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Leptogenesis: CPT,* CTP** & Flavour

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*Charge-Parity-Time Reversal **Closed Time Path Formalism

Outline

1. Standard Approach to Leptogenesis
2. CTP Formalism
3. CTP Approach to Leptogenesis
4. Flavoured Leptogenesis
5. Conclusions

1. Standard Approach to Leptogenesis

Baryon Asymmetry of the Universe

$$\frac{n_B}{s} = \begin{cases} (6{,}7 - 9{,}2)\cdot 10^{-11} & \text{Big Bang Nucleosynthesis} \\ (8{,}36 - 9{,}32)\cdot 10^{-11} & \text{Cosmic Microwave Background} \end{cases}$$

* Cannot be explained within the Standard Model (SM).
* An explanation is therefore mandatory for any cosmologically consistent extension of the SM.
* Many (more or less) plausible explanations: Electroweak Baryogenesis, SUSY flat directions, Leptogenesis, ...
* Theory challenge: accurate predictions within the hot early Universe

Sakharov Conditions

(1967)

1. $\cancel{B}$

2. $\cancel{C}$ & $\cancel{CP}$

3. $\cancel{\text{Eq.}}$ Necessary to make 2. effective due to the CPT theorem.

Leptogenesis

Fukugita, Yanagida (1986)

Lagrangian for simple variant of Leptogenesis:

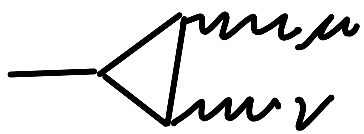
$$\mathcal{L} = \frac{1}{2}\bar{\psi}_{N_i}(i\partial\!\!\!/ - M_i)\psi_{N_i} + \underbrace{\bar{\psi}_\ell\, i\partial\!\!\!/\,\psi_\ell}_{\text{left-handed leptons}} + \underbrace{(\partial^\mu\phi^\dagger)(\partial_\mu\phi)}_{\text{SM-Higgs}} - Y_i^*\bar{\psi}_\ell\phi^\dagger P_R\psi_{N_i} - Y_i\bar{\psi}_{N_i}P_L\phi\psi_\ell$$

L-violating large Majorana mass for right-handed neutrinos N_i

Yukawa couplings for small see-saw left-handed neutrino masses & CP-violation

* L violation in conjunction with the weak sphaleron & chiral anomaly that violates B+L leads to B violation.

$$\partial^\mu j_\mu^{(B+L)} = -g_2^2\frac{2n_f}{32\pi^2}W^a_{\mu\nu}\tilde{W}^{a\mu\nu} =$$



chiral anomaly

Bell, Adler, Jackiw

$$\Delta(B+L) = -\frac{2n_f}{32\pi^2}g_2^2\int d^4x\, W^a_{\mu\nu}\tilde{W}^{a\mu\nu}$$

instanton assuming a particular winding number

e.g. $e_L + \mu_L + \nu_{\tau_L} \longrightarrow \bar{u}\bar{u}\bar{d} + \bar{c}\bar{c}\bar{s} + \bar{t}\bar{t}\bar{b}$

Belavin, Polyakov, Shvarts, Tyupkin (1975); 't Hooft (1976); Klinkhamer, Manton (1984), Kuzmin, Rubakov, Shaposhnikov (1985)

Leptogenesis

* CP-violation due to rephasing invariants within the M_i and Y_i.

* Out of equilibrium due to expansion of the Universe, small Y_i & absence of other interactions of the N_i.

In particular, Leptogenesis occurs at the time when $T \sim M_1$ and is most efficient if

$Y_i^2 M_1 \sim H \sim \frac{T^2}{m_{Pl}}$. $\longrightarrow$ strong & weak washout.

See-saw mechanism establishes link to masses of the left-handed neutrinos.

Boltzmann Equations for Leptogenesis

$$\frac{\partial}{\partial t}(n_\ell - \bar{n}_\ell) + 3H(n_\ell - \bar{n}_\ell)$$ in Maxwell-Boltzmann approximation

$$= \int \frac{d^3k}{(2\pi)^3 2\sqrt{\vec{k}^2+M_1^2}} \frac{d^3p}{(2\pi)^3 2|\vec{p}|^2} \frac{d^3q}{(2\pi)^3 2|\vec{q}|^2} (2\pi)^4 \delta^4(k-p-q)$$

$$* \Big\{ |\mathcal{M}_{N_1\to\ell\phi}|^2 f_{N_1}(\vec{k}) - |\mathcal{M}_{N_1\to\bar{\ell}\phi^*}|^2 f_{N_1}(\vec{k})$$
$$- |\mathcal{M}_{\ell\phi\to N_1}|^2 f_\ell(\vec{p}) f_\phi(\vec{q}) + |\mathcal{M}_{\bar{\ell}\phi^*\to N_1}|^2 f_{\bar{\ell}}(\vec{p}) f_{\phi^*}(\vec{q}) \Big\}$$

$$+ \int \frac{d^3k}{(2\pi)^3 2|\vec{k}|} \frac{d^3l}{(2\pi)^3 2|\vec{l}|} \frac{d^3p}{(2\pi)^3 2|\vec{p}|^2} \frac{d\,q}{(2\pi)^3 2|\vec{q}|^2} (2\pi)^4 \delta^4(k+l-p-q)$$

$$* \Big\{ |\mathcal{M}_{\bar{\ell}\phi^*\to\ell\phi}|^2 f_{\bar{\ell}}(\vec{k}) f_{\phi^*}(\vec{l}) - |\mathcal{M}_{\ell\phi\to\bar{\ell}\phi^*}|^2 f_\ell(\vec{k}) f_\phi(\vec{l}) \Big\}$$

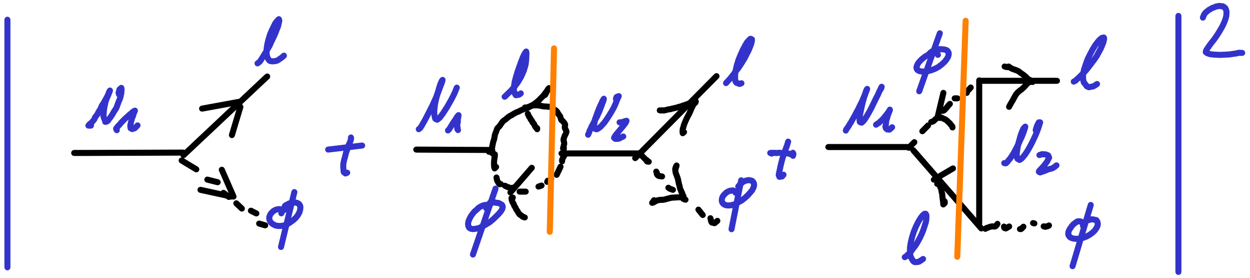
Expressed in terms of vacuum matrix elements, including loop effects, to provide CP-violation.

CP-Violating Vacuum Matrix Elements

$$\mathcal{L} = \frac{1}{2}\bar{\Psi}_{N_i}(i\partial\!\!\!/ - M_i)\Psi_{N_i} + \bar{\psi}_\ell i\partial\!\!\!/ \psi_\ell + (\partial^\mu\phi^\dagger)(\partial_\mu\phi) - Y_i^* \bar{\psi}_\ell \phi^\dagger P_R \psi_{N_i} - Y_i \bar{\psi}_{N_i} P_L \phi \psi_\ell$$

The CP-violation within $\mathcal{M}_{N_1\to\ell\phi,\bar{\ell}\phi^*}$ comes from interference of tree and loop diagrams:

$$\left|\mathcal{M}_{N_1\to\ell\phi}\right|^2 =$$

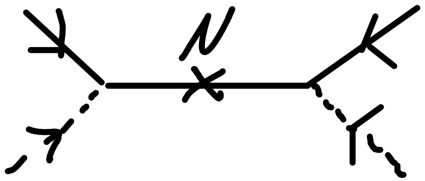


Note: It is the discontinuity of the loop diagrams that contributes to the CP asymmetry. Using Cutkosky's rules, the discontinuity arises as the interference of the indicated cut-diagrams.

Fukugita, Yanagida (1986); Covi, Roulet, Vissani (1996); Buchmüller, Plümacher (1996)

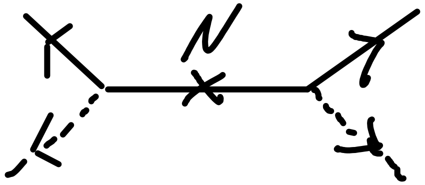
What the Vacuum Matrix Elements Can Do for Us:

$2 \longleftrightarrow 2$



$$|\mathcal{M}_{\ell\phi\to\bar{\ell}\phi^*}|^2$$

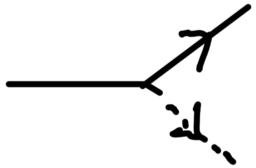
$$=$$



$$|\mathcal{M}_{\bar{\ell}\phi^*\to\ell\phi}|^2$$

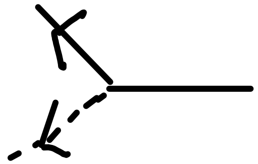
No asymmetry ever generated

$1 \longleftrightarrow 2$

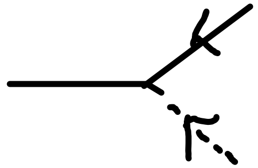


$$|\mathcal{M}_{N\to\ell\phi}|^2 \sim 1+\varepsilon$$

$$\overset{CPT}{=}$$

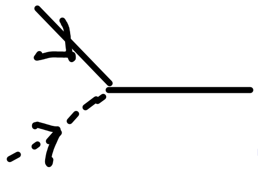


$$|\mathcal{M}_{\bar{\ell}\phi^*\to N}|^2 \sim 1+\varepsilon$$



$$|\mathcal{M}_{N\to\bar{\ell}\phi^*}|^2 \sim 1-\varepsilon$$

$$\overset{CPT}{=}$$



$$|\mathcal{M}_{\ell\phi\to N}|^2 \sim 1-\varepsilon$$

Naive multiplication* suggests asymmetry even in equilibrium:

$$\Gamma_{\bar{\ell}\phi^*\to\ell\phi} \sim 1+2\varepsilon$$

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*Do not try this at home: The unstable N cannot constitute elements of a unitary S-matrix.

RIS Subtraction

The contributions to the $2 \leftrightarrow 2$ processes where N is on-shell (Real Intermediate States) are accounted for already by the $1 \leftrightarrow 2$.

Subtracting the RIS from the $2 \leftrightarrow 2$ leads to Boltzmann equations that

- ✓ produce no asymmetry in equilibrium.
- ✓ generate an asymmetry for out-of-equilibrium N.
- ✓ predict a net asymmetry due to washout effects.

Kolb & Wolfram (1980)

Problems

* What happens when we account for Fermi-suppression factors of the N (cf. the original Boltzmann equations: we may straightforwardly introduce these factors for the $1 \leftrightarrow 2$, but what about the $2 \leftrightarrow 2$)?

* Is there a less heuristic (first principle) approach that is also suitable for further improvements in the predictions and addresses problems such as thermal effects?

2. CTP Formalism

(geared toward weakly interacting systems close-to-equilibrium)

Schwinger (1961); Keldysh (1964); Calzetta, Hu (1988);
Schmidt, Prokopec, Weinstock (2004)

Closed Time Path* Formalism

*a.k.a. Schwinger-Keldysh Formalism, in-in Formalism

- In the heuristic Boltzmann equations, the matrix elements are computed within the usual in-out framework: $\langle out | S | in \rangle$

- These are related to time-ordered expectation values, that can e.g. be calculated in the functional formalism:

$$Z[J] = N^{-1} \langle vac_{out} | vac_{in} \rangle_J = \int D\phi \, e^{i\int d^4x \left(\mathcal{L} + J(x)\phi(x)\right)}$$

$$\langle T[\phi(x)\phi(y)] \rangle = - \frac{\delta^2}{\delta J(x)\, \delta J(y)} \log Z[J] \Big|_{J=0}$$

CTP Formalism (continued)

- What if we want to calculate e.g. a charge density $q(x) = \lim_{y \to x} \langle \overline{\psi}(x) \gamma^0 \psi(y) \rangle$ in a finite-density medium?
- Or a retarded Green function (for linear responses) $i\Delta_\phi^R(x,y) = \theta(x^0 - y^0) \langle [\phi(x), \phi(y)] \rangle$?

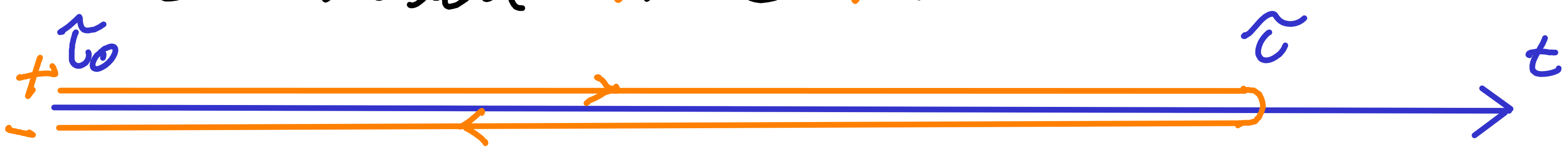
CTP Formalism (continued)

In-In generating functional: $\phi_{in}(\vec{x}) = \phi(\vec{x}, \tau_0)$

$$Z[J_+, J_-] = \int D\varphi\, D\phi^-_{in}\, D\phi^+_{in} \langle \phi^-_{in} | \varphi, \tau \rangle_{J_-} \langle \varphi, \tau | \phi^+_{in} \rangle_{J_+} \langle \phi^-_{in} | \varrho | \phi^+_{in} \rangle$$

$$= \int D\phi^+ D\phi^-\, e^{i \int d^4x \{ \mathcal{L}[\phi^+] + J_+ \phi^+ - \mathcal{L}[\phi^-] - J_- \phi^- \}} \langle \phi^-_{in} | \varrho | \phi^+_{in} \rangle$$

The Closed Time Path:



Path ordered Green-functions:

$$i \Delta^{ab}_{\phi}(u,v) = -\frac{\delta^2}{\delta J_a(u)\, \delta J_b(v)} \log Z[J_+, J_-] \Big|_{J_\pm = 0}$$

$$= i \langle \mathcal{C}[\phi^a(u)\, \phi^b(v)] \rangle$$

Path Ordered Green Functions

$$i\Delta_\phi^<(u,v) = i\Delta_\phi^{+-}(u,v) = \langle \phi(v)\phi(u)\rangle$$

$$i\Delta_\phi^>(u,v) = i\Delta_\phi^{-+}(u,v) = \langle \phi(u)\phi(v)\rangle$$

$$i\Delta_\phi^T(u,v) = i\Delta_\phi^{++}(u,v) = \langle T[\phi(u)\phi(v)]\rangle$$

$$i\Delta_\phi^{\bar{T}}(u,v) = i\Delta_\phi^{--}(u,v) = \langle \bar{T}[\phi(u)\phi(v)]\rangle$$

Free propagators in momentum space:

$$i\Delta_\phi^<(p) = 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)f_\phi(\vec{p}) + \vartheta(-p_0)(1+\bar{f}_\phi(-\vec{p}))\right]$$

$$i\Delta_\phi^>(p) = 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)(1+f_\phi(\vec{p})) + \vartheta(-p_0)\bar{f}_\phi(-\vec{p})\right]$$

$$i\Delta_\phi^T(p) = \frac{i}{p^2-m_\phi^2+i\varepsilon} + 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)f_\phi(\vec{p}) + \vartheta(-p_0)\bar{f}_\phi(-\vec{p})\right]$$

$$i\Delta_\phi^{\bar{T}}(p) = \frac{i}{p^2-m_\phi^2-i\varepsilon} + 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)f_\phi(\vec{p}) + \vartheta(-p_0)\bar{f}_\phi(-\vec{p})\right]$$

Charge & Number Densities

$f_\phi(\vec{p})$: distribution function of particles

$\bar{f}_\phi(\vec{p})$: distribution function of anti-particles

Then note:

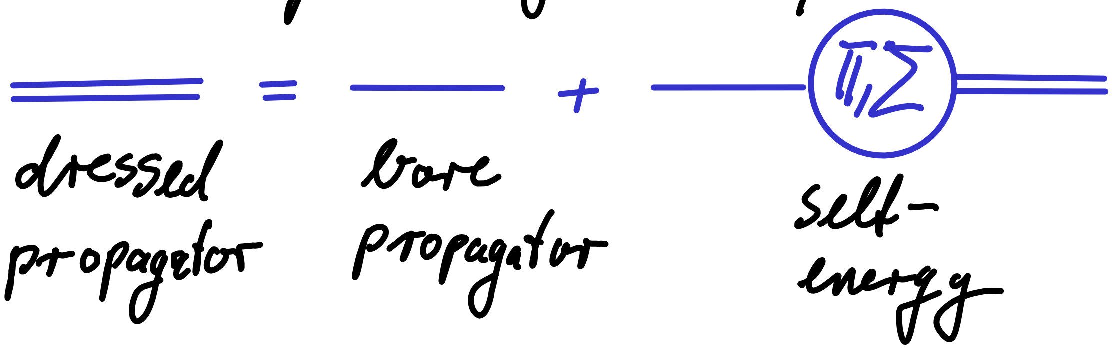
$$n_\phi - \bar{n}_\phi = \int \frac{d^3k}{(2\pi)^3}\left(f_\phi(\vec{k}) - \bar{f}_\phi(\vec{k})\right) = \int \frac{d^4k}{(2\pi)^4}\, 2k^0 \Delta_\phi^{<,>}(k)$$

Likewise, for fermions ($iS^{ab}(u,v) = \langle \mathcal{C}[\psi(u)\bar{\psi}(v)]\rangle$)

$$n_\psi - \bar{n}_\psi = \int \frac{d^3k}{(2\pi)^3}\left(f_\psi(\vec{k}) - \bar{f}_\psi(\vec{k})\right) = \frac{1}{2}\,\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\, i\gamma^0 S^{<,>}(k)$$

Kadanoff-Baym Equations

Schwinger-Dyson Equations:



- find an (approximate) self-consistent solution

The Kadanoff-Baym equations are the $<,>$ components:

$$(-\partial^2 - m^2)\Delta^{<,>} - \Pi^{H} \odot \Delta^{<,>} - \Pi^{<,>} \odot \Delta^{H} = \frac{1}{2}\left(\Pi^{>} \odot \Delta^{<} - \Pi^{<} \odot \Delta^{>}\right)$$

$\odot$: convolution — renormalise/neglect — collision term

- Wigner transformation & gradient expansion leads to the kinetic equations:

$$2k^0 \partial_t \, i\Delta^{<,>}(k,x) = \Pi^{>}(k,x)\Delta^{<}(k,x) - \Pi^{<}(k,x)\Delta^{>}(k,x)$$

Wigner Transformation

(For scalars & schematically, for simplicity)

$$(-\partial^2 - m^2)\Delta^{<,>} - \Pi^H \odot \Delta^{<,>} - \Pi^{<,>} \odot \Delta^H = \frac{1}{2}(\Pi^> \odot \Delta^< - \Pi^< \odot \Delta^>)$$

The convolution is understood as:

$$A \odot B = \int d^4w \, A(u,w)\, B(w,v)$$

The Wigner transform is:

$$A(k,x) = \int d^4r \, e^{ikr} A\left(x+\frac{r}{2}, x-\frac{r}{2}\right)$$

x → average coordinate — macroscopic evolution

k → relative coordinate — microscopic (quantum) properties

For the convolutions, can show that:

$$\int d^4r \, e^{ikr} \int d^4w \, A\left(x+\frac{r}{2}, w\right) B\left(w, x-\frac{r}{2}\right) = e^{-i\diamond}\{A(k,x)\}\{B(k,x)\}$$

where $\diamond\{\cdot\}\{\cdot\} = \frac{1}{2}\left(\partial_x^{(1)} \cdot \partial_k^{(2)} - \partial_k^{(1)} \cdot \partial_x^{(2)}\right)$

Gradient Expansion

When the system is slowly evolving, expand in powers of $\partial_x \cdot \partial_k \sim H/T \sim T/m_{Pl} \ll 1$

↳ typical momentum scale, i.e. T

↳ typical time-scale, i.e. Hubble time H^{-1}

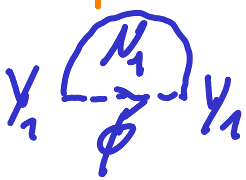
Leptogenesis is most efficient if

$$\Gamma_N = Y_1^2 \frac{1}{16\pi} M_1 \sim H \quad \text{and} \quad M_1 \sim T \quad \Rightarrow \quad Y_1^2 \frac{1}{16\pi} \sim H/T \ll 1$$

↷ We expand to linear order in $\partial_x \cdot \partial_k$ and Y_1^2

e.g. $$\not{\Sigma}_\ell \odot S_\ell \longrightarrow e^{-i\diamond}\{\not{\Sigma}_\ell(k,x)\}\{S_\ell(k,x)\} \approx \underbrace{\not{\Sigma}_\ell(k,x)\, S_\ell(k,x)}_{\text{local in time: "Markovian"}}$$

↳ convolution non-local in time



and $$-\partial^2 - m^2 \longrightarrow k^2 - \frac{1}{4}\partial^2 + ik\cdot\partial - m^2$$

Finite Width and Locality

In the present work, use zero width propagators.

Naive expectation for finite width:

$$\pi\,\delta(p^2-m^2) \longrightarrow \frac{\Pi^{\mathcal{A}}}{(p^2-m^2)^2+\Pi^{\mathcal{A}\,2}} \qquad \Pi^{\mathcal{A}} = \frac{i}{2}\left(\Pi^{>}-\Pi^{<}\right)$$

However, for constant Π, we find:

$$i\Delta^{<}(p) = \frac{2\Pi^{\mathcal{A}}}{(p^2-m^2)^2+\Pi^{\mathcal{A}\,2}}\, f^{eq}(p^0) + 2\pi\,\delta(p^2-m^2)\,\delta f(p^0)$$

Substituting this (i.e. the analogous solution for the propagator of N_1) leads to non-local terms in the kinetic equations.

Anisimov, Buchmüller, Drewes & Mendizabal (2010)

Finite Width and Locality

Gradient expansion only holds in the distributional sense:

$$\int \frac{dk^0}{2\pi} (\partial_t \Pi^>)(\partial_{k^0} \Delta^<) \simeq \int \frac{dk^0}{2\pi} \Delta^< \partial_{k^0} \frac{\Pi \Gamma^2}{k^0} \longrightarrow \text{higher order in } \vec{\nabla}$$

$$\hookrightarrow \partial_t \Pi^> \simeq -\frac{\Pi \Gamma^2}{k^0}$$

But:

$$(\partial_t \Pi^>)(\partial_{k^0} \Delta^<) \simeq \frac{\Pi \Gamma^2}{k^0} \frac{4\Pi\Gamma}{[(k^2-m^2)^2 + \Pi\Gamma^2]^2} (k^2-m^2) k^0 \longrightarrow \begin{array}{l}\text{leading order}\\ \text{at the pole}\end{array}$$

Backreaction is important!

Summation of these contributions leads to finite width for out-of-equilibrium part. (BG, in progress)

Problem is related to pinch singularities.

Is there a more efficient (re)summation procedure?

3. CTP Approach to Leptogenesis

Brief Summary of Recent Activity

* Buchmüller & Fredenhagen (2000):
  leptogenesis from Kadanoff-Baym equations; Wigner space; limit of classical statistics — find agreement with Boltzmann approach
* De Simone & Riotto (2007):
  give expression for source term in coordinate space; non-Markovian contributions but no evaluation
* Garny, Hohenegger, Kartavtsev & Lindner (2009-10):
  complete derivation & solution of kinetic equations from CTP formalism for scalar model; coordinate space; no need for RIS; resummation of wave-function corr.

Brief Summary of Recent Activity (continued)

* Anisimov, Buchmüller, Drewes & Mendizabal (2010):
  evaluation of lepton asymmetry with non-Markovian source — find sizable effects; include finite width of N_1 only; realistic initial conditions?

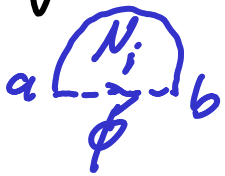
* Beneke, BG, Herranen, Schwaller & Fidler (2010):
  first principle derivation & solution for lepton asymmetry in a realistic model; Wigner space & gradient expansion $\longrightarrow$ Markovian kinetic equations; flavoured leptogenesis from first principles (derivation & solution of kinetic equations)

Tree-Level Collision Terms

Now, turn to leptogenesis.

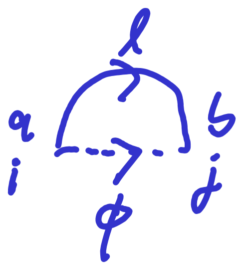
The leading order self energies are:

$$i\Sigma_\ell^{ab}(k) =$$

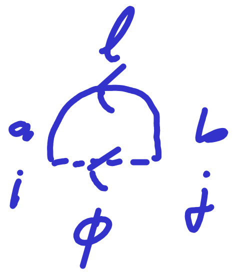


$$= |Y_i|^2 \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k-k'-k'') * P_R\, iS_{N_i}^{ab}(k')\, P_L\, i\Delta_\phi^{ba}(k'')$$

$$i\Sigma_{N_{ij}}^{ab}(k) =$$



$+$



$$= \overset{\text{2 from } SU(2)_L}{g_w} \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k-k'-k'')$$

$$* \Big\{ Y_i Y_j^* P_L\, iS_\ell^{ab}(k')\, P_R\, i\Delta_\phi^{ab}(k'') + Y_i^* Y_j\, C\left[P_L\, iS_\ell^{ab}(-k')\, P_R\right]^T C^\dagger\, i\Delta_\phi^{ba}(-k'') \Big\}$$

Tree-Level Collision Terms (continued)

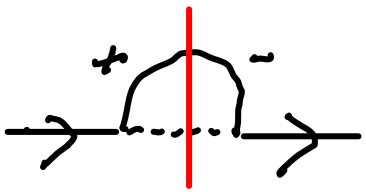
$$\frac{\partial}{\partial t}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\,\gamma^0 iS_\ell^{<,>}(k) = -\frac{\partial}{\partial t}(n_\ell-\bar n_\ell)$$

$$= -\int\frac{d^4k}{(2\pi)^4}\left[i\not\Sigma_\ell^{>}(k)P_L\, iS_\ell^{<}(k) - i\not\Sigma_\ell^{<}(k)P_L\, iS_\ell^{>}(k)\right]$$

$$= |Y_i|^2\int\frac{d^3k}{(2\pi)^3 2|\vec k|}\frac{d^3k'}{(2\pi)^3 2\sqrt{\vec k'^2+M_i^2}}\frac{d^3k''}{(2\pi)^3 2|\vec k''|}(2\pi)^4\delta^4(k'-k-k'')\,2k\cdot k'$$

$$*\Big\{\left[1-f_{N_i}(\vec k')\right]*\left[f_\ell(\vec k)f_\phi(\vec k'')-\bar f_\ell(\vec k)\bar f_\phi(\vec k'')\right]$$

$$-f_{N_i}(\vec k')*\left[(1-f_\ell(\vec k))(1+f_\phi(\vec k''))-(1-\bar f_\ell(\vec k))(1+\bar f_\phi(\vec k''))\right]\Big\}$$



$$\longrightarrow \left(\text{---}\!\!\!<\right)*\left(\text{---}\!\!\!<\right)^* * f_N(1-f_\ell)(1+f_\phi)$$

+ 3 additional combinations

KMS* Relations

*Kubo-Martin-Schwinger

A useful symmetry for simplifications close to equilibrium.

$$iS_\ell^>(p) = -e^{\beta p_0}\, iS_\ell^<(p) \qquad iS_{N_i}^>(p) = -e^{\beta p_0}\, iS_{N_i}^<(p) \qquad i\Delta_\phi^>(p) = e^{\beta p_0}\, i\Delta_\phi^<(p)$$

These relations generally hold for equilibrium Green functions, as can be shown in the imaginary time formalism.

Hence, they also should hold for the self-energies at all orders in perturbation theory:

$$i\Sigma_{\ell,N}^>(p) = -e^{\beta p_0}\, i\Sigma_{\ell,N}^<(p) \qquad i\Pi_\phi^>(p) = e^{\beta p_0}\, i\Pi_\phi^<(p)$$

KMS Relations (continued)

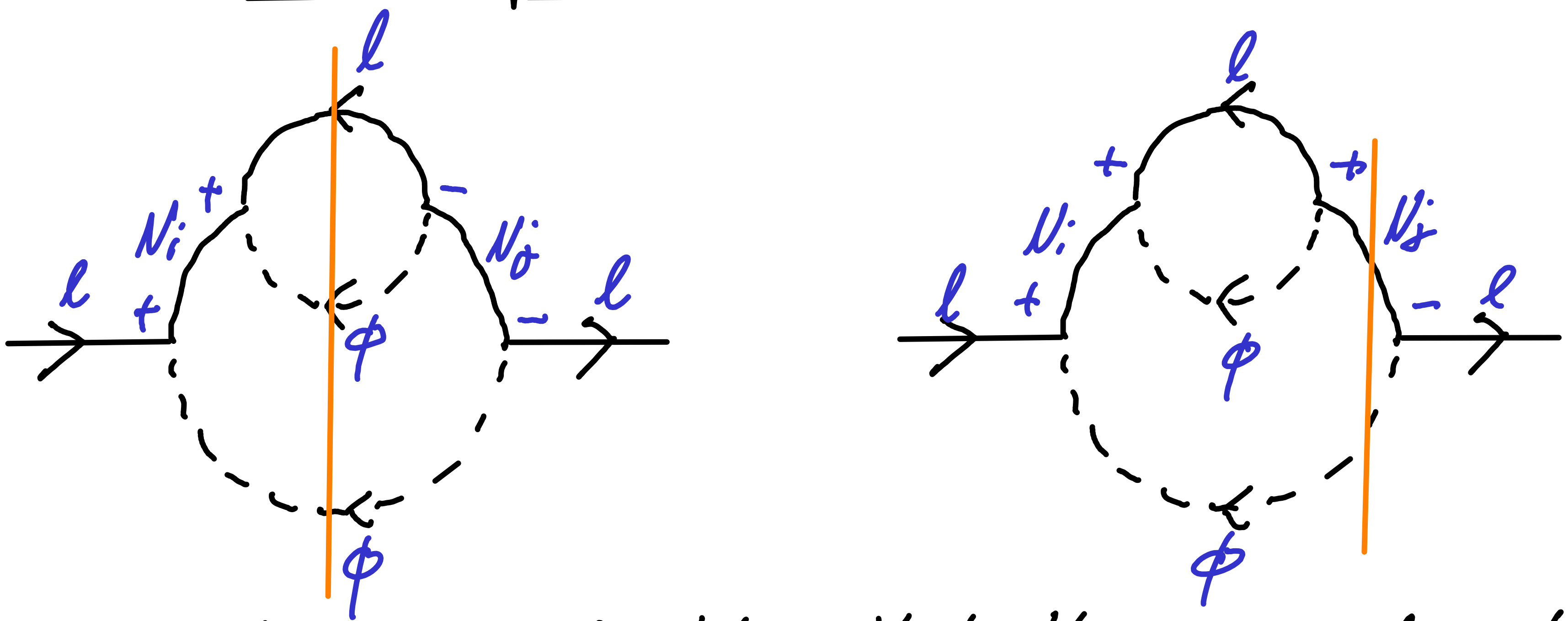
Note that KMS implies the vanishing of the collision term in thermal equilibrium:

$$i \not{\Sigma}_\ell^{>}(k) P_L \, i S_\ell^{<}(k) - i \underbrace{\not{\Sigma}_\ell^{<}(k)}_{-e^{-\beta k^0} \not{\Sigma}_\ell^{>}(k)} P_L \, i \underbrace{S_\ell^{>}(k)}_{-e^{\beta k^0} S_\ell^{<}(k)} = 0$$

↝ Within the CTP formalism, we get the vanishing of the asymmetry in equilibrium for free.

Yet, it is instructive to see how this is explicitly realised in perturbative calculations for leptogenesis.

Wave-function Contribution



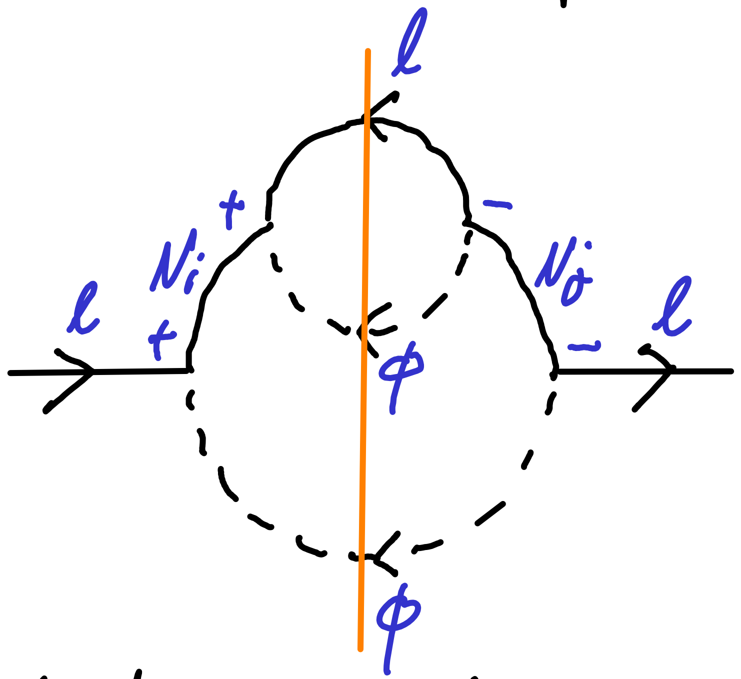
We can show explicitly, that the wave-function contributions to the collision term cancel, provided

$$iS^{>}_{N_i}(k)\, iS^{<}_{\ell}(k')\, i\Delta^{<}_{\phi}(k'') = iS^{<}_{N_i}(k)\, iS^{>}_{\ell}(k)\, i\Delta^{>}_{\phi}(k)$$

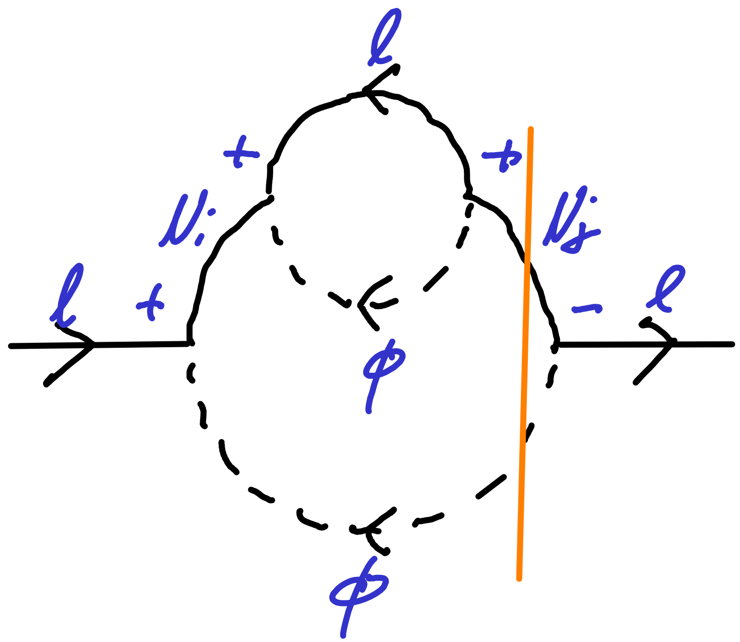
This holds in thermal equilibrium (KMS), but more generally when $\Gamma_{N_i\to\ell\phi} = \Gamma_{\ell\phi\to N_i}$ and $\Gamma_{N_i\to\bar{\ell}\phi^*} = \Gamma_{\bar{\ell}\phi^*\to N_i}$.

No asymmetry in local equilibrium, as required by CPT.

Wave-function Contribution (continued)



Interference between two s-channel scatterings.



Interference between loop and tree-level decays.

When neglecting quantum-statistical corrections (i.e. $(1-f_{\ell,N}) \to 1$ and $(1+f_\phi) \to 1$), and cutting as indicated, we recover the usual RIS-subtraction.

Result for Wave-Function Contribution

$$\int \frac{d^3k}{(2\pi)^3} \epsilon_\ell^{wf}(\vec{k}) = 4\, \mathrm{Im}\left[Y_1^2 Y_2^{*2}\right] \frac{M_1 M_2}{M_1^2 - M_2^2}$$

$$* \int \frac{d^3k'}{(2\pi)^3\, 2\sqrt{\vec{k}'^2 + M_1^2}}\, \delta f_N(\vec{k}') \frac{\Sigma_{N\mu}(\vec{k}')\, \Sigma_N^\mu(\vec{k}')}{g_w}$$

where we have the thermal decay rate:

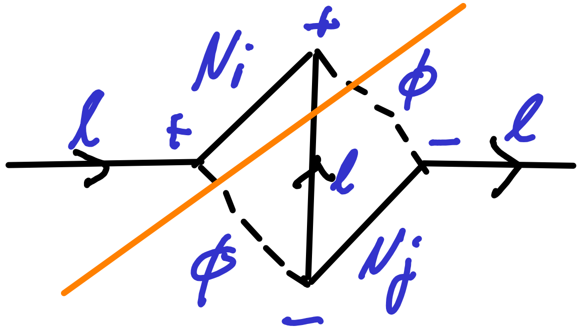
$$\Sigma_N^\mu(\vec{k}) = g_w \int \frac{d^3p}{(2\pi)^3\, 2|\vec{p}|} \frac{d^3q}{(2\pi)^3\, 2|\vec{q}|} (2\pi)^4 \delta^4(k-p-q)$$

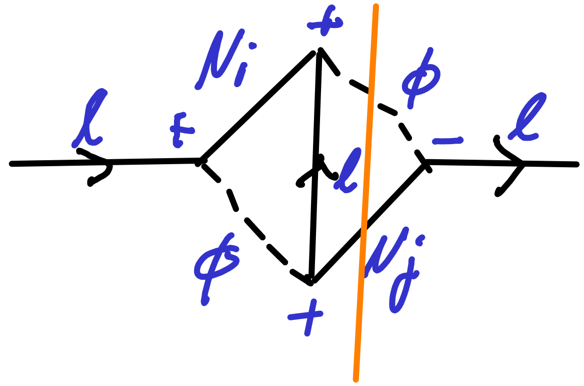
$$* p^\mu \left[1 - f_\ell(\vec{p}) + f_\phi(\vec{q})\right]$$

Note: We can evaluate Σ_N^μ analytically in terms of logarithms and dilogarithms.

$$\Sigma_N^\mu(\vec{k}) \xrightarrow[M_N \gg T]{} g_w \frac{k^\mu}{16\pi}$$ recover standard approximation

Vertex Contribution





Interference between s- and t-channel scatterings

Interference between loop and tree-level decays.

$$\int \frac{d^3p'}{(2\pi)^3}\, \epsilon^V(\vec{p}') = 4\,\mathrm{Im}\left[Y_1^2 Y_2^{*2}\right] \int \frac{d^3k}{(2\pi)^3\, 2\sqrt{\vec{k}^2+M_1^2}}\, \delta f_{N_1}(\vec{k})\, V(k)$$

$$V(k) = \int \frac{d^3p'}{(2\pi)^3\, 2|\vec{p}'|} \frac{d^3p''}{(2\pi)^3\, 2|\vec{p}''|} (2\pi)^4 \delta^4(k-p'-p'')\, p'^{\mu}\, \Gamma_\mu(k,p'') \left[1 - f_\ell(\vec{p}') + f_\phi(\vec{p}'')\right]$$

Thermal vertex function:

$$\Gamma_\mu(k,p'') = \int \frac{d^3k'}{(2\pi)^3\, 2|\vec{k}'|} \frac{d^3k''}{(2\pi)^3\, 2|\vec{k}''|} (2\pi)^4 \delta^4(k-k'-k'')\, k'_\mu\, \frac{M_1 M_2}{(k'-p'')^2 - M_2^2} \left[1 - f_\ell(\vec{k}') + f_\phi(\vec{k}'')\right]$$

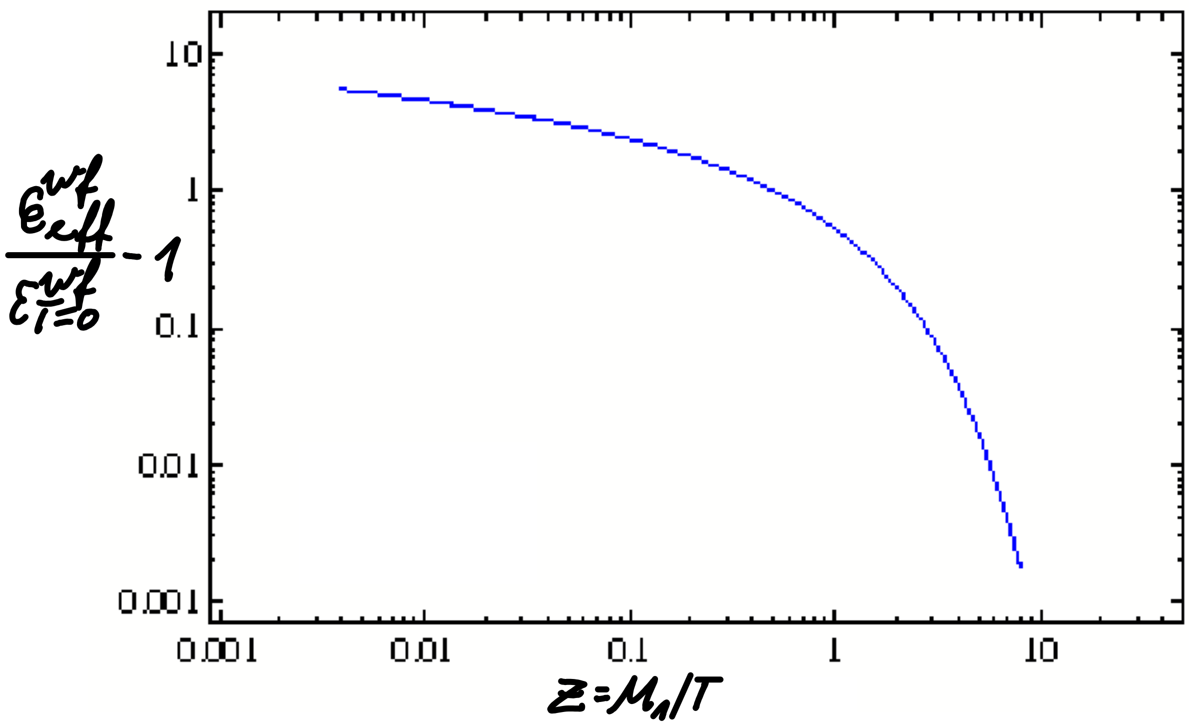
Effective CP-Violating Parameter

Consider limit $M_1 \ll M_2$

full result

$$\varepsilon_{\rm eff}^{wf} / \varepsilon_{T=0}^{wf} = \frac{16\pi}{g_w} \frac{\int d|\vec{k}| \frac{\vec{k}^2}{k_0} \Sigma^{\mu}_{N_1}(\vec{k}) \Sigma_{N_1 \mu}(\vec{k}) \, \delta f_{N_1}(\vec{k})}{\int d|\vec{k}| \frac{\vec{k}^2}{k_0} k_\mu \Sigma^{\mu}_{N_1}(\vec{k}) \, \delta f_{N_1}(\vec{k})}$$

quantum statistics in external states only – not in loops

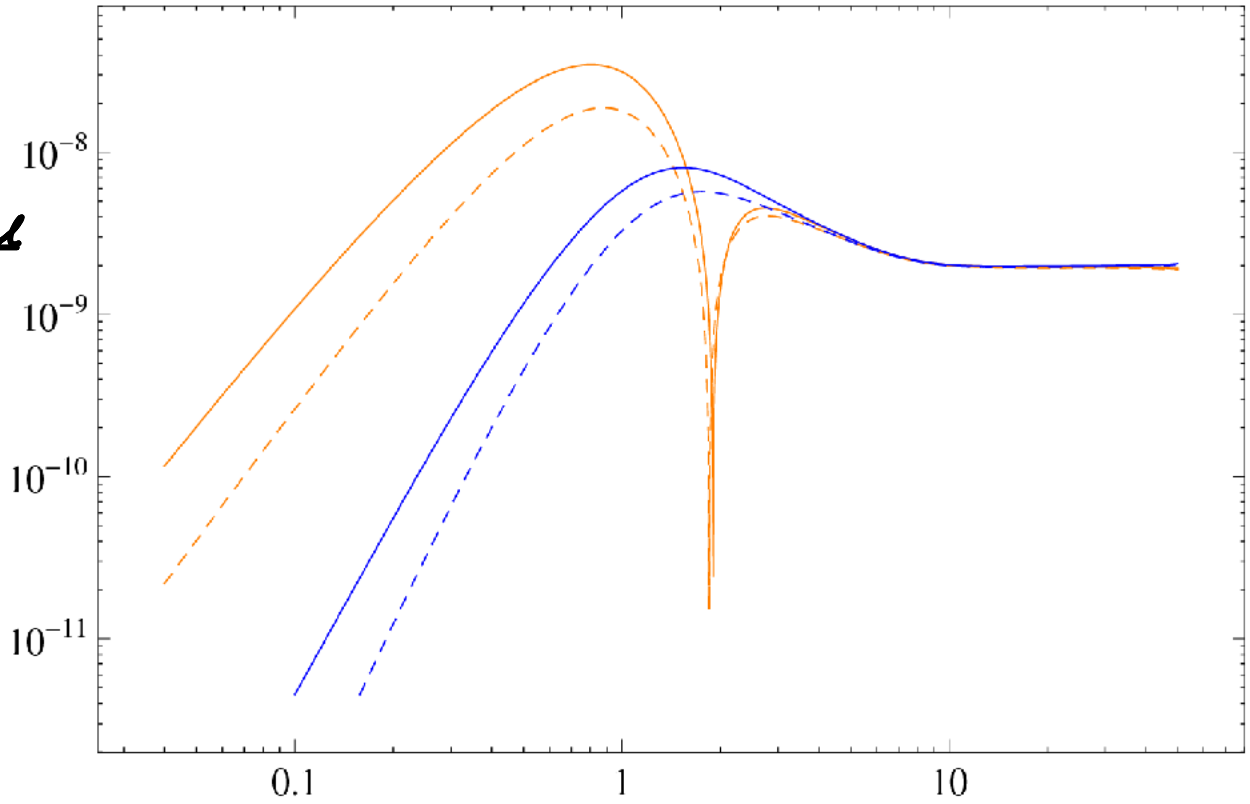


Expect large effect for weak washout,

small effect for strong washout.

Numerical Results: Strong Washout

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



$z = M_1/T$

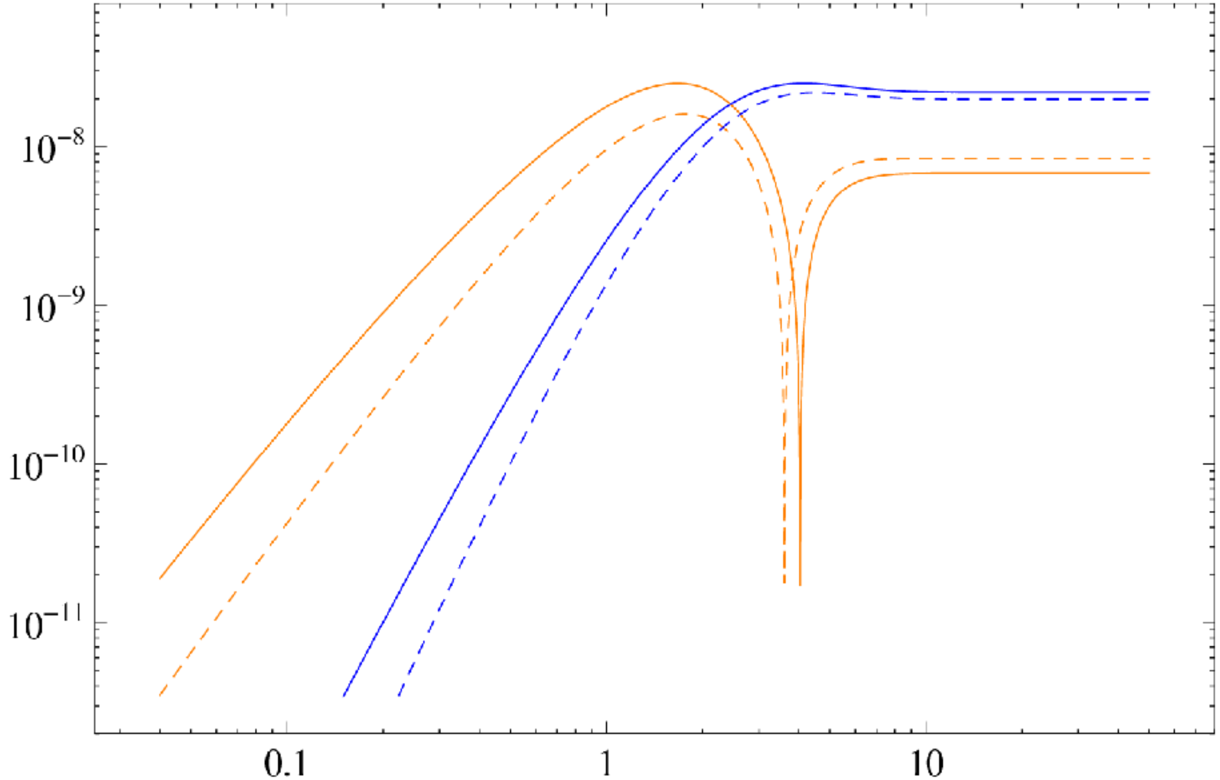
blue: thermal initial f_{N_1} red: zero initial f_{N_1}

solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 5 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

Numerical Results: Moderate Washout

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



$z = M_1/T$

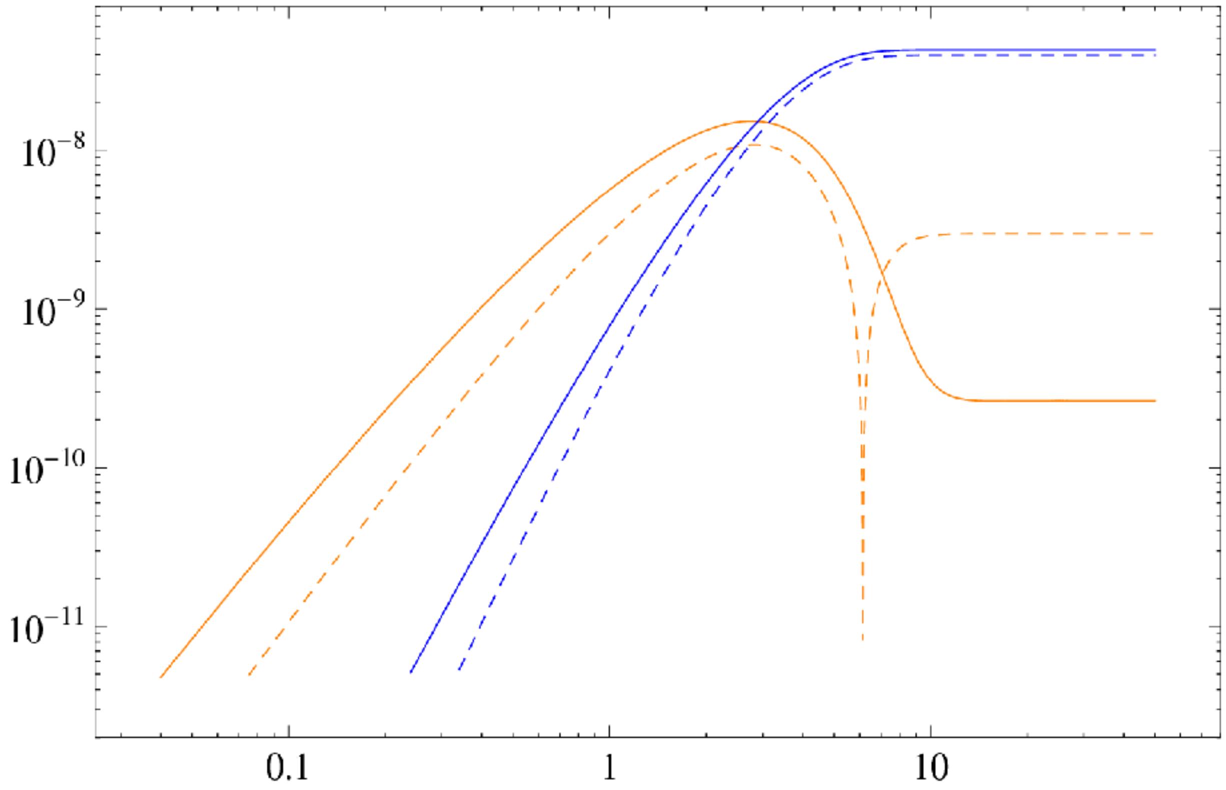
blue: thermal initial f_{N_1} red: zero initial f_{N_1}

solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 2 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

Numerical Results: Weak Washout

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



$z = M_1/T$

sign change for vanishing initial conditions

blue: thermal initial f_{N_1} red: zero initial f_{N_1}

solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 1 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

4. Flavoured Leptogenesis

Flavour Coherence

Add flavour:

$$\mathcal{L} = \frac{1}{2}\overline{\psi}_{N_i}(i\partial\!\!\!/ - M_i)\psi_{N_i} + \overline{\psi}_{\ell a}\, i\partial\!\!\!/\,\psi_{\ell a} + \overline{\psi}_{Rb}\, i\partial\!\!\!/\,\psi_{Rb} + (\partial^\mu \phi^\dagger)(\partial_\mu \phi)$$

$$- Y_{ia}^* \overline{\psi}_{\ell a} \phi^\dagger \psi_{N_i} - Y_{ia} \overline{\psi}_{N_i} \phi \psi_{\ell a} - h_{ab} \phi^\dagger \overline{\psi}_{Ra} P_L \psi_{\ell b} - h_{ab}^* \phi \overline{\psi}_{\ell b} \psi_{Ra}$$

In general, h_{ba} and Y_{ia} are linearly independent.

$\longrightarrow$ off-diagonal densities in flavour basis (where $h^\dagger h$ is diagonal)

$\longrightarrow$ off-diagonals are damped depending on the size of the h_{ab}.

Flavour Dynamics

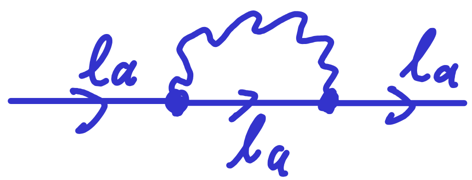
The Schwinger-Dyson equations on the CTP can easily be generalised in order to include flavour:

$$i\not\partial_u S^{fg}_{\ell ab}(u,v) = f\,\delta^{fg}\delta_{ab}\,\delta^4(u-v)P_R + \sum_{hc}\int d^4w\, \not\Sigma^{fh}_{\ell ac}(u,w)\, S^{hg}_{\ell cb}(w,v)$$

Flavour oscillations are induced through thermal masses within $\not\Sigma^H_\ell = \frac{1}{2}\left(\not\Sigma^T_\ell - \not\Sigma^{\bar T}_\ell\right)$:

$$\not\Sigma^H_\ell = P_R\left[\gamma^0\left(\bar\varsigma^{bl}+\bar\varsigma^{fl}\right) + \frac{\vec k\cdot\vec\gamma}{|\vec k|}\left(\varsigma^{bl}+\varsigma^{fl} - \mathrm{sign}(k^0)\left[\bar\varsigma^{bl}+\bar\varsigma^{fl}\right]\right)\right]P_L$$

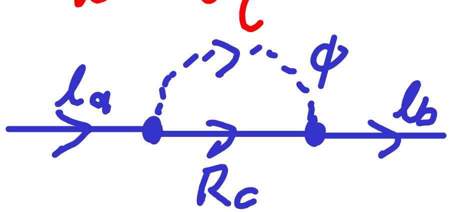
$$\varsigma^{bl}_{ab}(k^0,\vec k) = \delta_{ab}\,\varsigma^{bl}(k^0,\vec k)$$

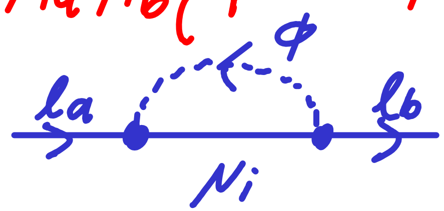


"blind"

$$\varsigma^{fl}_{ab}(k^0,\vec k) = h^\dagger_{ac}h_{cb}\,\varsigma^{fl,h}(k^0,\vec k) + Y^*_{ia}Y_{ib}\,\varsigma^{fl,Y}_i(k^0,\vec k)$$

"flavoured"





Flavour Oscillations

Σ^{fl}_{ab} is hermitian and can be diagonalised, $\Sigma^{fl}_D = U^\dagger \Sigma^{fl} U$.
The kinetic equations in the flavour-diagonal basis are:

$$i\partial_t\, i\gamma^0 S_\ell^{<,>} + i[\Xi, i\gamma^0 S_\ell^{<,>}] - \underbrace{[\not{\Sigma}_\ell^H \gamma^0, iS_\ell^{<,>}]}_{\text{flavour oscillations}} = \frac{1}{2}(i\mathcal{C}_\ell + i\mathcal{C}_\ell^\dagger)$$

compensates for time-dependent basis change:
$\Xi = U^\dagger \partial_t U$

Kadanoff-Baym Ansatz:

$$iS^<_{\ell ab} = -2\pi P_L \not{k} P_R\, \delta(k^2)\left[\vartheta(k^0) f^+_{\ell ab}(\vec{k}) - \vartheta(-k^0)(\mathbb{1}_{ab} - f^-_{\ell ab}(-\vec{k}))\right]$$

$$iS^>_{\ell ab} = -2\pi P_L \not{k} P_R\, \delta(k^2)\left[\vartheta(k^0)(\mathbb{1}_{ab} - f^+_{\ell ab}(\vec{k})) - \vartheta(-k^0) f^-_{\ell ab}(-\vec{k})\right]$$

thermal mass and finite-width correction in the mass-shell structure neglected $\longrightarrow$ higher order in gradients in the oscillation and the collision term

Kinetic Equilibrium & Number Densities

fast gauge interactions $\longrightarrow$ kinetic equilibrium

$$f^{\pm}_{\ell ab}(\vec{k}) = \left(\frac{1}{e^{\beta|\vec{k}| - \beta\mu^{\pm}} + 1} \right)_{ab}$$

$\mu^{\pm}_{ab}$: hermitian matrix of chemical potentials for (anti-)leptons

pair creations/annihilations impose $\mu^{-}_{ab} = -\mu^{+}_{ab}$ (more details below)

Number densities:

$$n^{\pm}_{\ell ab} = \int \frac{d^3k}{(2\pi)^3} f^{\pm}_{\ell ab}(\vec{k}) = \mp \int \frac{d^3k}{(2\pi)^3} \int_{0,-\infty}^{\infty,0} \frac{dk^0}{2\pi} \operatorname{tr}\left[i\gamma^0 S^{<}_{\ell ab} \right]$$

$$\delta n^{\pm}_{\ell ab} = n^{\pm}_{\ell ab} - n^{\pm eq}_{\ell ab} \approx \mu_{\ell ab} \frac{T^2}{12} \quad \text{for } \frac{|\mu|}{T} \ll 1$$

Charge number density:

$$q_{\ell ab} = \delta n^{+}_{\ell ab} - \delta n^{-}_{\ell ab}$$

Collision Terms

Flavour blind (gauge interactions):

$$\pm\frac{1}{2}\,\mathrm{tr}\int\limits_{0,-\infty}^{\infty,0}\frac{dk^0}{2\pi}\int\frac{d^3k}{(2\pi)^3}\left(\mathscr{C}^{bl}_{\ell ab}+\mathscr{C}^{bl\dagger}_{\ell ab}\right)=-\underbrace{\Gamma^{bl}}_{\sim g_2^4 T}\left(\delta n^+_{\ell ab}+\delta n^-_{\ell ab}\right)$$

Flavour sensitive: $\Gamma^{fl}\sim g_2^2 h^2 T$ — more complicated structure, but essentially damp off-diagonals

$\longrightarrow$ Kinetic Equations for Number Densities

eff $\equiv$ thermal momentum average

$$\frac{d}{dt}\delta n^\pm_{\ell ab}=-\Gamma^{\mathrm{eff}}_{ac}\,\delta n^\pm_{\ell cb}+\delta n^\pm_{\ell ac}\,\Gamma^{\mathrm{eff}}_{cb}\overbrace{\mp\Delta\omega^{\mathrm{eff}}_{\ell ab}\,\delta n^\pm_{\ell ab}}^{\text{flavour oscillations}}$$

$$\underbrace{-W_{ac}\,\delta n^\pm_{\ell cb}-\delta n^\pm_{\ell ac}W^\dagger_{cb}}_{\text{washout}}\underbrace{\pm S_{ab}}_{\text{source}}-\Gamma^{bl}\left(\delta n^+_{\ell ab}+\delta n^-_{\ell ab}\right)-\Gamma^{fl}_{ab}$$

flavour oscillations — $\Delta\omega$ is purely off-diagonal

Subtract "−" from "+" equation, to obtain equation for $q_{\ell ab}$.

Suppression of Flavour Oscillations

Essential dynamics is captured by the toy system

$$\frac{d}{dt}\delta g^+(t) = -i\Delta\omega\,\delta g^+(t) - \Gamma^{bl}\left[\delta g^+(t) + \delta g^-(t)\right]$$

$$\frac{d}{dt}\delta g^-(t) = +i\Delta\omega\,\delta g^-(t) - \Gamma^{bl}\left[\delta g^+(t) + \delta g^-(t)\right]$$

$\Gamma^{bl} \sim g_2^4 T$

$\Delta\omega \sim h_t^2 T \ll \Gamma^{bl}$

$\longrightarrow$ short & long modes: $\delta g_{s,l} \approx \delta g^+ \pm \left(1 \mp i\frac{\Delta\omega}{\Gamma}\right)\delta g^-$

$$\tau_{s,l}^{-1} = \Gamma^{bl} \pm \sqrt{\Gamma^{bl\,2} - \Delta\omega^2}$$

* identify long mode with q_l
* constrain $\delta g^+ + \delta g^- = 0$

$$\tau_s \approx \frac{1}{2\Gamma^{bl}}$$ pair creation/annihilation

$$\tau_l \approx \frac{2\Gamma^{bl}}{\Delta\omega^2} \sim \frac{g^4}{h_t^4 T} \gg \tau^{fl} \sim \frac{1}{g^2 h_t^2 T}$$

flavour oscillations over-damped because of fast pair creation/annihilation

$\longrightarrow$ Flavour sensitive damping dominates the dynamics of off-diagonal densities.

Creation/Annihilation vs. Majorana Constraint

Majorana condition: $C = i\gamma^0\gamma^2$; $\psi^c = C\bar{\psi}^t = C\gamma^{0t}\psi^*$

$$S^{+-}_{\alpha\beta ij}(x,y) = -\langle \bar{\psi}_{\beta j}(y)\,\psi_{\alpha i}(x)\rangle \overset{!}{=} -\langle \bar{\psi}^c_{\beta j}(y)\,\psi^c_{\alpha i}(x)\rangle$$

$$= \langle (\psi^t_{\gamma j}(y)\, C^\dagger_{\gamma\beta})\, C_{\alpha\delta}\,\gamma^0_{\sigma\delta}\,\psi^*_{\sigma i}(x)\rangle$$

$$= C_{\alpha\delta}\,\langle \psi^\dagger_{\gamma j}(y)\,\psi^*_{\delta i}(x)\rangle\, C^\dagger_{\gamma\beta} = \left[C\, S^{-+t}(y,x)\, C^\dagger\right]_{\alpha\beta ij}$$

$\Longrightarrow$ $f_{ij}(k^0,\vec{k}) = f_{ji}(-k^0,-\vec{k})$ transposition *and* sign flip in k^0

$\longrightarrow$ same oscillation frequency

Cf. fast gauge interactions:

$\delta f_{ij}(k^0,\vec{k}) = -\delta f_{ij}(-k^0,-\vec{k})$ sign flip in k^0, *no* transposition

$\longrightarrow$ opposite oscillation frequency

Flavoured Kinetic Equations

$$\boxed{\begin{aligned} \frac{\partial q_{\ell ab}}{\partial \eta} &= -\sum_c \left[W_{ac}\, q_{\ell cb} + q_{\ell ac}\, W_{cb} \right] + 2 S_{ab} - \Gamma^{\ell\ell}_{\ell ab} \\ \frac{\partial q_{Rab}}{\partial \eta} &= -\Gamma^{\ell\ell}_{Rab} \end{aligned}}$$

$\uparrow$ washout $\uparrow$ source

Can work in fixed basis, since oscillations are frozen in.

Take $h = \begin{pmatrix} h_{\tilde{t}} & 0 \\ 0 & 0 \end{pmatrix}$

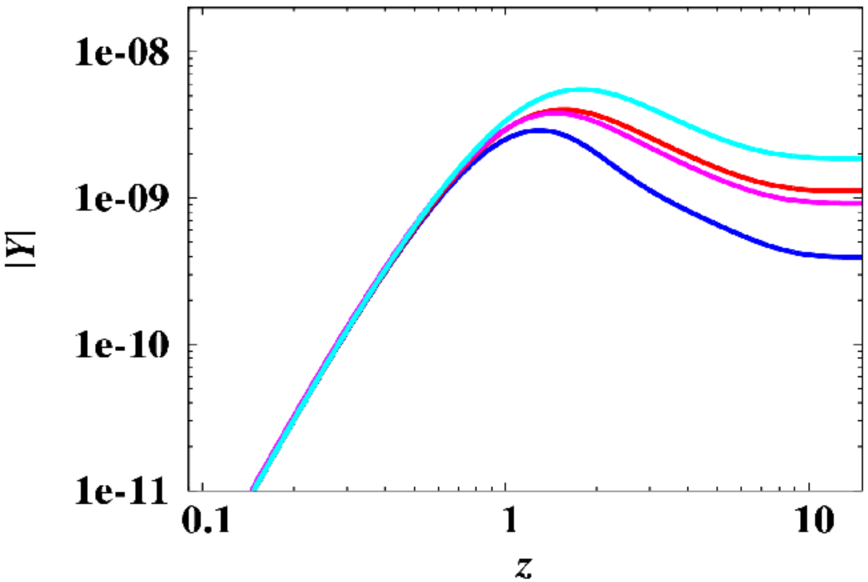
$$\Gamma^{\ell\ell}_{\ell} \sim h_{\tilde{t}}^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_\ell + q_\ell \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{R11} & 0 \\ 0 & 0 \end{pmatrix} \right]$$

$$\Gamma^{\ell\ell}_{R} \sim h_{\tilde{t}}^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_R + q_R \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{\ell 11} & 0 \\ 0 & 0 \end{pmatrix} \right]$$

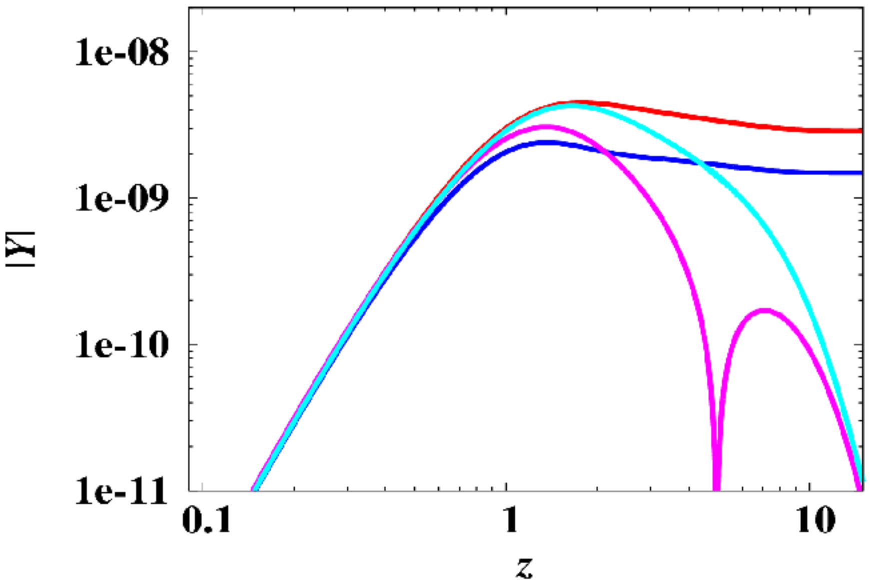
$\longrightarrow$ off-diagonals suppressed for $\Gamma^{\ell\ell}_{\ell,R} \gg H, \Gamma^{ID} = \Gamma_{\ell\phi \to N_1}$

Suppression of the off-Diagonals

$h_\tau = 0$



$h_\tau = 7 * 10^{-3}$



$\begin{pmatrix} Y_{\ell 11} & Y_{\ell 12} \\ Y_{\ell 21} & Y_{\ell 22} \end{pmatrix}$ lepton number to entropy ratio

$$Y = \left.\begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix}\right\} \text{r.h. neutrino}$$

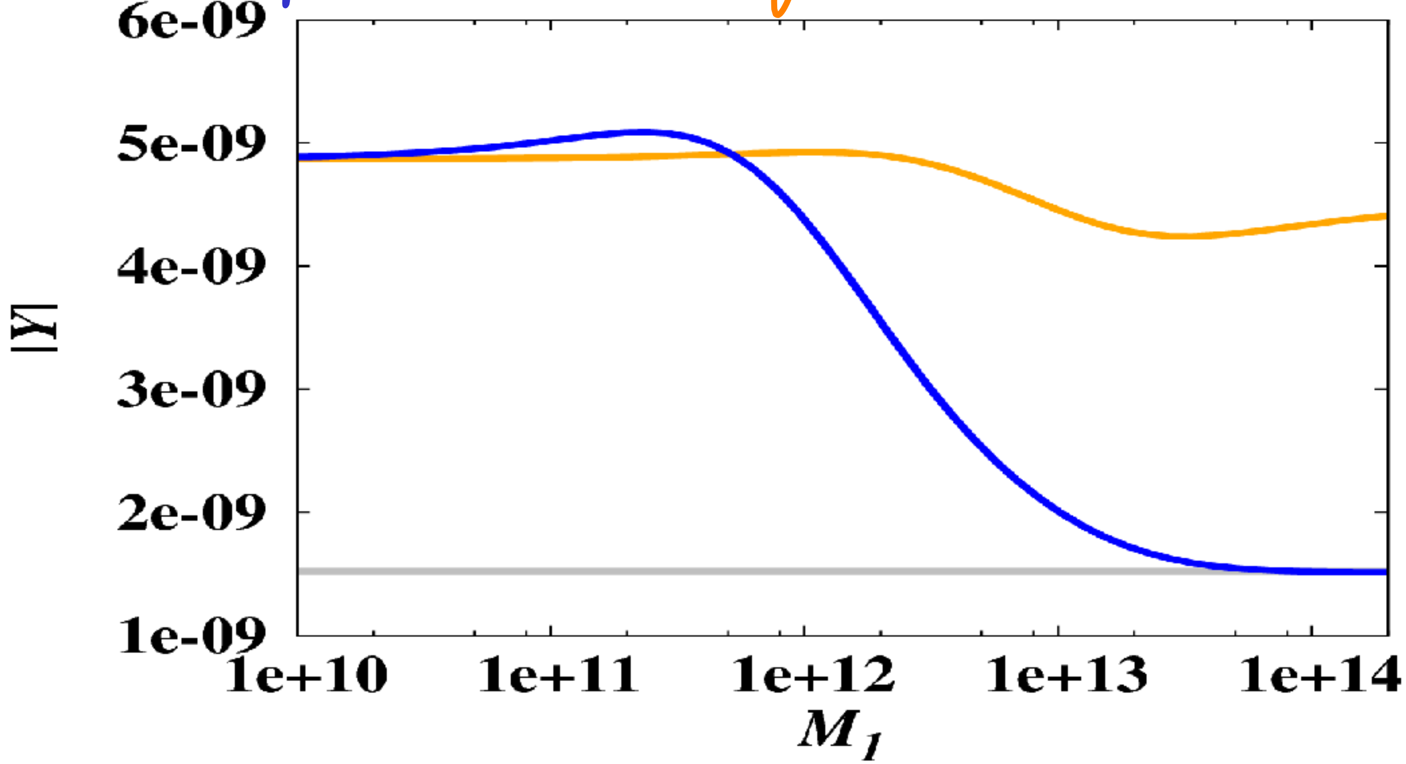
$\ell_\tau \equiv \ell_1 \qquad \ell_{\mu e} \equiv \ell_2$

$M_1 = 10^{12}\ \mathrm{GeV}$

$M_2 = 10^{14}\ \mathrm{GeV}$

Full Result Interpolates Between Flavoured/Unflavoured Limits

full result / fully flavoured / unflavoured



$h_\tau = 7 * 10^{-3}$

$M_1 \longrightarrow \alpha M_1$
$Y_{11} \longrightarrow \alpha Y_{11}$
$Y_{12} \longrightarrow \alpha Y_{12}$

fixed Y_ℓ in the unflavoured limit

$$Y = \begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix}$$

r.h. neutrino

$\ell_\tau \equiv \ell_1 \qquad \ell_{\mu,e} \equiv \ell_2$

5. Conclusions

- First principle description of leptogenesis within the CTP framework.
- Alternative – perhaps more satisfactory – manifestation of CPT invariance in the kinetic equation when compared to the usual RIS subtraction scheme.
- Full inclusion of quantum statistical corrections in loops & external states (contribute at the same level).
- First principle description of flavoured leptogenesis, gauge interactions suppress flavour oscillations
- Future: thermal corrections ($\longrightarrow$ weak washout), Markovian issue, quasi-degenerate r.h. neutrinos, ...

  Present work is a platform for more theoretical improvements.