Minimal Walking Technicolor on the Lattice

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arXiv:0910.4535, 1010.5909, 1011.0607, 1011.0864, . . .

Outline

Technicolor

What is the mechanism of Electroweak Symmetry Breaking?

- The Higgs boson?

Technicolor

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- The Higgs boson?
- Strong dynamics at the EW scale?

Technicolor:

- *Techniquarks* charged under a new gauge symmetry, which becomes strong at EW scale.
- Chiral symmetry of techniquarks breaks spontaneously.
- $\langle\overline{\psi}\psi\rangle$ breaks EW symmetry.

Gives masses to W and Z bosons, but not to SM fermions. Need *Extended Technicolor*.

Extended technicolor

New gauge bosons at a scale M_{ETC} couple SM fermions to techniquarks. Give mass to SM fermions:

$$m \sim \frac{\langle \overline{\psi} \psi \rangle}{M_{ETC}^2}$$

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- Require $M_{ETC} \sim 10\ \mathrm{TeV}$ to get right SM fermion masses.
- But to avoid FCNC, need $M_{ETC} > 100\ \mathrm{TeV}$!

Walking technicolor

$$m \sim \frac{\langle \overline{\psi}\psi \rangle_{ETC}}{M_{ETC}^2}$$

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But $\langle \overline{\psi}\psi \rangle$ is determined at TC scale, and runs to ETC scale:

$$\langle \overline{\psi}\psi \rangle_{ETC} = \langle \overline{\psi}\psi \rangle_{TC} \exp \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu}\gamma(\mu)$$

Can running be enhanced?

- Need $\gamma(\mu)$ large over a large range of scales.
  $\gamma > 1.0$ [Chivukula & Simmons]
- Unlike QCD, where $\gamma(\mu)$ falls rapidly above Λ_{QCD}.
- Need "walking": coupling runs slowly above Λ_{TC} so $\gamma(\mu)$ can be large.

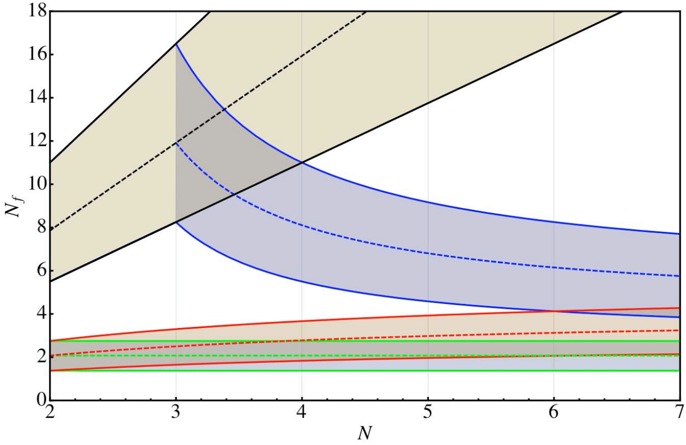
Walking technicolor

Conformal window:
If $\beta(g) = 0$ for $g \neq 0$ there will be an infrared fixed point (IRFP).
Expect this to happen for sufficiently large N_f.
For slightly lower N_f, $\beta(g)$ will be small – expect walking.

The existence of an IRFP, and the value of the associated critical exponents, is scheme-independent. However, walking, and the associated running of $\langle\overline{\psi}\psi\rangle$ are *scheme-dependent*. Can be changed by changing scheme.

Candidate theories

Phase diagram of possible theories (Sannino):



Minimal walking technicolor

Simplest model is *minimal walking technicolor*.

- $SU(2)$ technicolor gauge group
- Two techniquarks in adjoint representation
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Is this model phenomenologically viable?

- Does chiral symmetry breaking occur?
- Is γ large?
- S-parameter

Putting it on the lattice

Want to use lattice QCD experience as far as possible.

Differences:

- $SU(2)$ instead of $SU(3)$. Easy to change (usually).
- $N_f = 2$ instead of $N_f = 2$.
- Fermions in adjoint representation instead of fundamental. More difficult.

Don't know answers!

Adjoint fermions

To change representation of fermions, need copy of gauge field in rep. R.
E.g. Wilson-Dirac operator:

$$\sum_y D(x,y)\psi(y) = -\frac{1}{2a}(\sum_\mu [(1-\gamma_\mu)U^R_\mu(x)\psi(x+\mu) + (1+\gamma_\mu)U^R_\mu(x)^\dagger\psi(x-\mu)] - (8+2am)\psi(x))$$

$U_\mu(x)$ and $U^R_\mu(x)$ are related by:

$$U_\mu(x) = \exp(iA^a_\mu(x)T^a_f)\ ,\ \ U^R_\mu(x) = \exp(iA^a_\mu(x)T^a_R)$$

Need to alter HMC to include $\frac{\partial U^R_\mu(x)}{\partial U_\mu(x)}$.
Codes: *Chroma* and *HiRep*.

Observables

What can the lattice tell us?

- Phase: conformal or chiral symmetry breaking
- Running coupling
- Running mass, γ
- Spectrum
- S-parameter

All with controlled systematic errors.

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Warning: lattice and finite box break conformal symmetry. Fermions may break conformal symmetry or be doubled.

Running of coupling and mass are scheme-dependent except at fixed points.

Phase

How to determine phase?

- Directly measure $\langle\overline{\psi}\psi\rangle$.
- Running coupling has fixed point if conformal.
- Spectrum in chiral limit.
- Eigenvalues of Dirac operator

Best to combine more than one approach.

Running coupling

Running coupling can be measured in various schemes. Existence of a fixed point is scheme-independent. Walking is *not* scheme independent.

We use Schrödinger Functional scheme. Other groups using schemes based on Polyakov loops / Wilson loops.

Schrödinger Functional

Box of size aL with boundary conditions at top and bottom.

- Boundary conditions impose a chromoelectric field on the system.
- Coupling defined through response of the system to this field: $\overline{g}^2 = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1}$. To leading order, $\overline{g}^2 = g_0^2$.

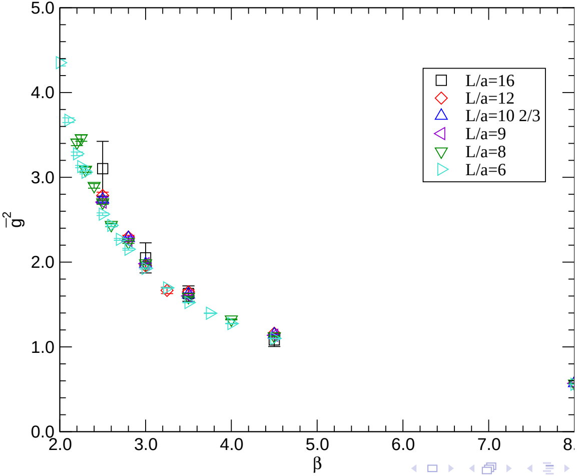
Use step-scaling: compare $\overline{g}^2$ on boxes of size L and $4L/3$. Do this at a range of couplings to get β-function over a range of scales.

SF Coupling

- Action: Wilson gauge action with unimproved Wilson quarks.
- Advantage of Schrödinger Functional boundary conditions: can work at zero quark mass, so no need for chiral extrapolations.
- Lattice sizes 6,8,12,16.
- Choose bare couplings to cover range $0.5 < \overline{g}^2 < 3$.

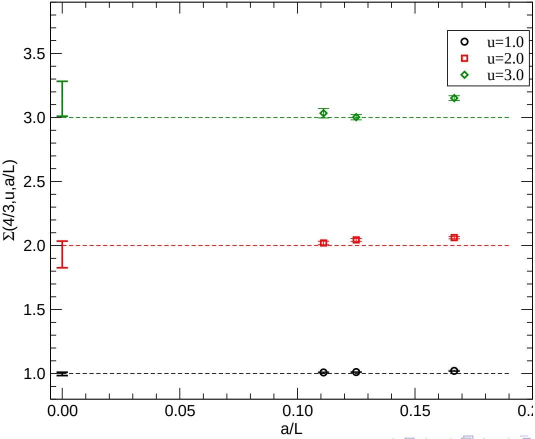
SF results

Running of coupling is slow:



Continuum limit

Fit as function of β, then extrapolate to $a/L = 0$ at constant $\overline{g}^2(\beta, L)$ to obtain continuum limit of step-scaling function.



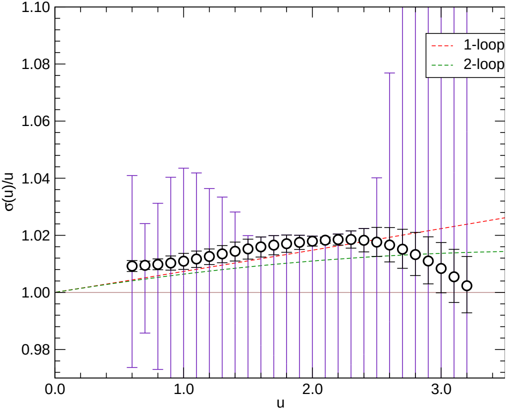
Errors

Overall fitting procedure quite complicated: several stages, choice of interpolation functions etc. Use several stages of bootstrapping to estimate statistical and systematic errors.

Validate by generating fake data and fitting it. Expect true value to lie within error 68% of the time.

Continuum running coupling

Systematic errors are large. Consistent with fixed point, but not conclusive.



Consistent with evidence for fixed point at $2.0 < \overline{g}^2 < 3.2$ from Hietanen et al. [arXiv: 0904.0864]. Also consistent with walking.

Running mass

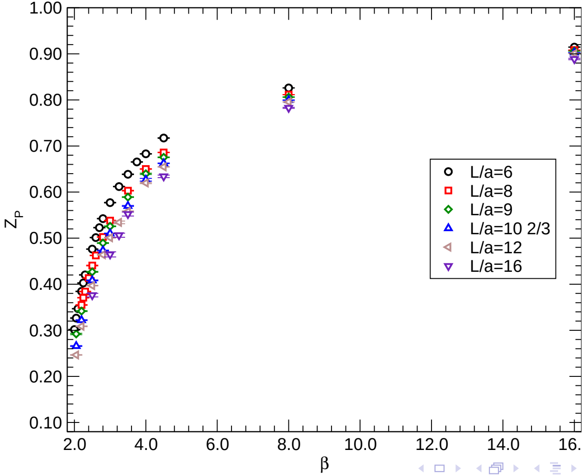
We measure the running of the mass by calculating pseudoscalar renormalisation constant Z_P.

$$Z_P(L) = \sqrt{3f_1}/f_P(L/2)$$

As for $\overline{g}^2(\beta, L)$, calculate on lattices of size L and $4L/3$ and extrapolate to continuum limit.

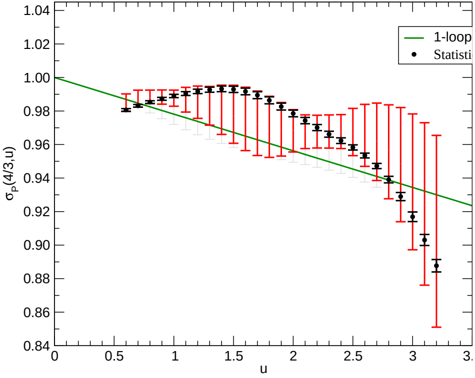
Z_P results

Running of Z_P is faster than coupling:



Continuum running mass

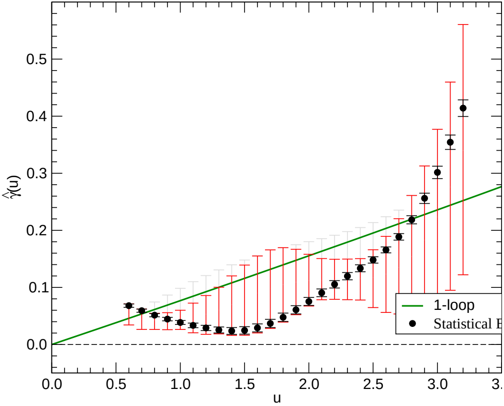
Analyse as for running coupling.



Errors not consistent with 1, so observe running.

Anomalous dimension

Convert to anomalous dimension: $\gamma = -\frac{\ln|\sigma_P(u,s)|}{\ln|s|}$



Anomalous dimension

Implications of running coupling and mass measurements:

- $\gamma < 1.0$ in this region. Not sufficient for technicolor.
- If indeed fixed point in $2.0 < \overline{g}^2 < 3.2$, this model probably ruled out. $0.05 < \gamma < 0.56$ in this range.
- But don't want fixed point anyway since then there is no chiral symmetry breaking.
- If it is walking in this range, γ still too small.
- Better if it is walking at stronger coupling. But difficult to measure there.

Can we determine phase by other methods?

Measuring masses

Measuring spectrum of low-lying states can give useful information on phase of the theory.

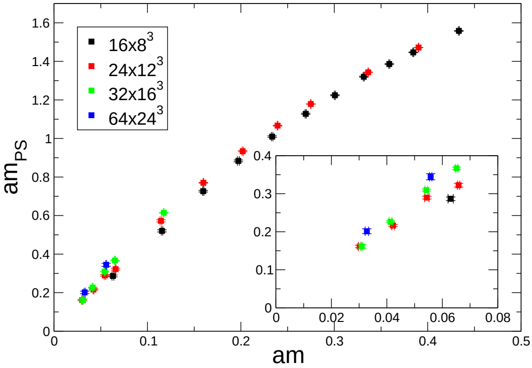
Behaviour in chiral limit:

- QCD-like: m_{PS} goes to zero, but other masses have finite limit.
- Conformal: all masses go to zero as $m_q^{1/(1+\gamma)}$.

Have to have big enough box size L for this to work.

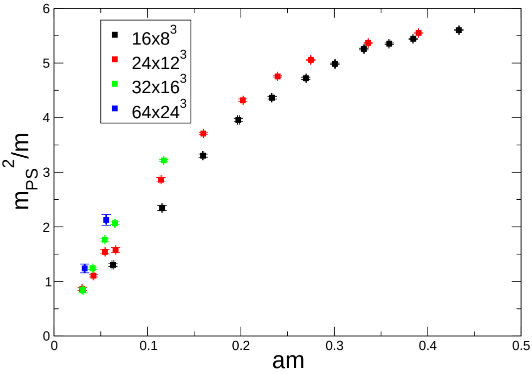
Pseudoscalar mass

Using high statistics and QCD smearing techniques, can get accurate results at low masses.



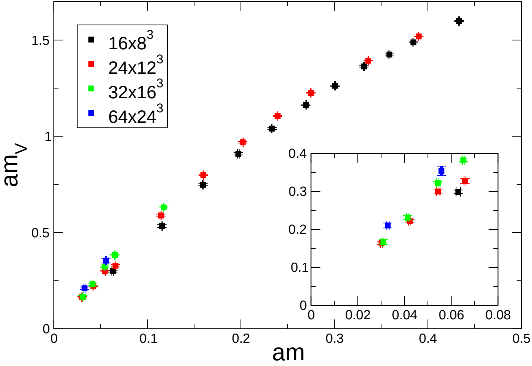
Scaling of pseudoscalar mass

m_{PS}^2/m_q appears to vanish in chiral limit, unlike QCD:



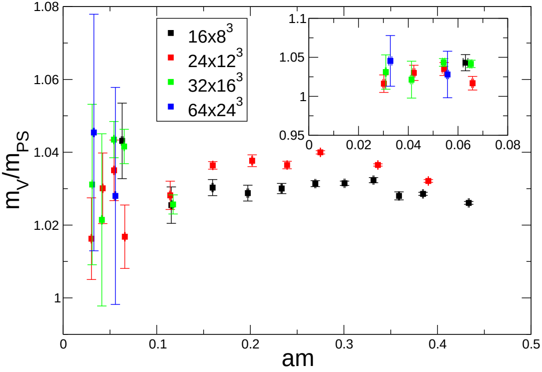
Vector mass

m_V also appears to vanish:



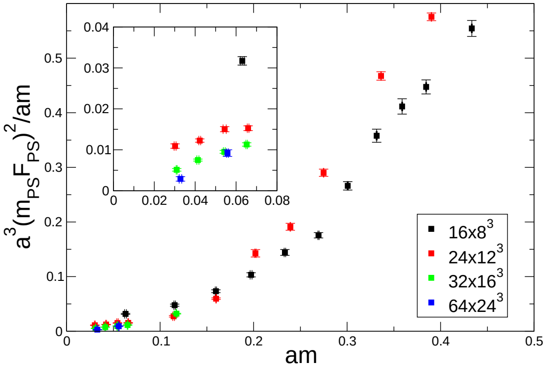
Scaling of vector mass

m_V/m_{PS} does not diverge:



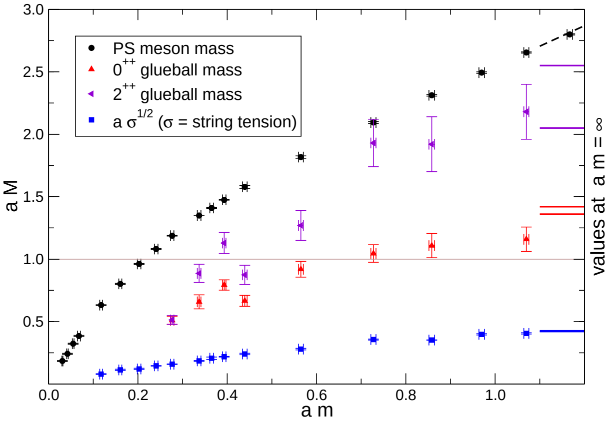
Chiral condensate from GMOR relation

Can also use Gell-Mann–Oakes–Renner relation to estimate chiral condensate:



Glueball masses

Glueball masses also vanish in chiral limit:



Spectrum implications

Spectrum consistent with conformality, very different from QCD.

Expect masses $\propto m_q^{1/(1+\gamma)}$. Difficult to fit, but consistent with a low value of γ.

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Finite-size scaling: expect scaling law $LM \propto F(L^{1/(1+\gamma)} m_q)$.
Fits give $\gamma < 0.5$.
Consistent with low values from direct measurements of running of mass.

Monte Carlo Renormalisation Group

The Monte Carlo Renormalistaion Group is another method for estimating the running of the coupling and the mass.

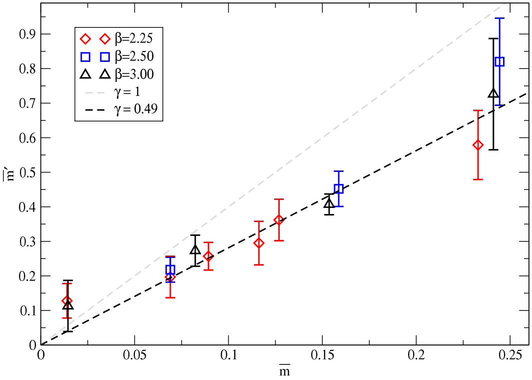
Uses repeated blocking transformations to identify values of coupling or mass that occur at scales differing by a factor of 2.

Specific method for coupling:

- Carry out simulations at bare coupling β and block n times
- Carry out simulations at bare coupling β' and block $n-1$ times.
- Measure observables on both blocked lattices and tuneβ' so that they match.
- Then step scaling function is $\Delta\beta = \beta - \beta'$.

Anomalous dimension

Again, anomalous dimension is easier than running of coupling.



Fit gives $\gamma = 0.49(13)$. Consistent with direct measurement, $\gamma = 1.0$ ruled out.

Conclusions

The lattice can provide non-perturbative answers about technicolor models, with systematic errors under control.

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The lattice can provide non-perturbative answers about technicolor models, with systematic errors under control.

- Consistent picture for Minimal Walking Technicolor, from a range of observables.
- MWT is conformal or near-conformal.
- Anomalous dimension is small.

Outlook

Outlook for the future:

- Other models: $SU(2)$ fundamental, $SU(3)$ sextet, $SU(3)$ fundamental. . .
- Other observables: S-parameter, chiral condensate.
- Anomalous dimension is a relatively easy way to tell if a model is worth further investigation.
- Improved simulations to reduce $\mathcal{O}(a)$ errors.
- 4-fermi interactions may be important; expect them anyway from ETC sector.