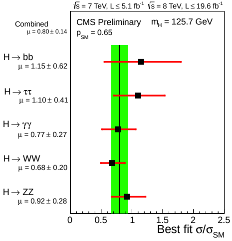
Towards event generation at NNLO

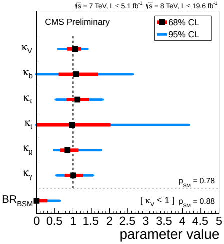
Emanuele Re
Rudolf Peierls Centre for Theoretical Physics, University of Oxford

University of Sussex, 19 May 2014

Status after LHC “run I”

- Scalar at 125 GeV found, study of properties begun





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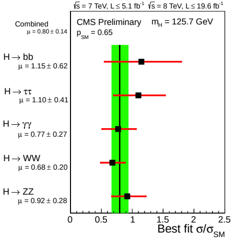
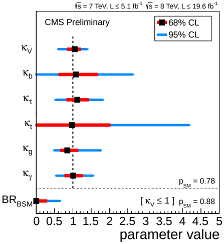
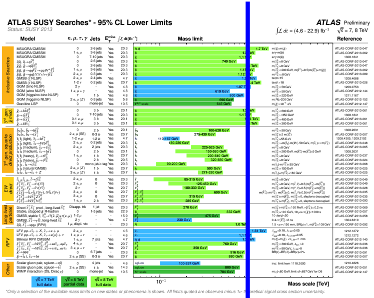
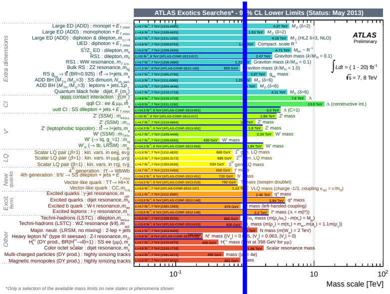




- In general no smoking-gun signal of new-physics





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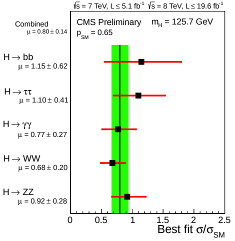

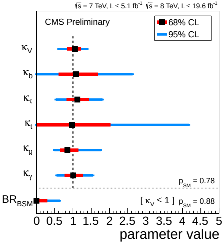

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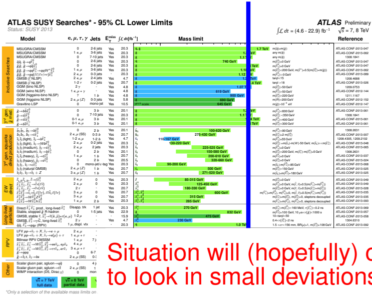
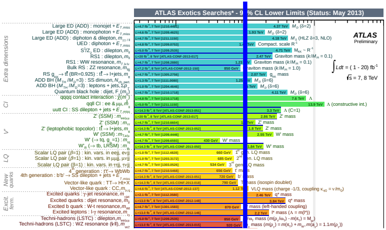


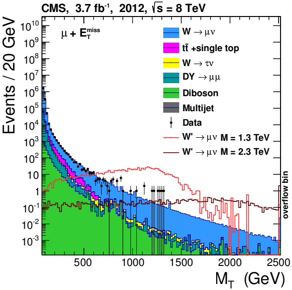


Situation will (hopefully) change at 13-14 TeV. If not, then we have to look in small deviations wrt SM: “precision physics”.

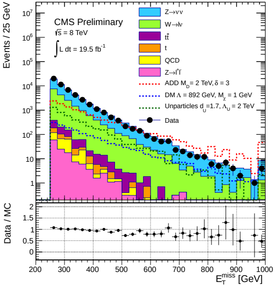
Search strategies and theory inputs

- examples of strategies to find new-physics / isolate SM processes:

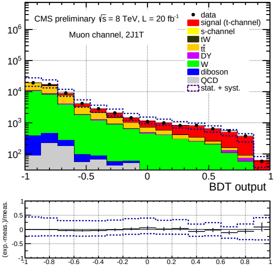
1 : s-channel resonance



2 : high-pt excess



3 : BDT output



- Higgs discovery belongs to 1, but Higgs characterization requires theory inputs (rates,shapes,binned x-sections,...)
- For 2 and 3, we need to control as much as possible QCD effects (i.e. rates and shapes, and also uncertainties!)
- Some analysis techniques (e.g. 3) heavily relies on using MC event generators to separate signal and backgrounds

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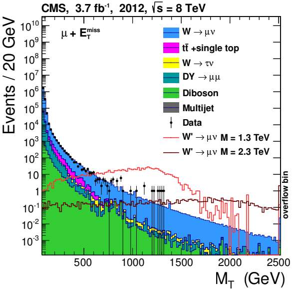

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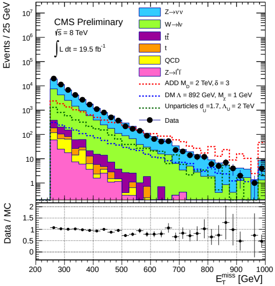

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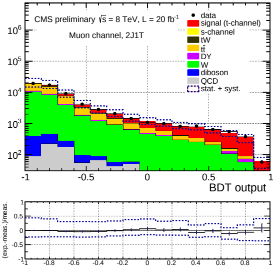


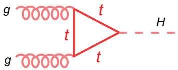
- at some level, MC event generators enter in almost all experimental analyses

precise tools ⇒ smaller uncertainties on measured quantities

⇓

"small" deviations from SM accessible

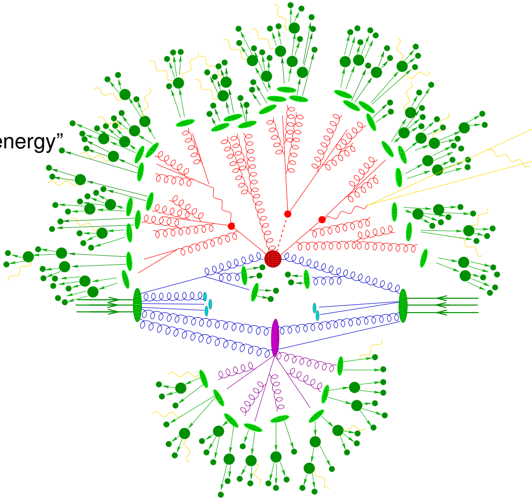
ideal world: high-energy collision and detection of elementary particles



Event generators: what they are?

ideal world: high-energy collision and detection of elementary particles
real world:

- collide non-elementary particles
- we detect e, μ, γ,hadrons, “missing energy”
- we want to predict final state
  - realistically
  - precisely
  - from first principles



[sherpa’s artistic view]

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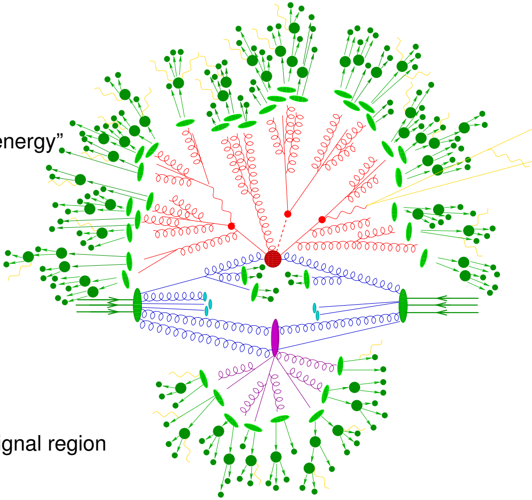


[sherpa's artistic view]

$\Rightarrow$ full event simulation needed to:
- compare theory and data
- estimate how backgrounds affect signal region
- test analysis strategies

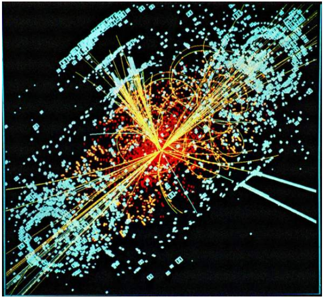
Event generators: what's the output?

- in practice: momenta of all outgoing leptons and hadrons:

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY
31	NU_E	12	1	29	22	0	0	60.53	37.24	-1185.0	1187.1
32	E+	-11	1	30	22	0	0	-22.80	2.59	-232.4	233.6
148	K+	321	1	109	9	0	0	-1.66	1.26	1.3	2.5
151	PI0	111	1	111	9	0	0	-0.01	0.05	11.4	11.4
152	PI+	211	1	111	9	0	0	-0.19	-0.13	2.0	2.0
153	PI-	-211	1	112	9	0	0	0.84	-1.07	1626.0	1626.0
154	K+	321	1	112	9	0	0	0.48	-0.63	945.7	945.7
155	PI0	111	1	113	9	0	0	-0.37	-1.16	64.8	64.8
156	PI-	-211	1	113	9	0	0	-0.20	-0.02	3.1	3.1
158	PI0	111	1	114	9	0	0	-0.17	-0.11	0.2	0.3
159	PI0	111	1	115	18	0	0	0.18	-0.74	-267.8	267.8
160	PI-	-211	1	115	18	0	0	-0.21	-0.13	-259.4	259.4
161	N	2112	1	116	23	0	0	-8.45	-27.55	-394.6	395.7
162	NBAR	-2112	1	116	23	0	0	-2.49	-11.05	-154.0	154.4
163	PI0	111	1	117	23	0	0	-0.45	-2.04	-26.6	26.6
164	PI0	111	1	117	23	0	0	0.00	-3.70	-56.0	56.1
167	K+	321	1	119	23	0	0	-0.40	-0.19	-8.1	8.1
186	PBAR	-2212	1	130	9	0	0	0.10	0.17	-0.3	1.0

Plan of the talk

1. review how these tools work
   - parton showers (LOPS)
   - fixed-order (NLO)
2. discuss how their accuracy can be improved
   - matching NLO and PS (NLOPS): POWHEG
   - NLOPS merging & MiNLO
3. explain how to build an event generator that is NNLO accurate (NNLOPS)
   - Higgs production at NNLOPS



Plan of the talk

Why going NNLO?

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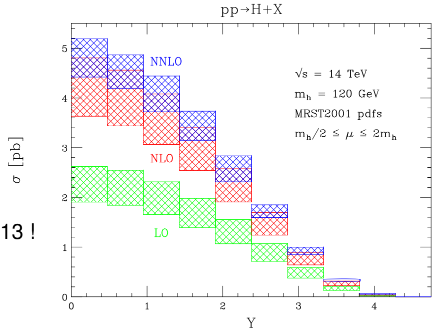
- "just" NLO sometimes not enough:
  - large NLO/LO "K-factor"
    [perturbative expansion "not (yet) stable"]
  - very high precision needed
- NNLO is the frontier:
  first $2 \to 2$ NNLO computations in 2012-13 !
- paramount example: Higgs production



[Anastasiou et al., '04-'05]

- the approach I'll discuss here works for "color-singlet" production processes at the LHC
- we used it for Higgs production [Hamilton,Nason,Zanderighi,ER '13]
- we are currently studying Drell-Yan production [Karlberg,Zanderighi,ER in progress]

parton showers and fixed order

Parton showers I

- connect the hard scattering ($\mu \approx Q$) with the final state hadrons ($\mu \approx \Lambda_{\text{QCD}}$)
- need to simulate production of many quarks and gluons

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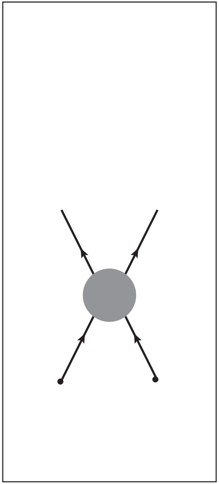

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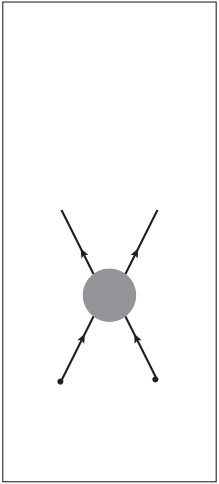
   $\Rightarrow$ they radiate (like photons off electrons)



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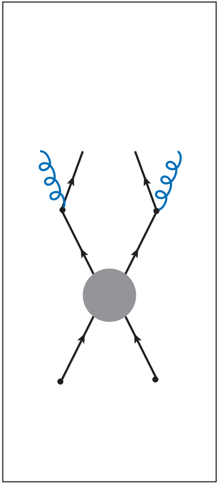

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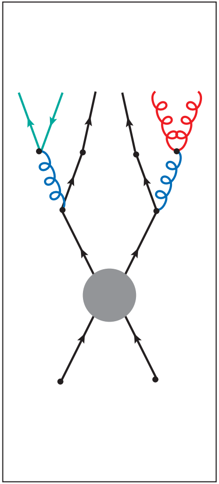

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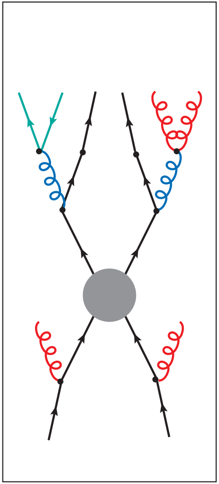

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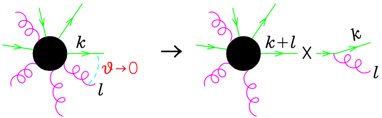
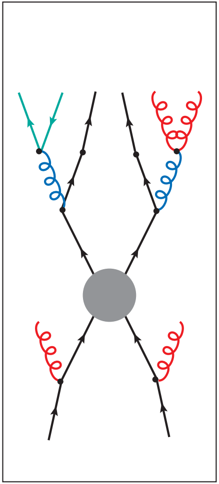
$$\frac{1}{(p_1+p_2)^2} = \frac{1}{2E_1E_2(1-\cos\theta)}$$

4. in soft-collinear limit, factorization properties of QCD amplitudes



$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \; \frac{\alpha_S}{2\pi}\frac{dt}{t}P_{q,qg}(z)dz\frac{d\varphi}{2\pi}$$

$z = k^0/(k^0+l^0)$	quark energy fraction
$t = \left\{(k+l)^2, l_T^2, E^2\theta^2\right\}$	splitting hardness
$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$	AP splitting function



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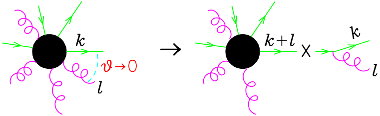
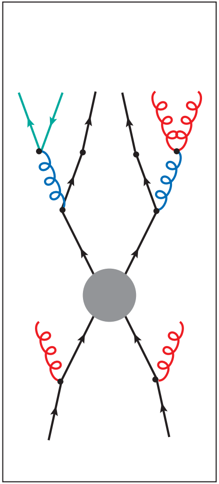




$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \quad \frac{\alpha_S}{2\pi}\frac{dt}{t} P_{q,qg}(z)dz\frac{d\varphi}{2\pi}$$

→ probabilistic interpretation!

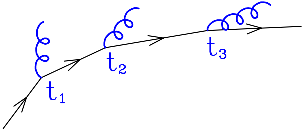
$$z = k^0/(k^0+l^0)$$ quark energy fraction

$$t = \left\{(k+l)^2, l_T^2, E^2\theta^2\right\}$$ splitting hardness

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$$ AP splitting function

5. dominant contributions for multiparticle production due to strongly ordered emissions

$$t_1 > t_2 > t_3 ...$$



6. at any given order, we also have virtual corrections: for consistency we should include them with the same approximation

- LL virtual contributions included by assigning to each internal line a Sudakov form factor:

$$\Delta_a(t_i, t_{i+1}) = \exp\left[-\sum_{(bc)} \int_{t_{i+1}}^{t_i} \frac{dt'}{t'} \int \frac{\alpha_s(t')}{2\pi} P_{a,bc}(z)\, dz\right]$$

- Δ_a corresponds to the probability of having no resolved emission between t_i and t_{i+1} off a line of flavour a

☞ resummation of collinear logarithms

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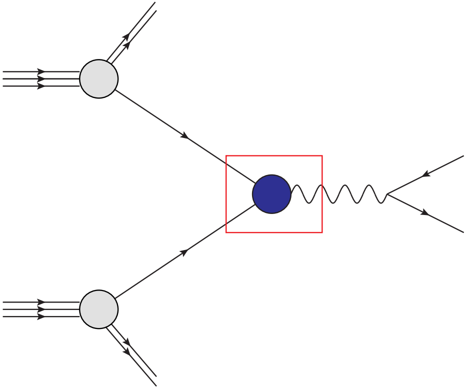
7. At scales $\mu \approx \Lambda_{\text{QCD}}$, hadrons form: non-perturbative effect, simulated with models fitted to data.

Parton showers: summary

$$d\sigma_{\rm SMC} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \Big\{ \qquad \Big\}$$



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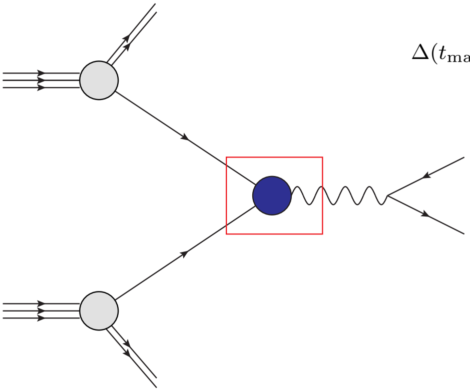
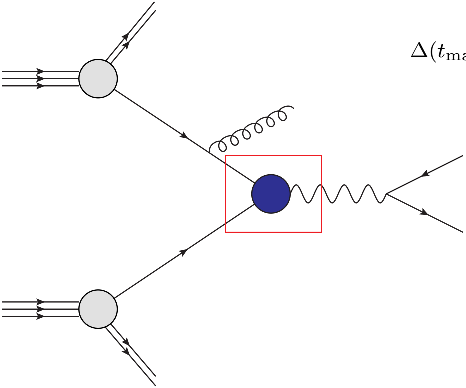


$$\Delta(t_{\max}, t) = \exp\left\{ - \int_t^{t_{\max}} d\Phi'_r \, \frac{\alpha_s}{2\pi} \frac{1}{t'} P(z') \right\}$$

$$d\sigma_{\text{SMC}} = \underbrace{|\mathcal{M}_B|^2 d\Phi_B}_{d\sigma_B} \Big\{ \Delta(t_{\max}, t_0) + \Delta(t_{\max}, t) \quad \underbrace{d\mathcal{P}_{\text{emis}}(t)}_{\frac{\alpha_s}{2\pi}\ \frac{1}{t} P(z)\ d\Phi_r} \qquad \Big\}$$



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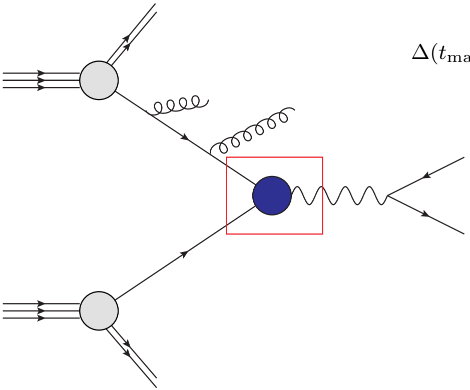

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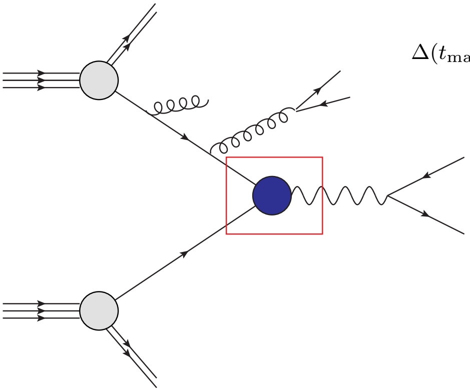

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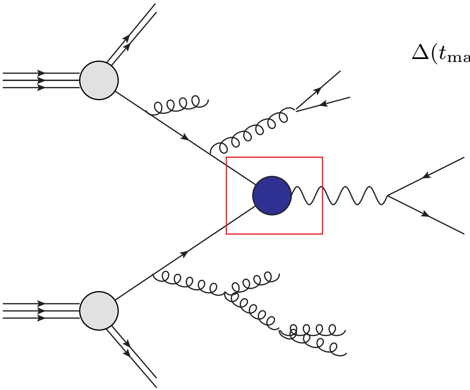

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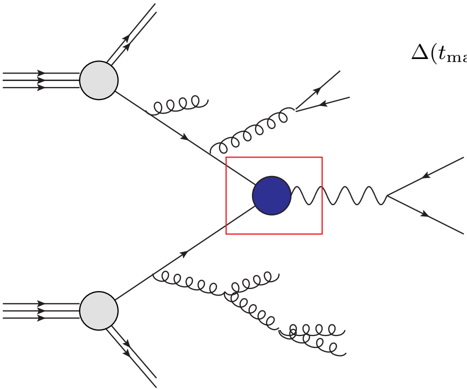

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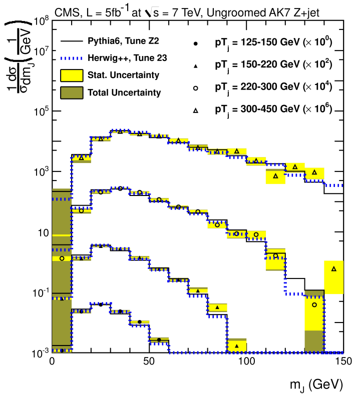
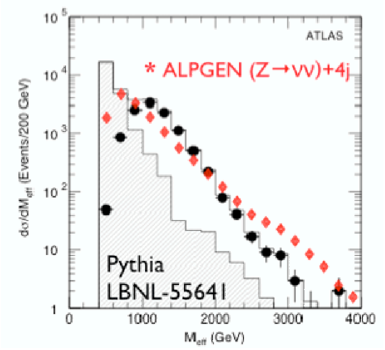
This is “LOPS”

- A parton shower changes shapes, not the overall normalization, which stays LO (*unitarity*)

Do they work?





[Gianotti,Mangano 0504221]

✓ ok when observables dominated by soft-collinear radiation

✗ Not surprisingly, they fail when looking for hard multijet kinematics

✗ they are only LO+LL accurate (whereas we can compute up to (N)NLO QCD corrections)

⇒ Not enough if interested in precision (10% or less), or in multijet regions

Next-to-Leading Order I

$\alpha_S \sim 0.1 \Rightarrow$ to improve the accuracy, use exact perturbative expansion

$$d\sigma = d\sigma_{\mathrm{LO}} + \left(\frac{\alpha_S}{2\pi}\right) d\sigma_{\mathrm{NLO}} + \left(\frac{\alpha_S}{2\pi}\right)^2 d\sigma_{\mathrm{NNLO}} + \ldots$$

LO: *Leading Order*
NLO: *Next-to-Leading Order*
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LO: *Leading Order*
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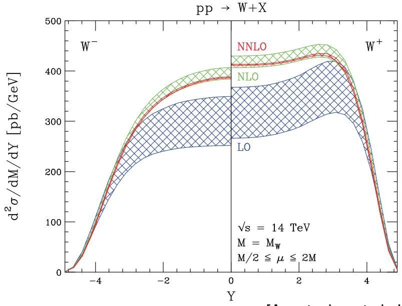
...

Why NLO is important?

- first order where rates are reliable
- shapes are, in general, better described
- possible to attach sensible theoretical uncertainties

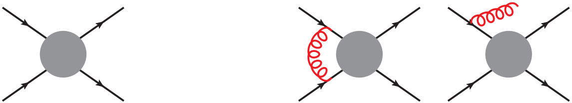
☞ when NLO corrections large, NNLO is desirable (as in Higgs production!)



[Anastasiou et al., '03]

Next-to-Leading Order II

NLO how-to



$$d\sigma = d\Phi_n \Big\{ \underbrace{B(\Phi_n)}_{\text{LO}} + \frac{\alpha_s}{2\pi}\Big[\underbrace{V(\Phi_n) + R(\Phi_{n+1})\, d\Phi_r}_{\text{NLO}} \Big] \Big\}$$

- Inputs: tree-level n-partons (B), 1-loop n-partons (V), tree-level $n+1$ partons (R)
- truncated series $\Rightarrow$ result depends on "unphysical" scales (can be used to estimate theoretical uncertainties)

Limitations:

- Results are at the parton level only ($5-6$ final-state partons is the frontier)
- In regions where collinear emissions are important, they fail (no resummation)
- Choice of scale is an issue when multijets in the final states

matching NLO and PS

- POWHEG (POsitive Weight Hardest Emission Generator)

NLO

- ✓ precision
- ✓ nowadays this is the standard
- ✗ limited multiplicity
- ✗ (fail when resummation needed)

parton showers

- ✓ realistic + flexible tools
- ✓ widely used by experimental coll's
- ✗ limited precision (LO)
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☞ can merge them and build an NLOPS generator?

<u>Problem:</u>

PS vs NLO

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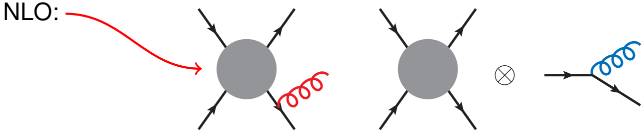
Problem: overlapping regions!



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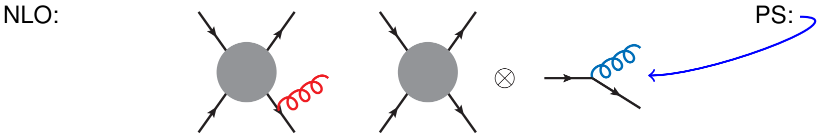

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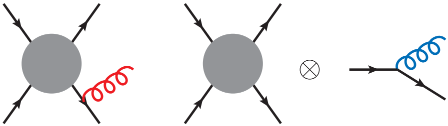
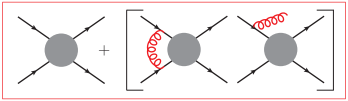


✓ 2 methods available to solve this problem:
MC@NLO and POWHEG [Frixione-Webber '03, Nason '04]

$$d\sigma_{\mathrm{POW}} = d\Phi_n \quad \bar{B}(\Phi_n) \quad \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\mathrm{min}}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r \right\}$$

NLOPS: POWHEG I

$$B(\Phi_n) \Rightarrow \bar{B}(\Phi_n) = B(\Phi_n) + \frac{\alpha_s}{2\pi}\left[V(\Phi_n) + \int R(\Phi_{n+1})\, d\Phi_r\right]$$



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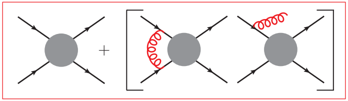

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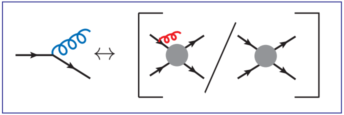

$$\Delta(t_{\mathrm{m}}, t) \Rightarrow \Delta(\Phi_n; k_{\mathrm{T}}) = \exp\left\{-\frac{\alpha_s}{2\pi} \int \frac{R(\Phi_n, \Phi_r')}{B(\Phi_n)} \theta(k_{\mathrm{T}}' - k_{\mathrm{T}})\, d\Phi_r'\right\}$$

NLOPS: POWHEG II

$$d\sigma_{\mathrm{POW}} = d\Phi_n \; \bar{B}(\Phi_n) \left\{ \Delta(\Phi_n; k_{\mathrm{T}}^{\mathrm{min}}) + \Delta(\Phi_n; k_{\mathrm{T}}) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \; d\Phi_r \right\}$$

[+ p_{T}-*vetoing subsequent emissions*, to avoid double-counting]

- inclusive observables: @NLO
- first hard emission: full tree level ME
- (N)LL resummation of collinear/soft logs
- extra jets in the shower approximation

This is “NLOPS”

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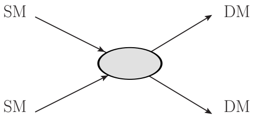
POWHEG BOX

[Alioli,Nason,Oleari,ER '10]

- large library of SM processes, (largely) automated
- widely used by LHC collaborations

Recently studied DM production at the LHC, including PS effects [Haisch,Kahlhoefer,ER '13]



✗ nothing to detect ⇒ not visible !

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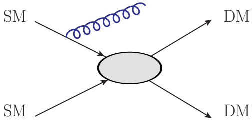

✓ can emit extra SM particle

interlude: BSM with POWHEG

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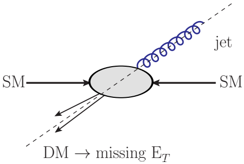

monojet signal !

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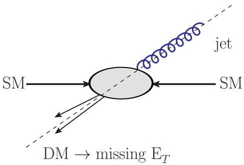

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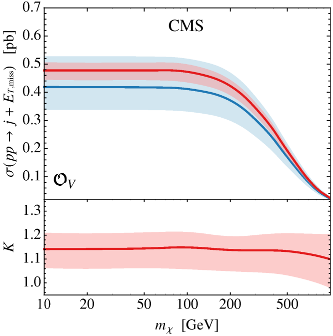

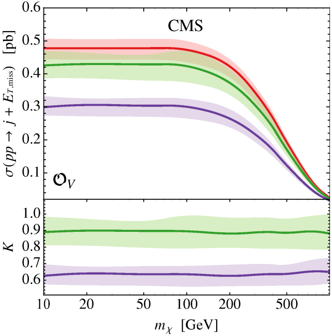

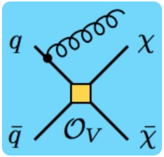

- H+j @ NLO, H+jj @ LO are needed for inclusive H @ NNLO
  $\hookrightarrow$ start from H+j @ NLOPS (POWHEG)

NLOPS: H+j

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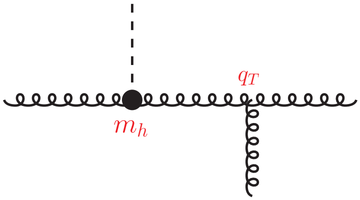

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☞ when doing $X+$ jet(s) @ NLO, $\bar{B}(\Phi_n)$ is not finite !
$\hookrightarrow$ need of a generation cut on Φ_n (or variants thereof)

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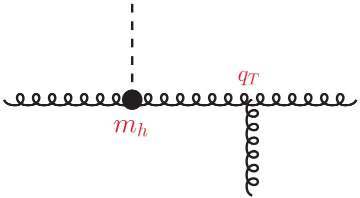

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☞ want to reach NNLO accuracy for *e.g.* y_H, *i.e.* when fully inclusive over QCD radiation

- need to allow the 1st jet to become unresolved
- above approach needs to be modified
- notice: H+j is a 2-scales problem ($\rightarrow$ choice of μ not unique!)

NLOPS merging

- MiNLO (Multiscale Improved NLO)

MiNLO: intro

- for processes with widely different scales (*e.g.* $X+$ jets close to Sudakov regions) choice of scales is not straightforward
- scale often chosen a posteriori, requiring typically
  - NLO corrections to be small
  - sensitivity upon scale choice to be minimal ($\rightarrow$ plateau in $\sigma(\mu)$ vs. μ)

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MiNLO: Multiscale Improved NLO [Hamilton,Nason,Zanderighi, 1206.3572]

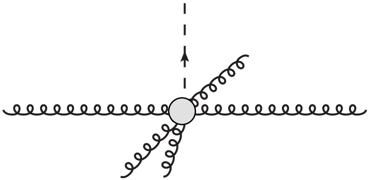
- aim: method to a-priori choose scales in NLO computation
- idea: at LO, the CKKW procedure allows to take these effects into account: modify the LO weight $B(\mathbf{\Phi}_n)$ in order to include (N)LL effects.

$\Rightarrow$ "Use CKKW" on top of NLO computation that potentially involves many scales

☞ Next-to-Leading Order accuracy needs to be preserved

From CKKW to MiNLO

- Find “most-likely” shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



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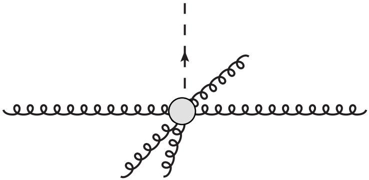

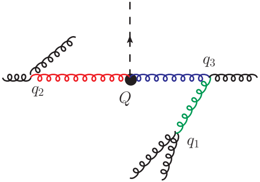

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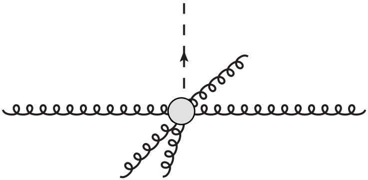

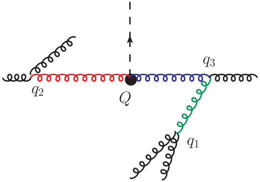

- Evaluate α_S at nodal scales

$$\alpha_S^n(\mu_R)B(\mathbf{\Phi}_n) \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B(\mathbf{\Phi}_n)$$

☞ scale compensation: use $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$ in V

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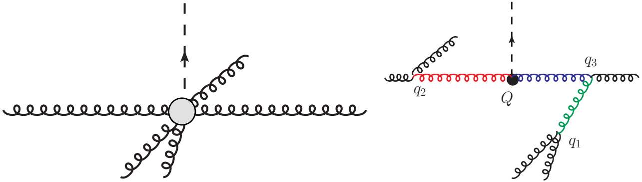

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- Sudakov FFs in internal and external lines of Born "skeleton"

$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n) \times \{\Delta(Q_0, Q)\Delta(Q_0, q_i)...\}$$

☞ recover NLO exactly: remove $\mathcal{O}(\alpha_S^{n+1})$ (log) terms generated upon expansion

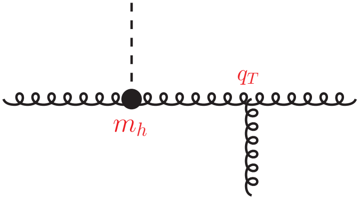
$$B(\mathbf{\Phi}_n) \Rightarrow B(\mathbf{\Phi}_n)\Big(1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + ...\Big)$$

MiNLO: example

Example, in 1 line: $H + 1$ jet

- Pure NLO:

$$d\sigma = \bar{B}\, d\boldsymbol{\Phi}_n = \alpha_{\rm S}^3(\mu_R)\Big[B + \alpha_{\rm S}^{\rm (NLO)} V(\mu_R) + \alpha_{\rm S}^{\rm (NLO)} \int d\Phi_{\rm rad} R\Big]\, d\boldsymbol{\Phi}_n$$



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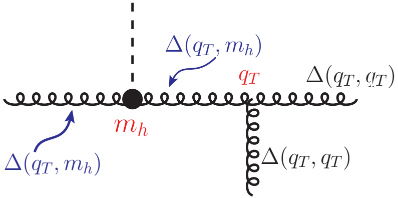
- MiNLO:

$$\bar{B} = \alpha_S^2(m_h)\alpha_S(q_T)\Delta_g^2(q_T, m_h)\Big[B\left(1 - 2\Delta_g^{(1)}(q_T, m_h)\right) + \alpha_S^{(\mathrm{NLO})} V(\bar{\mu}_R) + \alpha_S^{(\mathrm{NLO})}\int d\Phi_{\mathrm{rad}} R\Big]$$



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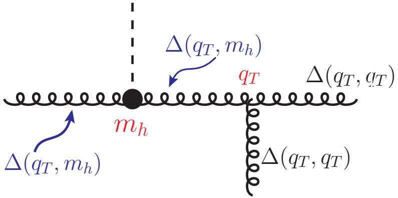

* $\bar{\mu}_R = (m_h^2 q_T)^{1/3}$
* $\log \Delta_{\mathrm{f}}(q_T, Q) = -\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi}\Big[A_f \log \frac{Q^2}{q^2} + B_f\Big]$
* $\Delta_{\mathrm{f}}^{(1)}(q_T, Q) = -\alpha_S^{(\mathrm{NLO})} \frac{1}{2\pi}\Big[\frac{1}{2} A_{1,\mathrm{f}} \log^2 \frac{Q^2}{q_T^2} + B_{1,\mathrm{f}} \log \frac{Q^2}{q_T^2}\Big]$
* $\mu_F = Q_0 (= q_T)$

☞ Sudakov FF included on Born kinematics

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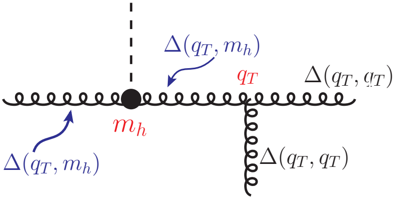

☞ $X+$ jets cross-section finite without generation cuts
↪ $\bar{B}$ with MiNLO prescription: ideal starting point for NLOPS (POWHEG) for $X+$ jets
↪ can be used to extend validity of $H+j$ POWHEG when jet becomes unresolved

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  - `HJ-MiNLO` describes inclusive observables at order α_S (relative to inclusive H @ LO)
  - to reach genuine NLO when inclusive, "spurious" terms must be of relative order α_S^2, *i.e.*

    $$O_{\mathrm{HJ-MiNLO}} = O_{\mathrm{H@NLO}} + \mathcal{O}(\alpha_S^{b+2}) \qquad (b = 2 \text{ for } gg \to H)$$

    if O is inclusive (H@LO $\sim \alpha_S^b$).
  - "Original MiNLO" contains ambiguous $\mathcal{O}(\alpha_S^{b+3/2})$ terms.

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  - to reach genuine NLO when inclusive, "spurious" terms must be of relative order α_S^2, *i.e.*

$$O_{\mathrm{HJ-MiNLO}} = O_{\mathrm{H@NLO}} + \mathcal{O}(\alpha_S^{b+2}) \qquad (b = 2 \text{ for } gg \to H)$$

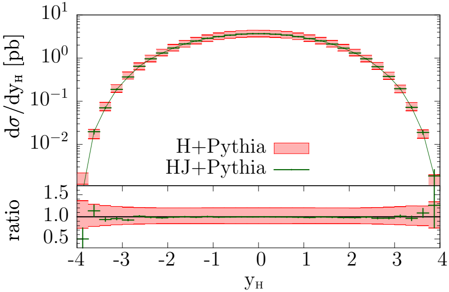
    if O is inclusive (H@LO $\sim \alpha_S^b$).
  - "Original MiNLO" contains ambiguous $\mathcal{O}(\alpha_S^{b+3/2})$ terms.

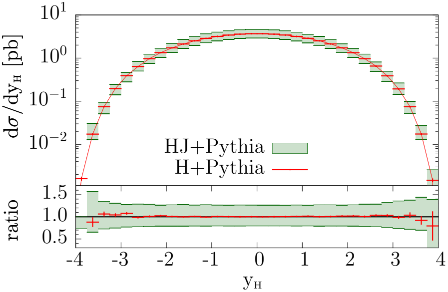
- Possible to improve `HJ-MiNLO` such that H @ NLO is recovered (NLO$^{(0)}$) , without spoiling NLO accuracy of H+j (NLO$^{(1)}$) .
  - proof based on careful comparisons between `MiNLO` and analytic resummation
  - need to include B_2 coefficient in `MiNLO`-Sudakovs
  - need to evaluate $\alpha_S{}^{(\mathrm{NLO})}$ in `HJ-MiNLO` at scale q_T, and $\mu_F = q_T$

Effectively as merging NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging different samples (no merging scale used: there is just one sample).

MiNLO merging: results

[Hamilton et al., 1212.4504]





- “H+Pythia”: standalone POWHEG ($gg \to H$) + PYTHIA (PS level) [7pts band, $\mu = m_H$]
- “HJ+Pythia”: HJ−MiNLO* + PYTHIA (PS level) [7pts band, μ from MiNLO]

✓ very good agreement (both value and band)

☞ Notice: band is $\sim 20 - 30\%$

matching NNLO with PS

- Higgs production at NNLOPS

- HJ−MiNLO* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

- thus, reweighting each event with this factor, we get NNLO+PS
  - obvious for y_H, by construction
  - α_S^4 accuracy of HJ−MiNLO* in 1-jet region not spoiled, because $W(y) = 1 + \mathcal{O}(\alpha_S^2)$
  - if we had NLO$^{(0)}$ $+ \mathcal{O}(\alpha_S^{2+3/2})$, 1-jet region spoiled because

$$[\mathrm{NLO}^{(1)}]_{\mathrm{NNLOPS}} = \mathrm{NLO}^{(1)} + \mathcal{O}(\alpha_S^{4.5}) \neq \mathrm{NLO}^{(1)} + \mathcal{O}(\alpha_S^5)$$

NNLO+PS I

- HJ−MiNLO* differential cross section $(d\sigma/dy)_{\mathrm{HJ-MiNLO}}$ is NLO accurate

$$W(y) = \frac{\left(\frac{d\sigma}{dy}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{dy}\right)_{\mathrm{HJ-MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \simeq 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

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---

* Variants for W are possible:

$$W(y, p_T) = h(p_T)\frac{\int d\sigma_A^{\mathrm{NNLO}}\delta(y - y(\mathbf{\Phi}))}{\int d\sigma_A^{\mathrm{MiNLO}}\delta(y - y(\mathbf{\Phi}))} + (1 - h(p_T))$$

$$d\sigma_A = d\sigma\; h(p_T), \qquad d\sigma_B = d\sigma\;(1 - h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

* $h(p_T)$ controls where the NNLO/NLO K-factor is spread
* β (similar to resummation scale) cannot be too small, otherwise resummation spoiled

NNLO+PS II

In 1309.0017, we used

$$W(y,p_T) = h(p_T)\frac{\int d\sigma^{\rm NNLO}\delta(y-y(\boldsymbol{\Phi})) - \int d\sigma_B^{\rm MiNLO}\delta(y-y(\boldsymbol{\Phi}))}{\int d\sigma_A^{\rm MiNLO}\delta(y-y(\boldsymbol{\Phi}))} + (1-h(p_T))$$

$$d\sigma_A = d\sigma\, h(p_T), \qquad d\sigma_B = d\sigma\,(1-h(p_T)), \qquad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}$$

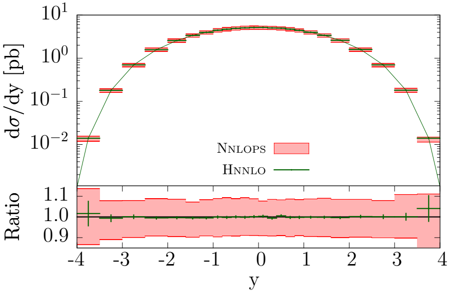
- one gets exactly $(d\sigma/dy)_{\rm NNLOPS} = (d\sigma/dy)_{\rm NNLO}$ (no α_S^5 terms)
- we used $h(p_T^{j_1})$ (hardest jet at parton level)

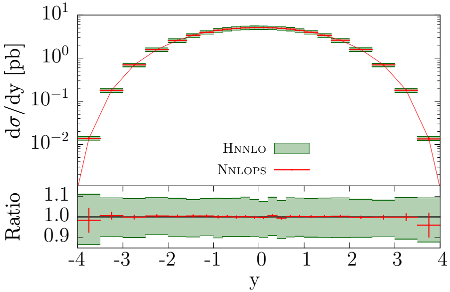
---

inputs for following plots:

- results are for 8 TeV LHC
- scale choices: NNLO input with $\mu = m_H/2$, HJ-MiNLO "core scale" m_H (other powers are at q_T)
- PDF: everywhere MSTW8NNLO
- NNLO always from HNNLO
- events reweighted at the LH level
- plots after $k_{\rm T}$-ordered PYTHIA 6 at the PS level (hadronization and MPI switched off)

- NNLO with $\mu = m_H/2$, HJ−MiNLO "core scale" m_H [NNLO from HNNLO, Catani,Grazzini]
- $(7_{\rm Mi} \times 3_{\rm NN})$ pts scale var. in NNLOPS, 7pts in NNLO



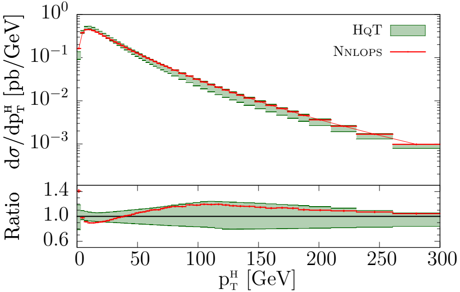


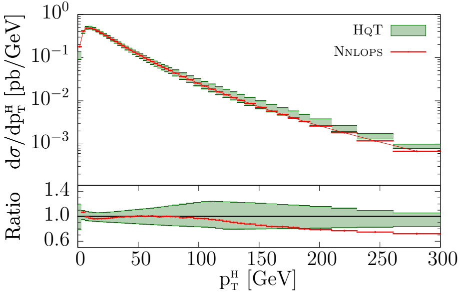
☞ Notice: band is 10%

[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don't affect y_H (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]

NNLO+PS (p_T^H)

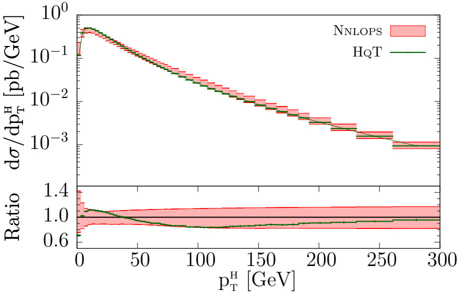
$\beta = \infty$ (W indep. of p_T) $\beta = 1/2$



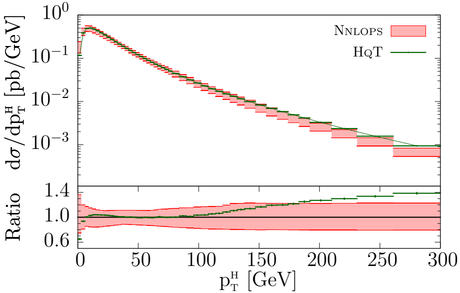


- HqT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\rm res} \equiv m_H/2$ [HqT, Bozzi et al.]
- ✓ $\beta = 1/2$ & ∞: uncertainty bands of HqT contain NNLOPS at low-/moderate p_T
- $\beta = 1/2$: HqT tail harder than NNLOPS tail ($\mu_{\rm HqT} <$ "$\mu_{\rm MiNLO}$")
- $\beta = 1/2$: very good agreement with HqT resummation ["∼ expected", since $Q_{\rm res} \equiv m_H/2$]

$\beta = \infty$ (W indep. of p_T)

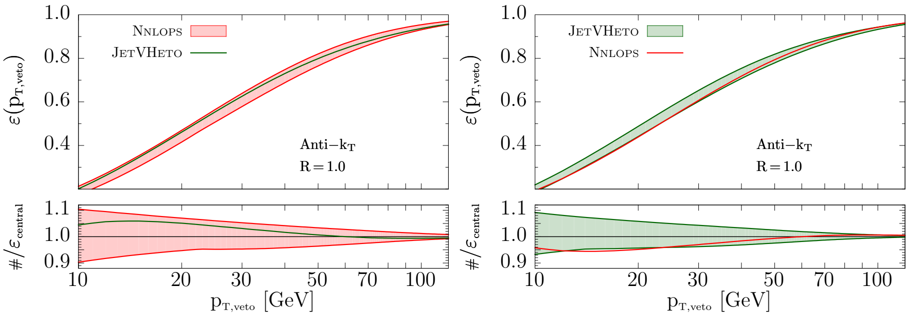


$\beta = 1/2$



- HQT: NNLL+NNLO, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\text{res}} \equiv m_H/2$
- $\beta = 1/2$: NNLOPS tail $\rightarrow$ NLOPS tail [$W(y, p_T \gg m_H) \rightarrow 1$]
  larger band (affected just marginally by NNLO, so it's $\sim$ genuine NLO band)

NNLO+PS ($p_T^{j_1}$)



$$\varepsilon\left(p_{\mathrm{T,veto}}\right) = \frac{\Sigma(p_{\mathrm{T,veto}})}{\sigma^{\mathrm{tot}}} = \frac{1}{\sigma^{\mathrm{tot}}}\int d\sigma\,\theta\left(p_{\mathrm{T,veto}} - p_{\mathrm{T}}^{\mathrm{j}_1}\right)$$

- JetVHeto: NNLL resum, $\mu_R = \mu_F = m_H/2$ [7pts], $Q_{\mathrm{res}} \equiv m_H/2$, (a)-scheme only

  [JetVHeto, Banfi et al.]

- nice agreement, differences never more than 5-6 %

☞ Separation of $H \to WW$ from $t\bar{t}$ bkg: x-sec binned in N_{jet}
0-jet bin (WW-dominated) ⇔ jet-veto accurate predictions needed !

Conclusions and Outlook

- Especially in absence of very clear singals of new-physics, accurate tools are needed for LHC phenomenology
- In the last decade, impressive amount of progress: new ideas, and automated tools

⇒ Shown results of merging NLOPS for different jet-multiplicities *without* merging scale

⇒ Shown first working example of NNLOPS

What next?

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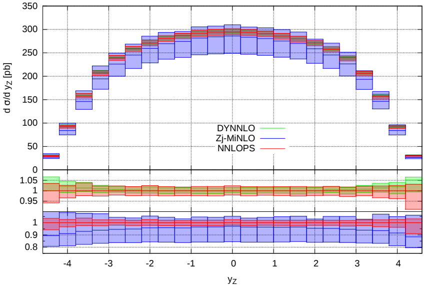
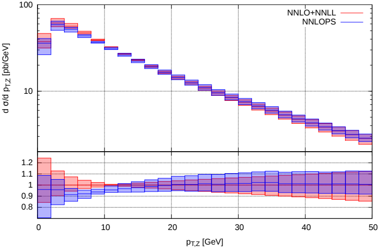
What next?

- Drell-Yan: conceptually the same as $gg \to H$, technically slightly more involved, phenomenologically important (standard candle + W mass extraction, pdfs,...)

- Born kinematics more complicated: $W(y) \to W(m_{ll}, y_Z, \theta_l)$; $\beta = 1$.





✓ reproduce NNLO

✓ very good agreement with analytic resummation

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- merging for higher multiplicity / NNLO matching for *e.g.* $t\bar{t}$...

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- merging for higher multiplicity / NNLO matching for *e.g.* $t\bar{t}$...

Thank you for your attention!

Extra slides

"Improved" MiNLO & NLOPS merging II

- Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2}\Big\{[C_{ga}\otimes f_a](x_A, q_T)\times[C_{gb}\otimes f_b](x_B, q_T)\times \exp S(q_T, Q)\Big\} + R_f$$

$$S(q_T, Q) = -2\int_{q_T^2}^{Q^2}\frac{dq^2}{q^2}\frac{\alpha_S(q^2)}{2\pi}\left[A_f \log\frac{Q^2}{q^2} + B_f\right]$$

- If $C_{ij}^{(1)}$ included and R_f is LO$^{(1)}$, then upon integration we get NLO$^{(0)}$
- Take derivative, then compare with `MiNLO` :

$$\sim \sigma_0 \frac{1}{q_T^2}[\alpha_S, \boxed{\alpha_S^2}, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L]\exp S(q_T, Q) + R_f \qquad L = \log(Q^2/q_T^2)$$

- highlighted terms are needed to reach NLO$^{(0)}$:

$$\int^{Q^2}\frac{dq_T^2}{q_T^2}L^m {\alpha_S}^n(q_T)\exp S \sim \left(\alpha_S(Q^2)\right)^{n-(m+1)/2}$$

(scaling in low-p_T region is $\alpha_S^2 L \sim 1$!)

- if I don't include B_2 in `MiNLO` Δ_g, I miss a term $(1/q_T^2)\ \boxed{\alpha_S^2}\ B_2 \exp S$
- upon integration, violate NLO$^{(0)}$ by a term of <u>relative</u> $\mathcal{O}(\alpha_S^{3/2})$
- "wrong" scale in $\alpha_S^{(\mathrm{NLO})}$ in `MiNLO` produces again same error

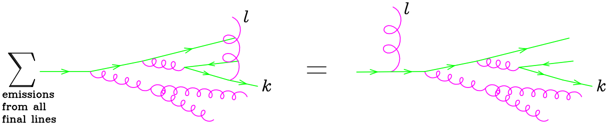
- Sudakov FF from infinitesimal emission probabilities

$$\mathcal{P}_i^{\text{no emiss}}(t \to t') \simeq \prod_{k=1}^{N} \left(1 - d\mathcal{P}_i^{\text{emiss}}(t_k)\right) = \prod_{k=1}^{N} \left(1 - \frac{\alpha_S(t_k)}{2\pi} \frac{\delta t}{t_k} \sum_{(jl)} \int P_{i,jl}(z)\, dz \, \frac{d\phi}{2\pi}\right)$$

  This reduces to the Sudakov form factor $\Delta_i(t, t')$ in the continuum limit $N \to +\infty$.
  We can state that, in Parton Showers, virtual corrections are included in a probabilistic way.

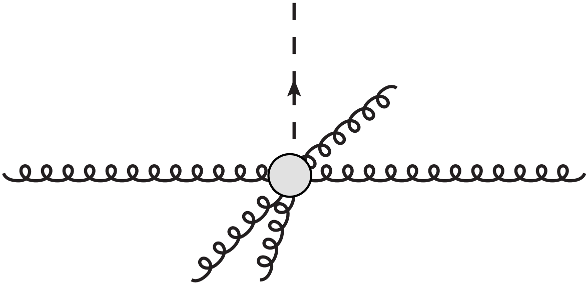
- Choice of the ordering variable affects double-log structure
  - angular ordering is the correct choice
  - exact in `HERWIG`, approximate in other generators
- The use of $\alpha_S = \alpha_S(p_T^2)$, in the radiation scheme, allows to include (part of) the 2-loop splitting kernels
- Nominal accuracy is LL, although it's common believe that in practice it's better.
- For some observables (*e.g.* low-p_T DY) NLL can be achieved.
- Momentum conservation (via reshuffling/recoil) is respected (and this is a NLL effect).

- Soft emissions from final-state-partons add coherently
- After azimuthal average, color coherence force emissions to be angular ordered

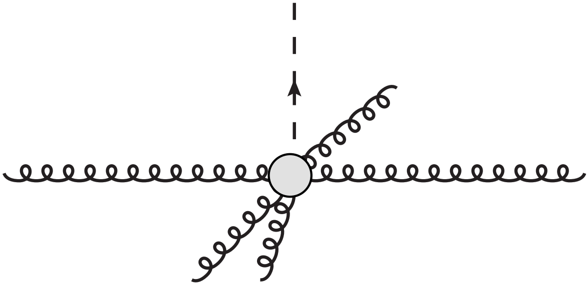


In the above figure, the soft large-angle gluon sees the net colour charge of the initial quark, and not the charges of each emitter.

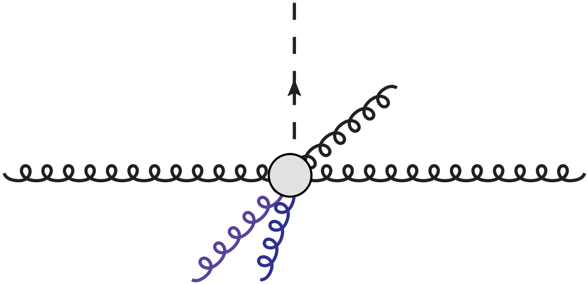
- In non angular-ordered Shower, this is not taken into account $\rightarrow$ need of corrections to the algorithm without spoiling the collinear accuracy.
- If the Shower is angular-ordered, the coherence is built-in: large-angle soft emissions are generated first.
- The hardest emission (highest p_T), in general, happens later.



- start from ME weight $B(\mathbf{\Phi}_n)$

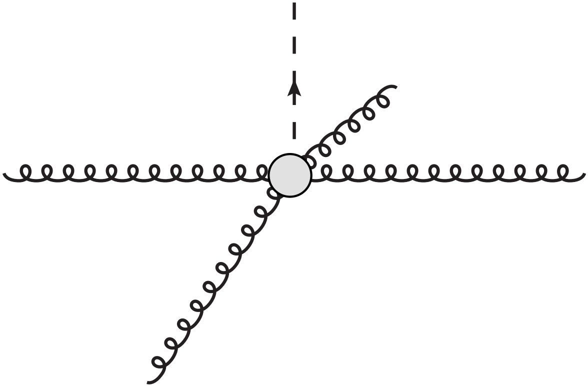


- find “most-likely” shower history (via k_T-algo)



- find "most-likely" shower history (via k_T-algo)
- clustering scale $q_1 = k_T$

CKKW in a nutshell I



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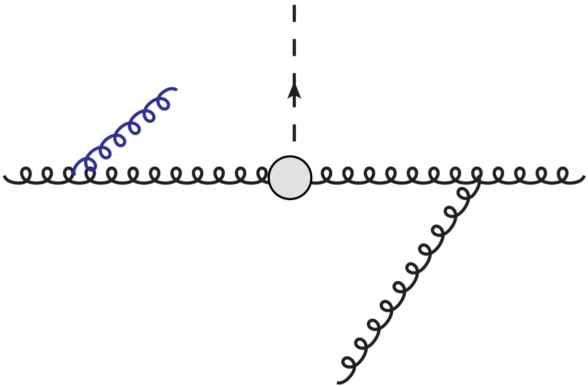
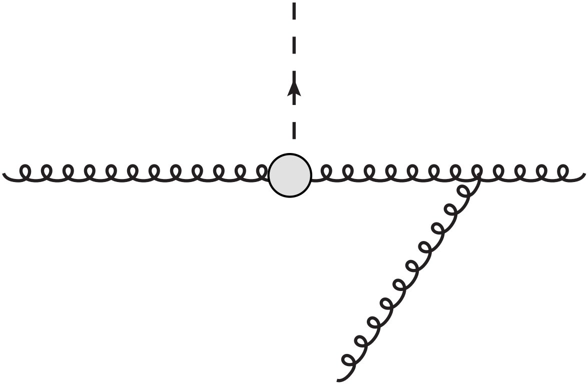


- find “most-likely” shower history (via k_T-algo)
- clustering scale $q_2 = k_T$

CKKW in a nutshell I



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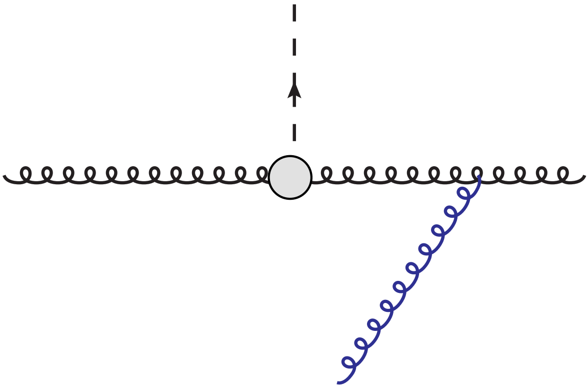
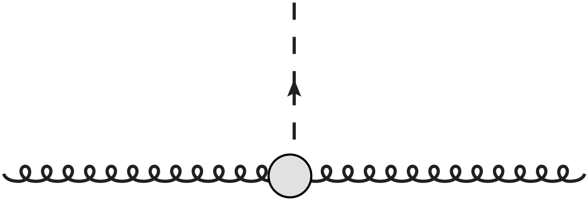
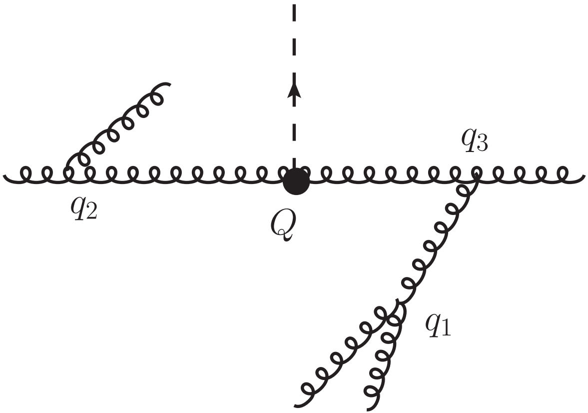


- find "most-likely" shower history (via k_T-algo)
- clustering scale $q_3 = k_T$



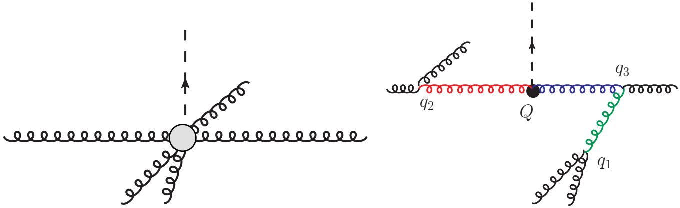
- Hard process scale Q



- most-likely shower history

CKKW in a nutshell

- ME weight $B(\mathbf{\Phi}_n) \Rightarrow$ "most-likely" shower history (via k_T-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$



- New weight:

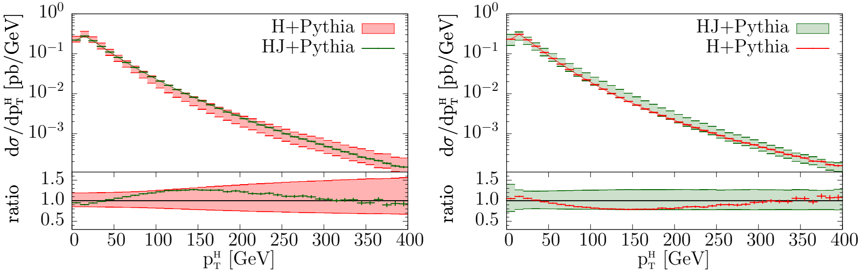
$$\alpha_S^5(Q)B(\mathbf{\Phi}_3) \quad\rightarrow\quad \alpha_S^2(Q)B(\mathbf{\Phi}_3)\,\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_2)}\,\frac{\Delta_g(Q_0,Q)}{\Delta_g(Q_0,q_3)}\,\frac{\Delta_g(Q_0,q_3)}{\Delta_g(Q_0,q_1)}$$
$$\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_2)\Delta_g(Q_0,q_3)\Delta_g(Q_0,q_1)\Delta_g(Q_0,q_1)$$
$$\alpha_S(q_1)\alpha_S(q_2)\alpha_S(q_3)$$

where typically

$$\log\Delta_f(q_T,Q) = -\int_{q_T^2}^{Q^2}\frac{dq^2}{q^2}\,\frac{\alpha_S(q^2)}{2\pi}\left[A_{1,f}\log\frac{Q^2}{q^2}+B_{1,f}\right]$$

- Fill phase space below Q_0 with vetoed shower

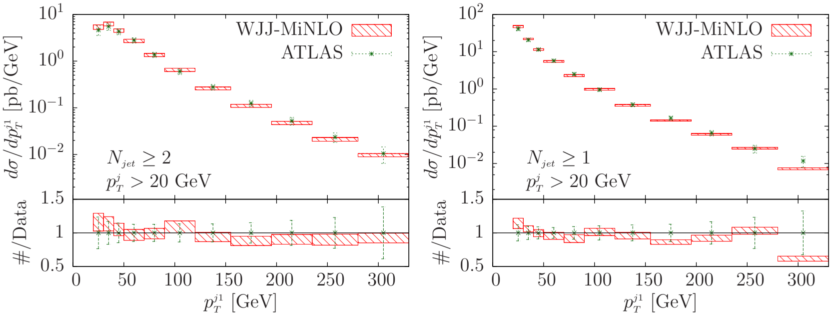
[Hamilton et al., 1212.4504]



- Good agreement
- At high p_T, bands as expected (LO vs NLO)
  (POWHEG $(gg \to H)$ with hfact $= m_H/1.2$, YR2)

MiNLO: results

VJJ−MiNLO [Campbell,Ellis,Nason,Zanderighi, 1303.5447]



☞ Start from $W + 2$ jets @ NLO, good agreement with data also when requiring $N_{\rm jet} \geq 1$!

This is not possible in a standard NLO...

MiNLO details

MiNLO: All α_S in Born term are chosen with CKKW (local) scales $q_1, ..., q_n$

$$\alpha_S^n(\mu_R)B \Rightarrow \alpha_S(q_1)\alpha_S(q_2)...\alpha_S(q_n)B$$

- Normal NLO structure ($\mu = \mu_R$):

$$\sigma(\mu) = \underbrace{\alpha_S^n(\mu)B}_{\text{Born}} + \underbrace{\alpha_S^{n+1}(\mu)\Big(C + nb_0 \log(\mu^2/Q^2)B\Big)}_{\text{Virtual}} + \underbrace{\alpha_S^{n+1}(\mu)R}_{\text{Real}}$$

- Explicit μ dependence of virtual term as required by RG invariance:

$$\alpha_S^n(\mu')B = \Big[\alpha_S(\mu) - nb_0\alpha_S^{n+1}(\mu)\log(\mu'^2/\mu^2)\Big]B + \mathcal{O}(\alpha_S^{n+2})$$

$$\text{Virtual}(\mu') = \text{Virtual}(\mu) + \alpha_S^{n+1}(\mu)nb_0\log(\mu'^2/\mu^2)\,B + \mathcal{O}(\alpha_S^{n+2})$$

$\Rightarrow \sigma(\mu') - \sigma(\mu) = \mathcal{O}(\alpha_S^{n+2})$

- In MiNLO “scale compensation” kept if

$$\Big(C + nb_0\log(\mu_R^2/Q^2)B\Big) \Rightarrow \Big(C + nb_0\log(\bar{\mu}_R^2/Q^2)B\Big)$$

with $\bar{\mu}_R^2 = (q_1 q_2 ... q_n)^{2/n}$

MiNLO details

Few technicalities for original MiNLO:

- $\mu_F = Q_0$ (as in CKKW)
- Cluster with CKKW also V and R kinematics
  - Actual implementation uses FKS mapping for first cluster of $\mathbf{\Phi}_{n+1}$
  - Ignore CKKW Sudakov for 1^{st} clustering of $\mathbf{\Phi}_{n+1}$ (inclusive on extra radiation)
- Some freedom in choice of $\alpha_S^{(\mathrm{NLO})}$ (entering V, R and $\Delta^{(1)}$):
  * suggested average of LO α_S
  * not free for "improved" `MiNLO`
- Used full NLL-improved Sudakovs (A_1, B_1, A_2)

Improved MiNLO: where are terms coming from when differentiating resum. formula?

$1/q_T^2$, always from integration in Sudakov
α_S from $C^{(0)} \times B_1$, ...
α_S^2 from $C^{(0)} \times B_2$, ...
...
$\alpha_S L$ from A_1 term in exponent
$\alpha_S L^2$ from A_2 term in exponent
...

p_T^H spectrum:

- "$\mu_{\mathrm{HJ-MiNLO}} = m_H, m_H, p_T$"
- At high p_T, $\mu_{\mathrm{HJ-MiNLO}} = p_T$
- If $\beta = 1/2$, NNLOPS $\rightarrow$ HJ-MiNLO at high p_{T}
- NNLO/NLO ~ 1.5, because HNNLO with $\mu = m_H/2$, $\mu_{\mathrm{HJ-MiNLO,core}} = m_H$