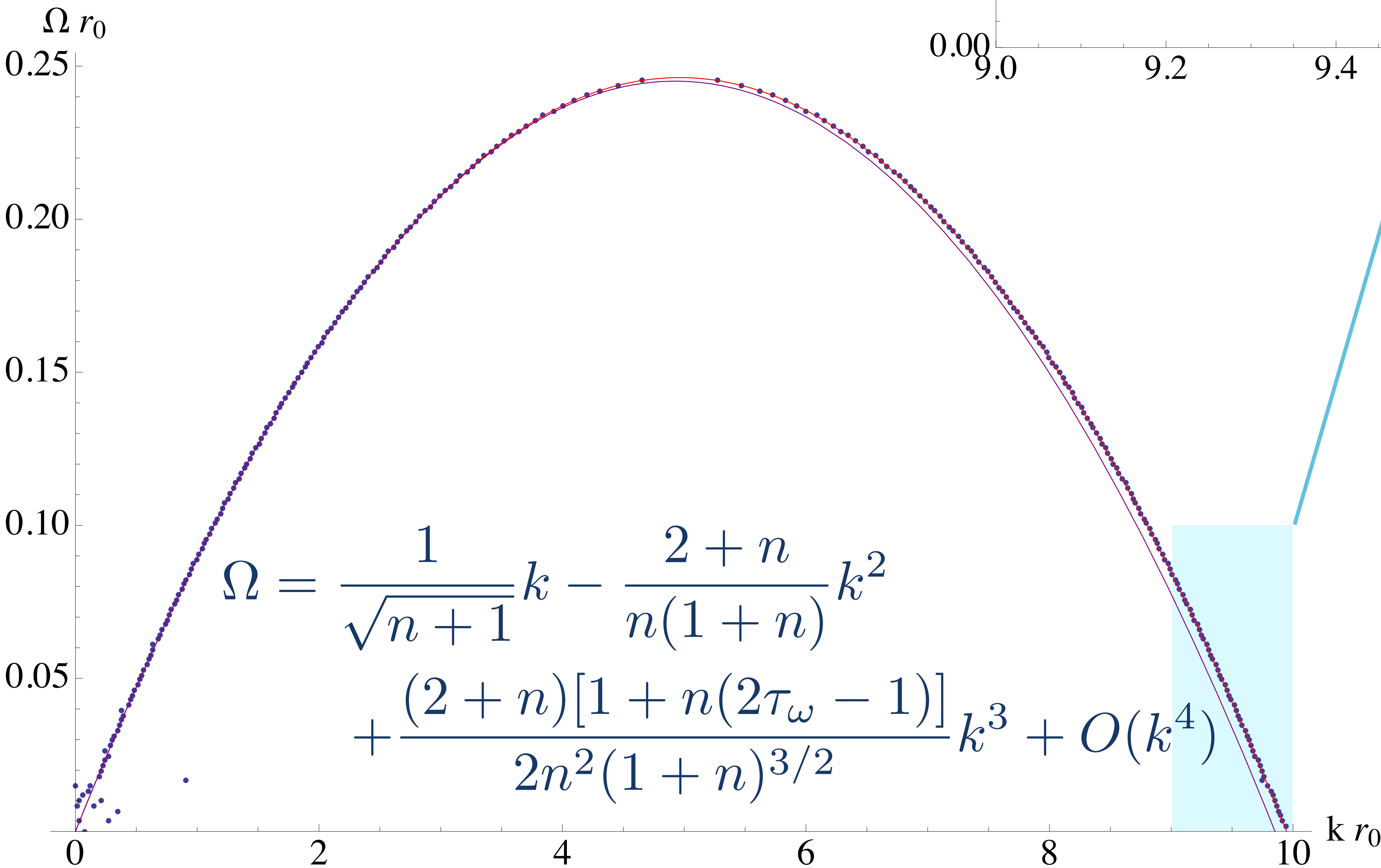
GL dispersion relation for n=100

.... numerical data (P. Figueras)

_____ first order (Camps, Emparan, Haddad '10)

_____ second order





$$\Omega = \frac{1}{\sqrt{n+1}}k - \frac{2+n}{n(1+n)}k^2 + \frac{(2+n)[1+n(2\tau_\omega - 1)]}{2n^2(1+n)^{3/2}}k^3 + O(k^4)$$

The Rindler/fluid correspondence

Bredberg et al '12, Compere et al '12, Eling et al '12

Black p-brane:

$$\mathrm{d}s_0^2 = -f(r)\mathrm{d}\tau^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega_{n+1}^2 + \mathrm{d}\vec{x}^2, \qquad f(r) = 1 - \frac{r_0^n}{r^n}$$

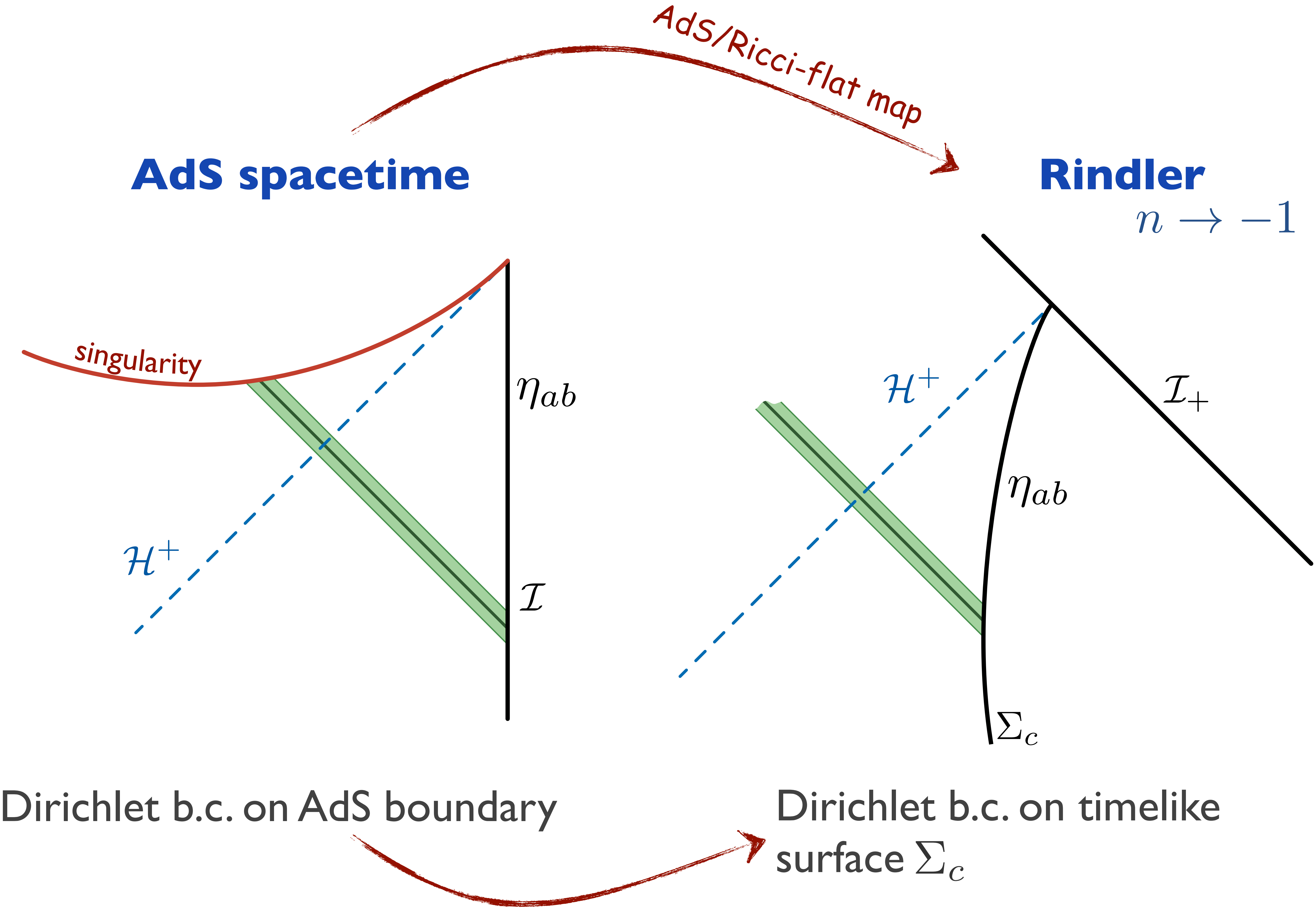
When $n \to -1$ the sphere collapse to a point and we obtain **Rindler spacetime in p+2 dimensions**:

$$\mathrm{d}s_0^2 = -\frac{\rho^2}{4r_0^2}\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \mathrm{d}\vec{x}^2, \qquad \rho^2 = 4r_0^2\,(1 - r/r_0)$$

The AdS boundary is mapped on a constant ρ hypersurface with induced metric η_{ab}

Taking carefully the $n \to -1$ limit of the gravity/fluid metric, we recover the hydrodynamic perturbations of Rindler spacetime and the associated stress energy tensor to second order in derivatives.

The Rindler/fluid correspondence



~ Conclusions ~

* **AdS/Ricci-flat correspondence** maps asymptotically locally **AdS** solutions on torus to **Ricci-flat** spacetimes

* **Holography for asymptotically flat spacetimes**
- Source for dual operators located at the location of a p-brane
- Stress energy tensor due to this p-brane is holographic

* Mapped **AdS fluid** metric to the **Ricci-flat blackfold** fluid
- Holographic stress tens. ➡ effective stress tens. of a p-brane
- "Hidden" conformal symmetry reflected in transport coeff.

* Ricci-flat spacetimes inherit a **generalized conformal structure**

* **AdS/Ricci-flat correspondence** maps asymptotically locally **AdS** solutions on torus to **Ricci-flat** spacetimes

* **Holography for asymptotically flat spacetimes**
  - Source for dual operators located at the location of a *p*-brane
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  - Holographic stress tens. → effective stress tens. of a *p*-brane
  - "Hidden" conformal symmetry reflected in transport coeff.

* Ricci-flat spacetimes inherit a **generalized conformal structure**

* **Turn on finite sources** to develop a full holographic dictionary

* **Implications of the hidden conformal invariance?**

* **Explore possible generalizations of the correspondence**

~ *Thank you!* ~