Self-Completeness and Thermodynamics of Generalized Uncertainty Principle Black Holes

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Based on:
M. Isi, J. Mureika (JM), P. Nicolini (PN), JHEP **1311:**139 (2013) [arXiv:1310.8153]
JM and PN, EPJ+ **128**:78 (2013) [arXiv:1206.4696]

Acknowledgments

- **<u>Collaborators:</u>**
  - **Piero Nicolini**
    - Frankfurt Institute for Advanced Studies
    - Goethe University, Frankfurt
  - **Maximiliano Isi**
    - Loyola Marymount University, Los Angeles & Caltech (as of Fall '14)
  - **Marco Knipfer**
    - Goethe University, Frankfurt

Historical Overview of GUP

- Nature defines a fundamental minimal length $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$

- Uncertainty determined by QM, but also gravitation:

$$\Delta x \sim \Delta x_Q + \Delta x_G \quad \Longrightarrow \quad \Delta x \sim \frac{\hbar}{\Delta p} + G\Delta p$$

  – Also emerges in scattering of strings

    [Veneziano, Europhys. Lett. **2**, 199 (1986); Amati, Ciafaloni, Veneziano, Phys. Lett. **B 197**, 81 (1987); Amati, Ciafaloni, Veneziano, Phys. Lett. **B 216**, 41 (1989)]

- Consequences at Planck scale:
  – *Critical point*: smallest BH = largest particle (Planck mass)
  – gravity is *self-complete*: singularities cannot be probed

    [Dvali and Gomez, arXiv:1005.3497]

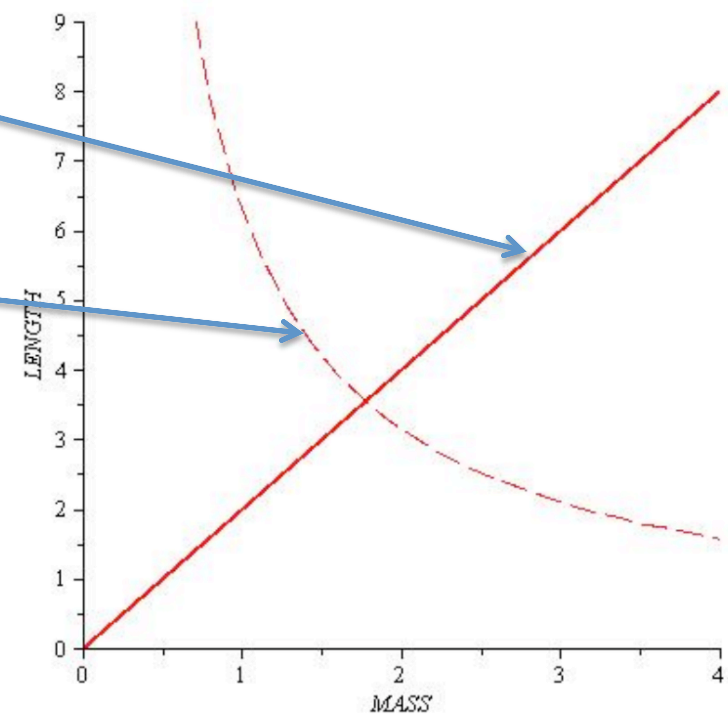
Gravitational Self-Completeness

[G. Dvali *et al.*, PRD **84**, 024039 (2011);
Dvali *et. al*. JHEP **1108** , 108 (2011)]

- The large- and short-scale characteristics of a black hole are defined by different theories

Large (IR): $r_g = 2GM_{\rm BH}$

Short (UV): $\lambda_C = \frac{1}{M_{\rm BH}}$



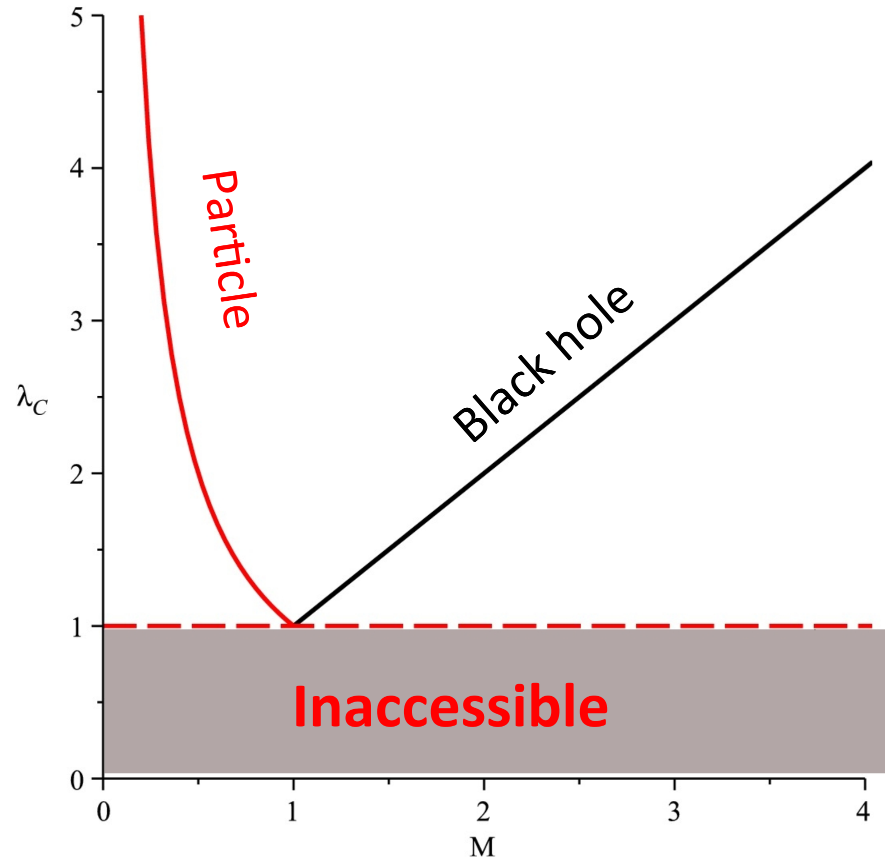
Gravitational Self-Completeness

[G. Dvali *et al.*, PRD **84**, 024039 (2011);
Dvali *et. al*. JHEP **1108** , 108 (2011)]

- **Black holes with $r_g \approx \lambda_C$ are critical points**

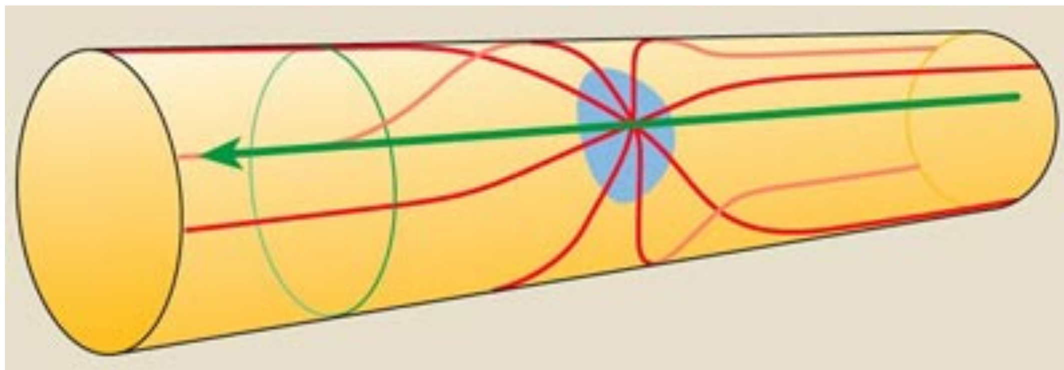
$$\lambda_C = r_g \quad \Longrightarrow \quad M_{\min} \sim \frac{1}{\sqrt{G}} = M_{\rm Pl}$$

Gravity *self-completes* itself in the UV where quantum effects take over.



Compactified Extra Dimensions

- There are *(3+d)* spatial dimensions that extend spacetime
    - **Compactified (ADD)**



- **Gravitation is *stronger* at scales *r* < R_c**

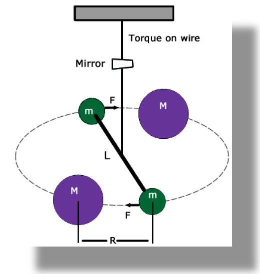
$$R_c = \left(\frac{G_D}{G_N}\right)^{\frac{1}{d}} = \left(\frac{M_{\rm Pl}^2}{M_D^{d+2}}\right)^{\frac{1}{d}}$$

- **Solves the hierarchy problem; renormalizability**

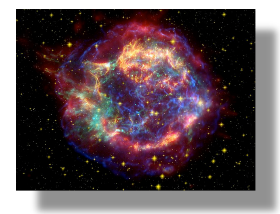
Consequences of Extra Dimensions

- **Modifications to Newtonian gravity**

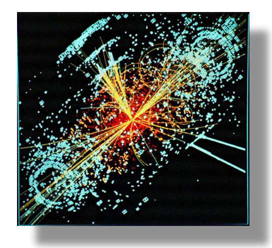
$$F = \frac{G_N m M}{r^2} \quad \Longrightarrow \quad \frac{G_D m M}{r^{d+2}}$$



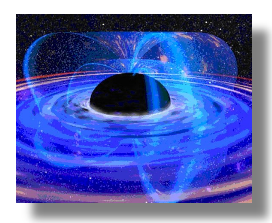
- **Anomalous cooling of supernovae**
    - If nova energy exceeds M_D, gravitons are produced and escape into extra dimensions; nova will cool quicker than expected



- ***Missing energy* in particle collisions**
    - Conservation of 4-momentum will be violated



- **Mini *black holes* in particle collisions!**
    - If energy is compressed into volume 10^{-19} m in diameter



Extra-Dimensional Self-Completeness

[JM and PN, EPJ+ **128**:78 (2013)]

- Adjust r_H to its *(d+1)-D* equivalent:

$$r_{d+1} = (2G_D M_{\rm BH})^{\frac{1}{d-2}}$$

- Self-completeness condition:

$$\lambda_C = r_{d+1} \quad \Longrightarrow \quad M_{\rm BH} \sim \left(\frac{1}{G_D}\right)^{\frac{1}{d-1}}$$

- Dimensionful coupling:

$$G_D \sim \frac{1}{M_D^{d-1}} \quad \rightarrow \quad M_{\rm BH} \sim M_D$$

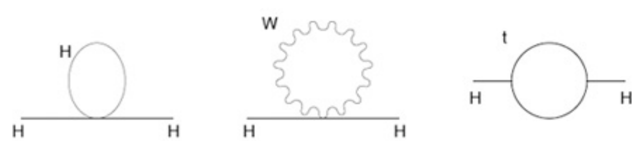
Gravity is self-complete in *all* (d+1)-D, d ≥ 3

Dimensional Reduction

- **There is evidence to suggest that the effective dimension of spacetime *reduces* at high energy / short length scales**

- **Advantages:**
  - **Gravitation**
    - Renormalizable
    - No hierarchy problem!
    - Exactly solvable quantum theory in (1+1)-D

  - **Quantum field theory**
    - Divergences in radiative corrections (*e.g.* Higgs) are tamed



$$\Delta\mu_H^2 \sim \sum_i \int^{\Lambda} \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m_i^2} = F_d(\Lambda) \; .$$

$$d = 4 \quad \Longrightarrow \quad F_4(\Lambda) \;\sim\; \Lambda^2$$

$$d = 3 \quad \Longrightarrow \quad F_3(\Lambda) \;\sim\; \Lambda$$

$$d = 2 \quad \Longrightarrow \quad F_2(\Lambda) \;\sim\; \log(\Lambda)$$

Dimensional Reduction Techniques

- Spectral (CDTs/EDTs, Hořava-Lifshitz, non-commutative)

  [Ambjorn, Jurkiewicz, Loll, PRD**72**:064014 (2005);
  Horava, PRL**102**, 161301 (2009);
  Modesto and Nicolini, PRD**81**:104040 (2010)]

- Evolving (Vanishing) Dimensions

  [Anchordoqui et al., PRD**83,**114046 (2011);
  JRM and Stojkovic, PRL **106,** 101101 (2011);
  Anchordoqui et al., MPLA**27**,1250021 (2012)]

- Brane-world

  [C. Callan, J. Maldacena, Nucl.Phys. B513, 198 (1998),
  N. Constable, R. Myers, O. Tafjord, PR**D61:**106009 (2000)]

- Multifractal gravity

  [Calcagni, PRL **104**, 251301(2010);
  Calcagni PLB**697**, 251 (2011);
  Arzano *et al.*, PRD**84**:125002 (2011)]

Spectral Dimensional Reduction

[Modesto and PN, PRD**81**:104040 (2010)]

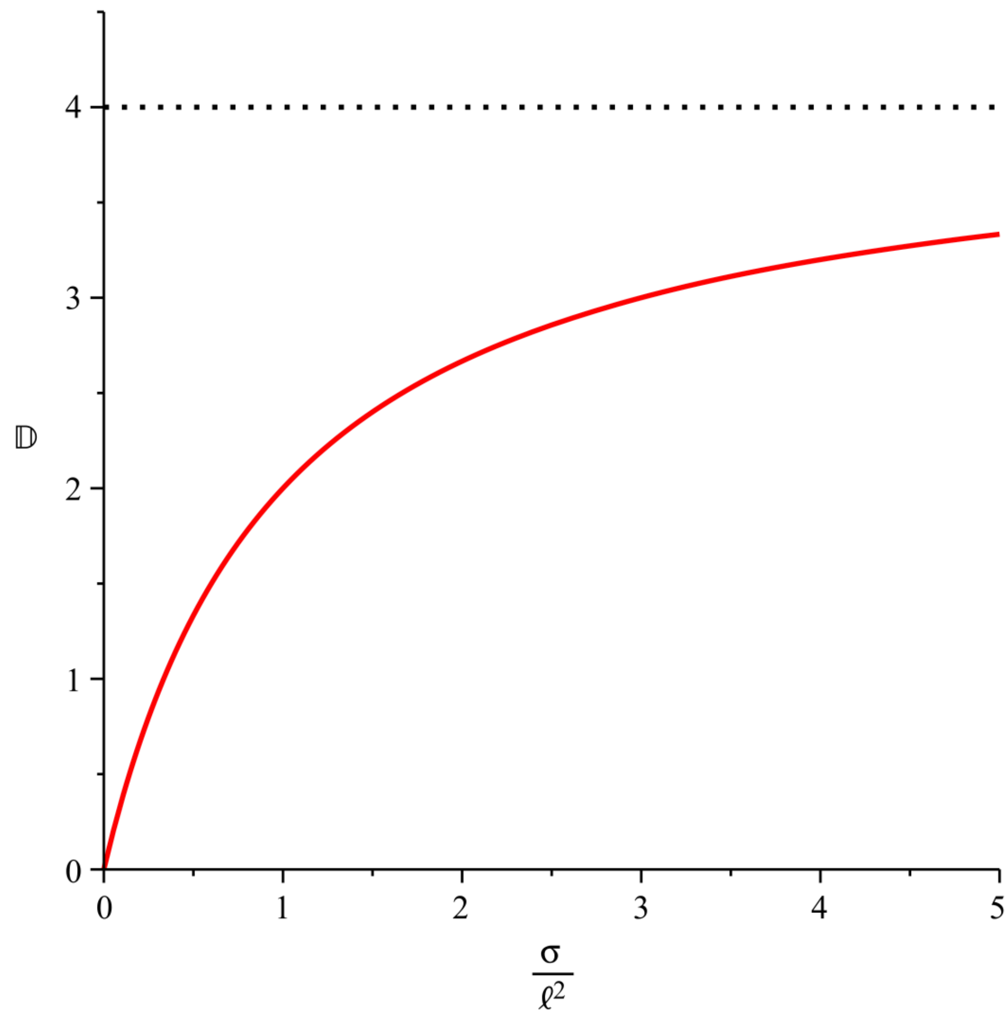
$$\Delta K_\ell(x,y;s) = \frac{\partial}{\partial s} K_\ell(x,y;s)$$
$$K_\ell(x,y;0) = \rho_\ell(x,y)$$

Minimal length constraint

$$K_\ell(x,y;s) = \frac{e^{-\frac{(x-y)^2}{4(s+\ell^2)}}}{\left[4\pi\left(s+\ell^2\right)\right]^{d/2}}$$

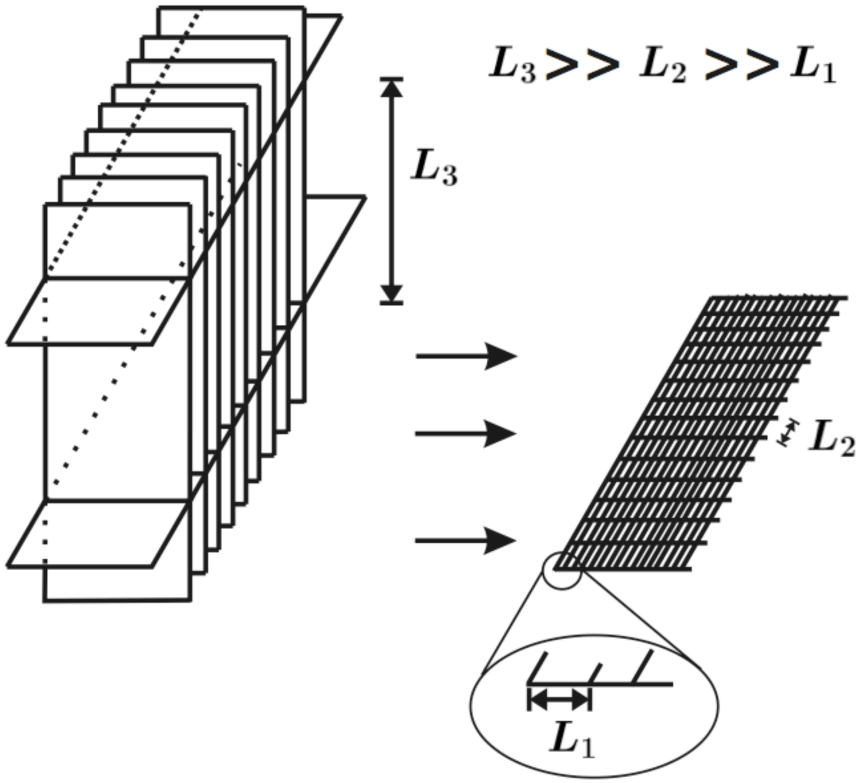
Spectral dimension for diffusion timestep *s*

$$\mathbb{D} = \frac{s}{s+\ell^2}\, d.$$



Vanishing Dimensions

[Anchordoqui et al., PRD**83,**114046 (2011);
JRM and Stojkovic, PRL **106,** 101101 (2011);
Anchordoqui et al., MPLA**27**,1250021 (2012)]



Dimension depends on interaction / energy scale of associated phenomena

d < (3+1) at small scales / high energy
d = (3+1) at macroscopic scales / low energy
d > (3+1) at very large scales / very low energy

Vanishing Dimensions Model

[Afshordi and Stojkovic, in prep.;
JRM and Stojkovic, in prep.]

$$L_{\rm mass} = \sum_{i=1}^{n} M(m_i, T)^2 X^i X_i =$$
$$m_0^2 e^{-m_1/T} X^1 X_1 + m_0^2 e^{-m_2/T} X^2 X_2 +$$
$$m_0^2 e^{-m_3/T} X^3 X_3 + \ldots + m_0^2 e^{-m_n/T} X^n X_n$$

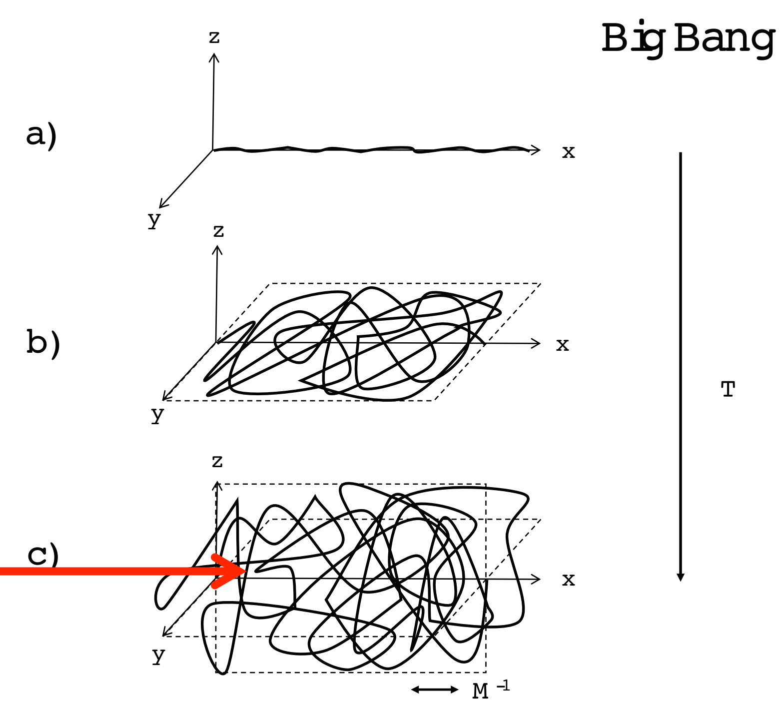
$$X^\mu = \begin{cases} 1.\ \text{Physical coords.} \\ 2.\ \text{Fields on string worldsheet} \end{cases}$$

m_0 = fundamental scale

As T cools (<< m_i) , fields X_i become excited and effectively massless

String intersections = lattice spacing

$$\ell_{\rm eff} = M^{-1}$$



Lower-Dimensional Self-Completeness

[JM and PN, EPJ+ **128**:78 (2013)]

- What happens in spacetimes with dim < (3+1)?

(2+1)-D: No local curvature and **no black holes unless *Λ* < 0**

(1+1)-D: Gravity is dilatonic; G_1 is dimensionless

[R. B. Mann and S. Ross, CQG 10 (1993) 1405]

$$S_{1+1} = \int d^2x\sqrt{-g}\left\{\left(\frac{1}{8\pi G_1}\psi R - \frac{1}{2}(\nabla\psi)^2\right) + \mathcal{L}_m^{(1+1)}\right\}$$

$$ds^2 = (2G_1M|x| - C)\,dt^2 - \frac{dx^2}{2G_1M|x| - C}$$

$$x_g = \frac{C}{2G_1M_{\rm BH}}$$

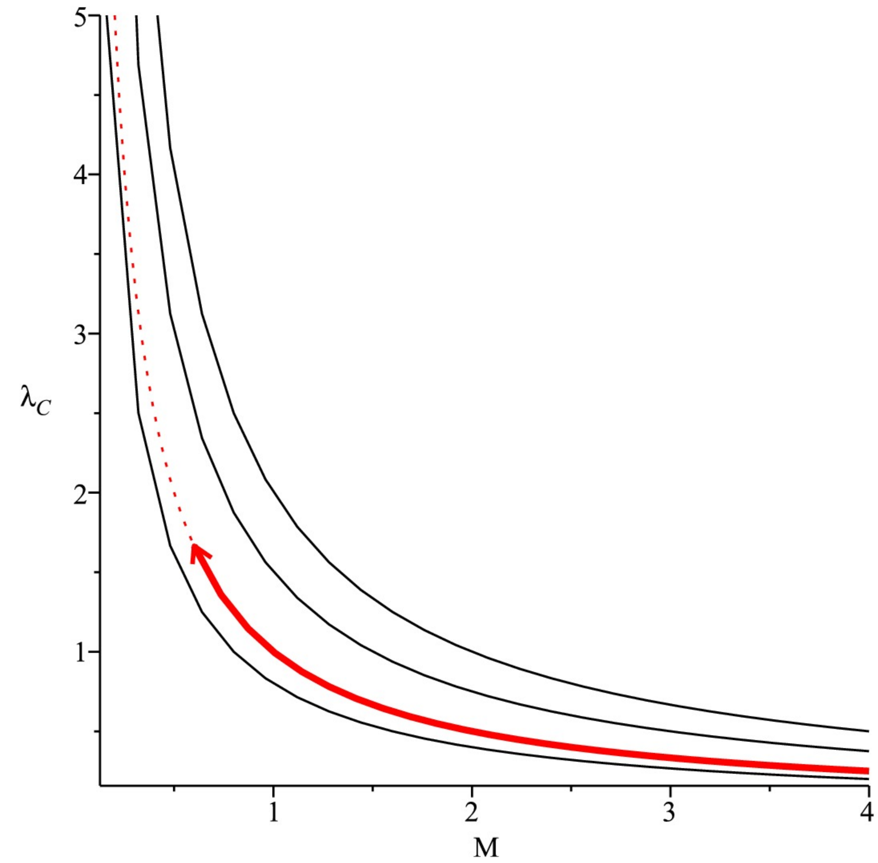
(1+1)-Dimensional Self-*In*Completeness

[JM and PN, EPJ+ **128**:78 (2013)]

- Self-completeness condition:
$$\lambda_C = x_g \quad \Longrightarrow \quad C = 2G_{1+1} = 8\pi^2$$

- **No fundamental scale**
- **Particles are particles**
- **Black holes are black holes**
- **Gravity is "naturally" QM?**

(1+1)-D gravity is self-*IN*complete (?)



Hawking Temperature from HUP

The Hawking Temperature can be derived heuristically:
Consider a photon scattered from a BH:

$$\Delta p \sim \frac{\hbar}{\Delta x} \quad , \quad \Delta x = \frac{2GM}{c^2}$$

Momentum of photon radiated from BH

$$\Longrightarrow \quad \Delta p \sim \frac{\hbar c^2}{2GM}$$

$$T = \Delta p c \quad \Longrightarrow \quad T = \frac{\hbar c^3}{2GM}$$

Hawking Temperature from GUP

The localization of a particle is limited by two factors:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \ell_{\rm Pl}^2 \frac{\Delta p}{\hbar} \quad \Longrightarrow \quad \Delta x \geq \frac{\hbar}{\Delta p} + \frac{G \Delta p}{c^3}$$

Necessitates minimum resolution as $(\Delta x)_{\rm min} \sim \ell_{\rm Pl} = \sqrt{\frac{\hbar G}{c^3}}$

Photon argument:

$$\Delta p = \frac{\hbar \Delta x}{2\ell_P^2} \left(1 \pm \sqrt{1 - 4\frac{\ell_P^2}{\Delta x^2}} \right)$$

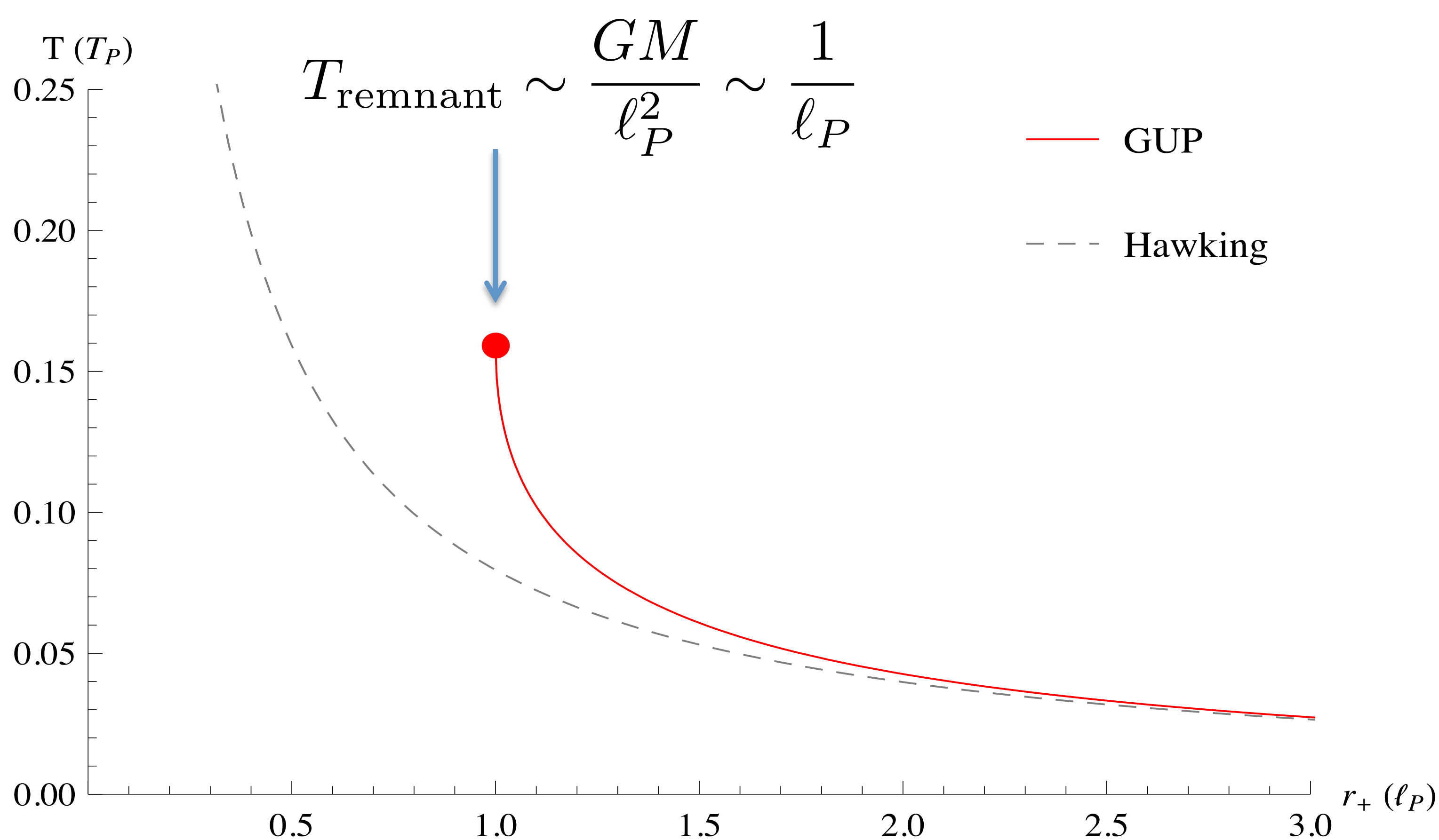
$$T = \frac{GM\hbar c}{\ell_P^2} \left(1 \pm \sqrt{1 - \frac{\ell_P^2}{\Delta x^2}} \right)$$

Adler and Chen, Nucl. Phys. Proc. Suppl. **124** (2003) 103

GUP Black Hole Remnants

The minima of the GUP relation are

$$\frac{d(\Delta x)}{d(\Delta p)} = 0 \quad \Longrightarrow \quad \Delta p_{\min} = \frac{\hbar}{\ell_P} \ , \ \Delta x_{\min} = 2\ell_P$$



Adler and Chen, Nucl. Phys. Proc. Suppl. **124** (2003) 103

Problems

- Temperature cannot be derived from any known metric or surface gravity
- Remnant is VERY **hot** (Planckian)
- Remnant is **unstable** ($C < 0$ before remnant)
- Not viable candidate for anything (*e.g* dark matter)
- BUT: *all* results are derived heuristically – self-completeness is an *a priori* condition
- Is self-completeness a *result* of some formalism?
- **Solution: find metric solution for GUP**

Generalized Uncertainty Principle

[Kempf, Mangano, Mann, PR**D52**, 1108 (1995)]

Canonical commutation relations subject to a minimum length:

$$[\mathbf{x}_i, \mathbf{x}_j] = 2i\hbar\beta\,(\mathbf{p}_i\mathbf{x}_j - \mathbf{p}_j\mathbf{x}_i)$$

which results in a modified uncertainty relation:

$$\Delta x\ \Delta p \geq \frac{\hbar}{2}\left(1 + \beta(\Delta p)^2\right)$$

Derived from deformation in momentum-space measure:

$$\int \frac{d^n p}{1 + \beta\vec{p}^2} |p\rangle\langle p| = 1$$

What is influence on metric?

Given a non-local action $$S \sim \int d^4x\sqrt{-g}\, G^{\mu\nu}\, \frac{\mathcal{A}^{-1}(\Box)}{\Box}\, R_{\mu\nu}$$

[Barvinski, PLB**710**, 12 (2012); Modesto,PRD**86**:044005 (2012)]

Can modify stress tensor via non-local modifications (entire function)

$$\mathcal{A}^2(\Box)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 8\pi G T_{\mu\nu}$$

$$\Rightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \mathcal{T}_{\mu\nu}$$

$$\mathcal{T}_{\mu\nu} \equiv \mathcal{A}^{-2}(\Box)T_{\mu\nu}.$$

[Modesto, Moffat, PN, PLB**695**, 397 (2011)]

We choose the entire function to reflect the desired momentum measure:

$$\int \frac{d^n p}{1+\beta\vec{p}^2}|p\rangle\langle p| = 1 \quad \Rightarrow \quad \mathcal{A}(\Box) = (1-\Box)^{1/2}$$

GUP Static Metric

Evaluate the deformation of the stress-energy components:

$$T^0_0 = -\frac{M}{4\pi r^2}\delta(\vec{x}) \quad \Rightarrow \quad \mathcal{T}^0_0 = -\frac{M}{4\pi r^2}\mathcal{A}^{-2}(\Box)\delta(\vec{x})$$

[Balasin, Nachbagauer, CQG **10**, 2271 (1993)]

This can be written as the Fourier integral (in local coordinates)

$$\mathcal{A}^{-2}(\Box)\delta(\vec{x}) = \mathcal{A}^{-2}(\nabla^2)\delta(\vec{x}) = (2\pi)^{-3}\int \frac{d^3p}{1+\beta\vec{p}^2}e^{i\vec{x}\cdot\vec{p}} \ ,$$

whose solution yields

$$\mathcal{T}^0_0 = -\frac{M}{\beta}\frac{e^{-|\vec{x}|/\beta}}{4\pi|\vec{x}|}$$

GUP Metric

[PN, PRD**52**, 1202.2102 [hep-th];
M. Isi, JM, PN, JHEP **11**:139 (2013)]

The deformed mass distribution is thus

$$\mathcal{M}(r) = -4\pi \int_0^r dr' r'^2 \, \mathcal{T}^0{}_0 \quad \Rightarrow \quad \mathcal{M}(r) = M\gamma\left(2; \frac{r}{\sqrt{\beta}}\right)$$

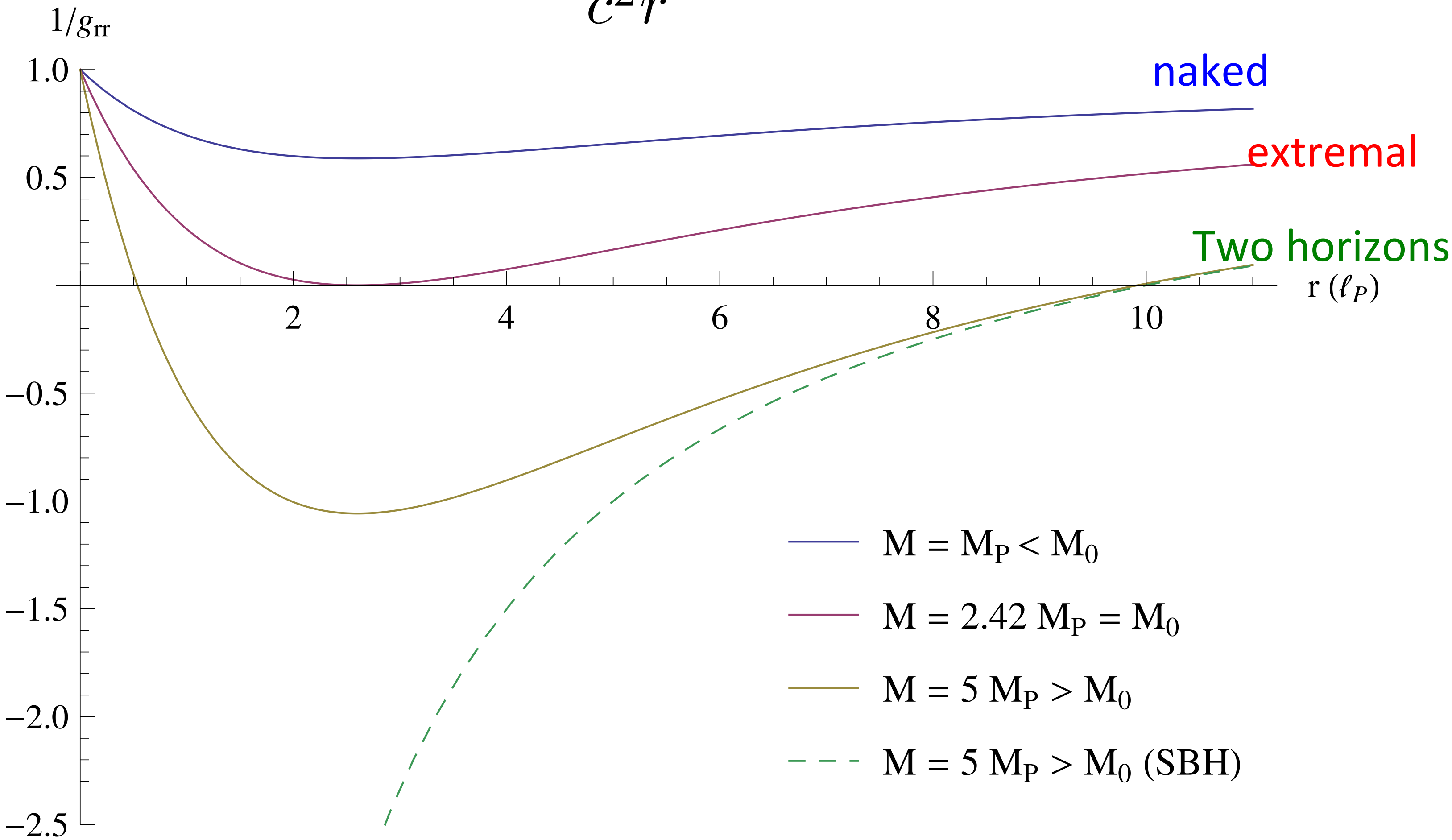
The metric is thus

$$ds^2 = -\left(1 - \frac{2GM}{r}\gamma\left(2; \frac{r}{\sqrt{\beta}}\right)\right) dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}\gamma\left(2; \frac{r}{\sqrt{\beta}}\right)} + r^2 d\Omega^2$$

Characteristics:

- Schwarzschild-like for $r \gg \beta$
- Still singular at $r = 0$
- Remnants possible $M = M_0$

$$f(r) = 1 - 2\frac{GM}{c^2 r}\gamma(2; r/\sqrt{\beta})$$



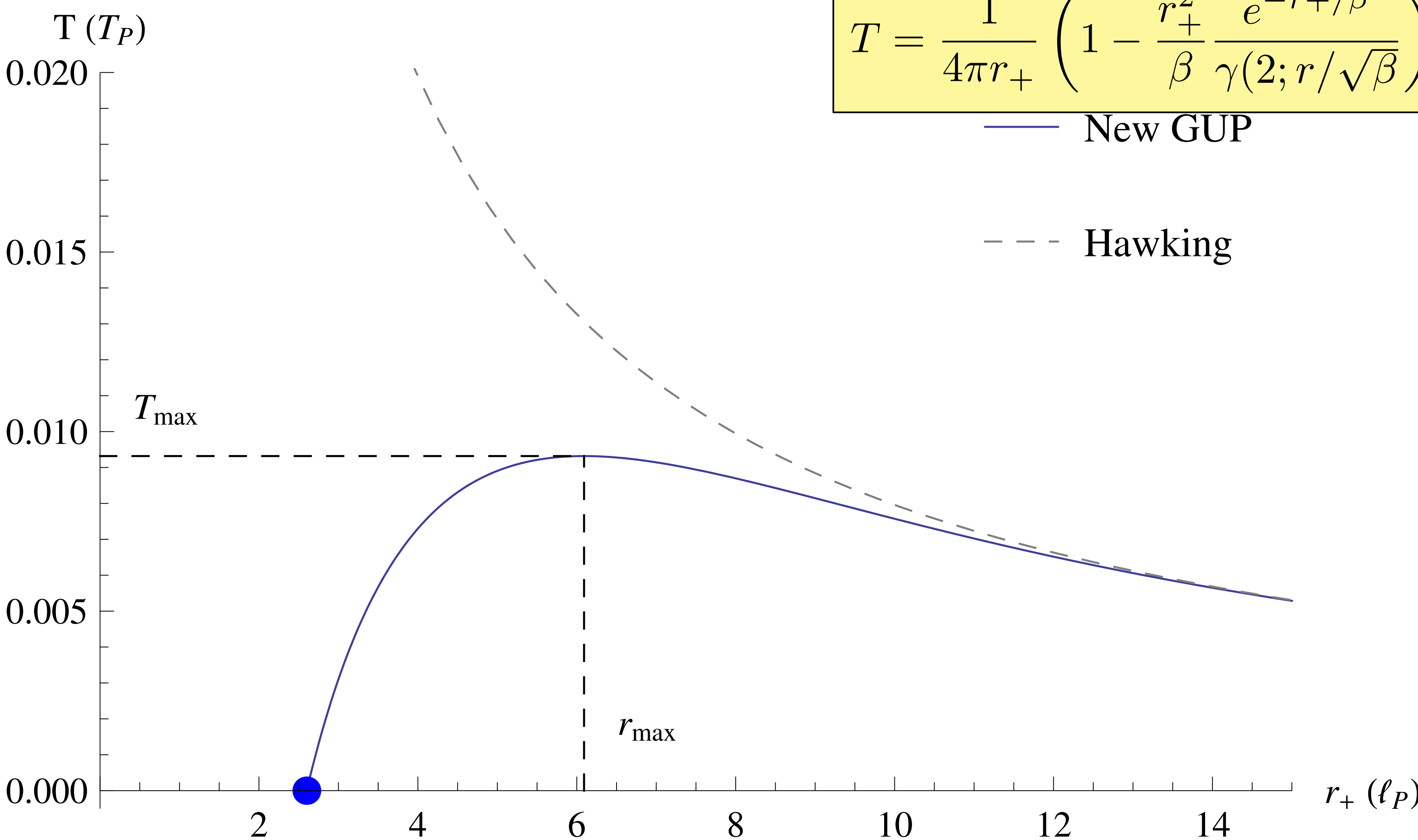
$$\text{Extremal} \longrightarrow \quad M_0 \approx 1.66\sqrt{\beta}c^2/G \quad , \quad r_0 \approx 1.73\sqrt{\beta}$$

GUPBH Thermodynamics

[PN, 1202.2102 [hep-th];
M. Isi, JM, PN, JHEP **11**:139 (2013)]

$$T \sim f'(r_+) \quad , \quad f(r) = g_{rr}^{-1}$$

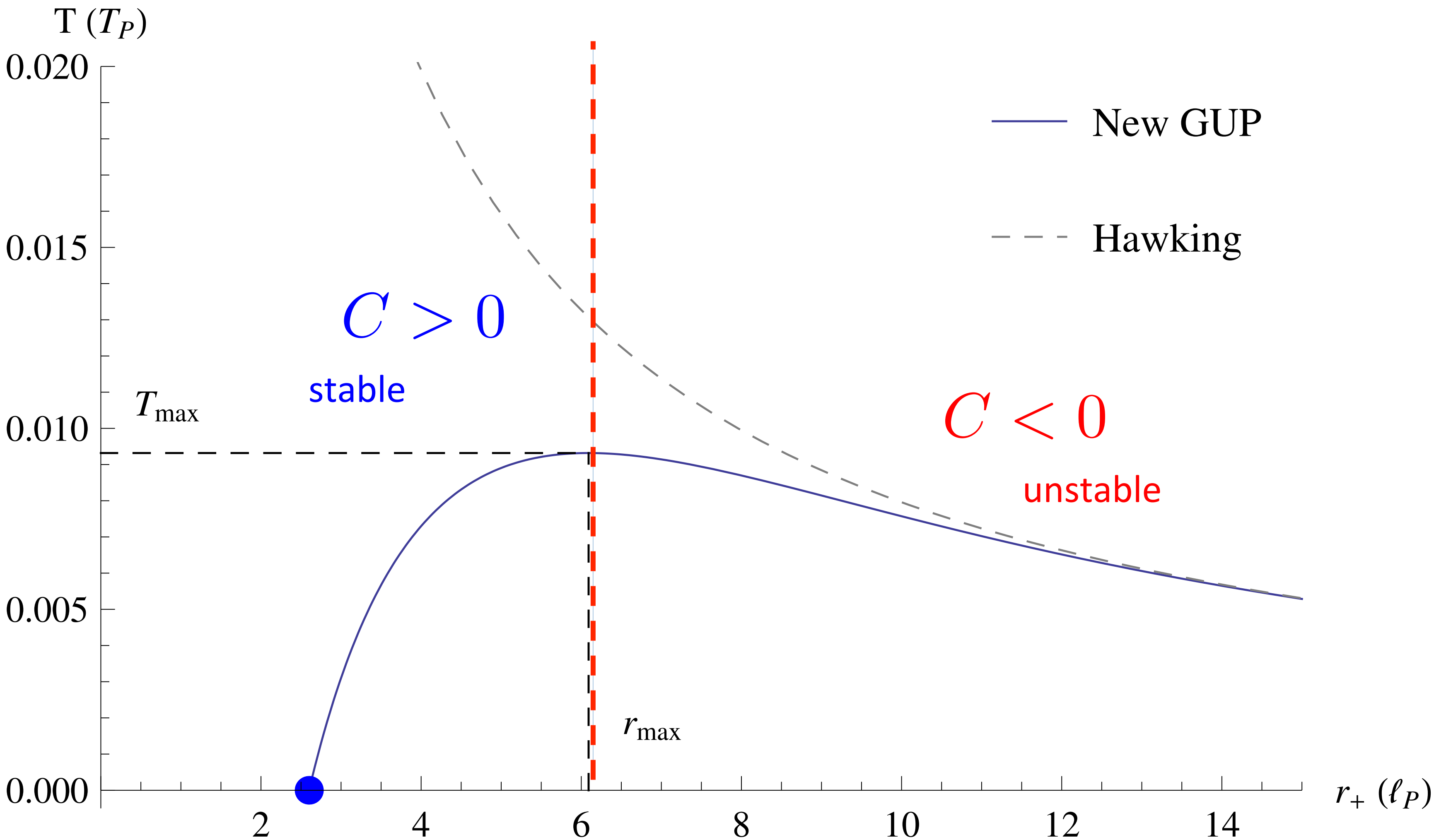
$$T = \frac{1}{4\pi r_+}\left(1 - \frac{r_+^2}{\beta}\frac{e^{-r+/\beta}}{\gamma(2; r/\sqrt{\beta})}\right)$$



Heat Capacity

[PN, 1202.2102 [hep-th];
M. Isi, JM, PN, JHEP **11**:139 (2013)]

$$C = \left(\frac{\partial M}{\partial r_+}\right)\left(\frac{\partial T}{\partial r_+}\right)^{-1}$$



GUP and Self-Completeness

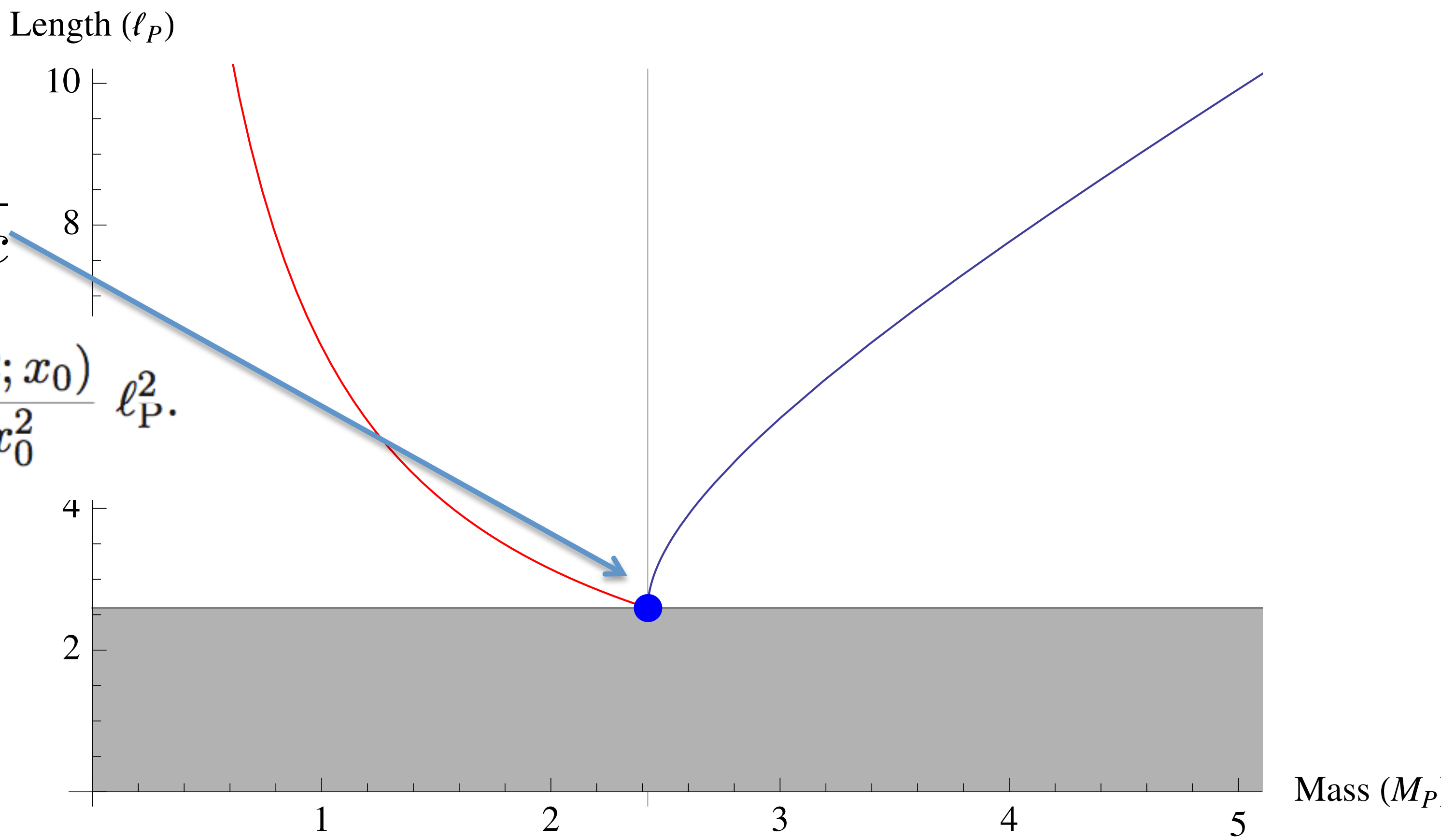
Since singular behavior persists, GUP gravity is not UV complete.

But is it *self-complete?*

No BHs can exist below the remnant scale; they are particles

$$r_0 = \frac{\hbar}{M_0 c}$$

$$\beta = 4\pi \, \frac{\gamma(2; x_0)}{x_0^2} \, \ell_{\mathrm{P}}^2.$$

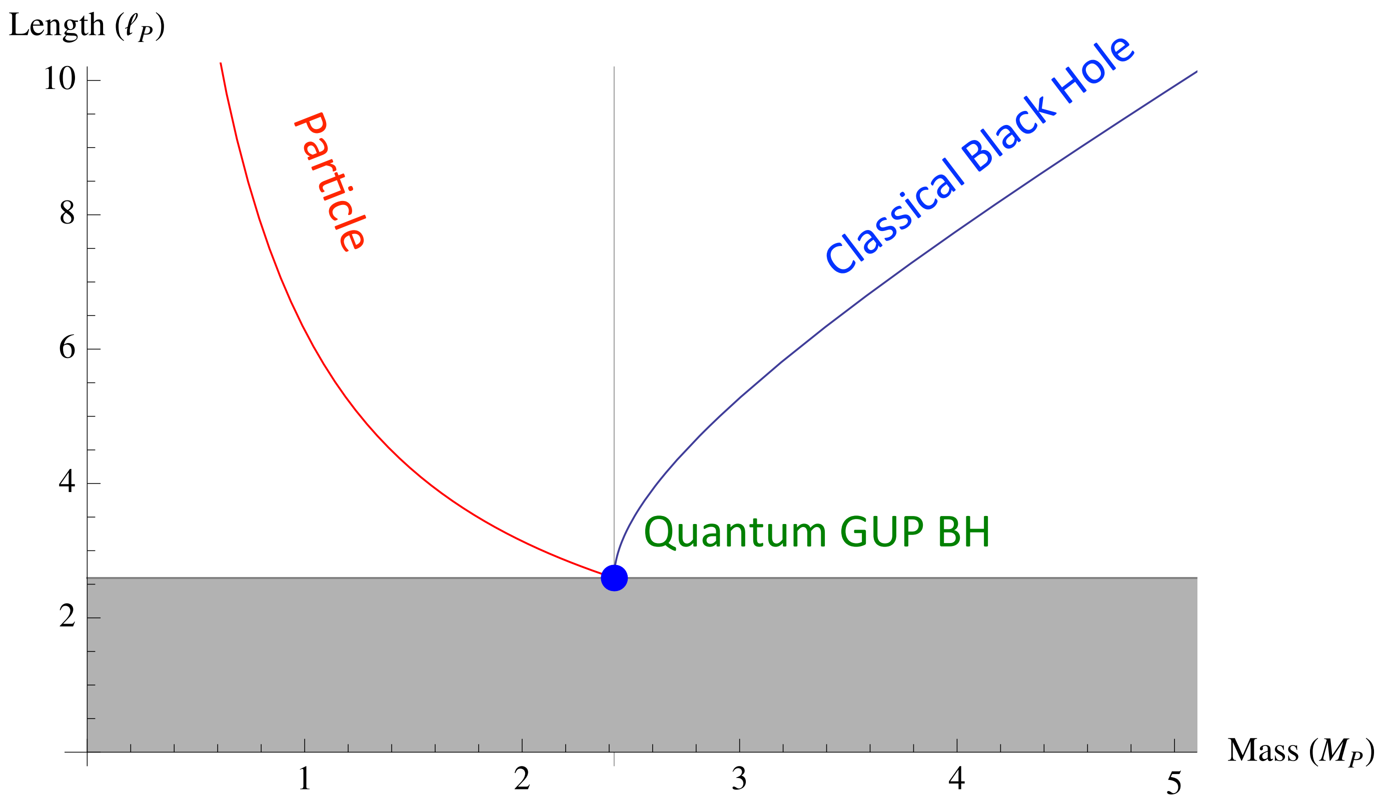


GUP and Self-Completeness

The GUP can now be written

$$\Delta x \sim 2\pi \frac{\hbar}{Mc} + 2\frac{GM}{c^2}\gamma\left(2; \Delta x/\sqrt{\beta}\right)$$

which defines three "phases":



Extra Dimensional GUP BHs

[M. Isi, M. Knipfer, JM, PN, in prep.]

We can extend the metric formalism to (n+1)-dimensions:

$$\mathcal{T}_0^0 = M \frac{1}{(2\pi)^n} \int \frac{d^n p\, e^{-ixp}}{1+\beta p^2}$$

which evaluates to

$$\boxed{\mathcal{T}_0^0(||x||) = M \frac{1}{(2\pi)^{n/2}} \beta^{-\frac{n}{4}-\frac{1}{2}} ||x||^{1-\frac{n}{2}} \cdot K_{\frac{n}{2}-1}\left(\frac{||x||}{\sqrt{\beta}}\right)}$$

$$\frac{\mathcal{M}(r)}{M} = \frac{1}{M}\int_{B_r} d^N x\, \mathcal{T}_0^0 = \frac{1}{M} A_{n-1} \int_0^r R^{n-1} \mathcal{T}_0^0(R)\, dR$$

$$= \frac{2\pi^{n/2}}{\Gamma(n/2)} \frac{1}{(2\pi)^{n/2}} \beta^{-\frac{n}{4}-\frac{1}{2}} \int_0^r dR\, K_{n/2}\left(\frac{R}{\sqrt{b}}\right) R^{\frac{n}{2}}$$

$$\boxed{\frac{\mathcal{M}(r)}{M} = 1 - \frac{2^{1-\frac{n}{2}}}{\Gamma(n/2)}\beta^{-n/4} r^{n/2} K_{n/2}\left(\frac{r}{\sqrt{\beta}}\right)}$$

- Recovers previous results for *n = 3*
- Schwarzschild for: $r \longrightarrow \infty$

$$\lim_{\beta \to 0} \frac{\mathcal{M}(r)}{M} = 1$$

Metric function:

$$f(r) = 1 - \frac{16\pi G}{(n-1)A_{n-1}} \frac{M}{r^{n-2}} \left(1 - \frac{2^{1-\frac{n}{2}}}{\Gamma(n/2)}\beta^{-n/4} r^{n/2} K_{n/2}\left(\frac{r}{\sqrt{\beta}}\right)\right)$$

Conclusions

- GUP BH derived from first principles; corrects earlier heuristic approach
- Momentum space deformation yields correction to self-completeness paradigm by introducing curvature corrections near the fundamental scale
- Extra-dimensional GUP-BH
  - *In preparation! (M. Isi, M. Knipfer, JM, PN)*
- Additional effects (NCBH, unparticle) by modeling entire function

Thank you!

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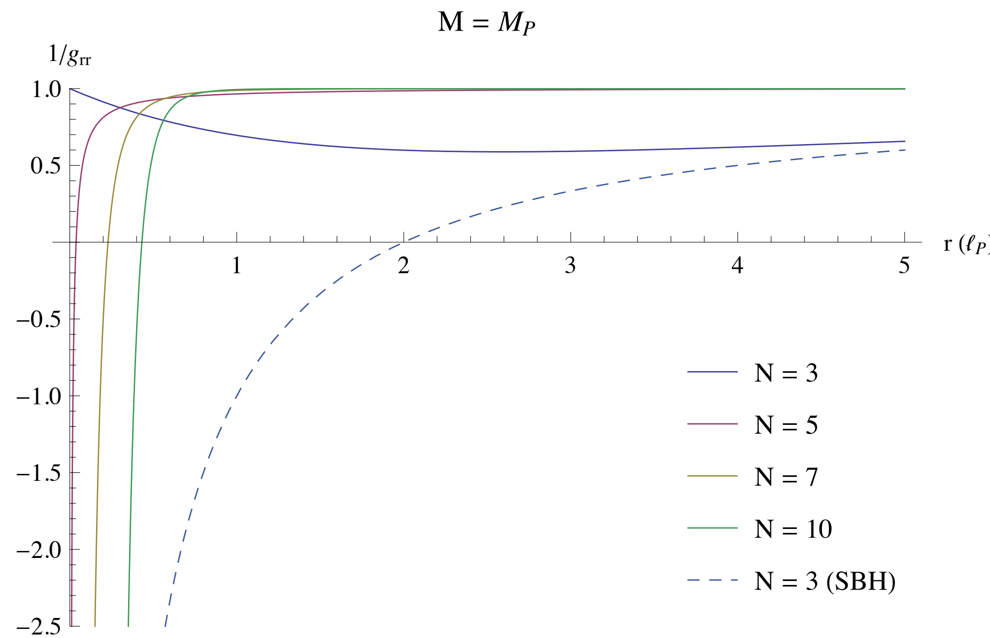
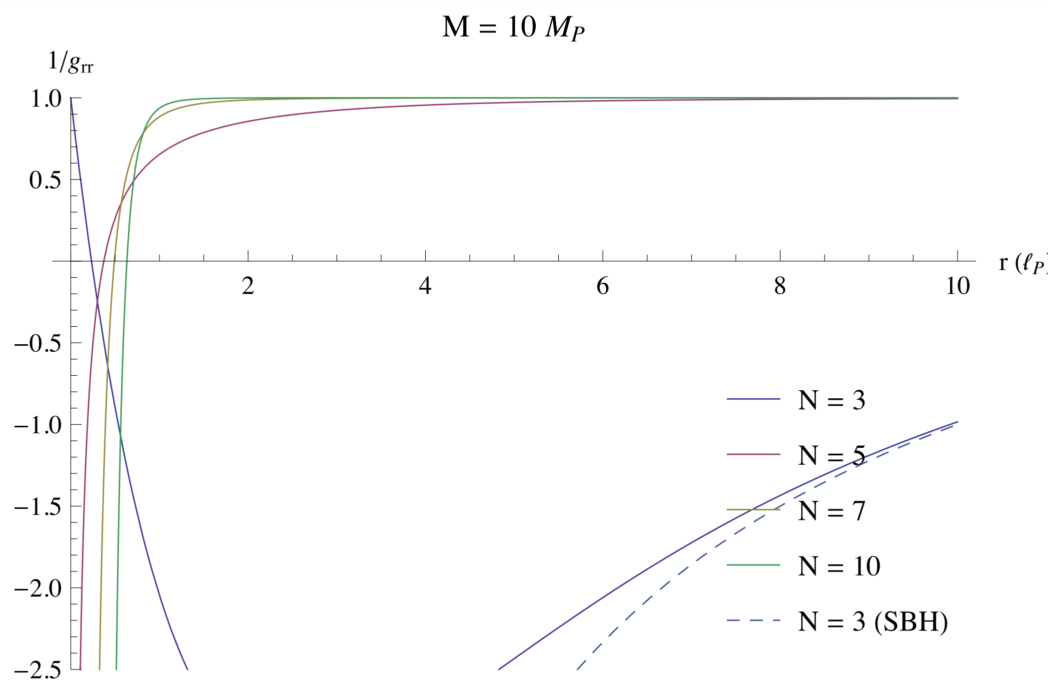
$f(r) \quad vs. \quad r$





GUP Metric

[PN, PRD**52**, 1202.2102 [hep-th];
M. Isi, JM, PN, JHEP **11**:139 (2013)]

Encode deformation via coordinate effects into the source term

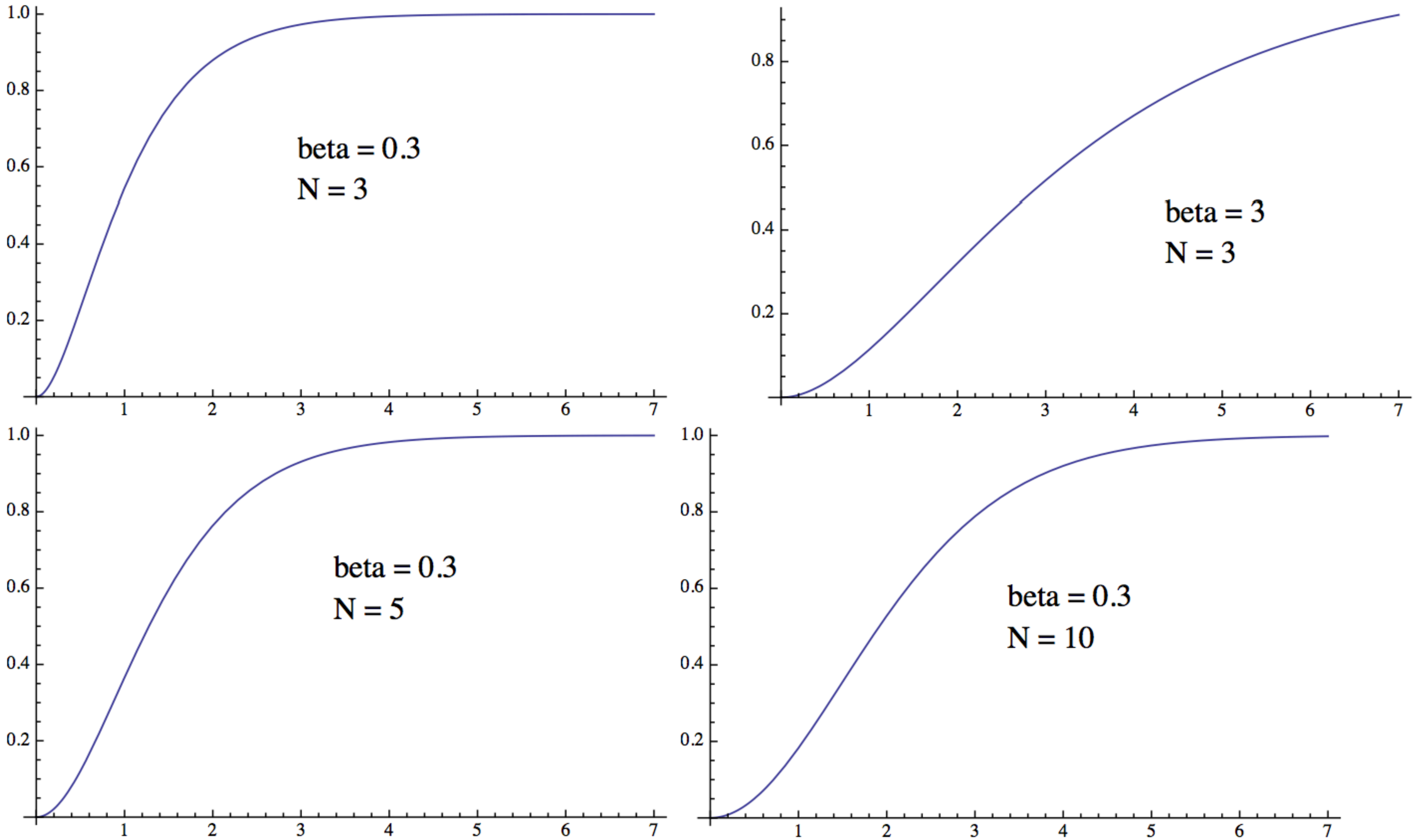
$$\mathcal{A}^2(\Box)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 8\pi G T_{\mu\nu}$$

$$\longrightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \mathcal{T}_{\mu\nu}$$

$$\mathcal{T}_{\mu\nu} \equiv \mathcal{A}^{-2}(\Box)T_{\mu\nu}.$$

We choose the entire function to reflect the desired momentum measure:

$$\int \frac{d^n p}{1+\beta\vec{p}^2}|p\rangle\langle p| = 1 \quad \Longrightarrow \quad \mathcal{A}(\Box) = (1-\Box)^{1/2}$$



$$\mathcal{M}(r)/M \quad \text{vs} \quad r$$