Particle spectra and the QCD phase transition

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Contents

Outlines

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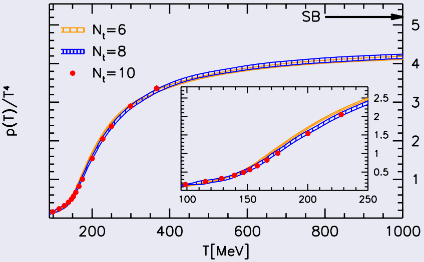
The QCD equation of state

pressure from MC simulation



(Sz. Borsanyi et al, JHEP 1011 (2010) 077)

The QCD equation of state

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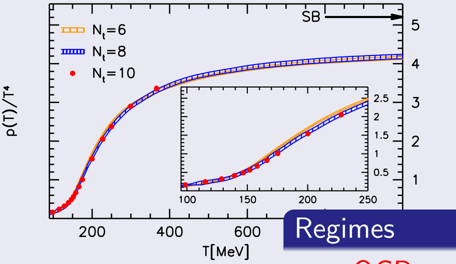


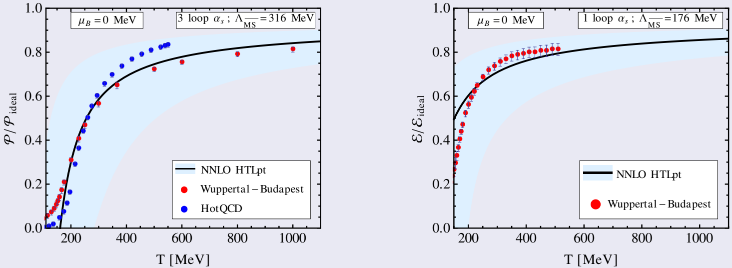
(Sz. Borsanyi et al, JHEP 1011 (2010)

Regimes

- QGP at high T: 8 gluon + 3 quark dof
- hadrons at low T: Hadron Resonance Gas (HRG)
- in between continuous crossover phase transition (PT) with "T_c" $= 156$ MeV

At high temperature: Quark Gluon Plasma (QGP)

3-loop HTL calculation



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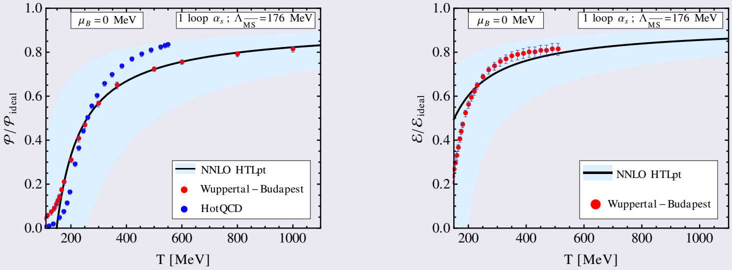

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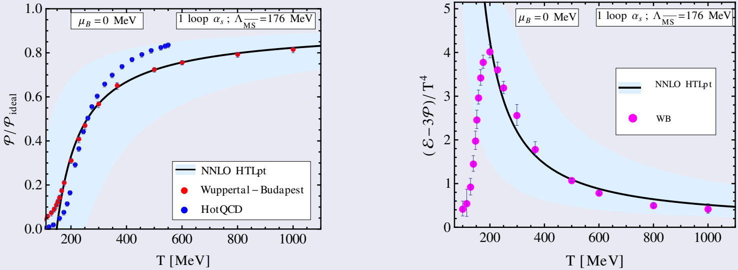

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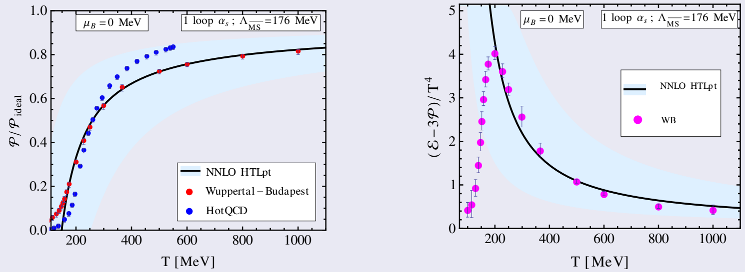

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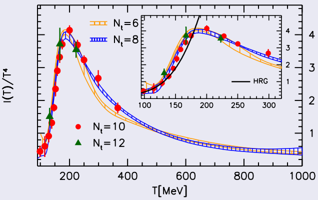
Lesson

Correct description for temperatures $T \gtrsim 2T_c \approx 300\,\mathrm{MeV}$.

At low temperature: hadrons

HRG: free hadrons with fixed ($T = 0$) masses from experiments

Thermodynamics



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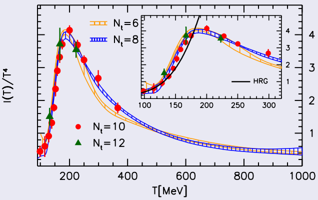

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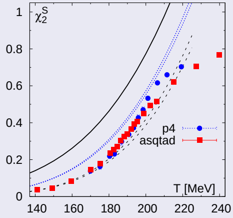
Chiral susceptibility



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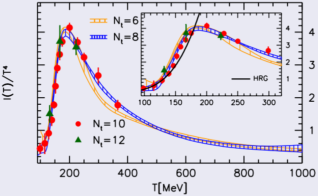

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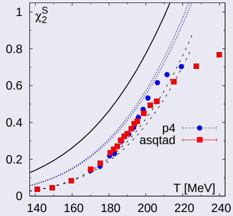

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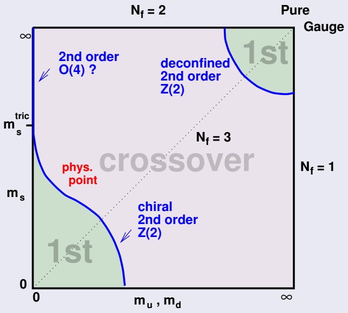
Lesson

HRG describes thermodynamics at $T < 150 - 180\,\mathrm{MeV}$

Phase transition region

Temperature range of $150\,\mathrm{MeV} \lesssim T \lesssim 300\,\mathrm{MeV}$.

Columbia plot



Mechanisms of the PT

deconfinement: 1st order PT

- hadrons become unstable
- order parameter: Polyakov-loop
- valid at $m_{u,d,s} \to \infty$ (quenched)

chiral phase transition: 1st order PT

- chiral condensate unstable
- order parameter: $\langle \bar{\Psi}\Psi \rangle$
- valid at $m_{u,d,s} \to 0$ (chiral case)

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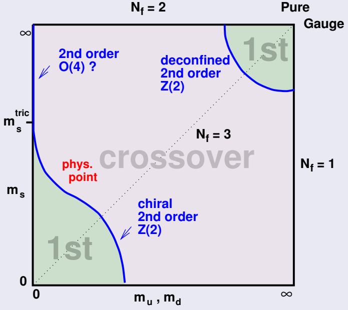

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Physical point

What happens in the crossover regime?

What happens with the hadrons at T_c?

- HRG contains infinitely many dof $\Rightarrow$ $P_{SB} = \infty$
  singularity in P_{SB} at T_H Hagedorn temperature.

  (R. Hagedorn, Nuovo Cim. Suppl. **3**, 147 (1965); W. Broniowski, *et.al.* PRD **70**, 117503 (2004))

  $\Rightarrow$ we must get rid of the hadrons before T_H.
- no change of ground state (1st or 2nd order phase transition)

  $\Rightarrow$ hadrons must not disappear at once

  (J. Liao, E.V. Shuryak PRD73 (2006) 014509 [hep-ph/0510110])
- MC: hadronic states are observable even at $T \sim 1.2\text{-}1.5\,T_c$!

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Proposal

$150\,\mathrm{MeV} \lesssim T \lesssim 300 - 400\,\mathrm{MeV}$ is the melting hadron phase (hadron fluid phase). Quarks appear gradually with the disappearance of the hadrons. (ionization-recombination, chemistry, Gribov)

Outlines

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What is melting?

Heuristically: disappearance of a particle species

Usual approaches in the literature

- fast growing (thermal) mass (J. Liao, E.V. Shuryak PRD73 (2006) 014509)
  Would explain why we do not see quarks at low energy and hadrons at high energy
  $\Rightarrow$ in contradiction with lattice results
- FRG: all states are present, but with different wave fct. renormalization
  $\Rightarrow$ but Z drops out from pressure

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Question

How can a particle state disappear?

What is a particle? Free systems.

$\exists$ conserved particle number operator: $\hat{N} = \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}}, \quad [\hat{H}, \hat{N}] = 0$

definition

particle: energy (and momentum) eigenstate in $N = 1$ sector.

Moreover:

- one particle spectrum contains a single line at $E = E(\mathbf{p})$ (dispersion relation)
- time evolution $|t, E, n\rangle = e^{-iEt} |0, E, n\rangle$ is unique from any initial condition
- in particular linear response function G_r has the same time dependence, also at $T > 0$
- particles are also thermodynamical degrees of freedom, eg. $P_{SB} = \frac{\pi^2 T^4}{90}\left(N_b + \frac{7}{8} N_f\right)$ is the Stefan-Boltzmann limit.

Identifications

Since these are true in free particle case, we intuitively identify the following concepts:

- particle number operator
- spectral line (energy eigenstate)
- general time evolution
- linear response theory
- linear response theory at $T > 0$
- statistical/thermodynamical definition

They all mean "particle".

Identifications

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They all mean "particle".

Warning

These all mean different things in an interacting theory!

...and we get mixed up, when these definitions are contradicting

Outlines

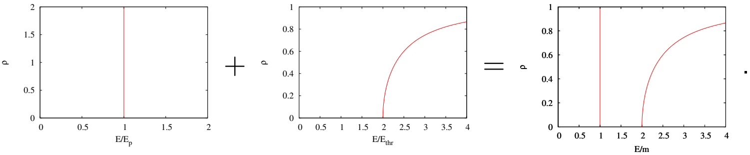
Spectrum at zero temperature

- Usually in interacting systems $\nexists$ enough conserved quantities to fully describe the system

  $\Rightarrow$ $\nexists$ particle definition through particle number

  Exception: integrable systems

- energy levels of different N sectors mix together! At $T = 0$



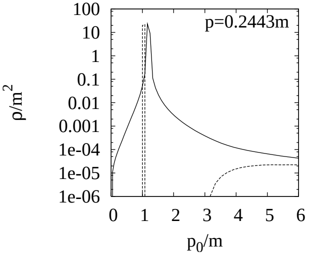
- multiple energy levels, non-unique time dependence
- BUT $\exists$ discrete E-level

  $\Rightarrow$ linear response for long times: $Ze^{-iEt} + Ct^{-3/2}e^{-iE_{thr}t}$

  $\Rightarrow$ define particles as asymptotic particle states

General case

no clear distinction between particle and continuum states, if



(AJ, PRD76 (2007) 125004 [hep-ph/0612268])

- zero mass excitation (no gap)
- unstable particles
- $T > 0$ environment

$\Rightarrow$ $\nexists$ asymptotic states
(in practically all realistic cases...)

- linear response: $\varrho(t) = Ze^{-iEt-\gamma t} + f_{bckg}(t) = \mathrm{pole} + \mathrm{cut}$
- for large Z and small γ: complex pole dominates long time evolution $\Rightarrow$ quasiparticles

Quasiparticles and thermodynamics

- In QM quasiparticles give fundamental particle-like contribution to free energy (Beth, Uhlenbeck)

$$\delta Z \sim \int_0^\infty \frac{d\omega}{\pi} \frac{\partial \delta}{\delta \omega} e^{-\beta\omega} \sim e^{-\beta E}$$

since $\delta_\ell(\varepsilon)$ phase shift jumps π-t at pole $\omega = E$

(Landau, Lifsitz V.)

- true also for bound states
- In QFT this is true only for well separated quasiparticle peaks

(R.F Dashen, R. Rajaraman, PRD10 (1974), 694.)

- In scattering theory: quasiparticles are included in S-matrix as Breit-Wigner resonances with complex amplitudes
unitarity $\Rightarrow$ constraints

(H. Feshbach, Ann. Phys. 43, 110 (1967); L. Rosenfeld, Acta Phys. Polonica A38, 603 (1970); M. Svec, PRD64, 096003 (2001) [hep-ph/0009275].)

The particle concept

concept of free particles can be saved as quasiparticles

- spectral definition $\Rightarrow$ broadened spectral line
- linear response theory $\Rightarrow$ unique long time dependence
- thermodynamical degree of freedom $\Rightarrow$ for well separated case

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- thermodynamical degree of freedom $\Rightarrow$ for well separated case

there are important differences

- Quasiparticles are not energy eigenstates!
- collective excitations with environment dependent spectral weights $\Rightarrow$ mass, width environment dependent
  $\Rightarrow$ they may give not particle-like contribution to P.
- . . .

Outlines

Condition for unitarity

Local Hamiltonian? Exponential damping $\Rightarrow$ $\hat{H} \to \hat{H} - i\gamma$
$\Rightarrow$ loss of unitarity!

Solution

For consistent description one has to take into account the complete spectrum, not just the quasiparticle peak!

Physics: quasiparticle $\nexists$ independently of environment.

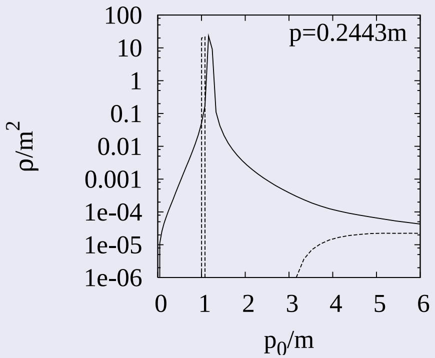
(Ward, Luttinger, Phys.Rev. 118 (1960) 1417; G. Baym, Phys. Rev. 127 (1962) 1391; Cornwall Jackiw, Tomboulis, Phys.Rev. D10 (1974) 2428-2445;J. Berges and J. Cox, Phys. Lett. B 517 (2001) 369)

From where can we take the spectrum?

- Φ-derivable (2PI) or SD approach: $G^{-1} = G_0^{-1} - \Sigma(G)$.
- We can also use experimental inputs for ϱ.

Typical spectral functions

Φ^4 model 2 loop 2PI, $T = m$



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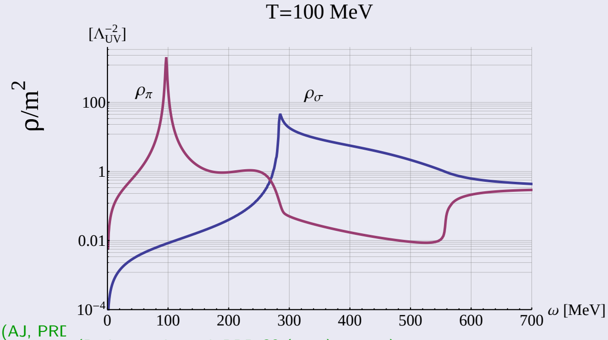
(AJ, PRD

meson model FRG



(R.-A. Tripolt *et.al.* PRD **89** (2014) 034010)

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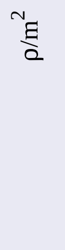

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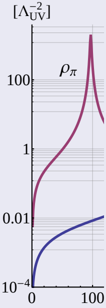
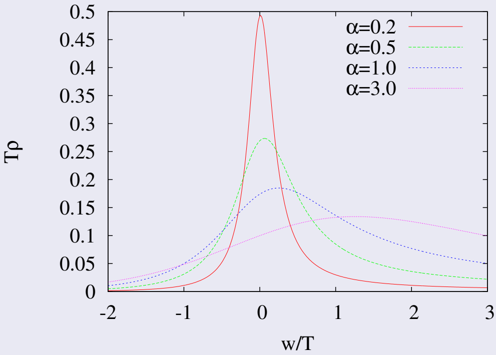


(R.-A. Tripolt

Bloch-Nordsieck model at finite T



(A.J, P. Mati, PRD **87** (2013) 125007 [arXiv:1301.1803])

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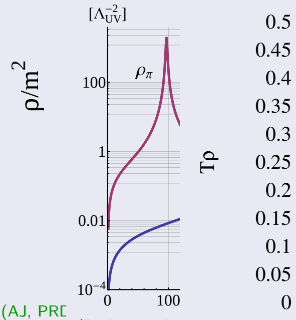
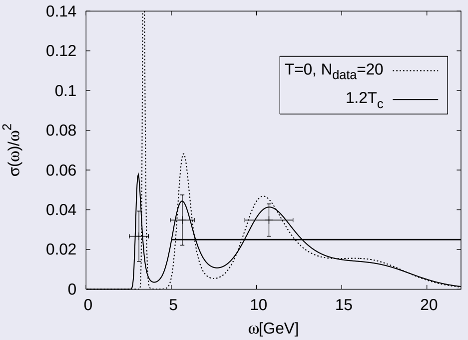
Bloch-N



(AJ, PRD

(R.-A. Tripolt

J/Ψ spectrum MC simulation, MEM



(AJ., P. Petreczky, K. Petrov, A. Velytsky, PRD75 (2007) 014506)

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Lagrangian representation of general spectral functions

$$\mathcal{L} = \frac{1}{2}\Phi^*(p)\mathcal{K}(p)\Phi(p)$$

- unique $\varrho \to \mathcal{K}$ relation:

$$G_{ret}(p) = \int \frac{d\omega}{2\pi} \frac{\varrho(\omega,\mathbf{p})}{p_0-\omega+i\varepsilon}, \qquad \mathcal{K} = \mathrm{Re}\, G_R^{-1}$$

- defines a consistent nonlocal field theory: unitary, causal, Lorentz-invariant, $E, \mathbf{p}$ conserving (just like in 2PI case)

(AJ. Phys.Rev. D86 (2012) 085007 [arXiv:1206.0865])

Thermodynamics from the spectral function

Technically:

$\to$ energy-momentum tensor from Noether currents

$\to$ energy density $\varepsilon = \frac{1}{Z}\,\mathrm{Tr}\, e^{-\beta\hat{H}}\,\hat{T}_{00}$

$\to$ averaging with KMS relations

$\to$ free energy, pressure from thermodynamical relations

Result:

$$\varepsilon = \int \frac{d^4p}{(2\pi)^4} E(p) n(p_0) \varrho(p), \qquad E(p) = p_0 \frac{\partial \mathcal{K}}{\partial p_0} - \mathcal{K}$$

- plausible: sum up $n(p)$ weighted energy values
- classical mechanical analogy: $\mathcal{K}$ quadratic kernel "Lagrangian" with $p_0 \sim \dot{q} \quad \Rightarrow \quad E(p)$ energy.
- but: energy values depend on $\mathcal{K}$ and so on ϱ
  ε is a nonlinear functional of ϱ!
- ε does not depend on the normalization of ϱ.

Outlines

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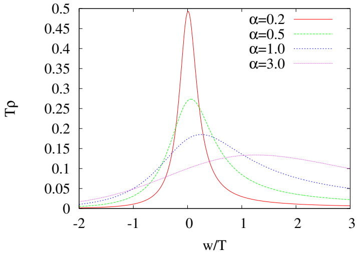
1. Introduction
   - Phases of QCD
2. The particle concept
   - Particles in free systems
   - Particles in interacting systems
   - Mathematical treatment of quasiparticles
3. Physical applications
   - Formation time of a quasiparticle
   - Gibbs paradox: indistinguishability of particles
   - Particle melting
4. QCD thermodynamics
   - Statistical model of QCD excitations
5. Conclusions

Spectral function in gauge theories

Gauge theories are complicated – solvable simplification: resummation of all photon contribution in 1-component QED (Bloch-Nordsieck resummation)

One can compute the spectral function at finite temperature. In comoving frame: (A.J, P. Mati, Phys.Rev. D87 (2013) 125007 [arXiv:1301.1803])

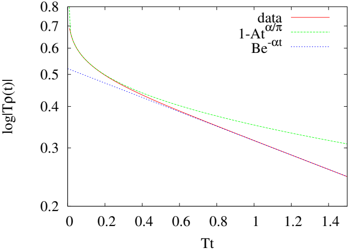
$$\varrho(w) = \frac{N_\alpha \beta \sin\alpha \, e^{\beta w/2}}{\cosh \beta w - \cos\alpha} \left| \Gamma\left(1 + \frac{\alpha}{2\pi} + i\frac{\beta w}{2\pi}\right)\right|^{-2},$$

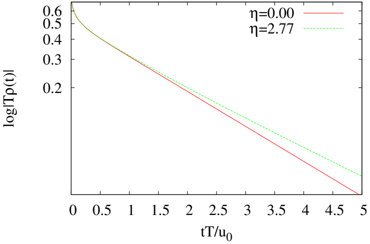


- $\alpha = e^2/(4\pi)$ structure consant
- function of $w = p_0 - m$
- Near the peak: Lorentzian with width $\gamma = \alpha T$
- $p_0 \gg m$ power law: $\sim p_0^{-1-\alpha/\pi}$
- $p_0 \ll m$ exponential: $\sim e^{2\beta w}$

Real time dependence

Fourier transform of the result: $\varrho(t) = e^{-imt}\bar{\varrho}(t)$

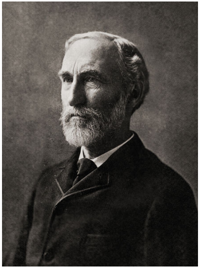




- for long times $Tt \gg 1$: $\sim e^{-\alpha_{eff}(u)Tt}$ quasiparticle behaviour
- for short times $Tt \ll 1$: $\sim 1 - c(Tt)^{\alpha/\pi}$ not quasiparticle-like!
- formation time of the quasiparticle: $t \sim \beta$!
- at $T \to 0$ $\varrho(t) \to e^{-imt}$, but we have to wait long to see the QP behaviour.

Outlines

The Gibbs paradox



J.W.Gibbs (1839-1903)

(J.W.Gibbs, 1875-1878; E.T.Jaynes, 1996)

take two containers with (ideal) gases:

initially $n_1,\ V_1\ n_2, V_2,\ p_1 = p_2,\ T_1 = T_2$

mix them: $V = V_1 + V_2,\ n = n_1 + n_2$

entropy difference $(f = n_1/n_2)$

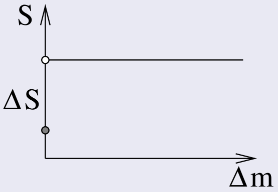
$$\begin{aligned}\Delta S &= nR \log V - R(n_1 \log V_1 - n_2 \log V_2) \\ &= -nR(f \log f + (1-f)\log(1-f)) \\ &\Rightarrow -nR \log 2, \ \text{for } n_1 = n_2,\ V_1 = V_2.\end{aligned}$$

Independent of the gas properties, provided they are different

e.g. let the two gases have the same quantum numbers, but different masses

$\Rightarrow$ discontinuity at $\Delta m = 0$

discuntinuous entropy



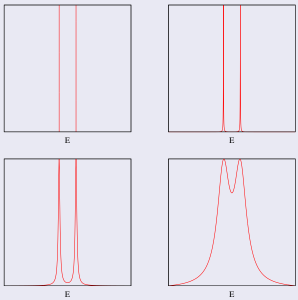
Understanding Gibbs paradox

- indistinguishability from Fock-space construction:
  $|0\rangle$ vacuum, $a^\dagger_{\mathbf{p}}$ p-momentum particle creation operator
  $\Rightarrow \quad |\mathbf{p}_1, n_1, \dots \mathbf{p}_i, n_i \dots\rangle = a^{\dagger n_1}_{\mathbf{p}_1} \dots a^{\dagger n_i}_{\mathbf{p}_i} \dots |0\rangle$
  multiparticle state $\Rightarrow$ single state, permutation $\pm$ sign
- several gases: $a^{(1)\dagger}_{\mathbf{p}}, a^{(2)\dagger}_{\mathbf{p}}, \dots$ we assign new creation operators for all species
  we have to fix the number of species in advance!
- But in Gibbs paradox Δm is the control parameter $\Rightarrow$ we should be able to compute $S(\Delta m)$
- To describe Gibbs paradox number of particle species must be a dynamical parameter
  (integer number?)

Gibbs paradox in interacting systems

Without interaction the energy levels (spectral lines) are infinitely thin lines. In interacting gases the spectral lines broaden.

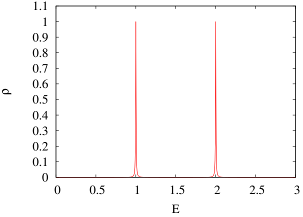
spectrum in interacting gases



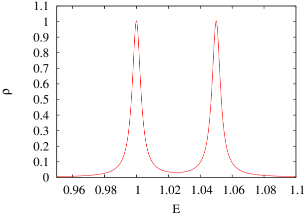
- 1st plot: 2 lines
  4th plot: one broad peak
- Gibbs: particles are distinguishable, if a mixed gas can be separated by some means. Going from case 1 to 4 this is harder and harder!
- Γ width sets resolution $\Rightarrow$ in case $\Gamma \gtrsim \Delta m$ we do not see separate peaks!
- real question is quantitative: how does it appear in thermodynamics?

Thermodynamics

Change of spectrum:



⇒



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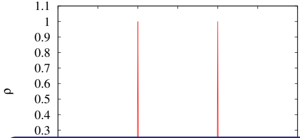

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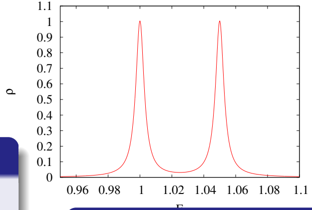
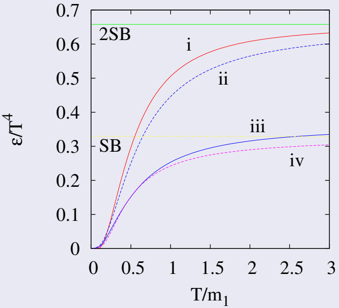


energy density



Curves

$m_1 = 1$, $m_2 = 2$, $\Gamma = 0$ or 0.2

- i. $\Gamma = 0$
- ii. independent, finite Γ
- iii. $\Gamma/\Delta m = 0.2$
- iv. one free particle

Thermodynamics

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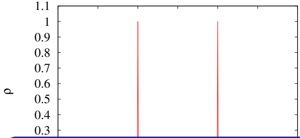

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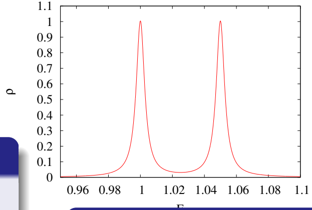

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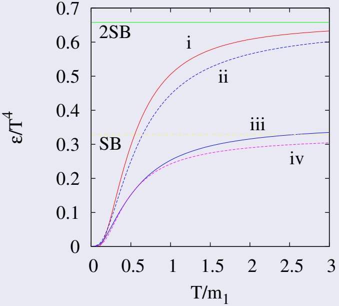

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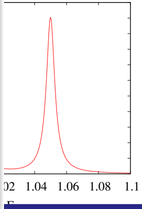
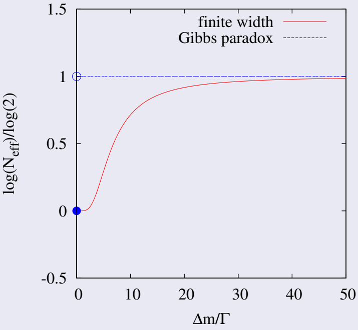
degrees of freedom disappear continuously!

Thermodynamics

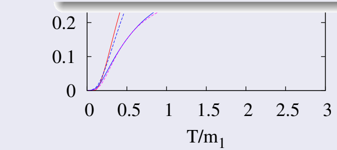
Char



number of species



energ



= 2, Γ = 0 or 0.2

$= 0$

independent, finite Γ

- iii. $\Gamma/\Delta m = 0.2$
- iv. one free particle

degrees of freedom disappear continuously!

Outlines

1. Introduction
   - Phases of QCD
2. The particle concept
   - Particles in free systems
   - Particles in interacting systems
   - Mathematical treatment of quasiparticles
3. Physical applications
   - Formation time of a quasiparticle
   - Gibbs paradox: indistinguishability of particles
   - Particle melting
4. QCD thermodynamics
   - Statistical model of QCD excitations
5. Conclusions

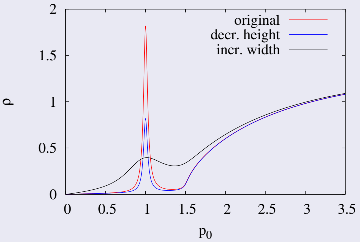
Temperature dependence of a typical spectral function

Spectrum in QFT: QP peak(s) and multiparticle continuum.

At finite T

- continuum height increases, and so
  - QP width grows
  - quasiparticle peaks merge into the continuum
  - relative height of quasiparticle peak decreases (sum rule)
- Lorentz-invariance is broken
- thermal mass
- T-dependent couplings

T-variation of a spectrum



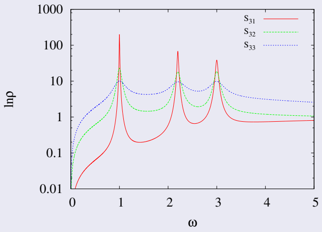
Strategy:
compute thermodynamics for generic $\varrho_Q(p_0, |\mathbf{p}|; T, \mu)$.

Increasing continuum, fixed mass

Trial spectral functions with 3 QP peaks and continuum

⇒ typical for bound states

Spectra



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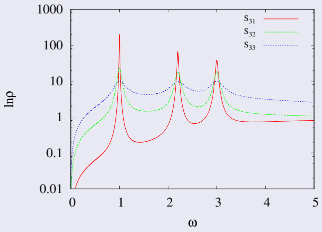
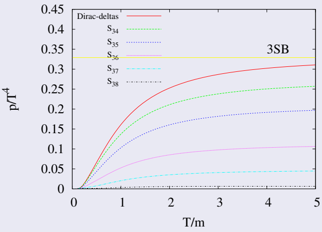


thermodynamics



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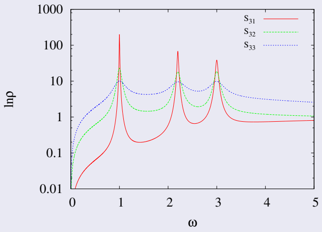

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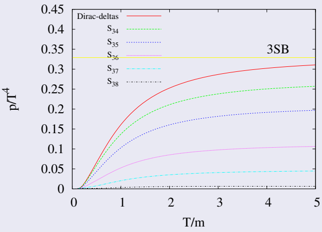

General behaviour

Pressure decreases for increasing continuum height; for pure continuum the pressure is very small!

Effective number of degrees of freedom

Characterization: pressure is roughly proportional to the free gas pressure

$$\Rightarrow \quad N_{eff}(T) = \frac{P(T)}{P_0(T)} \text{ is appr. } T\text{-independent}$$

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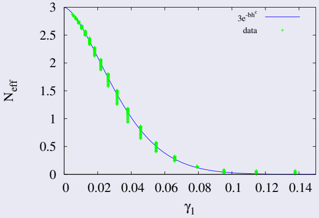
effective ndof



- T-variation: green band
- fit a streched exponential
  $$e^{-(\gamma/\gamma_0)^c}$$
  where $\gamma_0 = 0.38, \; c = 1.6$.

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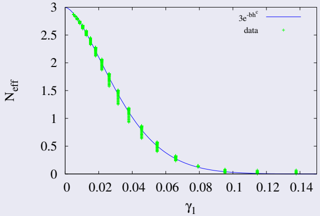

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Physics

We describe vanishing particle species! $\Rightarrow$ melting

Momentum dependence of the spectral function

Extreme case of spatial momentum dependence:

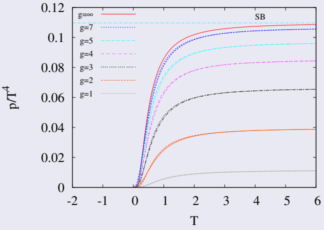
- for small momenta: Dirac-delta (free particle)
- for large momenta: very broad spectral function

Model

- simplified model for hadrons
- for quarks (asymptotic freedom) we expect inverse behaviour
- broad spectral function gives no contribution to P
  $\Rightarrow$ effective cutoff of spatial integration
- for simplicity we choose $\Lambda_{eff} = gT$
  (g can be T-dependent)

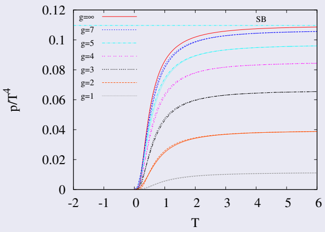
Effective number of dof

Pressure

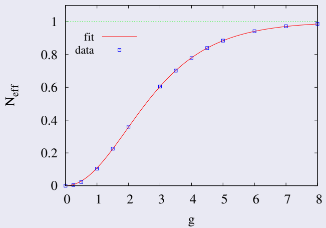


Effective number of dof

Pressure



N_{eff}



fit function: $\frac{1}{1 + x^{-2} e^{-(bx)^a}}$ $(a = 1.79, \quad b = 0.58)$

Outlines

Outlines

Statistical description

- HRG: huge # of hadronic contributions, each small!

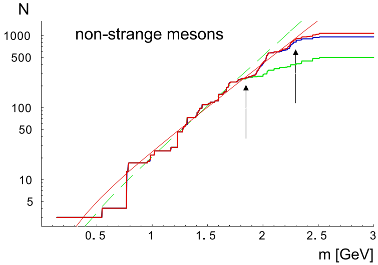
  $\Rightarrow$ statistical description is needed

- we need spectra. . . hard to obtain

$\Rightarrow$ idealized, simplified picture for hadron masses and widths.

Hadron masses: Coulomb spectrum of QCD

QCD bound state dynamics cannot be solved...
experimental evidence: exponentially rising energy level density



(W. Broniowski, W. Florkowski and L. Y. .Glozman, Phys. Rev. D **70**, 117503 (2004) [hep-ph/0407290].)

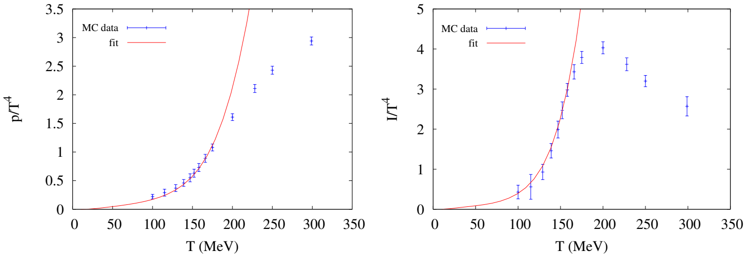
statistical description:

$$\varrho_{hadr}(m) \sim (m^2 + m_0^2)^a e^{-m/T_H}$$

Hagedorn spectrum

several fits (also $a = 0$) possible

Thermodynamics with free hadrons



- MC data from BMW collaboration

  (Sz. Borsanyi et al, JHEP 1011 (2010) 077)

- Hagedorn fit: 5000 hadronic resonances,
  $m_1 = 120\,\mathrm{MeV},\ T_H = 240\,\mathrm{MeV},\ a = 0$
- for infinitely many resonances: divergent at $T > T_H$
- overestimates pressure above $\approx 200\,\mathrm{MeV}$.

Melting: number of hadronic/partonic excitations

Pressure

$$P_{hadr}(T) = e^{-G_{eff}^{(hadr)}} \sum_{n\in\text{hadrons}}^{N} P_0(T, m_n), \qquad G_{eff}^{(hadr)} = aT^b,$$

$$P_{QGP}(T) = e^{-G_{eff}^{(part)}} \sum_{n\in\text{partons}} P_0(T, m_n), \qquad G_{eff}^{(part)} = G_0 + ce^{-dG_{eff}^{(hadr)}}.$$

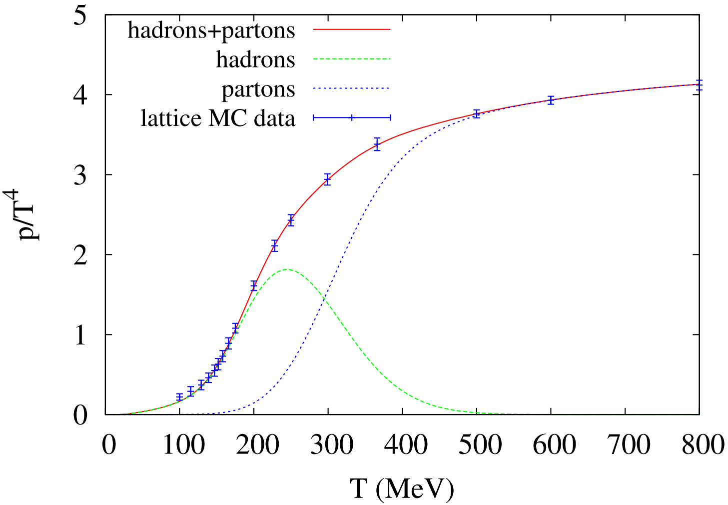
We use oversimplified description

- continuum height increases with # of decay channels
  $\Rightarrow$ effective cut-off in hadron mass

  (J. Cleymans, D. Worku, Mod. Phys. Lett. A **26**, 1197 (2011).)
- assumed same width, height for all hadronic/partonic channels
- most simple choice for hadronic $G_{eff} \sim \gamma^b$ (stretched exponential) and $\gamma \sim T$
- for partons: take into account the number of hadronic modes
  correlated parton-hadron description

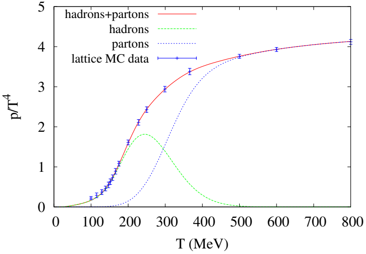
Full QCD pressure

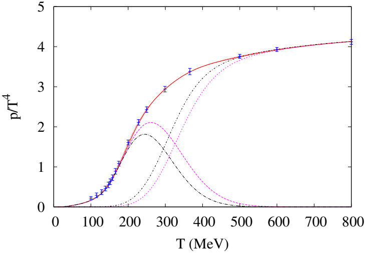
Fit the model parameters to MC data ⇒ good agreement



Full QCD pressure

Different fits



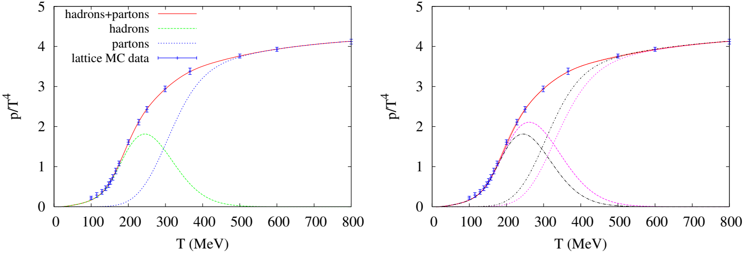


properties

- $T \lesssim T_c$: HRG fully describes thermodynamics
- $[T_c, 2T_c]$, $[2T_c, 3T_c]$: hadron or parton dominated QCD thermodynamics; both dof are present
- $T \gtrsim 3T_c$: QGP

Full QCD pressure

Different fits



Corollary

- T_c is not a hadron – QGP transition temperature: partons just start to appear there
- full QGP only for $T \gtrsim 2.5 - 3T_c$: $\nexists$ mechanism which could do it faster

Outlines

Conclusions

- excitations can be characterized by their spectra
- not necessarily particle-like:
  - non-exponential time dependence
  - Gibbs paradox, melting: (continuous) disappearance of species
- QCD thermodynamics at physical point at $\mu = 0$
  - at $T_c \approx 156$ MeV partons start to appear
  - $T \lesssim T_c$: hadrons
  - $T \in [T_c, 3T_c]$: mixed phase
  - $T \gtrsim 3T_c$: QGP
- hadron phyiscs + melting + QGP

  $\Rightarrow$ perturbative QCD thermodynamics?